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THE COMPLEMENT OF CREDIBILITY

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Abstract

This paper explains the most commonly used complements of credibility and offers a comparison of the effectiveness of the various methods. It includes numerous examples. It covers credibility complements used in excess ratemaking as well as those used in first dollar ratemaking. It also offers six criteria for judging the effectiveness of various credibility complements. One criterion, statistical independence, has not previously been covered in the actuarial literature. This paper should explain all the common credibility complements to the actuarial student.

1. INTRODUCTION

Many actuarial papers discuss credibility. Actuaries use credibility when data is sparse and lacks statistical reliability. Specifically, actuaries use it when historical losses have a large error around the underlying expected losses (average of the distribution of potential loss costs) the actuary is estimating. They use a loss estimate such as

$$\text{estimate} = Z \times \text{historical losses} + (1 - Z) \times \text{ancillary statistic},$$

where Z is the credibility associated with the historical losses.

In those circumstances, the ancillary statistic that receives the remainder of the credibility can be more important than the data's credibility. For example, if the ratemaking statistic varies around the true expected losses with a standard deviation equal to its mean, it will probably receive a very low credibility. Therefore, the vast majority of the rate (in this context, expected loss estimate) will come from whatever statistic receives the complement of credibility. So, it is very important to use an effective statistic for the ancillary statistic (hereafter called the complement of credibility).

This paper will first discuss six desirable qualities for the complement: accuracy, unbiasedness, independence, availability of data, ease of computation, and an explainable relationship to the subject loss costs. It will do so in light of four basic areas: practical issues, competitive market issues, regulatory issues, and statistical issues. Then it will discuss several complements actuaries often use for first dollar (low or no deductible) losses. It will include a practical example of each complement. Also, it will discuss how well each complement possesses the six desirable qualities. Last, it will discuss several complements commonly used with losses excess of a high deductible or retention (with examples and discussions of how well each possesses the six desirable qualities).

2. FUNDAMENTAL PRINCIPLES—WHAT SHOULD THE ACTUARY CONSIDER?

There are four types of issues that any actuary must consider when choosing the complement: practical issues, competitive market issues, regulatory issues, and statistical issues.

Practical Issues

The easiest statistic to use is one that is readily available. Since some statistics require more complicated programming or expensive processing than others, some statistics are more readily available than others.

Ease of computation is another factor to consider. If a statistic is easy to compute, it is often easier to explain to management and customers. Since few actuaries have unlimited budgets, they usually weigh the time involved in computing a very accurate statistic against the increase in accuracy it generates. Also, when computations are easy to do, there is less chance of error.

Competitive Market Issues

Rates are rarely made in a vacuum. Generally, whatever rate the actuary produces will be subject to market competition. If the rate is too high, competitors can undercut the rate and still make a profit. That will cost the insurer customers and profit opportunities. If the rate is too low, the insurer will lose money. Therefore, in mathematical terms, the rate should be unbiased (neither too high nor too low over a large number of loss cost estimates) and accurate (the rate should have as low an error variance as possible around the future expected losses being estimated). Also, the difference between unbiasedness and accuracy is important. An unbiased statistic varies randomly about the following year's losses over many successive years, but it may not be close. An accurate statistic may average higher or lower than the following year's losses, but it is always close. Ultimately, the complement of the credibility should help make the rate as unbiased and accurate as possible.

Regulatory Issues

Usually, rates require some level of approval from insurance regulators. The classic rate regulatory law requires that rates be "neither inadequate, excessive, nor unfairly discriminatory." The principles of adequacy and non-excessiveness imply that rates should be as unbiased as possible.

Those principles could be stretched to imply that rates should be accurate. The argument goes as follows: Inaccurate rates create a greater risk of insolvency by causing random inadequacies. The law seeks to prevent insolvencies. Therefore, the law suggests rates should be as accurate as possible. Also, for most purposes, actuaries interpret "unfairly discriminatory" in the ratemaking context as "unbiased." Supposedly, if a rate truly reflects a class's probable loss experience, it is fair by definition.

A complement should have some logical relationship to the loss costs of the class or individual being rated. It is easier to explain to a regulator a rate for a class or individual that is consistent with the related loss costs.

Statistical Issues

Clearly, the actuary must attempt to produce the most accurate rate that is practical, but in doing so, the actuary must consider all the types of error that make up the prediction error. (The prediction error is the squared difference between the credibility weighted prediction and actual results.) There are, of course, the natural year-to-year variations in losses about the true mean due to process variance. There may also be errors because the predictor has a different mean than the losses (bias).

The error of the predictor may stem from the error of its components. The historical losses (the usual base statistic), when trended and developed, will contain prediction errors because the factors used to bring losses to a fully developed current cost level are different than what actually will happen (loss development and trend variance). When mathematical models of losses are used as complements, there may be errors in both the type of model used (model variance) and the specific parameters selected for the model (parameter variance). All of these (including any process error and bias of the complement) contribute to prediction error and reduce the accuracy of the prediction.

If the complement of the credibility is accurate in its own right and relatively independent of the base statistic (which receives the credibility), the resulting rate will be more accurate. The rationale involves statistical properties of credibility-weighted estimates. As Appendix A shows, if the optimum credibility for two unbiased statistics is used, then the prediction error (the variance of an estimate around next year's actual loss costs once they are known) of the credibility-weighted estimate is

$$\frac{\tau_1^2\tau_2^2(1-\rho^2)}{\tau_1^2+\tau_2^2-2\rho\tau_1\tau_2},$$

where

τ_1^2 is the average squared error (inaccuracy) of the base statistic as a stand-alone predictor of next year's mean loss costs (i.e., the expected squared difference between the base statistic and next year's actual eventual loss costs);

τ_2^2 is the average squared error (inaccuracy) of the complement of the credibility as a stand-alone predictor of next year's mean loss costs; and

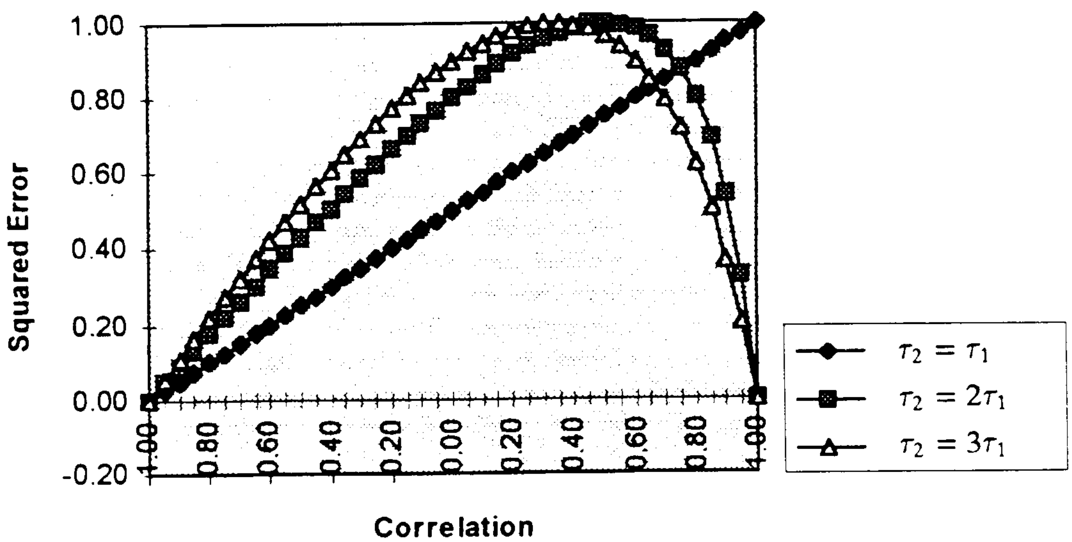
ρ is the correlation (interdependence) between the first statistic's prediction error (error in predicting next year's mean loss costs) and the second statistic's prediction error.

