THE COMPETITIVE MARKET EQUILIBRIUM RISK LOAD FORMULA FOR CATASTROPHE RATEMAKING

GLENN G. MEYERS

Abstract

The catastrophic losses caused by Hurricane Andrew and the Northridge Earthquake are leading many actuaries to reconsider their pricing formulas for insurance with a catastrophe exposure. Many of these formulas incorporate the results of computer simulation models for catastrophes. In a related development, many insurers are using a geographic information system to monitor their concentration of business in areas prone to catastrophic losses. While insurers would like to diversify their exposure, the insurance-buying public is not geographically diversified. As a result, insurers must take on greater risk if they are to meet the demand for insurance. This paper develops a risk load formula that uses a computer simulation model for catastrophes and considers geographic concentration as the main source of risk.

1. INTRODUCTION

Hurricane Andrew and the Northridge earthquake caused unprecedented catastrophic losses to the U.S. insurance industry and its reinsurers. These events revealed significant weaknesses in insurance practices in the United States. This paper will discuss a way to correct some of these weaknesses. It will focus on risk management practices from the point of view of the insurance company and suggest where these practices may lead.

Hurricane Andrew and the Northridge earthquake revealed that some insurers have been doing a poor job of diversifying their exposure to catastrophic losses. In response to this, a number of firms with sophisticated geographic mapping software have entered the market and are being kept very busy by insurers seeking to diversify their exposures.

However, the insurance-buying population itself is not geographically diversified. Therefore, insureds who live in densely populated areas will find it harder to obtain insurance, and hence the price of insurance will be higher for densely populated areas than for lightly populated areas. Since an insurer assumes a higher risk in writing geographically concentrated business, the portion of the price that varies by population density could well be called a "risk" load. This paper will propose a formula for calculating such a risk load. This formula will be called the Competitive Market Equilibrium (CME) risk load formula.

As we develop this risk load formula, it will become clear that an insurer who follows the strategy of geographically diversifying its exposure will have lower capital needs. However, the administrative expense involved in such diversification may discourage all but the very large insurers. Reinsurance can provide an economical alternative to direct diversification for smaller insurers. This paper will analyze the effect of various reinsurance strategies. Also, this paper will illustrate the use of some alternatives to reinsurance.

The insurance problems discussed here are certainly old ones, but this paper will cast new light on these problems through the use of geographic mapping technology and the resulting risk load formula.[1]

[1] Schnieper [13] addresses many of the same problems as this paper. However, Schnieper assumes that the losses of individual insureds are uncorrelated. Many of the results of this paper reduce to Schnieper's results for uncorrelated losses.

2. GEOGRAPHIC INFORMATION SYSTEMS AND INSURANCE RATEMAKING

Catastrophic events happen so infrequently that the traditional actuarial methodology of extending past experience into the future is largely irrelevant. For example, no hurricane has made a direct hit on Miami in recorded insurance history. The same is true for Orlando. However, since Miami is on the coast and Orlando is well inland, no reasonable insurer would charge the same windstorm rates for the two cities. Moreover, data from past hurricanes is of questionable relevance since building practices have changed and the population density in coastal regions has increased in recent years. One can imagine making rates based on insured losses from the 1811–1812 New Madrid Earthquake, or the 1906 San Francisco Earthquake.

Recently, a number of firms have attempted to combine meteorological information, geological information, engineering expertise and insurance loss information to make insurance rates. The results usually take the form of computer simulated events. Exhibits 1 and 2 show the kind of information that typically goes into such an effort.

A geographic information system is a comprehensive database of geographical information. Typically, a geographic information system operates by taking an address and estimating its latitude and longitude. With the latitude and longitude, the system can link the address to other information such as distance to the ocean or distance from known seismic fault lines.

The computer simulated events can be combined with geographic exposure information provided by the insurer to produce a size of loss distribution for the insurer's book of business. This information can be used to evaluate its riskiness; price potential reinsurance contracts; and, as this paper will demonstrate, calculate a risk load.

3. ASSUMPTIONS ABOUT THE INSURANCE ENVIRONMENT

The CME risk load formula makes the following assumptions about the insurance environment.

