DISCUSSION OF PAPER PUBLISHED IN VOLUME LXXIX

PARAMETRIZING THE WORKERS' COMPENSATION EXPERIENCE RATING PLAN

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1. INTRODUCTION

Mr. Gillam's paper provides an excellent explanation of the detailed actuarial study that led the National Council on Compensation Insurance (NCCI) to revise the Experience Rating Plan for Workers' Compensation. This actuarial study is an example of a practical application of credibility theory to the situation where parameter uncertainty and risk heterogeneity are important.

As shown in Exhibit 1, the revised plan shares many of the features of the prior plan. Administratively, the plans are the same. There have been important actuarial changes. As Mr. Gillam states, the revised plan is a single split plan rather than a multi-split plan, and the credibilities that are determined by the parameters of the two plans are very different.

2. ACTUARIAL FORMULAS UNDERLYING EXPERIENCE RATING

Mr. Gillam's Formula 1.5 is used in both the prior plan and the revised plan in order to calculate the experience modification:

$$M = 1 + Z_p (P_t - \mathrm{E}[P_t]) + Z_x (X_t - \mathrm{E}[X_t]) ,$$

[1] The NCCI study was also explained in Venter [1].

where:

M = experience modification;

P = primary loss divided by expected losses = A_p/E ;

X = excess loss divided by expected losses = A_x/E ;

t = (past) time period;

Z_p = primary credibility;

Z_x = excess credibility; and

E [Y] = expected value of Y.

This formula for the experience modification can also be written following Snader [2] as:

$$M = 1 + Z_p\left(\frac{A_p}{E} - \frac{E_p}{E}\right) + Z_x\left(\frac{A_x}{E} - \frac{E_x}{E}\right)$$

$$= \frac{(1 - Z_p)\,E_p + Z_pA_p + (1 - Z_x)\,E_x + Z_xA_x}{E} .$$

The credibilities are given by:

$$Z_p = \frac{E}{E + K_p} \text{ , and}$$

$$Z_x = \frac{E}{E + K_x} ,$$

where E is the expected losses, and K_p and K_x are the credibility parameters to be determined.

Under the revised plan, the credibility parameters have the form $E\,\frac{\text{Linear}}{\text{Linear}}$ or in Mr. Gillam's notation $K = E\left(\frac{CE + D}{E + F}\right)$. The NCCI determined the particular coefficients used in the revised plan by the empirical testing described by Mr. Gillam.

It follows from the formulas for the credibility parameters that under the revised plan the credibilities as a function of the size of risk are of the form $\frac{\text{Linear}}{\text{Linear}}$. This can be written as:

$$Z = \frac{E + I}{JE + I + K}, \text{ where } 0 \le I,\ 1 \le J, \text{ and } 0 \le K,$$

with one formula for primary credibility and one formula for excess credibility, each with different constants I, J, and K. As explained by Mr. Gillam, this is the form of credibility one expects if both parameter uncertainty and risk heterogeneity are important.[2] The more familiar formula for credibility is a special case of this formula with $I = 0$ and $J = 1$.

In the more familiar formula $Z = E/(E + K)$ the parameter K is a "scale parameter." Changing K changes the overall scale of the credibility curve without changing its shape. As will be discussed below, K, and thus the scale of the curve, depends on a state-specific inflation-sensitive parameter.

In the formula used in the revised plan, there are two additional parameters I and J which are "shape parameters." Changing I and/or J changes the shape of the credibility curve. The size of the parameter I relative to the parameter K adjusts the shape of the credibility curve for small risks. The minimum credibility is given by $I/(I + K)$, which is determined by the ratio of I to K. The parameter J adjusts the shape of the credibility curve for large risks. The maximum credibility is given by $1/J$.

[2] See Equation 1.6 in Mahler [3]. What was denoted as K there is denoted as $I + K$ here. This is a matter of notation rather than substance. The notation used here allows K to have the same underlying source in both the credibility formula in the revised experience rating plan and the more familiar formula for credibility.

Thus the revised plan uses a more general formula for credibility, which is better able to approximate those credibilities that would have performed well in the past and thus are expected to work well in the future. As shown in Mahler [3], one could derive an even more general formula than that used in the revised Experience Rating Plan. As a function of the size of risk, the credibilities given by formulas in Mahler [3] are of the form $\frac{\text{Quadratic}}{\text{Quadratic}}$.

As discussed in Appendix B, if one assumes the covariance of excess and primary losses is not extremely important, these formulas for the credibilities reduce to the form $\frac{\text{Linear}}{\text{Linear}}$ used in the revised Experience Rating Plan.[3]

This more general formula for credibility is somewhat better able to approximate those credibilities that would have performed well in the past. The two additional parameters can be selected so as to adjust the shape of the credibility curve for medium-size risks. In any given application, one has to decide whether the extra generality introduced by these additional parameters is worth the extra complications also introduced.

The specific formulas for Z_p and Z_x used in the revised plan are:

$$Z_p = \frac{E + 0.0028S}{1.1E + 0.01308S}\text{, and}$$

$$Z_x = \frac{E + 0.0204S}{1.75E + 0.8357S}\text{,}$$

where S is the State Reference Point.[4]

These formulas can also be stated in terms of the parameter g:[5]

[3] These covariances are discussed in more detail in a later section.

[4] The State Reference Point is calculated as 250 times the average cost per case in the particular state.

[5] The parameter g is calculated as the average cost per case in the particular state divided by 1,000; g is rounded to the nearest 0.05.

$$Z_p = \frac{E + 700g}{1.1E + 3{,}270g}\text{, and}$$

$$Z_x = \frac{E + 5{,}100g}{1.75E + 208{,}925g}.$$

Thus, under the revised plan, the primary and excess credibilities are each given by the formula $Z = (E + I)/(JE + I + K)$, with the following parameters:

	Primary	Excess
I	$0.0028S = 700g$	$0.0204S = 5{,}100g$
J	1.1	1.75
K	$0.01028S = 2{,}570g$	$0.8153S = 203{,}825g$

If $S = \$500{,}000$ and $g = 2$, for example,[6] then the parameters would be:

	Primary	Excess
I	\$1,400	\$10,200
J	1.1	1.75
K	\$5,140	\$407,650

Note that the curves for primary and excess credibilities under the revised plan have a significantly different scale from each other due to their vastly different values of the parameter K. As is shown in Exhibit 2, the two curves also have significantly different shapes due to their different values of the parameter J and different ratios of I to K.