Reviewing that error expression shows that greater inaccuracy in either the base statistic or the complement of credibility will yield greater inaccuracy in the resulting prediction. The expression is symmetric in the two errors. Therefore, the accuracy of the complement of credibility is just as important as the accuracy of the base statistic.

The benefits of independence are more subtle. As it turns out, independence is most important when credibility is most important. That is, independence is most important for the intermediate credibilities (Z between 10% and 90%). As shown in Appendix B, that occurs when the largest standard predicting

FIGURE 1

PREDICTION ERROR AS A FUNCTION OF CORRELATION



error ($\sqrt{\text{inaccuracy}} = \tau$) is within two to three times[1] the smaller error. Consider the following graphs of the total prediction error by correlation for τ_2 = one, two, and three times τ_1.

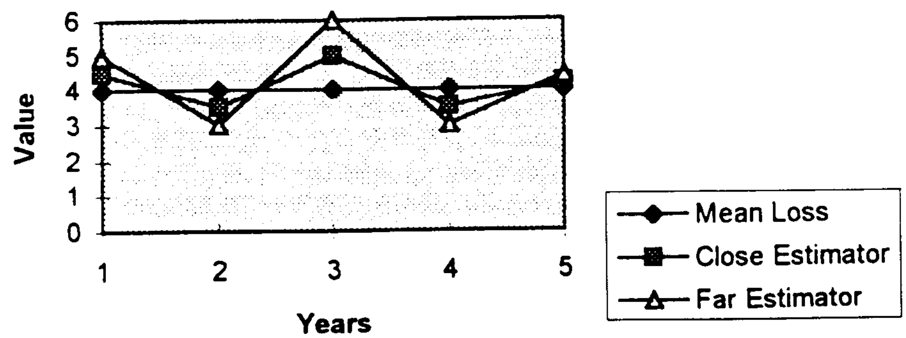
As one can see in Figure 1, the predictions are generally best when there is actually a negative correlation between the two errors (that is, they offset), but that rarely occurs in practice. Generally, the complement of credibility will have some weak correlation with the base statistic. In that range, the prediction error is clearly lowest as the correlation is smaller. Further, the graph beyond the maximum error (correlations near unity) is misleading. Appendix B shows that the downward slope near unity brings negative credibilities. Those negative credibilities are clearly outside the general actuarial philosophy of credibility.

In fact, the example below illustrates the case of negative credibility.

[1] Since Boor [1] shows that credibility is roughly proportional to the relative τ^2s, these examples cover credibilities between 10% and 90%. That range covers instances where credibility matters most.

FIGURE 2

NEGATIVELY CORRELATED ESTIMATORS



As one can see in Figure 2, the close estimator is always between the mean loss and the far estimator. In fact, the far estimator is always twice as far away as the close estimator. Thus, setting

$$\text{estimator} = x - (y - x) = 2x + (-1)y = Zx + (1 - Z)y$$

yields a perfect predictor with negative credibility. Of course, it is extremely rare for the errors of the two statistics to be as correlated as that of these two. In practice, one would rarely assume negative credibility.

Therefore, a complement of credibility is best when it is statistically independent of (that is, not related to) the base statistic.

Summary of Desirable Qualities

The previous sections show six desirable qualities for a complement of credibility:

1. accuracy as a predictor of next year's mean loss costs (i.e., low variance around next year's mean loss costs);
2. unbiasedness as a predictor of next year's mean subject expected losses (i.e., the differences between the predictor and the subsequent loss costs should average out near zero when the predictor is used a number of times);

3. independence from the base statistic;
4. availability of data;
5. ease of computation; and
6. explainable relationship to the subject loss costs.

3. FIRST DOLLAR RATEMAKING

First dollar ratemaking (as opposed to pricing above a very high deductible) generally uses historical loss data for the base statistic. Further, in first dollar ratemaking the historical losses are usually roughly the same magnitude as the true expected losses. Regulators are very concerned with whether or not a complement for first dollar ratemaking is related to the subject loss costs. They are usually less concerned about complements used in excess ratemaking.

There are a wide variety of techniques actuaries use to develop credibility complements. The following pages discuss some of the major methods in use. They are:

- using loss costs from a larger group including the class;
- using loss costs of a larger related class;
- Harwayne's method;
- trending present rates;
- applying the rate change from a larger group to present rates; and
- using competitors' rates.

Loss Costs of a Larger Group Including the Class–Bayesian Credibility

The most basic credibility complement comes from the most classic casualty actuarial technique: Bayesian credibility. In

Bayesian credibility, actuaries are typically either making rates for a large group of classes or making rates for a number of large insureds that belong to a single class. The classes (or individual insureds) do not contain enough exposure units for their historical loss data to reliably predict next year's mean loss costs. Therefore, actuaries supplement the classes' historical loss data by credibility weighting them with the loss costs of the entire group. Sometimes, as in Hurley [5], actuaries weight a class's losses with the losses of the same class in different states.

In mathematical terms, Hurley's loss cost estimate is

$$Z(L_c/E_c) + (1-Z)\left(\sum_i L_i \Big/ \sum_i E_i\right),$$

where

L_c is the historical loss costs for the subject class, c;

E_c is the historical exposure units for class c;

L_i is the historical loss costs for the ith class in the group;

E_i is the historical exposure units for the ith class in the group; and

Z is the credibility.

(For the rest of this paper, P_c will denote the historical loss rate for class c (L_c/E_c). P_g will do the same for the group's historical loss cost rate.)

A. *Complement's Qualities*

This complement has problems in two areas, accuracy and unbiasedness. The group mean loss costs may be the best available substitute. They may be unbiased with respect to all the information the actuary has when making the rate (e.g., historical loss data—the real means remain unknown). On the other hand, the actuary should believe that the true expected class losses will take a different value than the group expected losses. Therefore, this method contains an intrinsic bias and inaccuracy that is unknown.

This complement generally has some independence from the base statistic. As long as the base class does not predominate in the whole group, the process errors of all the other classes should be independent from that of the base class. Also, the error created by using the group mean instead of the class mean is independent of the base class process variance (error). To the extent that the actuary uses the same loss development, trend, and current level factors on the class and group, the error from those factors is interdependent between the class and group loss costs. On the other hand, one could view the ratemaking process as first estimating undeveloped, untrended historical expected losses at previous rates, and then applying adjustment factors. In the first part of that process, the predicting errors are nearly independent.

This complement performs well on availability and ease of computation. Generally, actuaries compute the group mean and group rate indication as the first stage of the pricing process for the entire line of business.

As long as all the classes in the group have something in common, a logical connection between the class's loss costs and those of the group is formed. However, that does not totally eliminate controversy from this credibility complement. Customers with good loss histories may complain that they are treated "just like everyone else." Overall, this complement has an average degree of relationship to the expected subject losses.