1. An insurer's capital is a function of its insurance risk. The CME risk load formula is derived from the assumption that the amount of capital needed to support an insurer is a function of the variance of the insurer's total insurance portfolio. To write an additional insurance contract, the insurer must raise additional capital. However, the amount of capital that must be raised for a particular insurance contract may vary by insurer.
2. Each insurer will choose to write an insurance contract that will maximize the return on its required additional (or marginal) capital.
3. Insurers operate in a competitive market. The price for a particular insurance contract will be the same regardless of who insures it.

The CME risk load is then defined as the cost of the marginal capital needed to write the insurance contract.

The assumption that an insurer's capital is a function of the variance of its total insurance portfolio has precedent in both economic and actuarial theory. In their derivation of the Capital Asset Pricing Model (CAPM), Copeland and Weston [4, p. 187] assume that an investor's "utility is a function of the mean and variance of his end-of-period cash flows." In the CAPM, the role of the investor in selecting securities is very similar to the role of the insurer in selecting insurance contracts.

In a more direct treatment of insurance pricing, Ang and Lai [2] write: "The insurer's optimization problem can be written as one of maximizing its mean variance utility $U(E,V)$ subject to the budget constraint … ". They go on to derive a formula very similar to the CME.

Since the willingness of an insurer to take on risk increases with its capital, the role of the insurer's capital is similar to the role of the investor's, or the insurer's, utility function. For example, Kreps [8] assumes that the insurer's capital is proportional to the standard deviation (i.e., the square root of the variance) of the insurer's total loss distribution.

4. THE INSURER BEHAVIOR ASSUMPTIONS

In the course of doing business, an insurer gets the opportunity to expand its business by adding any one of a number of insurance contracts to its portfolio. For each contract it adds, it must add a given amount of capital. Let R be the risk load associated with a given contract. Since the insurer wants to maximize its marginal rate of return on capital, it will choose the contract for which

$$\frac{R}{\Delta\text{Capital}} \tag{4.1}$$

is a maximum.

Since the required capital is assumed to be a function of the variance of the total portfolio, we can rewrite Equation 4.1 to obtain:

$$\frac{R}{\Delta\text{Variance}} \cdot \frac{\Delta\text{Variance}}{\Delta\text{Capital}} \tag{4.2}$$

is a maximum.

Let the capital as a function of variance be given by $C(\text{Variance})$. If the marginal capital required for the insurance contract is small compared to the total variance, we can write:

$$\frac{\Delta\text{Capital}}{\Delta\text{Variance}} \approx C'(\text{Variance}).$$

Then we can approximate Equation 4.2 by:

$$\frac{R}{\Delta\text{Variance}} \cdot \frac{1}{C'(\text{Variance})} \tag{4.3}$$

is a maximum.

The increase in the variance of an insurer's portfolio brought on by the addition of an insurance contract could depend upon the other contracts in the portfolio. The amount of capital required for a given insurer should also depend on other factors, such as the quality of its assets and the variability of its loss reserves. Thus we allow this marginal variance to vary by insurer. The other uses of capital should not present any difficulties if we allow the function C(Variance) to differ by insurer.

At this point, we derive a general expression for the marginal variance due to an individual insurance contract.

Let: X_i = random losses for the ith group of existing contracts; and

Y = random losses for the additional contract under consideration.

Consider the following covariance matrix.

$$\begin{matrix} \mathrm{Cov}[X_1,X_1] & \cdots & \mathrm{Cov}[X_1,X_n] & \mathrm{Cov}[X_1,Y] \\ \vdots & \vdots & \vdots & \vdots \\ \mathrm{Cov}[X_n,X_1] & \cdots & \mathrm{Cov}[X_n,X_n] & \mathrm{Cov}[X_n,Y] \\ \\ \mathrm{Cov}[Y,X_1] & \cdots & \mathrm{Cov}[Y,X_n] & \mathrm{Cov}[Y,Y] \end{matrix}$$

The variance of the sum of random variables is the sum of the covariances in the covariance matrix of the variables. The sum of the covariances in the single framed box represents the total variance before introducing the new contract. The sum of the covariances in the double framed box represents the marginal variance of the new contract. Thus:

$$\Delta\text{Variance} = \mathrm{Var}[Y] + 2 \cdot \sum_{i=1}^{n} \mathrm{Cov}[X_i, Y]. \qquad (4.4)$$

Since covariances are additive, the marginal variance does not depend upon the grouping of the X_is.