3. IMPLEMENTING THE ACTUARIAL FORMULAS

The values for the credibilities underlying actual experience ratings may differ slightly from those calculated using the formulas given above, due to the rounding process involved in establishing a

[6] These correspond to an average claim of \$2,000.

table of W and B values. Also, they will differ for small risks (those with expected losses below about \$20,000) because of the minimums imposed on the parameters W, K_p and K_x.[7]

As stated by Gillam, for the smaller risks, there are maximum values imposed on the experience rating modification under the revised plan.

Expected Losses	Maximum Modification
0 to \$5,000	1.6
\$5,000 to \$10,000	1.8
\$10,000 to \$15,000	2.0

The maximum debit and credit for small risks are compared in Exhibit 6.

The NCCI's reduction in the maximum swing for smaller risks not only makes practical sense, but also is sound from a theoretical standpoint. The inclusion of the parameter I in the credibility formula produces the larger than desired credibilities for smaller risks. However, this was based on a consideration of risk heterogeneity. Such considerations become inapplicable as risks become too small to have separate and distinct subunits.[8] Thus a credibility formula parametrized based on all sizes of risks may not fit well for the very smallest risks.

Under both plans, the W and B values vary with the expected losses and are displayed in a table. However, the formulas used to determine W and B are significantly different under the two plans. An example of W and B values for both plans is shown in Exhibit 5.

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[7] The imposition of minimums on K_p and K_x reduces the credibility assigned to very small risks (those with expected losses below about \$6,000). The imposition of a minimum on W increases the credibility assigned to the excess losses of small risks.

[8] This is explained in Mahler [3].

The W and B values determine the credibility parameters and credibilities under both experience rating plans following the development in Snader [2].

Let $Z_p = \frac{E}{E+B}$, and

$$Z_x = \frac{E}{E + \frac{B+(1-W)\,E}{W}} = \frac{WE}{E+B} = WZ_p \,.$$

These equations can be compared to the equations given by Gillam using the credibility parameters:

$$Z_p = \frac{E}{E+K_p}, \text{ and}$$

$$Z_x = \frac{E}{E+K_x} \quad .$$

The credibility parameters K_p and K_x can be calculated from the expected losses E, W, and B:

$$K_p = B \text{ , and}$$

$$K_x = \frac{B+(1-W)\,E}{W} .$$

As stated by Gillam, under the prior plan:

$$B = (1-W)\;20{,}000, \text{ and}$$

$$W = \begin{cases} 0 & E \le 25{,}000 \\ \dfrac{E-25{,}000}{S-25{,}000} & S \ge E \ge 25{,}000 \\ 1 & E \ge S \end{cases} ,$$

where S is the self-rating point.

Under the revised plan, the values of the credibility parameters K_p and K_x are given via formula, and then B and W follow from them. The formulas in terms of the State Reference Point S are:

$$K_p = E\left[\frac{0.1E + 0.01028S}{E + 0.0028S}\right],$$

where K_p is subject to a minimum of 7,500 (K_p subject to this minimum is labeled B by the NCCI) and

$$K_x = E\left[\frac{0.75E + 0.8153S}{E + 0.0204S}\right],$$

where K_x is subject to a minimum of 150,000 (K_x subject to this minimum is labeled C by the NCCI).

These equations can also be stated in terms of g.[9] These equations are the ones used by the NCCI:[10]

$$K_p = E\left[\frac{0.1E + 2{,}570g}{E + 700g}\right], \text{ and}$$
$$K_x = E\left[\frac{0.75E + 203{,}825g}{E + 5{,}100g}\right].$$

By solving the set of equations, one can express W and B in terms of K_p and K_x. These equations are used to determine W and B from K_p and K_x:[11]

$$B = K_p, \text{ and}$$

$$W = \frac{E + K_p}{E + K_x},$$

[9] The state specific parameter g is defined by the NCCI as the average claim cost in the state divided by 1,000; g is rounded to the nearest 0.05.

[10] The NCCI has written these formulas in a slightly different form. For example, $K_p = E\,[0.1 + (2{,}500g/(E + 700g))]$.

[11] The NCCI actually defines B as K_p subject to the minimum. The NCCI defines C as K_x subject to the minimum. Then $W = (E + B)/(E + C)$.

where W is subject to a minimum of 0.07.

4. CREDIBILITIES: PRIOR PLAN VS. REVISED PLAN

The credibilities under the revised plan differ significantly from the prior plan. Therefore, the switch in experience rating plans has led to very significant impacts on individual insureds.[12] The credibilities assigned to the primary[13] and excess losses are each significantly different, as can be seen in Exhibits 3, 4, and 5:

1. For small risks, primary credibilities are larger.
2. For large risks, primary credibilities are smaller. The maximum primary credibility is 91%, rather than 100% as under the prior plan.
3. For small risks, excess credibilities are a little larger. Even very small risks have a small non-zero excess credibility, as opposed to zero under the prior plan.
4. For large risks, excess credibilities are much smaller. The maximum excess credibility is 57%, rather than 100% under the prior plan.

Thus one important change is that under the revised plan there are no longer self-rated risks. Since the primary losses are assigned a maximum credibility of 91%, while the excess losses are assigned a maximum credibility of 57%, the maximum credibility assigned to any risk is approximately 70%.[14]

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[12] As shown in Exhibit 3, a risk's credibility can change by up to 40%. For example, if a risk with a 0.6 mod had its credibility decline by 40%, it would now get a 0.76 mod, all other things being equal. Its standard premium would then increase by 27% ($1.27 = 0.76/0.6$).

[13] Under the revised plan the definition of primary losses is changed. Thus the D-ratios, which measure the expected portion of the losses that will be primary, have to be recalculated with the adoption of the revised plan. In one state (Massachusetts) the average D-ratio decreased from about 0.35 to about 0.30. The results will vary by state, depending on the size of loss distribution, which depends heavily on the particular state workers' compensation law.