B. Choosing the Larger Group

When choosing a larger group, actuaries often use more years of data, a group of related classes (or all classes) within the same state or region, data from more insurers, or data from the same class for all of a state or region. An actuary should be careful when choosing which larger group to use. For example, given a choice between using same class data from other states (provinces) or other class data from the same state, the actuary should consider: Are the differences by state in loss levels more significant than the differences between class costs in the same state? (Usually, class differences are larger.) Can the other state's class data be adjusted to reflect the base state's loss levels (reducing bias)? Is there a group of classes in the state that the actuary would expect to have about the same loss costs (small bias)? All these factors merit consideration. The actuary should attempt to find the larger group statistic that has the least expected bias.

C. *Example*

Consider the data in Table 1.

TABLE 1

DATA FOR BAYESIAN CREDIBILITY COMPLEMENT

		Last Year's Data			Last Three Year's Data		
Rate Group	Class	Exposures	Losses	Pure Premium	Exposures	Losses	Pure Premium
A	1	100	5,000	$50	250	16,000	$64
	2	300	20,000	$67	850	55,000	$65
	3	400	19,000	$48	1,100	55,000	$50
	Subtotal	800	44,000	$55	2,200	126,000	$57
B	Subtotal	600	29,000	$48	1,700	55,000	$32
C	Subtotal	500	36,000	$72	1,400	120,000	$86
D	Subtotal	800	75,000	$94	2,300	200,000	$87
Total		2,700	184,000	$68	7,600	501,000	$66

If one is making rates for Class 1 in Rate Group A, one must first consider whether to use one-year or three-year historical losses. One must consider that the three-year pure premiums will be less affected by process variance (year-to-year fluctuations in experience due to small samples from the distribution of potential claims). On the other hand, sometimes the exposure base is large enough to minimize process variance, and societal events are causing pure premiums to change. In that situation, the one-year pure premiums are preferable.

Suppose one chooses the one-year pure premium ($50) for historical data. Using the three-year pure premium of the class ($64) for the complement would be inappropriate because the three-year pure premium is heavily interdependent with the one-year pure premium. Also, presumably the actuary has already decided that the three-year data is biased because of changes in loss cost levels. Thus, the actuary believes the three-year data does not add accuracy to the prediction. For the same reasons, the three-year rate group and grand total pure premiums would be inappropriate complements.

The next choice is between the rate group and grand total pure premiums. The decision involves a tradeoff between bias reduction and process variance reduction. The rate group data should reflect risks that are more similar to Class 1. Therefore, it should have less bias. On the other hand, the grand total data is spread over more risks, so it has less process variance.

In this example, the choice is more difficult. The one-year and three-year rate group pure premiums are very similar ($55 versus $57). For the other rate groups, there are more pronounced inconsistencies (i.e., $32 versus $48 for Rate Group B). The grand total shows it has little process variance, but it appears to contain roughly $15 of bias. The one-year Rate Group A pure premium of $55 is probably the best choice.

One could also consider using the three-year pure premium for historical losses. That does not preclude using the one-year rate group data as a complement. Using the one-year Rate Group A pure premium would simply assume that the total Rate Group A exposures were sufficient to minimize process variance. All things considered, it may be appropriate to use one-year data as a complement to three-year data.

Loss Costs of a Larger Related Class

Actuaries sometimes use the loss costs of a larger, but related class for the complement of credibility. For example, if a company writes very few picture framing stores but writes a large number of art stores, the actuary may choose to use the art store loss costs for the framing store complement of credibility. He may or may not make some adjustments to the art store loss costs to make them more applicable to framing stores. For example, he may wish to adjust for the minor woodworking exposure. Actuaries pricing general liability often use this "base class" (meaning the larger related class in this context) approach. See Lange [6] for an example of this.

A. *Complement's Qualities*

This approach has qualities similar to the large group complement. It is biased (though the bias and its direction are unknown), and thus it is inaccurate. The more the actuary adjusts the related class loss data to match the loss exposure in the subject base class, the more the bias is reduced. The independence may be less if the factor relating the classes generates high losses for the two classes simultaneously, but the actuary must be careful that this seeming independence is not just a simultaneous shift in the expected losses (which is not prediction error); it is usually an increase in expected losses.

This complement does not fare quite as well as the group mean in other categories. Data is not as readily available for this complement as the group mean, but if the company writes some related class, data should be available and already computed for that class's rates.

The computations involved in adjusting related class data may be more difficult. Any loss cost adjustments will require some extra work. Since there is some relationship between the base class and the related class (they must be related some way by definition), explaining this complement may be easier than explaining the larger group complement.

B. *Example*

Consider the case of the framing stores. Suppose the actuary wishes to estimate a fire rate for framing stores and already has a well-established rate for art stores. Perhaps the actuary sees that the only visible difference in exposure is the presence of wood and sawdust. He might choose to add a judgmental ten percent of the excess of the fire rate for lumberyards over the fire rate for art stores.

Harwayne's Method

Sometimes individual class data for a given state may be sparse, but the data for that class in other states may be affected by differences in the legal environment, traffic density, or other differences in the other state's overall level of loss costs. Harwayne's method [3] attempts to adjust for those state differences by adjusting the ancillary data from other states for differences in state loss costs levels. It is used extensively for making state class rates for workers compensation.

Harwayne's method uses a specific type of data from a related class. Usually, it is also a case of using loss costs from the larger group. In Harwayne's method, actuaries use countrywide (excepting the base state being reviewed) class data to supplement the loss cost data for each class, but they adjust countrywide data to remove overall loss cost differences between states (or provinces).

The process is as follows. First, the actuary determines what the total countrywide average pure premium would be if the countrywide data had the same percentage mixture of classes (class distribution) as the base state. The result reflects the base state class distribution but probably also reflects the differences in overall loss costs differences between states.

Next, actuaries use that difference in overall loss costs to adjust the countrywide class data to match the base state overall loss cost levels. They determine the ratio of overall state loss costs to overall (all classes) adjusted countrywide loss costs. Then they multiply that ratio times the countrywide base class loss costs to get the complement of credibility.

That is Harwayne's basic method. In a variant form, actuaries may adjust each state's loss costs individually to the base state level to eliminate biases due to different state distributions between classes. (Harwayne used this variant.) Then, actuaries compute the average class complement by weighting the individual state's adjusted loss costs. In another variant, actuaries adjust other states' historical loss ratios by class to match the base state's overall loss ratio. In either variant, the basic principles are the same.

A. *Formulas*

The simplified formula for Harwayne's method is as follows. Let:

$L_{c,s}$ denote the historical losses for class c in the base state s;

$E_{c,s}$ denote the associated exposure units;

$P_{c,s}$ denote the state pure premium for class c;

$L_{i,j}$ denote the historical losses for an arbitrary class i in some state j;

$E_{i,j}$ denote the associated exposure units; and

$P_{i,j}$ denote the state j pure premium for class i.

Actuaries compute the countrywide pure premium adjusted to the state class distribution. The first step is to compute the "state s" average pure premium (rate)

$$P_s = \sum_i L_{i,s} \Big/ \sum_i E_{i,s}.$$

The next step is to compute the countrywide rates by class

$$P_i = \sum_{j \neq s} L_{i,j} \Big/ \sum_{j \neq s} E_{i,j}.$$

Then, actuaries compute the countrywide rate using the state s distribution of exposures

$$\overline{P} = \sum_i E_{i,s} P_i \Big/ \sum_i E_{i,s}.$$

Thus, the overall pure premium adjustment factor is

$$F = P_s/\overline{P},$$

and the complement of the credibility for class c is assigned to $F \times P_c$.