Combining Equations 4.3 and 4.4 yields the choice of insurance contracts for which

$$\frac{R}{\mathrm{Var}[Y] + 2 \cdot \sum_{i=1}^{n} \mathrm{Cov}[X_i, Y]} \cdot \frac{1}{C'(\mathrm{Variance})} \qquad (4.5)$$

is a maximum.

5. THE EFFECT OF GEOGRAPHIC CONCENTRATION

Suppose an insurer wants to start writing property insurance in areas with a catastrophe exposure. In accordance with Equation 4.1, a simple strategy would be to find the area where the marginal rate of return is the highest, and write as much as possible in that area. In this section, we argue that insurers will not do this. Instead, we argue that an insurer can maximize its marginal rate of return by spreading its writings geographically.

To illustrate, suppose one area has prospective insureds subject to a random loss, $U_1, U_2, \ldots$. Suppose further that another area has prospective insureds subject to a random loss, $V_1, V_2, \ldots$. We assume that all the Us are independent of the Vs, and that the Us and Vs are both independent of the losses arising from any other contracts the insurer is writing. Let the risk loads for writing a contract in the two areas be R_U and R_V respectively.

According to Equation 4.5, an insurer with no contracts in either area will decide to write its first contract by comparing[2]

$$\frac{R_U}{\mathrm{Var}[U_1]} \quad \text{and} \quad \frac{R_V}{\mathrm{Var}[V_1]}.$$

[2] We need not consider the term $1/C'(\mathrm{Variance})$ since it will be the same for each comparison.

Suppose that writing the Us gives the greatest return on marginal capital, and so the insurer writes the first U. Now let's suppose the insurer proceeds to write n Us. To decide what to write for its $n+1$st contract, the insurer compares

$$\frac{R_U}{\mathrm{Var}[U_{n+1}] + 2\cdot\sum_{i=1}^{n}\mathrm{Cov}[U_i, U_{n+1}]} \qquad \text{and} \qquad \frac{R_V}{\mathrm{Var}[V_1]}.$$

Since all the Us are in the same area, we should expect them to have similar experience when a catastrophe hits. Thus $\mathrm{Cov}[U_i, U_j]$ will be positive for any i and j. As a result, the marginal rate of return will decrease as the insurer writes more Us. Thus, for some n, the marginal rate of return will be greater for writing a V.

We can extend this argument to many areas and lines of business, with the consequence that the insurer will seek to write the insurance contract that gives the greatest marginal rate of return. The process continues until:

$$\frac{R}{\Delta\mathrm{Capital}} = \frac{R}{\Delta\mathrm{Variance}}\cdot\frac{1}{C'(\mathrm{Variance})} = K \tag{5.1}$$

for all prospective insurance contracts.

K is the rate of return on the marginal capital to write the latest insurance contract. One should expect K to vary by insurer. If the insurer is new to the business, K could initially be very high. But a high K will attract more capital, enabling the insurer to expand its writings. As the insurer expands, it will eventually increase its concentration in all the areas in which it writes. As described above, the insurer's return on marginal capital will eventually decrease. When the insurer's volume has reached the point where it can no longer attract new capital, it will stop expanding.

Assume that K is the lowest rate at which the insurer can attract capital. It will then compete to write an insurance contract with risk load R and random loss Y if:

$$\frac{R}{\mathrm{Var}[Y] + 2 \cdot \sum_{i=1}^{n} \mathrm{Cov}[X_i, Y]} \cdot \frac{1}{C'(\mathrm{Variance})} \geq K. \qquad (5.2)$$

In a world of perfect competition, the needs of an individual insurer do not set the risk load, R. Instead, it is set by the insurance market. However, the insurer can control its concentration of business in a given area, and concentration is the relevant variable for the insurer seeking a competitive rate of return on marginal capital.

Back in the real world, insurance regulators have some influence on the insurance market. In addition to their traditional regulation of rates, some insurance regulators are putting restrictions on an insurer's withdrawal of coverage.