[14] Assuming a D-ratio of D, the maximum credibility is $(D \times 91\%) + ((1 - D) \times 57\%)$. For $D = 0.50$ the maximum credibility is 74%. For $D = 0.35$ the maximum credibility is 69%. For $D = 0.20$ the maximum credibility is 64%.

5. COVARIANCE OF EXCESS AND PRIMARY LOSSES

As discussed previously, the equations for credibility by size of risk underlying the revised experience rating plan can be derived from theoretical considerations, provided one assumes that the covariance of excess and primary losses is not extremely important. If this covariance is important, i.e., if excess and primary losses are highly correlated, then one expects a more complex relationship of credibilities with size of risk. (See Appendix B for the derivation of equations.)

Recall that under revised experience rating, the first \$5,000 of a loss is considered primary and the remainder of the loss that enters into the experience rating calculation is excess.[15] A simple special case will illustrate why one would expect the excess and primary losses to be significantly correlated. Assume half the losses were of size \$30,000 (with primary portion \$5,000 and excess portion \$25,000), while the other half were of size \$3,000 (with primary portion \$3,000 and excess portion of zero). Then the excess and primary losses are perfectly correlated.

While, in actuality, there are claims of all sizes, the large losses will all have \$5,000 in primary losses, while the smallest losses will all have no excess and less than \$5,000 of primary losses. Thus some positive correlation should exist. This should carry over to an examination of all the losses for an insured. For a constricted example in Mahler [3, p. 141], the primary and excess losses were highly correlated. The actual size of these correlations for actual insureds can be examined empirically. These covariances can be estimated from the data used for experience rating.

As an illustrative example, the covariances were estimated using three years of data from one state. The estimation process is described in detail in Appendix A. While there was insufficient data to arrive at a definitive conclusion, the results are interesting and should point the

[15] Recall that for very large claims, the maximum amount that enters into the calculation of the experience modification is 10% of the State Reference Point.

way for further research. As expected, the primary and excess losses were found to be significantly correlated. The "between" and "within" correlations were each greater than 50%.

6. CREDIBILITIES TAKING INTO ACCOUNT COVARIANCES

The credibilities are determined in Appendix A by using the estimated variances and covariances in the theoretical formula for the split experience rating plan. The resulting credibilities differ significantly from those under revised experience rating. As shown in Exhibit 7, the calculated primary credibilities are all 100% while the excess credibilities range from about 10% to about 45%. Both the primary and excess credibilities are significantly larger than those indicated by revised experience rating.

The data was too limited to draw any detailed information about the behavior of credibilities with size of risk, beyond the expectation that the excess credibilities increase with size of risk.[16] There are a number of reasons why the credibilities calculated here may differ from those for revised experience rating.

First, the calculation here explicitly considered the covariances between primary and excess losses.

Second, the calculation here relied upon a limited number of intrastate-rated risks from just one state from just one point in time. The credibilities are a relative measure of the informational value of the expected losses and actual losses. The informational value of the expected losses calculated from the expected loss rates depends in turn on the precision of the classification relativities. This precision will vary by state and over time depending on many factors. In addition, while most of the parameters are scaled to the average claim cost by state, the split between primary and excess losses is a fixed $5,000. Thus, the proportions of claim dollars that are primary and excess vary among states based on their differing average claim costs.

[16]Not only do the calculated excess credibilities exhibit fluctuation error, but also there is no useful information on the very largest risks.

It is likely that the variance/covariance structure also varies among these states.

Third, only three years of data were analyzed. Experience rating involves predicting a future year of experience using data generally from two, three, and four years distant. As discussed in Mahler [4] and Mahler [5], as this distance in time gets greater, the phenomenon of shifting risk parameters becomes more important. This phenomenon would act to lower the credibilities from those calculated here.

Fourth, the revised Experience Rating Plan was parametrized via an examination of which credibilities would have performed well in the past.[17] Also, the criterion used to decide which credibilities performed better differs from the least squares criterion. The "Quintiles Test" used by the NCCI and described by Gillam is a refinement of the Dorweiler criterion.[18]

For all of the above reasons, one should not draw any definitive conclusions from the work done here.

7. POSSIBLE FURTHER RESEARCH

It would be interesting to compare the more general credibility formula $\frac{\text{Quadratic}}{\text{Quadratic}}$ versus the $\frac{\text{Linear}}{\text{Linear}}$ formula using the same types of tests as performed by the NCCI.

Another area for possible research is the number of years of data used in the experience period. Currently, three years are given equal weight.[19] One could test whether some other combination of number of years and weights could produce a more accurate result.[20] Appen-

[17]This was not possible to do here due to the limited data available.

[18]These criteria are contrasted in Mahler [4].

[19]Actually since more recent years have more payroll on average, due to inflation, the most recent year on average has somewhat more weight.

[20]As pointed out in Mahler [3], the optimal set of years and weights will depend on to what extent the risk parameters of an insured are shifting over time. This subject was explored in Mahler [4] and Mahler [5].

dix D displays an example of the type of analysis needed. This preliminary analysis indicates that further investigation would be worthwhile.

8. SUMMARY

The revised Experience Rating Plan is based on significantly different credibility formulas than the prior plan. The change results in a significantly more responsive plan for small risks and a significantly less responsive plan for large risks.

While the revised Experience Rating Plan, as explained by Mr. Gillam, has a firmer theoretical and empirical basis than the prior plan, there remain areas for further actuarial research.[21]

[21]The examination of the NCCI [8] performed by the actuarial consulting firm of Milliman & Robertson, Inc. for the NAIC contains a very interesting section on further areas of research on experience rating.

REFERENCES

[1] Venter, Gary G., "Experience Rating—Equity and Predictive Accuracy," *NCCI Digest,* Vol. II, Issue I, April 1987, p. 27.

[2] Snader, Richard H., "Fundamentals of Individual Risk Rating and Related Topics," Part I of Study Note, Casualty Actuarial Society, 1980.