Harwayne's more complicated (and more accurate) formula replaces the overall adjustments to countrywide data with separate adjustments for each state. That is, actuaries compute state overall means with the base state (s) class distribution

$$\overline{P}_j = \frac{\sum_m E_{m,s} P_{m,j}}{\sum_m E_{m,s}}.$$

Then, they compute individual state adjustment factors

$$F_j = P_s/\overline{P}_j.$$

Next, actuaries adjust each state's class c historical rates using the F_j's. That is, they compute the adjusted "state j" rates

$$P'_{c,j} = F_j P_{c,j}.$$

Then, actuaries weight them with the countrywide distribution among states

$$\text{Complement} = C = \frac{\sum_j E_{c,j} P'_{c,j}}{\sum_j E_{c,j}}.$$

The result is Harwayne's more complicated complement of credibility.

B. Complement's Qualities

This complement has very high statistical quality. Because Harwayne's method uses data from the same class in other states and attempts to adjust for state-to-state differences, it is very unbiased. It is also reasonably accurate as long as there is sufficient countrywide data to minimize process variance. Since the loss costs are from other states, their prediction errors (remaining bias) should be mostly independent of the base class process error in the base state. One exception might be where there is an across-the-board jump in all classes' loss costs in state s that alters the adjustment to the state experience level. On the other hand, across-the-board jumps usually flow through into the next year's expected losses, so they are rarely prediction errors.

This complement has a mixed performance on the less mathematical qualities. Data are usually available for this process, but the computations do take time and are complicated. Thankfully, they do bear a much more logical relationship to class loss costs in individual states than unadjusted countrywide statistics. On the other hand, this may be harder to explain because of complexity.

C. Example

Consider the data in Table 2. We will use it for Harwayne's method on Class 1 in State S.

For Harwayne's full method, one first computes

$$\overline{P}_T = \frac{100 \times 3.67 + 180 \times 4.00}{100 + 180} = 3.88,$$

and

$$\overline{P}_U = \frac{100 \times 2.22 + 180 \times 4.09}{100 + 180} = 3.42.$$

Then, one computes the state adjustment factors: $F_T = 2.86/3.88 = .737$ and $F_U = 2.86/3.42 = .836$. The next step is to compute the other states' adjusted Class 1 rates: $P'_{1,T} = .737 \times 3.67 = 2.70$ and $P'_{1,U} = .836 \times 2.22 = 1.86$. The last step is to weight the

TABLE 2

DATA FOR HARWAYNE'S METHOD

State s	Class c	Exposure E	Losses L	Pure Premium P
S	1	100	200	2.00
	2	180	600	3.33
	Subtotal	280	800	2.86
T	1	150	550	3.67
	2	300	1,200	4.00
	Subtotal	450	1,750	3.89
U	1	90	200	2.22
	2	220	900	4.09
	Subtotal	310	1,100	3.55
All	1	340	950	2.79
	2	700	2,700	3.86
	Total	1,040	3,650	3.51

two states' adjusted rates with their Class 1 exposures to produce

$$C = \frac{2.70 \times 150 + 1.86 \times 90}{150 + 90} = 2.39.$$

That is Harwayne's complement of the credibility.

Trended Present Rates

In some cases, especially countrywide rate indications, there is no larger group to use for the complement. Then, actuaries use present rates adjusted for inflation (trend) since the last rate change. If there is a difference between the last actuarial indication and the charged rate, actuaries build that in too. Essentially, this test allows some credibility procedure to dampen swings in the historical loss data, yet still forces the manual rates to keep up with inflation. This method was used to develop a second complement of credibility in Harwayne [3].

A. *Formula for the Complement*

The formula for this complement of credibility is

$$T^t \times R_L \times P_L \div P_C,$$

where:

T is the annual trend factor, expressed as one plus the rate of inflation. (This will usually be the same as the trend factor in the base indication.)

t is the number of years between the original target effective date of the current rates (not necessarily the date they actually went into effect) and the target effective date of the new rates. (This will often be different than the number of years in the base class trend. It is also usually different than the number of years between the experience period and the effective date of the new rates.)

R_L represents the loss costs presently in the rate manual.

P_L represents the last indicated pure premiums (loss costs).

P_C represents the pure premiums actually being charged in the current manual. This may differ from R_L because P_L and P_C may be taken over a broader group.

B. *Complement's Qualities*

This complement is not as desirable as the previous complements, but sometimes it may be the only alternative. It is less accurate for loss costs with high process variance. Process variance is presumably reflected in last year's rate. That is why it is primarily used for countrywide indications or state indications with voluminous data. It is unbiased in the sense that pure trended loss costs (i.e., with no updating for more current loss costs) are unbiased. Since it includes no process variance, it is mostly independent of the base statistic.

On the less mathematical side, this statistic performs fairly well. Everything an actuary needs to compute it is already in the base rate filing. Therefore, it is available and easy. There is one exception to this. Should one wish to analyze the effects of rate changes the company did not achieve at the level of individual classes, it may require more data than companies typically maintain. This statistic is also very logically related to the loss costs being analyzed. After all, the present rates are based on this complement.

C. Example

Consider the following data for 1996 policy rates:

Present pure premium rate—$120
Annual inflation (trend)—10%
Amount requested in last rate change—+20%
Effective date requested for last rate change—1/1/94
Amount approved by state regulators—+15%
Effective date actually implemented—3/1/94

The complement of the credibility would be

$$C = \$120 \times 1.1^2 \times \frac{1.20}{1.15} = \$152.$$

Rate Change from the Larger Group Applied to Present Rates

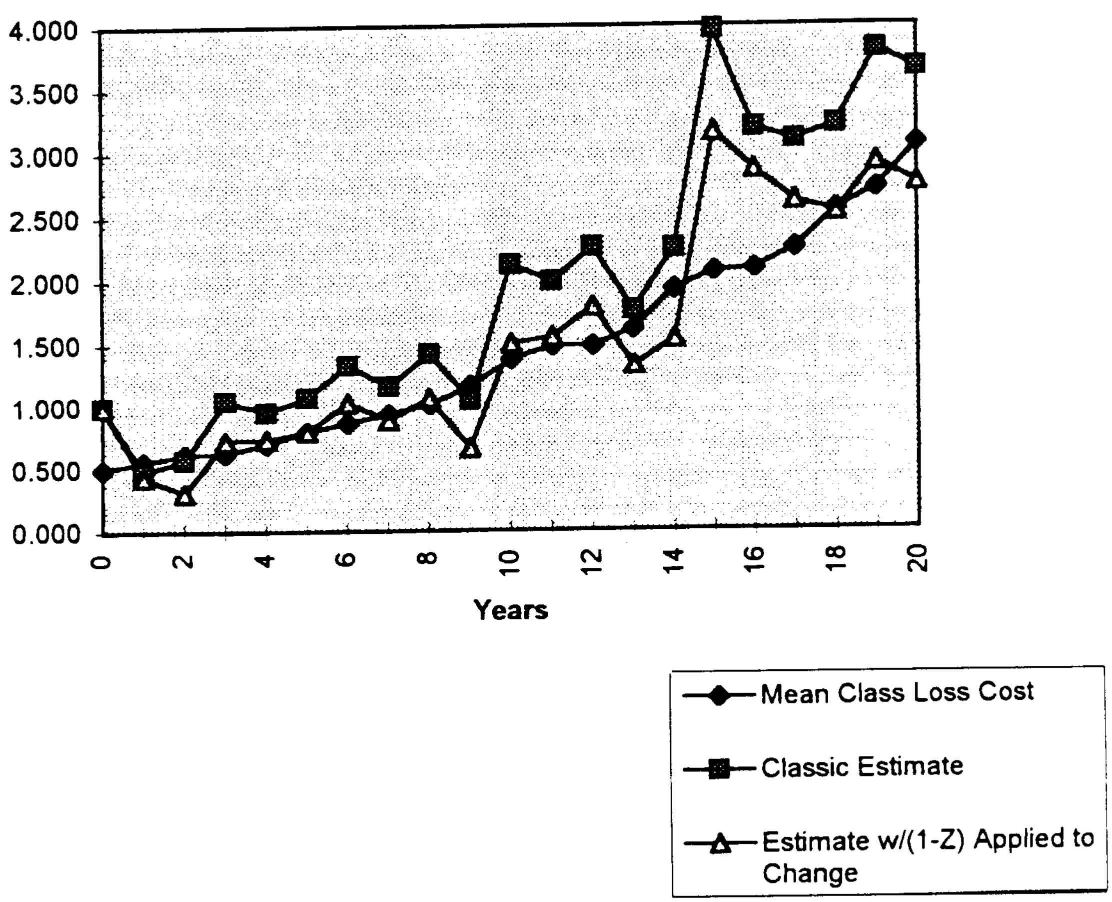
This complement is very similar to the Bayesian complement, but it does not have the substantial (though unknown) bias of the Bayesian complement. That is because the true class expected losses may be very different from the large group expected losses. This larger group test uses the large group rate change applied to present rates, instead of the large group historical loss data (Bayesian complement). Presumably, present rates are an unbiased predictor of the prior (i.e., before changes reflected in current ratemaking data) loss costs. As long as both rates need reasonably small changes, any bias in the overall larger group rate change as a predictor of the class rate change should be small. Also, using large group rate changes instead of straight trend allows the rate to mirror broad changes in loss cost levels that may not be reflected in trend.