Equation 5.2 may provide an adequate description of insurer behavior for a given risk load, but it gives no hint about what an appropriate risk load might be. We now turn to that question.

6. THE COMPETITIVE MARKET ASSUMPTION

As almost everyone knows, any attempt to predict the behavior of the insurance market is dangerous. We make no claim of immunity from these dangers. However, thinking about the problem is better than ignoring it.

Suppose m insurers are competing for a given insurance contract. Let:

Y = random losses for the insurance contract under consideration;

X_{ij} = random losses for the existing contract of insurer j in group i;

R = risk load for the insurance contract, which we assume to be equal for all m insurers; and

$\lambda_j = K_j \cdot C'(\text{Variance}_j)$ for insurer j.

From Equation 5.2 we have

$$\frac{R}{\lambda_j} = \text{Var}[Y] + 2 \cdot \sum_{i=1}^{n} \text{Cov}[X_{ij}, Y].$$

Summing over the m insurers and dividing by m yields

$$R = \overline{\lambda} \cdot \left(\text{Var}[Y] + 2 \cdot \sum_{i=1}^{n} \text{Cov}[\overline{X}_i, Y] \right) \tag{6.1}$$

where

$$\overline{\lambda} = \frac{1}{\frac{1}{m} \cdot \sum_{j=1}^{m} \frac{1}{\lambda_j}} \qquad \text{and} \qquad \overline{X}_i = \frac{1}{m} \cdot \sum_{j=1}^{m} X_{ij}.$$

Equation 6.1 is the competitive market equilibrium risk load formula.[3]

$\overline{\lambda}$ is called the risk load multiplier. As a consequence of Equation 5.2, the risk load multiplier is a function of the marginal rate of return, measured by K_j, and the marginal capital, measured by $C'(\text{Variance}_j)$, of each competitor.[4] The risk load also depends upon how the business written by competitors is related to, or covaries with, the contract under consideration.

7. THE RISK LOAD MULTIPLIER

Equation 6.1 shows that the risk load multiplier, $\overline{\lambda}$, depends upon the competition. Now it might be difficult for an insurer

[3]This formula gets its name from Meyers [10] although, on the surface, the derivation appears quite different. It was Heckman [6] who showed that the original Meyers formulation is equivalent to the return on marginal capital formulation used in this paper.

[4]Kreps [8] presents an alternative way to derive risk loads from marginal capital.

to obtain the λ_j of each of its competitors so, in practice, more informal competitive considerations might well be used. This section provides a formula to aid in the selection of a risk load multiplier.

Let K_j = expected total return of the jth insurer; and
C_j = capital of the jth insurer.

We now make two additional assumptions about the competing insurers:

1. The marginal return on capital is the same for all insurers. That is, $K_j \quad K$.
2. $C_j = C(\text{Variance}_j) = T \cdot \sqrt{\text{Variance}_j}$.

From the definition of λ_j, we obtain

$$\begin{aligned}\lambda_j &= K \cdot C'(\text{Variance}_j) \\ &= K \cdot \frac{T}{2 \cdot \sqrt{\text{Variance}_j}} \\ &= \frac{K}{C_j} \cdot \frac{T^2}{2}. \qquad (7.1)\end{aligned}$$

It follows from Equations 6.1 and 7.1 that

$$\begin{aligned}\overline{\lambda} &= \frac{1}{\frac{1}{m} \cdot \sum_{j=1}^{m} \frac{1}{\lambda_j}} \\ &= \frac{m \cdot K \cdot T^2}{2 \cdot \sum_{j=1}^{m} C_j} \\ & \quad \frac{K \cdot T^2}{2 \cdot \overline{C}}, \qquad (7.2)\end{aligned}$$

where $\overline{C} = 1/m \cdot \sum_{j=1}^{m} C_j$.

Thus under the additional assumptions of this section, it follows that the risk load multiplier is a function of:

K—the annual rate of return (before taxes);
$\overline{C}$—the average capital of the competitors; and
T—the coefficient of the capitalization function.

K and $\overline{C}$ can be estimated from publicly available data.