[3] Mahler, Howard C., Discussion of Meyers: "An Analysis of Experience Rating," *PCAS* LXXIV, 1987, p. 119.

[4] Mahler, Howard C., "An Example of Credibility and Shifting Risk Parameters," *PCAS* LXXVII, 1990, p. 225.

[5] Mahler, Howard C.,"The Credibility of a Single Private Passenger Driver," *PCAS* LXXVIII, 1991, p. 146.

[6] Meyers, Glenn G., "An Analysis of Experience Rating," *PCAS* LXXII, 1985, p. 278.

[7] Venter, Gary G., "Credibility," *Foundations of Casualty Actuarial Science,* Casualty Actuarial Society, New York, 1990, Chapter 7, pp. 432-433.

[8] Biondi, Richard S., Grannan, Patrick J., Mulvaney, Mark W., and Pestcoe, Marvin, *NAIC Examination of NCCI,* Volume IX-Section IIB -Part 7, December 5, 1991.

EXHIBIT 1

COMPARISON OF WORKERS' COMPENSATION EXPERIENCE RATING PLANS

Prior	Revised
Primary and Excess Losses	Primary and Excess Losses
Multi-Split Plan: Primary portion of a loss is determined via formula[1] or from a table.	Single Split Plan: Primary portion of a loss is the first $5,000.
Experience Modification depends on a comparison of actual losses to expected losses, taking into account credibilities.	Experience Modification depends on a comparison of actual losses to expected losses, taking into account credibilities.
W and *B* values are shown in a table, and depend on the expected losses for the risk.	*W* and *B* values are shown in a table, and depend on the expected losses for the risk.
The table of *W* and *B* values depends on a state-specific value, the *Self-Rating* Point (SRP).	The table of *W* and *B* values depends on a state-specific value, the *State Reference* Point (SRP).[2]
The per claim accident limitation is 10% of the state's *Self-Rating* Point.	The per claim accident limitation is 10% of the *State Reference* Point.
The State Multiple Claim Accident Limitation is twice the State Per Claim Accident Limitation.	The State Multiple Claim Accident Limitation is twice the State Per Claim Accident Limitation.

[1] $A_p = 10{,}000\,A/(A + 8{,}000)$. For losses less than 2,000, the whole loss is considered primary.

[2] The State Reference Point is equal to 250 times the average claim cost in the particular state. The NCCI uses the state-specific parameter g, which is defined as the average claim cost in the state divided by 1,000; g is rounded to the nearest 0.05; $g = \text{SRP}/250{,}000$.

EXHIBIT 2

NCCI REVISED EXPERIENCE RATING
PRIMARY AND EXCESS CREDIBILITIES

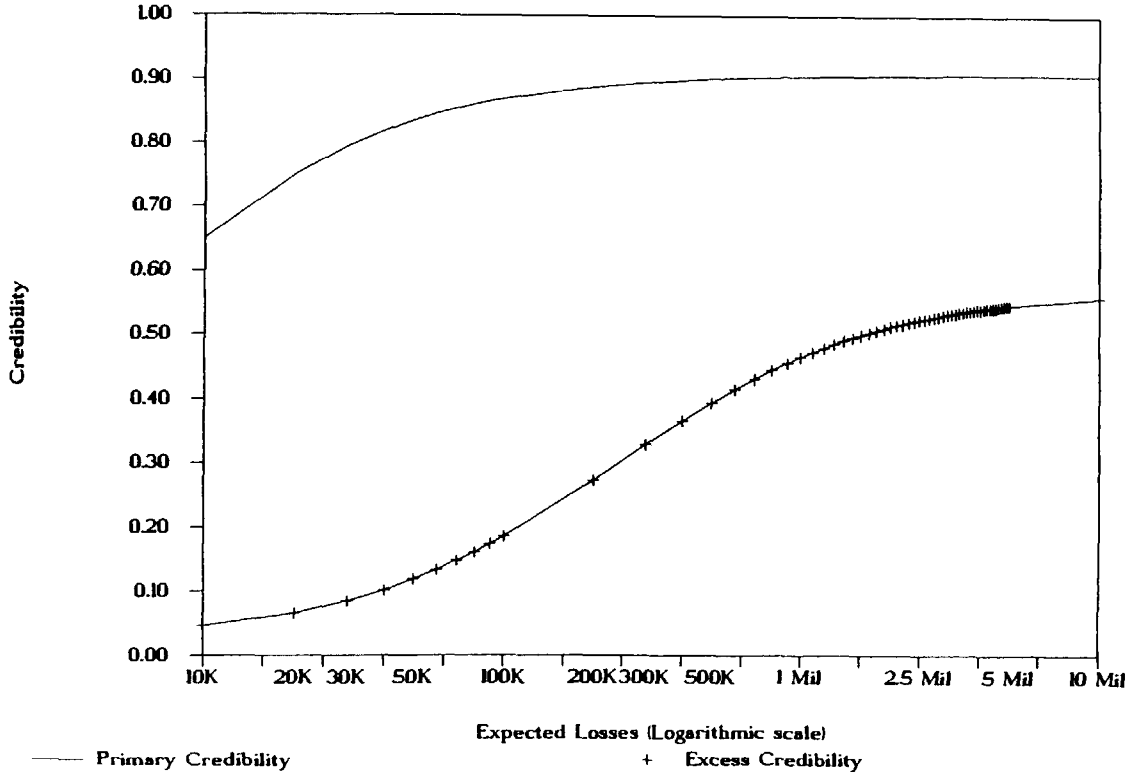

EXHIBIT 3

WORKERS' COMPENSATION EXPERIENCE RATING

	Credibilities (Weighted Average of Primary and Excess Credibilities)		
Expected Losses ($000)	Prior*	Revised**	Revised Minus Prior***
3****	5%	10%	5%
5	7	14	7
7.5	10	18	8
10	12	20	9
15	15	24	9
20	18	26	9
25	19	28	9
50	27	33	7
75	31	37	6
100	34	39	5
125	36	41	5
150	39	43	4
200	43	46	3
300	51	50	-1
400	58	53	-5
500	66	55	-11
750	83	58	-24
1,000	100	59	-41
2,000	100	63	-37
3,000	100	65	-35
4,000	100	65	-35
5,000	100	65	-35
7,500	100	66	-34
10,000	100	66	-34
∞	100	67	-33

* NCCI Experience Rating Plan prior to revision, assuming a Self-Rating Point of $1,000,000 and a *D*-ratio of 0.35.