A. *Example*

An example may help to illustrate how eliminating bias improves rate accuracy over time. Figure 3 shows group experience after simulation by successively applying $N(10\%, 0.25\%)$ (normal distribution with a mean of 10% and a standard deviation of

FIGURE 3

VALUE BEING ESTIMATED AND ESTIMATORS BY YEAR OF ITERATION



$\sqrt{0.25\%} = 5\%$) trends to a value starting at one. The true class expected losses were set at exactly half the group expected losses each and every year (a slightly unrealistic assumption). The historical class losses have a standard deviation of one-third the true expected losses for the class. A detailed chart of the values actually simulated is in Appendix C.

As Figure 3 shows, the Bayesian complement results in rates with consistent bias above the true expected losses. The complement based on applying group changes to present rates starts too high but very quickly becomes unbiased. It is almost always a better estimate.

B. *Formula*

This complement has a fairly straightforward formula. It is

$$R_c \times \left\{ 1 + \frac{(P_g - R_g)}{R_g} \right\},$$

where

R_c is the present manual loss cost rate for class c;

P_g is the present indicated loss cost rate for the entire group of classes; and

R_g is the present average loss cost rate for the entire group.

C. *Complement's Qualities*

This complement is a significant improvement over the Bayesian complement. It is largely unbiased. If the year-to-year changes are fairly small, it is very accurate over the long term (though often not as accurate as Harwayne's complement in practice). Also, since the complement is based on group variance, it is fairly independent. Since this complement requires a group rate change that must be calculated anyway, it is both available and easy to compute. Since it includes the present rate, it has a logical relationship to the class loss costs.

D. Numerical Example

Consider the data in Table 3. Using this data, the complement for Class 1 would be

$$\$750 \times (1 + (\$750 - \$782)/\$782) = \$719.$$

TABLE 3

DATA FOR APPLYING GROUP RATE CHANGE TO CLASS DATA

Class	Exposure	Losses	Indicated Pure Premium	Present Pure Premium	Underlying Losses
1	100	$ 70,000	$700	$750	$ 75,000
2	200	$180,000	$900	$920	$184,000
3	300	$200,000	$667	$700	$210,000
Total	600	$450,000	$750	$782	$469,000

Notes:—Both indicated and present pure premiums are at current cost levels.
—Underlying losses are extension of exposures by present premiums.
—Total present premium is ratio of total underlying to total exposures.

Using Competitors' Rates

New companies and companies with small volumes of data often find their own data too unreliable for ratemaking. Their actuaries use competitors' rates for the complement of credibility. They rationalize that if the competitor has a much larger number of exposures, the competitors' statistics have less process error. An actuary in this situation must consider that manual rates reflect marketing considerations, judgment, and the effects of the regulatory process, as well as loss cost statistics. Thus, competitors' rates have significant inaccuracies. They are also affected by differences in underwriting and claim practices between the subject company and its competitors. Competitors' rates probably have systematic bias as well. The actuary will often attempt to correct for those differences by using judgment, but those corrections and their size and direction may generate controversy. However, using competitors' rates may be the best viable alternative in some situations.

A. Complement's Qualities

Competitors' rates generally have prediction errors that are independent of the subject class loss costs. That is because their errors stem more from inter-company differences that are unrelated to subject company loss cost errors. They are often available from regulators, although the process may take some work. They are harder to use since they usually must be posted manually.

Regulators may complain that competitors' rates are unrelated to the subject company's own loss costs, but, if the company's own data is too unreliable, competitors' rates may be the only alternative.

B. Example

Consider a competitor's rate of \$100. Suppose a Schedule P analysis suggests the competitor will run a 75% loss ratio. Further, suppose one's own company has less underwriting expertise. Suppose one's own company expects ten percent more losses per exposure than the competitor. The complement would be $\$100 \times .75 \times 1.1 = \$83$.

Loss Ratio Methods

This paper discussed all the previous complements in terms of pure premium ratemaking. All the methods except the loss costs from a larger related class and competitors' rates also work with loss ratio methods. All the actuary needs to do is consider earned premium to be the exposure base. Replacing the exposure units with earned premium yields usable formulas.

4. SPECIFIC EXCESS RATEMAKING

Complements for excess ratemaking are structured around the special problems of excess ratemaking. Since specific excess policies only cover losses that exceed a very high per claim deductible (attachment point), there usually are very few actual claims in the historical loss data. Hence, actuaries will try to predict the volume of excess loss costs using the loss costs below the attachment point. For liability coverages, the loss development of excess claims may be very slow. That accentuates the sparsity of ratemaking data. Also, the inflation inherent in excess layers is different (usually higher) than that of total limits losses (see [2]). Since the "burning cost" (historical loss data) is an unreliable predictor, the statistic that receives the complement of credibility is especially important.

This paper will discuss four methods for determining excess credibility complements: using increased limits factors; derivation from a lower limits analysis; analysis reflecting the policy limits sold by the insurer; and the use of fitted curves.

Increased Limits Factors

When loss costs for the first dollar coverage up to the insurer's limit of liability are available, actuaries may use an increased limits factor approach. Actuaries multiply the "capped" loss costs by the increased limits factor for the attachment point plus the limit of liability. Then, they divide the result by the increased limits factor for the attachment point. That produces an estimate of loss costs from the first dollar up to the limit of liability. Then actuaries subtract the loss costs below the original attachment point. The remainder estimates the expected losses in the specific excess layer.