One possible way to choose T is so that S times the required capital is equal to Z standard deviations of the total loss distribution. That is:

$$S \cdot C_j = Z \cdot \sqrt{\text{Variance}_j},$$

which yields

$$T = \frac{Z}{S}. \tag{7.3}$$

In the examples below, $K = 20\%$, $\overline{C} = \$500{,}000{,}000$, $Z = 2$, and $S = 20\%$. This yields $\overline{\lambda} = 2 \cdot 10^{-8}$.

Here are some important caveats on the choice of the risk load multiplier.

1. While the capitalization function given in Assumption 2, above, is mathematically convenient, by no means is it universally recognized as the best. Other possible capitalization functions are based on the "probability of ruin" and the "expected policyholder deficit."[5]

2. An insurer must hold capital to write an insurance contract as long as potential liabilities remain. One year is usually sufficient for property insurance contracts, but for longer tailed lines of insurance, insurers must often hold some capital for several years. In this case, some modifications must be made to the formula for calculat-

[5] See, for example, Daykin, Pentikainen and Pesonen [5, p. 157], and the American Academy of Actuaries Property/Casualty Risk Based Capital Task Force [3, p. 123].

ing the risk load multiplier. This paper does not cover these modifications. Suffice it to say that the risk load multiplier should be higher for long tailed lines.

8. CALCULATING THE CATASTROPHE RISK LOAD

As described in Section 2, computer models can generate prospective catastrophe losses. To calculate the CME risk load, the information obtained from such a model should be organized in the following manner. Denote

h as the natural event causing the catastrophe indexed from 1 to s, and

i as the insured group indexed from 1 to n. Each group will have a class of business such as homeowners—wood frame houses, and a geographic unit such as ZIP code, associated with each i. (An alternative is to use two indices instead of one.) The class of business should be sufficiently homogeneous and the geographic unit should be small enough so that all properties in the insured group will have similar loss experience for a given event.

For each h and i, let

p_h = the probability of the event h happening in a given year;

d_{hi} = the loss per unit of exposure for insured group i, caused by event h; and

$\bar{e}_i$ = the average number of exposure units in insured group i. This average is to be taken over all insurers competing for the insurance contract under consideration.

Assume that: (1) each event is independent of the other events; and (2) each event can happen at most once in a given year. These assumptions seem reasonable in light of the time needed to repair the property damage caused by a catastrophe, the shortness of the hurricane season, and the physical properties of earthquakes.

Let:

N_h = The random number of occurrences (either 0 or 1)[6] of event h; and

y_h = The damage caused by event h to the property being insured.

Define the random variables

$$Y = \sum_{h=1}^{s} y_h \cdot N_h \qquad \text{and} \qquad \overline{X}_i = \sum_{h=1}^{s} d_{hi} \cdot \overline{e}_i \cdot N_h.$$

Now derive the formula for the catastrophe risk load:

$$E[Y] = \sum_{h=1}^{s} y_h \cdot p_h \tag{8.1}$$

$$\begin{aligned}\mathrm{Var}[Y] &= \sum_{h=1}^{s} y_h^2 \cdot \mathrm{Var}[N_h] \\ &= \sum_{h=1}^{s} y_h^2 \cdot p_h \cdot (1 - p_h)\end{aligned} \tag{8.2}$$

$$\begin{aligned}\mathrm{Cov}[\overline{X}_i, Y] &= \sum_{h=1}^{s} \mathrm{Cov}[\overline{X}_i, Y_h] \\ &= \sum_{h=1}^{s} y_h \cdot d_{hi} \cdot \overline{e}_i \cdot \mathrm{Cov}[N_h, N_h] \\ &= \sum_{h=1}^{s} y_h \cdot d_{hi} \cdot \overline{e}_i \cdot p_h \cdot (1 - p_h).\end{aligned} \tag{8.3}$$

[6]Alternatively, N_h could have a Poisson distribution. But since catastrophic events are rare, the results would hard to distinguish from the chosen binomial model.

Combining Equations 6.1, 8.2 and 8.3 yields

$$R = \overline{\lambda} \cdot \left(\sum_{h=1}^{s} y_h^2 \cdot p_h \cdot (1 - p_h) + 2 \cdot \sum_{i=1}^{n} \sum_{h=1}^{s} y_h \cdot d_{hi} \cdot \overline{e}_i \cdot p_h \cdot (1 - p_h) \right) \tag{8.4}$$

as the formula for the catastrophe risk load.