** Revised Experience Rating Plan, assuming a State Reference Point of $500,000 and a *D*-ratio of 0.30.

*** Result may differ slightly due to intermediate rounding.

**** Eligibility requirements vary by state. In most states $3,000 in expected losses is currently close to the minimum size risk ever experience rated.

EXHIBIT 4

WORKERS' COMPENSATION EXPERIENCE RATING CREDIBILITIES

Expected	Primary		Excess	
Losses ($000)	Prior*	Revised**	Prior*	Revised**
3***	13%	29%	0%	2%
5	20	40	0	3
7.5	27	50	0	4
10	33	57	0	5
15	43	67	0	6
20	50	73	0	7
25	56	77	0	8
50	72	83	2	12
75	80	86	4	15
100	84	87	7	18
125	87	88	9	21
150	90	88	12	24
200	92	89	17	28
300	95	90	27	33
400	97	90	37	37
500	98	90	48	40
750	99	90	73	44
1,000	100	90	100	46
2,000	100	91	100	52
3,000	100	91	100	54
4,000	100	91	100	54
5,000	100	91	100	54
7,500	100	91	100	55
10,000	100	91	100	55
∞	100	91	100	57

* NCCI Experience Rating Plan prior to revision, using Self-Rating Point of $1,000,000 (assumes average serious case of $40,000).

** Revised Experience Rating Plan, using State Reference Point of $500,000 (assumes average case of $2,000).

*** Eligibility requirements vary by state. In most states $3,000 in expected losses is currently close to the minimum size risk ever experience rated.

EXHIBIT 5

WORKERS' COMPENSATION EXPERIENCE RATING
W AND *B* VALUES

Expected	*B* ($00)		*W*	
Losses ($000)	Prior*	Revised**	Prior*	Revised**
3***	200	75	0	0.07
5	200	75	0	0.08
7.5	200	75	0	0.08
10	200	75	0	0.08
15	200	75	0	0.09
20	200	75	0	0.09
25	200	75	0	0.10
50	194	99	0.03	0.14
75	190	124	0.05	0.18
100	184	149	0.08	0.21
125	180	174	0.10	0.24
150	174	200	0.13	0.27
200	164	250	0.18	0.31
300	144	350	0.28	0.37
400	124	450	0.38	0.41
500	102	550	0.49	0.44
750	52	800	0.74	0.49
1,000	0	1,050	1.00	0.51
2,000	0	2,050	1.00	0.57
3,000	0	3,050	1.00	0.59
4,000	0	4,050	1.00	0.60
5,000	0	5,050	1.00	0.60
7,500	0	7,550	1.00	0.61
10,000	0	10,050	1.00	0.61

* NCCI Experience Rating Plan prior to revision using a Self-Rating Point of $1,000,000 (assumes average serious case of $40,000).

** Revised Experience Rating Plan, using State Reference Point of $500,000 (assumes average case of $2,000).

*** Eligibility requirements vary by state. In most states $3,000 in expected losses is currently close to the minimum size risk ever experience rated.

EXHIBIT 6

WORKERS' COMPENSATION EXPERIENCE RATING
REVISED EXPERIENCE RATING PLAN*

Expected	Maximum Credit**			Maximum
Losses ($000)	*D*-ratio = 0.40	*D*-ratio = 0.30	*D*-ratio = 0.20	Debit
3***	13%	10%	7%	60%
4	15	12	9	60
5	18	14	11	60
6	20	16	12	80
7	22	17	13	80
8	23	18	14	80
9	24	19	14	80
10	26	20	15	80
11	27	21	16	100
12	28	22	16	100
13	28	23	17	100
14	29	23	17	100
15	30	24	18	100
16	31	25	19	No Limit

* Revised Experience Rating Plan, using State Reference Point of $500,000 (assumes average case of $2,000).

** The maximum credit depends on the particular *D*-ratio. The maximum credit is the credibility which is equal to:

$$[D \times \text{primary credibility} + (1 - D) \times \text{excess credibility}].$$

*** Eligibility requirements vary by state. In most states $3,000 in expected losses is currently close to the minimum size risk ever experience rated.

EXHIBIT 7

CREDIBILITIES ESTIMATED TAKING INTO ACCOUNT THE COVARIANCES OF PRIMARY AND EXCESS LOSSES COMPARED TO THOSE FROM REVISED EXPERIENCE RATING

Expected	Credibilities from Appendix A[1]		Credibilities from Revised Experience Rating[2]	
Losses ($000)	Z_p	Z_x	Z_p	Z_x
7.5	100%	10%	40%	3%
12.5	100	21	48	3
17.5	100	13	53	3
22.5	100	16	58	4
37.5	100	12	66	5
62.5	100	31	74	6
87.5	100	16	78	8
125	100	35	81	10
200	100	40	84	13
375	100	27	87	19
750	100	46	89	28

[1] Based on variances and covariances estimated from experience rating data in Massachusetts. Primary credibility limited to no more than 100%.

[2] For a State Reference Point of $1,750,000 ($g = 7$), which is the current value in Massachusetts.

APPENDIX A

MASSACHUSETTS EXPERIENCE RATING DATA

The Data

The data examined consisted of a subset of the risks experience rated in Massachusetts during policy year 1991. Only intrastate-rated[1] risks with three years of data were examined. In addition, only risks whose expected primary losses in each of the three years were at least equal to 20% of the three year total were examined.[2] The resulting data set consisted of three years of information on each of about 16,000 risks. The information by year was expected and actual losses, each split between primary and excess losses.[3]

Estimation of Variances and Covariances

As per Gillam, we computed:

P = primary loss divided by expected loss = A_p/E ;

X = excess loss divided by expected loss = A_x/E .