Actuaries use a variety of sources for increased limits factors. The Insurance Services Office publishes tables of estimated increased limits factors for products, completed operations, premises and operations liability, and manufacturers and contractors liability. The National Council on Compensation Insurance publishes excess loss pure premium factors that allow actuaries to compute increased limits factors for workers compensation. The *Proceedings of the Casualty Actuarial Society* may contain tables of property losses by ratio to probable maximum loss. Those can be converted to increased limits factors by using the factors for the ratio of the attachment point to the probable maximum loss (and the ratio of the attachment point plus the limit of liability to the probable maximum loss). Actuaries may compute increased limits factor tables using a company's own data (if the company sells enough specific excess). Actuaries may modify industry tables to reflect their company's loss cost history. Competitor prices may allow actuaries to estimate increased limits factors for obscure coverages. Actuaries would consider the ratios between competitor prices for various limits of liability.

A. *Formula*

The formula is as follows:

$$(P_A \times \mathrm{ILF}_{A+L} \div \mathrm{ILF}_A) - P_A \qquad \text{or} \qquad P_A \times \left(\frac{\mathrm{ILF}_{A+L}}{\mathrm{ILF}_A} - 1\right).$$

In this case

P_A is the loss cost capped at the attachment point A (by convention, it is usually premium capped at the attachment point multiplied by the loss ratio the actuary projects);

ILF_{A+L} is the increased limits factor for the sum of the attachment point and the limit of liability L; and

ILF_A is the increased limits factor for the attachment point.

B. *Complement's Qualities*

As long as the insured being rated has a different loss severity distribution than the norm, this complement contains bias. In that likely event, it is also inaccurate. Further, it is not based on the individual insurer's own data. Actuaries must weigh those facts against the greater inaccuracy of burning cost statistics. When pricing specific excess insurance, actuaries must usually settle for less accurate and potentially biased estimators. That is because there are few highly accurate estimators available.

This complement's error is mostly independent of the burning cost error. This complement tends to contain a systematic (parameter-type) error rather than the process error inherent in burning cost. It is dependent on burning cost only to the extent that both are highly related to the losses below the attachment point.

Very few specific excess statistics are readily available or easy to compute. Considering the alternatives, the availability of industry increased limits tables (in the United States) makes this the easiest specific excess complement to compute. Also, the data for this test is available as long as premiums or loss costs capped at the attachment point are available.

The excess loss cost estimates this complement produces are more logically related to the losses below the attachment point than those above. That can be controversial with customers. It is a common problem with excess insurance pricing. However, burning cost is unreliable in isolation. Further, that problem is common to all excess complements.

C. *Example*

Consider Table 4.

TABLE 4

INCREASED LIMITS FACTORS

Limit of Liability	Increased Limits Factor
$ 50,000	1.00
$ 100,000	1.50
$ 250,000	1.90
$ 500,000	2.50
$1,000,000	3.50

Suppose one wishes to estimate the layer between $500,000 and $1,000,000 given losses of $2,000,000 capped at $500,000 each.

The complement using increased limits would be

$$C = \$2{,}000{,}000 \times \left(\frac{3.5}{2.5} - 1\right) = \$800{,}000.$$

Lower Limits Analysis

Sometimes the historical losses near the attachment point may be too sparse to be reliable. So, an actuary may wish to base his complement on basic limits losses, where the basic limit is some low loss cap. In this case, the formula is almost exactly the same as that of the previous analysis. The actuary simply multiplies the historical basic limits losses by a difference of increased limits factors. Specifically, he multiplies basic limits losses by the difference between the increased limits factor for the attachment point plus the limit of liability and the increased limits factor for the attachment point. The result is the complement of credibility.

A. Formula

The formula is

$$P_b \times (\text{ILF}_{A+L} - \text{ILF}_A),$$

where

P_b represents the historical loss data with each loss capped at the basic limit b; and

ILF_{A+L} and ILF_A are as before.

Alternately, the actuary might choose to use a low capping limit d that is different from the basic limit underlying the increased limits table. Then, the formula would be

$$P_d \times \left(\frac{\text{ILF}_{A+L}}{\text{ILF}_d} - \frac{\text{ILF}_A}{\text{ILF}_d}\right).$$

B. Complement's Qualities

Actuaries must usually use judgment to decide whether loss costs capped at the attachment point or some lower limit are more accurate and unbiased predictors of the excess loss. Estimates made using the lower cap are more prone to bias. That is because using losses far below the attachment point accentuates the impact of variations in loss severity distributions. On the other hand, when there are few losses near the attachment point, historical losses limited to the attachment point may be unreliable and inaccurate predictors of future losses. Consequently, using higher loss caps may produce even more inaccurate predictors of excess losses.

By an argument similar to that of the previous test, this complement's errors are mostly independent of those of burning cost.

Generally, this complement features more available statistics and a slightly greater complexity. Basic limits losses may need to be coded for statistical reporting. They may be readily available for this complement. On the other hand, since insureds and reinsureds may place a higher priority on accounting for the total losses they retain, they are not as available as losses limited to the attachment point. The calculations are no more complicated for basic limits analysis than retained limits (attachment point) analysis. The only exception would be where actuaries must manually compute the loss costs between basic limits and the attachment point from a claims list.

As with the straight increased limits factor approach, this complement may generate controversy with customers because it is not based on actual burning cost.

C. Example

Suppose an actuary is estimating the losses between $500,000 and $1,000,000, and the actuary feels he can only rely on historical losses limited to $100,000. The estimated historical losses limited to $100,000 are $1,000,000. Then, using the increased limits factors from Table 4, he would calculate the complement at

$$C = \$1{,}000{,}000 \times \left(\frac{3.5}{1.5} - \frac{2.5}{1.5}\right) = \$666{,}667.$$

Limits Analysis

The previous approaches work well when losses limited to a single capping point are available, but sometimes they are not. Reinsurance customers generally sell policies with a wide variety of policy limits. Some of the policy limits will fall below (not expose) the attachment point. Some limits may extend beyond the sum of the attachment point and the reinsurer's limit of liability. In any event, each subject (first dollar) policy limit will require its own increased limits factor.

Therefore, actuaries analyze each limit of coverage separately. Generally, they assume that all the limits will experience the same loss ratio. Actuaries multiply the all limits combined (total limits) first dollar loss ratio times the premium in each first dollar limit to estimate the loss costs for that limit. Then, actuaries perform an increased limits factor analysis on each first dollar limit's loss costs separately.

A. Formula

The formula is as follows:

$$LR_T \times \sum_{d \geq A} W_d \frac{(\mathrm{ILF}_{\min(d,A+L)} - \mathrm{ILF}_A)}{\mathrm{ILF}_d},$$

where

LR_T is the estimated total limits loss ratio;

The "d" are all the policy limits the customer sells that exceed the attachment point ($\geq A$);

Each W_d is the premium volume the customer sells with policy limits of d; and

The ILF's have the same meaning as previously.

B. *Complement's Qualities*

Actuaries use this approach because it may be all that is available. Reinsureds may be unable to split their historical losses any more finely than losses that would have pierced the cover in the past versus all other losses. Since the total limits loss costs (which are almost always available, at least as an estimate) may include claims beyond the layer, it may be impossible to calculate losses limited to the attachment point. In any event, if some of the reinsured's policy limits are below the attachment point, they do not expose the layer and should be excluded from an increased limits factor calculation. Therefore, this may be the only available complement with low bias.

It is biased and inaccurate to the same extent that the previous increased-limits-factor-based complements were biased or inaccurate. It is more time-consuming to compute (unless the alternative is computing limited claims from claims lists). Further, it generates the same controversy as the other methods since it is not the same as the actual burning cost.