9. AN ILLUSTRATIVE EXAMPLE

This section gives an example to illustrate some consequences of the risk load formula. Later, we will use this example to formulate hypotheses about the catastrophe exposure and propose ways to manage the catastrophe risk. It will require further work with a validated catastrophe model and real exposures to verify these hypotheses and justify the proposals.

Begin with a description of an imaginary state and the hurricanes that inflict damage on the property of its residents.

The State of Equilibrium is a rectangular state organized into 50 territories. It has an ocean on its east side and is isolated on its remaining three sides. Its property insurance is spread among various insurers that compete for business in every territory. Exhibit 3 provides a schematic map giving the average number of exposure units per insurer ($1,000's of insured value). Exhibit 3 shows that this state has a reasonable array of metropolitan areas, suburbs, and rural areas. The average number of exposure units per insurer is 2,500,000.

The State of Equilibrium is exposed to hurricanes that move in a westward path. Hurricanes occur at a rate of one out of every two years and come in various strengths. The damage caused by the hurricane can span a width of either one or two territories. Each landfall has the same probability of being hit. The losses due to each hurricane decrease as the storm goes inland, with the loss cost decreasing to 70% of the loss cost of the territory bordering on the east. The overall statewide average loss cost is $4 per $1,000 of insurance.

The appendix gives the parameters, p_h and d_{hi}, of the hurricanes.

Using Equation 8.4, risk loads are calculated for a \$100,000 property for each territory. The risk load multiplier, $\overline{\lambda}$, is set equal to 2×10^{-8}. Exhibit 3 shows these risk loads expressed as percentages of the expected losses.

Here are some general comments about these risk loads.

1. Higher risk loads are associated with the more densely populated territories. For example, Territory 25 has a higher risk load than Territory 15, even though the expected loss for a single exposure in each of these two territories is the same.
2. Proximity to a densely populated territory increases the risk load. For example, Territory 20 has the same population density as Territory 15, yet Territory 20 has a higher risk load than Territory 15. This is because some hurricanes hit both Territories 25 and 20, but no hurricanes hit both Territories 25 and 15.
3. Distance from a densely populated territory does not guarantee a lower risk load. For example, Territory 21 has a higher risk load than Territory 11, even though each territory is geographically isolated from a major population center. This is because Territory 21 is behind Territory 25, and these two territories are exposed to the same storm paths.
4. The risk load decreases slightly as a percentage of expected loss as we move inland. Equation 6.1 shows that we can divide the risk load into two parts:

$$\overline{\lambda} \cdot \mathrm{Var}[Y] \qquad \text{and} \qquad \overline{\lambda} \cdot 2 \cdot \sum_{i=1}^{n} \mathrm{Cov}[\overline{X}_i, Y].$$

   The risk load percentage due to the first part decreases from 0.03% to 0.01% as we move inland. The risk load percentage due to the second part remains the same as we move inland.

The magnitude of the risk loads in this hypothetical example are much larger than the customary "cost of capital" provisions in property (primary) insurance rates. The overall average risk load for this example is 172% of the expected loss. The remainder of this section discusses what one should expect as the overall average magnitude of the risk load.

Probably the most debatable part of the formula comes with the selection of the risk load multiplier. The risk load multiplier used depends on admittedly arbitrary risk based capital requirements presented in Equation 7.3. But, as Section 7 shows, the risk load multiplier also depends upon the properties of the competitors, the return on marginal capital, and the amount of time the insurer must hold capital to fulfill the obligations of the insurance contract. Unless the set of competitors differs noticeably by line of insurance, the risk load multiplier should not depend upon the line.

We argue that a catastrophe exposure can have a much larger overall risk load than a normal exposure. To see this, compare the variance added by a well-diversified insurer in the above example with the variance added by a fire insurer.