The covariances and variances were estimated as suggested in Venter [7]:

Let $X_{it} = X$ for risk i in year t ,

$P_{it} = P$ for risk i in year t,

N = number of risks,

n = number of years,

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[1] Complete data on interstate-rated risks was not available from this source.

[2] This limitation was imposed in order to obtain a more reliable estimate of the variation from year to year in the observed results for a given risk; i.e., to make the estimates of the within variances more reliable.

[3] The split between primary and excess losses used the definition of revised experience rating, which was in effect in Massachusetts for policy year 1991. (The expected losses were computed based on the Expected Loss Rates and D-ratios in effect in Massachusetts for policy year 1991.)

$$X_{i.} = \frac{1}{n} \sum_{t} X_{it}$$

$$X_{..} = \frac{1}{nN} \sum_{i,t} X_{it} .$$

Then one estimates the excess variances as follows (with the analogous equations for primary variances):

$$\text{estimated within variance of excess losses} = \frac{\sum_{i} \sum_{t} (X_{it} - X_{i.})^2}{(n-1)N} ;$$

estimated between variance of excess losses

$$= \frac{\sum_{i} (X_{i.} - X_{..})^2}{N-1} - \frac{\begin{pmatrix}\text{estimated within variance} \\ \text{of excess losses}\end{pmatrix}}{n} .$$

One estimates the covariances as follows:

estimated within covariance of excess and primary losses

$$= \frac{\sum_{i} \sum_{t} (X_{it} - X_{i.})(P_{it} - P_{i.})}{(n-1)N} ;$$

estimated between covariance of excess and primary losses

$$= \frac{\sum_{i} (X_{i.} - X_{..})(P_{i.} - P_{..})}{N-1} - \frac{\begin{pmatrix}\text{estimated within covariance} \\ \text{of excess and primary losses}\end{pmatrix}}{n} .$$

The estimated variances and covariances are shown in Exhibit A.1. In each case, risks between a certain minimum and maximum size (based on expected losses) were examined separately. While there is an overall pattern observed as the risk size varies, it is clear that the limited number of risks and years of data have produced significant fluctuation errors in the individual estimates.

Estimated Credibilities

Using the estimated variances and covariances, and the equations in Appendix B, one obtains the estimated credibilities shown in Exhibit A.2. It should be noted that the estimated credibilities are for the use of three years of data, as is currently used in experience rating. The within variances and covariances estimated from the data are for a single year of data; for use in estimating credibilities, these quantities have been divided by three.[4]

Exhibit A.2 displays three different sets of credibilities. The first is the Bühlmann credibility; i.e., the least squares credibility estimated separately for primary and excess losses, ignoring their correlation. The second set of credibilities is calculated via Equations B.2 and represents the least squares credibility taking into account the correlation of primary and excess losses. The third and final set of credibilities is similar to the second set; but it has had the primary credibility set equal to unity, as discussed in Appendix C.

[4] When using many years of data or when parameters shift significantly over time, a different adjustment than performed here would be appropriate.

EXHIBIT A.1

Expected Losses ($000)*		Number	Estimated Variances				Estimated Covariances		Estimated Correlations	
			Between		Within**					
Min	Max	of Risks	Primary	Excess	Primary	Excess	Between	Within**	Between	Within
5	10	2,731	0.053	1.218	0.106	6.395	0.179	0.465	0.702	0.565
10	15	3,536	0.040	1.341	0.068	3.810	0.209	0.262	0.898	0.514
15	20	2,080	0.051	0.636	0.048	2.776	0.128	0.208	0.713	0.567
20	25	1,396	0.036	0.664	0.040	2.398	0.127	0.173	0.816	0.561
25	50	3,154	0.024	0.401	0.028	1.855	0.074	0.134	0.753	0.590
50	75	1,246	0.034	0.549	0.016	0.977	0.122	0.070	0.888	0.566
75	100	568	0.035	0.220	0.012	0.782	0.070	0.059	0.793	0.609
100	150	590	0.018	0.397	0.010	0.599	0.061	0.046	0.712	0.601
150	250	470	0.022	0.339	0.006	0.428	0.064	0.030	0.752	0.574
250	500	265	0.008	0.110	0.004	0.234	0.022	0.017	0.710	0.566
500	1,000	79	0.017	0.201	0.002	0.202	0.043	0.015	0.748	0.655

Note: Based on intrastate-rated risks in Massachusetts, as explained in the text.

* The sum of expected losses for three years of data used for experience rating.

**While all within variances and covariances were estimated using individual years of data, the values listed here have been divided by three to adjust them for the use of three years of data for experience rating.

EXHIBIT A.2

Expected Losses ($000)		Between Variance Divided by Between Variance Plus Within Variance[1]		Least Squares Credibility[2]		Alternate Credibility[3]	
Min	Max	Primary	Excess	Primary	Excess	Primary	Excess
5	10	33%	16%	109%	9%	100%	10%
10	15	37	26	164	15	100	21
15	20	51	19	157	7	100	13
20	25	48	22	184	8	100	16
25	50	47	18	166	6	100	12
50	75	69	36	280	9	100	31
75	100	75	22	222	1	100	16
100	150	65	40	181	27	100	35
150	250	77	44	221	25	100	40
250	500	69	32	192	17	100	27
500	1,000	87	50	230	28	100	46

Note: Credibilities computed based on the variances and covariances in Exhibit A.1.

[1] Bühlmann credibility, estimated least squares credibility ignoring any correlation between primary and excess losses.

[2] Estimated least squares credibility taking into account the correlation between primary and excess losses.

[3] Primary credibility limited to 100%.

APPENDIX B

DEPENDENCE OF CREDIBILITY ON SIZE OF RISK

In this appendix, the variation of credibility with size of risk will be discussed. Equations B.10 are those used in the revised experience rating plan. The theoretical underpinnings of these formulas, as well as the more general Equations B.11, are discussed.

Following the development in Mahler [3], let

a = total variance of the primary losses,

b = total variance of the excess losses,

c = variance of the hypothetical means of the primary losses ,

= "between" variance of primary losses,

d = variance of the hypothetical means of the excess losses,

= "between" variance of excess losses,

r = total covariance of hypothetical means of the primary and excess losses, and

s = covariance of hypothetical means of the primary and excess losses

= "between" covariance of primary and excess losses.