C. *Example*

Suppose an actuary is estimating the losses in a layer between \$250,000 and \$500,000. Breakdowns of losses by size are unavailable, but the actuary believes the loss ratio of the customer's entire business to be 70%. He does have a breakdown of premiums by limit of liability. Using that breakdown and the increased limits factors from Table 4, he computes the losses in the layer (see Table 5). He estimates the losses in the layer at \$86,400.

Fitted Curves

The problem with most of the previous complements is that they do not give special attention to the claims above or near the attachment point. As a result, they miss differences in loss severity distributions between insureds, but of course that must

TABLE 5

LIMITS ANALYSIS FOR LAYER BETWEEN $250,000 AND $500,000

Limit of Liability	Premium	Times 70% Loss Ratio	Increased Limits Factor	% in Layer	Loss in Layer
$ 250,000	$ 600,000	$420,000	1.9	0.00%	$ —
$ 500,000	$ 300,000	$210,000	2.5	24.00%	$50,400
$1,000,000	$ 300,000	$210,000	3.5	17.14%	$36,000
Total	$1,200,000	$840,000			$86,400

be counterbalanced against the fact that individual insureds' large claims histories usually lack credibility.

By fitting a family of loss severity curves to the distribution, actuaries make the most of the large claim data that is available. If the loss history shows no claims beyond the attachment point but many claims that are very near to the attachment point, a fitted curve will usually reflect that and project high loss costs in the subject layer. On the other hand, if there are few large claims close to the attachment point, the fitted curve will project low loss costs for the layer.

The details of how to fit curves are beyond the scope of this paper (see [4]), but it will provide an outline of how to use fitted curves in practice. After fitting and trending the curve, an actuary will use the curve to estimate what percentage of the curve's total loss costs lie in the subject layer. He may do this by evaluating the difference between the limited mean function $\int_{-\infty}^{L} xf(x)dx + (1-F(L))L$ at the attachment point and the attachment point plus the limit of liability. He would then divide the result by the total mean (or the mean when claims are capped at the typical policy limit) to get the percentage of the total loss costs that lie in the layer. Multiplying that percentage by the total claims cost yields the estimate of claim costs in the layer (for details, see [4]).

Of course, excess values from curve fits need extensive loss development as do burning costs. Actuaries may use excess loss development factors such as those published by the Reinsurance Association of America, or they may triangulate the fitted loss costs.

A. *Complement's Qualities*

This method is generally unbiased (except for concerns that the general shape of a family of curves may predispose the results for the family to estimated costs in particular layers that are either too high or too low). When there are few large claims, it is more accurate than burning cost. It is often more accurate than increased limits factors simply because it does a better job reflecting any general tendency towards large or small claims. On one hand, fitting curves forces data into a mold that may not fit the data. The actual loss severity distribution will almost certainly look very different from all the members of the family of curves. This "super-parameter" risk introduces error of its own. The "super-parameter" risk is totally distinct from process risk, and that makes the complement mostly independent. On the other hand, the presence or absence of burning cost claims in the layer can influence the curve fit heavily. Thus, this complement has somewhat more dependent (relative to burning cost) errors than the increased limits approaches.

Data availability and computational complexity are problems here. To fit a loss severity curve, an actuary must either use a detailed breakdown of all the claims into size ranges or use a listing of every single claim. Usually, that data is not readily available. Further, the processing required to fit curves requires fairly complex mathematical calculations. Besides the fact that complex calculations require special personnel, the complexity makes the results difficult to explain to lay people.

On one hand, this complement uses more of the insured's own data in and near the layer than any other excess complement. On the other hand, its complexity may make that fact difficult to communicate.

5. SUMMARY

The complement of the credibility deserves at least as much actuarial attention as the base statistic (historical loss data). Actuaries owe special attention to its unbiasedness and accuracy. In some cases, interdependence must be avoided. Further, any actuarial method must be implemented using reasonable labor on available statistics. Meeting those qualities may require statistics that make less explainable sense to lay people, but explainability must be considered, too.

This paper has detailed several statistics that are commonly used for the complement of credibility. Their use improves many actuarial projections considerably.

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APPENDIX A

THE ERROR IN CREDIBILITY ESTIMATES

This appendix will show that the error in an optimum credibility weighted estimate is

$$\Phi(x_1,x_2) = \frac{\tau_1^2\tau_2^2(1-\rho^2)}{\tau_1^2+\tau_2^2-2\rho\tau_1\tau_2}.$$

The proof involves three equations from Boor [1]:

$$\Phi(x_1,x_2) = Z\tau_1^2 + (1-Z)\tau_2^2 + (Z^2-Z)\delta_{1,2}^2 \tag{A.1}$$

(p. 182, the simplified error of the credibility-weighted estimate);

$$Z = \frac{\tau_2^2 - \tau_1^2 + \delta_{1,2}^2}{2\delta_{1,2}^2} \tag{A.2}$$

(p. 183, the formula for the optimum credibility); and

$$\delta_{1,2}^2 = \tau_1^2 + \tau_2^2 - 2\mathrm{Cov}(x_1,x_2) \tag{A.3}$$

(p. 179, the formula relating $\delta_{1,2}^2$ to the correlation).

In this case, τ_1, τ_2, and ρ are the same as they were in the body of the paper (the prediction errors of burning cost and the credibility complement and their correlation); $\Phi(x_1,x_2)$ is the minimum possible average squared prediction error from credibility weighting burning cost (x_1) and the credibility complement (x_2); and $\delta_{1,2}^2$ is the average squared difference between burning cost and the credibility complement.

Simple algebra on (A.1) allows one to pull out several terms that will create the numerator of (A.2).

$$\Phi(x_1,x_2) = -Z(\tau_2^2 - \tau_1^2 + \delta_{1,2}^2) + \tau_2^2 + Z^2\delta_{1,2}^2$$

$$= -Z^2 2\delta_{1,2}^2 + \tau_2^2 + Z^2\delta_{1,2}^2 = \tau_2^2 - Z^2\delta_{1,2}^2.$$

Using the definition of Z (Equation A.2) once again with some algebra gives

$$= \tau_2^2 - \frac{(\tau_2^2 - \tau_1^2 + \delta_{1,2}^2)^2}{4\delta_{1,2}^2}.$$

Using (A.3) and the relationship between the covariance and correlation gives

$$= \tau_2^2 - \frac{(\tau_2^2 - \tau_1^2 + \tau_1^2 + \tau_2^2 - 2\mathrm{Cov}(x_1, x_2))^2}{4\delta_{1,2}^2}$$

$$= \tau_2^2 - \frac{(\tau_2^2 - \tau_1^2 + \tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2)^2}{4\delta_{1,2}^2}$$

$$= \tau_2^2 - \frac{(2\tau_2^2 - 2\rho\tau_1\tau_2)^2}{4\delta_{1,2}^2}$$

$$= \tau_2^2 - \frac{(\tau_2^2 - \rho\tau_1\tau_2)^2}{\delta_{1,2}^2}.$$

Then, more algebra gives

$$= \tau_2^2 \left(1 - \frac{(\tau_2 - \rho\tau_1)^2}{\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2}\right)$$

$$= \frac{\tau_2^2}{\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2} \times (\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2 - \tau_2^2 + 2\rho\tau_1\tau_2 - \rho^2\tau_1^2)$$

$$= \frac{\tau_2^2}{\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2} \times (\tau_1^2 - \rho^2\tau_1^2)$$

$$= \frac{\tau_2^2\tau_1^2(1 - \rho^2)}{\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2},$$

and that is the error formula we sought to prove.