Then the optimum least squares credibilities Z_p and Z_x are derived in Appendix F of Mahler [3] and given in Equations 5.3 and 5.4 of that paper as:

$$Z_p = \frac{(c+s)b - (d+s)r}{ab - r^2}, \text{ and} \tag{B.1.a}$$

$$Z_x = \frac{(d+s)a - (c+s)r}{ab - r^2} . \tag{B.1.b}$$

Thus, both the primary and excess credibilities can be written in terms of variances and covariances.

Therefore, the dependence of the credibilities on the size of the risk can be derived from the dependence of the various variances and covariances on the size of the risk.

Again following Mahler [3], let

$t = a - c =$ process variance of the primary losses

$=$ "within" variance of primary losses,

$u = b - d =$ process variance of the excess losses

$=$ "within" variance of excess losses, and

$v = r - s =$ process covariance of the primary and excess losses

$=$ "within" covariance of primary and excess losses.

Then substituting into Equations B.1, one gets:

$$Z_p = \frac{(c+s)(u+d) - (d+s)r}{(t+c)(u+d) - (v+s)^2}, \text{ and} \tag{B.2.a}$$

$$Z_x = \frac{(d+s)(t+c) - (c+s)r}{(t+c)(u+d) - (v+s)^2}. \tag{B.2.b}$$

The NCCI credibility parameters K_p and K_x are defined so that:

$$Z = \frac{E}{E+K}$$

and, therefore,

$$K = E\left(\frac{1}{Z} - 1\right). \tag{B.3}$$

Substituting into Equation B.3 the expressions for Z_p and Z_x given in Equations B.2, one obtains:

$$K_p = E\,\frac{tu + td + vd - su - sv - v^2}{cu + su + cd - s^2 - sv - dv}, \tag{B.4.a}$$

$$K_x = E\frac{tu + uc + vc - st - sv - v^2}{dt + st + cd - s^2 - sv - cv}. \tag{B.4.b}$$

If the covariances between the primary and excess losses are zero, $v = s = 0$,[1] i.e., if there is no useful information about the primary losses contained in the excess losses and vice versa, then these equations are greatly simplified:

$$K_p = E\frac{t}{c}, \tag{B.5.a}$$

$$K_x = E\frac{u}{d}. \tag{B.5.b}$$

Each of the two separate pieces, which are assumed to be uncorrelated with each other, has credibility parameter given by the familiar Bühlmann result.

It is Equations B.5 that form the theoretical bases of the credibilities used by the NCCI in the revised experience rating plan, rather than the more complicated but more general Equations B.4.

It is generally assumed that process variances and covariances (so-called "within" variances and covariances) such as t, u, and v, increase proportionally with E, the size of risk:

$$t \sim E, \tag{B.6.a}$$

$$u \sim E, \tag{B.6.b}$$

$$v \sim E. \tag{B.6.c}$$

However, as shown in Meyers [6], when the phenomenon of parameter uncertainty is important, Equations B.6 do not hold. Instead, t, u, and v increase partially proportionally with E and partially pro-

[1] In fact, the covariances are observed to be significantly different from zero. The total covariance of primary and excess losses, $r = s + v$, is generally positive in actual applications.

portionally with E squared.[2] When parameter uncertainty is important:

$$t \sim E \text{ Linear } [E]\,, \tag{B.7.a}$$

$$u \sim E \text{ Linear } [E]\,, \tag{B.7.b}$$

$$v \sim E \text{ Linear } [E]\,. \tag{B.7.c}$$

It is generally assumed that variances and covariances of the hypothetical means (so-called "between" variances and covariances) such as c, d, and s increase proportionally with the square of E, the size of risk:

$$c \sim E^2, \tag{B.8.a}$$

$$d \sim E^2, \tag{B.8.b}$$

$$s \sim E^2. \tag{B.8.c}$$

However, as shown in Mahler [3], in the presence of risk heterogeneity, Equations B.8 do not hold. Instead, c, d, and s increase partially proportionally with E and partially proportionally with E squared.[3] When risk heterogeneity is important:

$$c \sim E \text{ Linear } [E], \tag{B.9.a}$$

$$d \sim E \text{ Linear } [E], \tag{B.9.b}$$

$$s \sim E \text{ Linear } [E]. \tag{B.9.c}$$

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[2] As discussed in Mahler [3], the portion of the process variance or covariance that is proportional to the square of E represents the variation of the parameters due to the different states of the universe.

[3] As discussed in Mahler [3], the portion of the variance or covariance of the hypothetical means that is proportional to E represents the variation caused by grouping subunits together to form a single risk. For example, several separate factories might belong to a single insured.

One can substitute the behavior of the variances and covariances with size of risk into the equations for the credibility parameters K. The revised experience rating plan is based on Equations B.5, with parameter uncertainty (Equations B.7) and risk heterogeneity (Equations B.9). Substituting Equations B.7 and B.9 into Equations B.5 gives:

$$K_p \sim E \frac{\text{Linear } [E]}{\text{Linear } [E]}, \tag{B.10.a}$$

$$K_x \sim E \frac{\text{Linear } [E]}{\text{Linear } [E]}. \tag{B.10.b}$$

This is the form of the credibility parameters used in the revised Experience Rating Plan shown in the main text.[4] This form of the credibility parameters leads directly to the form of the credibilities in the main text.

If, instead of the special case Equation B.5, one starts with the more general Equations B.4, one gets a different form for the credibility parameters. Substituting Equations B.6 and B.8 into Equations B.5 gives the following general form of the credibility parameters with parameter uncertainty and risk homogeneity:[5]

$$K_p \sim E \frac{\text{Quadratic } [E]}{\text{Quadratic } [E]}, \tag{B.11.a}$$

$$K_x \sim E \frac{\text{Quadratic } [E]}{\text{Quadratic } [E]}. \tag{B.11.b}$$

Equations B.10 are a special case of Equations B.11. Therefore, Equations B.11 will always perform at least as well as and usually perform better than Equations B.10 in any empirical tests, including

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[4] This is the form for the no-split plan with parameter uncertainty and risk heterogeneity given at Mahler [3, p. 178].

5 This is the form for the split plan with parameter uncertainty and risk heterogeneity given in Mahler [3, p. 178].

the type of studies conducted by the NCCI in its development of the revised Experience Rating Plan. Practical considerations will determine whether in a particular application the extra generality represented by Equations B.11 is worth the extra complication introduced by the additional parameters contained in Equations B.11.

APPENDIX C

PRIMARY CREDIBILITY EQUAL TO UNITY

As pointed out in Mahler [3], the use of the credibilities given by Equations B.1 in Appendix B can lead to calculated credibilities greater than unity. This is the case for the Massachusetts data discussed in Appendix A.

For that data, the primary losses have a significantly smaller variance than the excess losses. In addition, the primary and excess losses are significantly positively correlated. Therefore, the observed primary losses are of value not only to predict future primary losses but also to predict future excess losses. Thus, in some sense, a portion of the credibility applied to the observed primary losses is predicting the future excess losses. Since the expected excess losses are usually greater than the expected primary losses,[1] this addition to the credibility applied to the primary losses due to taking into account the correlation with the excess losses can be very significant. It can easily result in primary credibilities greater than unity.[2]

In circumstances where the calculated primary credibility is greater than unity, one could reasonably set the primary credibility equal to unity. One can then solve for the optimal (least squares) value for the excess credibility.

Following Mahler [3], we have the following value for the efficiency:[3]

$$\text{Efficiency} = \frac{2\,Z_p(c+s) + 2Z_x(d+s) - Z_p^2 a - Z_x^2 b - 2Z_p Z_x r}{c + d + 2s},$$

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[1] For example, a D-ratio of 0.33 is equivalent to expected excess losses being twice expected primary losses.

[2] This same phenomenon was noted in the example discussed in Sections 6 to 9 of Mahler [3].

[3] The efficiency of an experience rating plan is defined in Meyers [6] as the reduction in the expected squared error. The higher the efficiency, the more accurate the Experience Rating Plan.

where a, b, c, d, r, and s are the variances and covariance defined in Appendix B.

If $Z_p \equiv 1$, then

$$\text{Efficiency} = \frac{2(c+s) + 2Z_x(d+s) - a - Z_x^2 b - 2Z_x r}{c+d+2s}.$$

The least squares credibility is obtained by maximizing the efficiency. Taking the derivative of the efficiency with respect to Z_x and setting it equal to zero gives:[4]

$$\frac{2(d+s) - 2Z_x b - 2r}{c+d+2s} = 0.$$

Solving for the excess credibility gives:

$$Z_x = \frac{d+s-r}{b}.$$

This equation is used in Appendix A in order to calculate the alternate credibilities with $Z_p = 1$. It may be of interest to use the fact that the total covariance equals the sum of the between and within covariances to rewrite this equation as:

$$\text{(Alternate) Excess Credibility} = \frac{\begin{array}{c}\text{Between}\\ \text{Variance of}\\ \text{Excess Losses}\end{array} - \begin{array}{c}\text{Within Covariance}\\ \text{of Primary and}\\ \text{Excess Losses}\end{array}}{\text{Total Variance of Excess Losses}}.$$

Except for the inclusion of the term involving the within covariance, this equation is the usual Bühlmann formula for credibility ignoring the correlation between primary and excess losses. Since that covariance is generally large and positive, this equation will produce lower excess credibilities than the usual Bühlmann formula, to go along with the higher primary credibility of 100%.

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[4] If Z_p were not constrained to be unity, one would set the partial derivatives of the efficiency with respect to Z_p and Z_x equal to zero, and solving, obtain Equations B.1.

APPENDIX D

SHIFTING RISK PARAMETERS OVER TIME

The phenomenon of shifting risk parameters over time can significantly alter the credibility assigned to data.[1] As shown in Mahler [4], the first step in determining the importance of this phenomenon is to examine the correlations between different years of data. If the correlations decline significantly as the separation increases, then the phenomenon of shifting risk parameters is significant.

Correlations were estimated for the three years of Massachusetts experience rating data described in Appendix A. For primary losses[2], the computed correlations were 0.22 between adjacent years and 0.16 between years with a year between. For excess losses[3], the computed correlations were 0.09 between adjacent years and 0.06 between years with a year between. Since the correlations are somewhat lower for years further apart, the phenomenon of shifting risk parameters has some significance.

Exhibit D.1 displays a more detailed breakdown by year and size category. While the overall pattern is maintained, there is significant fluctuation, particularly for excess losses.

In order to draw any conclusions, one should study more risks over a longer time span.

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[1] See Mahler [3], Mahler [4], and Mahler [5].

[2] Actually $P = A_p/E$ as defined in Appendix A

[3] Actually $X = A_x/E$ as defined in Appendix A.

EXHIBIT D.1
ESTIMATED CORRELATIONS

Expected Losses ($000)* Min	Expected Losses ($000)* Max	Number of Risks	Primary: Year 1 and Year 2	Primary: Year 2 and Year 3	Primary: Year 1 and Year 3	Excess: Year 1 and Year 2	Excess: Year 2 and Year 3	Excess: Year 1 and Year 3
5	10	2,731	0.16	0.16	0.12	0.06	0.11	0.02
10	15	3,536	0.19	0.18	0.14	0.14	0.11	0.07
15	20	2,080	0.23	0.32	0.23	0.04	0.08	0.09
20	25	1,396	0.30	0.20	0.20	0.09	0.06	0.10
25	50	3,154	0.27	0.24	0.16	0.06	0.08	0.06
50	75	1,246	0.48	0.45	0.35	0.23	0.13	0.12
75	100	568	0.45	0.55	0.49	0.11	0.05	0.11
100	150	590	0.36	0.48	0.33	0.16	0.23	0.16
150	250	470	0.55	0.55	0.49	0.17	0.24	0.22
250	500	265	0.37	0.49	0.42	0.17	0.15	0.08
500	1,000	79	0.70	0.67	0.72	0.33	0.26	0.30

* The sum of expected losses for three years of data used for experience rating.