APPENDIX B

For Correlations Near Unity, Credibility is Negative

This appendix will show that whenever the correlation exceeds the point of maximum error, the credibility of one statistic is negative. To explain this principle, reviewing the graph of error by correlation will help.

As one can see in Figure 1, the prediction error is initially minimized when the correlation is negative. Then it increases until the error is maximized. Then the error decreases again beyond that maximum point. This section will show that the one credibility is actually negative beyond that maximum point.

As it happens, when $\tau_2 \geq \tau_1$, that maximum point is where $\rho = \tau_1/\tau_2$. And all correlations beyond that yield negative credibility for the complement. Alternately, when $\tau_1 \geq \tau_2$, $\rho = \tau_2/\tau_1 \leq 1$ is the point of maximum prediction error. Beyond that, the burning cost's credibility will be negative. But, this appendix must prove that.

It is easy to show that Φ has a maximum where $\rho = \tau_1/\tau_2$. One need only note that the function $\Phi(\rho)$ has a maximum where

$$0 = \frac{\partial \Phi}{\partial \rho} = \frac{2\rho(\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2) - 2\tau_1\tau_2(1-\rho^2)}{(\tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2)^2}$$

(using the definition of $\Phi(\rho)$ from Appendix A). Using some algebra, that is equivalent to

$$0 = 2\rho\tau_1^2 + 2\rho\tau_2^2 - 4\rho^2\tau_1\tau_2 - 2\tau_1\tau_2 + 2\rho^2\tau_1\tau_2; \qquad \text{or}$$

$$0 = (\tau_1 - \rho\tau_2)(\tau_2 - \rho\tau_1).$$

So, the maximum is at τ_1/τ_2 or τ_2/τ_1, whichever is less than one.

To show that correlations beyond that maximum point result in negative credibilities, it suffices to show that they fulfill Boor's condition for negative credibility ([1], p.183):

$$\tau_2^2 \geq \tau_1^2 + \delta_{1,2}^2.$$

But that follows directly from Boor's equation relating the credibility and covariance ([1], p. 179). That is, since

$$\delta_{1,2}^2 = \tau_1^2 + \tau_2^2 - 2\mathrm{Cov}(x_1, x_2) = \tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2,$$

Boor's condition is equivalent to

$$\tau_2^2 \geq \tau_1^2 + \tau_1^2 + \tau_2^2 - 2\rho\tau_1\tau_2.$$

Or,

$$\rho \geq \frac{\tau_1}{\tau_2};$$

that is, Boor's condition for negative credibility is fulfilled and fulfilled only for ρ beyond the point of maximum error. So, the correlations near unity yield negative credibilities.

APPENDIX C

DATA FOR EXAMPLE APPLYING COMPLEMENT TO GROUP RATE CHANGE

(a) Year	(b) Group N(.1,.0025) Trend	(c) Group Loss Cost	(d) Mean Class Loss Cost	(e) Class with Process Variance $N(0,((d)/3)^2)$	(f) Classic Z	(g) Classic Estimate	(h) Estimate with $(1-Z)$ Applied to Change
0	0.115	1.000	0.500	0.188	0.692	1.000	1.000
1	0.101	1.115	0.558	0.256	0.692	0.481	0.438
2	0.021	1.228	0.614	0.825	0.692	0.572	0.306
3	0.107	1.254	0.627	0.695	0.692	1.044	0.724
4	0.137	1.389	0.694	0.782	0.692	0.954	0.731
5	0.091	1.579	0.790	1.037	0.692	1.065	0.792
6	0.082	1.723	0.862	0.747	0.692	1.324	1.025
7	0.082	1.865	0.932	1.034	0.692	1.153	0.885
8	0.143	2.017	1.009	0.468	0.692	1.418	1.056
9	0.188	2.305	1.153	1.759	0.692	1.039	0.659
10	0.075	2.739	1.369	1.393	0.692	2.119	1.498
11	0.000	2.945	1.472	1.653	0.692	1.988	1.545
12	0.093	2.946	1.473	0.992	0.692	2.256	1.782
13	0.192	3.220	1.610	1.516	0.692	1.753	1.315
14	0.075	3.839	1.919	3.501	0.692	2.244	1.527
15	0.009	4.128	2.064	2.358	0.692	3.966	3.162
16	0.077	4.167	2.083	2.213	0.692	3.193	2.862
17	0.136	4.487	2.244	2.225	0.692	3.096	2.616
18	0.062	5.096	2.548	2.733	0.692	3.214	2.525
19	0.133	5.411	2.705	2.394	0.692	3.806	2.917
20	0.093	6.128	3.064	2.819	0.692	3.654	2.752

Notes:—Column (g) is [(f)*(previous (e)) + (1 − (f))*(previous (c))]* (1.10).
—Column (h) is {(f)*[previous (e) +(1 − (f))*[previous (h)*[1 + (previous (c)−1.1*next previous (c))/previous (c)]]}*(1.10).

Errata for

"The Complement of Credibility"

By Joseph A. Boor, FCAS

***Proceedings of the Casualty Actuarial Society*, LXXXIII, 1996**

Updated: 4 August 2004

Page 19

Under Section B, Complement's Qualities, the last sentence of the first paragraph has been replaced with the two highlighted sentences below.

B. Complement's Qualities

This complement is not as desirable as the previous complements, but sometimes it may be the only alternative. It is less accurate for loss costs with high process variance. Process variance is presumably reflected in last year's rate. That is why it is primarily used for countrywide indications or state indications with voluminous data. It is unbiased in the sense that pure trended loss costs (i.e., with no updating for more current loss costs) are unbiased. **As long as the base statistic's data is not already reflected in the present loss costs, the present loss costs should not have any of the base statistic's process variance. Therefore, the present loss costs should be mostly independent of the base statistic.**

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In the seventh line of the last paragraph, "1 + F" was changed to "1 – F" (see next page).

TABLE 5

LIMITS ANALYSIS FOR LAYER BETWEEN $250,000 AND $500,000

Limit of Liability	Premium	Times 70% Loss Ratio	Increased Limits Factor	% in Layer	Loss in Layer
$ 250,000	$ 600,000	$420,000	1.9	0.00%	$ —
$ 500,000	$ 300,000	$210,000	2.5	24.00%	$50,400
$1,000,000	$ 300,000	$210,000	3.5	17.14%	$36,000
Total	$1,200,000	$840,000			$86,400

be counterbalanced against the fact that individual insureds' large claims histories usually lack credibility.

By fitting a family of loss severity curves to the distribution, actuaries make the most of the large claim data that is available. If the loss history shows no claims beyond the attachment point but many claims that are very near to the attachment point, a fitted curve will usually reflect that and project high loss costs in the subject layer. On the other hand, if there are few large claims close to the attachment point, the fitted curve will project low loss costs for the layer.

The details of how to fit curves are beyond the scope of this paper (see [4]), but it will provide an outline of how to use fitted curves in practice. After fitting and trending the curve, an actuary will use the curve to estimate what percentage of the curve's total loss costs lie in the subject layer. He may do this by evaluating the difference between the limited mean function $\int_{-\infty}^{L} xf(x)dx + (1-F(L))L$ at the attachment point and the attachment point plus the limit of liability. He would then divide the result by the total mean (or the mean when claims are capped at the typical policy limit) to get the percentage of the total loss costs that lie in the layer. Multiplying that percentage by the total claims cost yields the estimate of claim costs in the layer (for details, see [4]).