For the insurer with exposures equal to those given in Exhibit 3, expected losses are \$10,000,000 and variance of the loss is 4.28×10^{14}. Consider a claim severity distribution with an expected loss of \$8,000 and a standard deviation of \$24,000. This claim severity distribution is typical of that for fire insurance.[7] If the insurer expects 1,250 claims, the expected loss will be \$10,000,000. For simplicity, assume both the hurricane losses and the fire losses are independent of the other losses the insurer

[7] This distribution is from the "Total" Column of Exhibit 5 in Ludwig [9] and scaled to a homeowners policy with \$100,000 of insurance. The mean and standard deviation of the distribution are rounded to the nearest \$1,000.

anticipates. Then the relative risk load between the hurricane and fire exposures equals the quotient of their respective variances (see Table 1).

TABLE 1

Parameter Risk	Fire Insurance Variance[8]	Relative Cat/Fire Risk Load
None	8.00×10^{11}	535
Low	2.80×10^{12}	153
Moderate	4.80×10^{12}	89

If these examples are anywhere near realistic, one must conclude that either fire risk loads should be near zero, or that catastrophe risk loads are very large. In practice, the catastrophe risk loads could be significantly smaller—or larger—than the risk loads in this example.

10. ALLOCATING SURPLUS

An alternative to using a risk load is to allocate surplus to an individual contract, and include the cost of this allocated capital in place of the risk load. This practice is controversial because it implies a monoline auto insurer with a surplus of $X is equivalent to a multiline insurer with a surplus of $X allocated to auto insurance. Many, including this author, believe this is not a valid comparison. However, a proper use of allocated surplus does provide a way to pass down the insurer's goal for its overall rate of return to individual product managers. Given its increasing popularity, it should be addressed.

The purpose of this section is (1) to demonstrate that for any risk load formula, there is an equivalent surplus allocation for-

[8]These variances are calculated with Equation 4.4 in Meyers [10], using $b = 0$ and $c =$ 0.00, 0.02, and 0.04 respectively.

mula; and (2) to derive the surplus allocation formula that is equivalent to the CME risk load formula.

Let R_i be the risk load for the ith insurance contract. Let R be the total risk load charged by the insurer, that is:

$$R = \sum_{i=1}^{n} R_i.$$

An insurer with capital C can then "allocate" C_i of its capital to the ith contract in proportion to the risk load R_i, that is:

$$C_i \quad C \cdot \frac{R_i}{R}. \tag{10.1}$$

Conversely, an insurer that derives its allocated capital C_i and its overall risk load R from a different source can then calculate the "risk load" for the ith contract by setting:

$$R_i \quad R \cdot \frac{C_i}{C}. \tag{10.2}$$

By design, these formulas make the return on allocated capital equal to the return on total capital.

This equivalence of risk load formulas with surplus allocation formulas is not a deep thought. It is merely a tautology designed to bring together two schools of thought on pricing for risk.

Now according to Equation 5.1:

$$R_i = K \cdot C'(V) \cdot \Delta V_i \tag{10.3}$$

where V is the variance of the insurer's loss portfolio and ΔV_i is the marginal variance due to the ith insurance contract. Since $K \cdot C'(V)$ is constant across all insurance contracts, allocating surplus in proportion to the risk loads is equivalent to allocating surplus in proportion to the marginal variances.

The actual allocation formulas can now be derived.

According to Equation 4.4:

$$\Delta V_i = \mathrm{Var}[X_i] + 2 \cdot \sum_{\substack{j=1 \\ j \neq i}}^{n} \mathrm{Cov}[X_j, X_i].$$

It can be demonstrated that:

$$\sum_{i=1}^{n} \Delta V_i = V + 2 \cdot \sum_{i=1}^{n} \sum_{j=1}^{i-1} \mathrm{Cov}[X_i, X_j], \tag{10.4}$$

and therefore:

$$C_i = C \cdot \left(\frac{\mathrm{Var}[X_i] + 2 \cdot \sum_{\substack{j=1 \\ j \neq i}}^{n} \mathrm{Cov}[X_j, X_i]}{V + 2 \cdot \sum_{i=1}^{n} \sum_{j=1}^{i-1} \mathrm{Cov}[X_i, X_j]} \right). \tag{10.5}$$

11. MANAGING THE CATASTROPHE RISK

To compete effectively in the insurance market, an insurer must provide its product for the lowest cost. This cost includes the cost of capital, which is provided by the risk load. Reinsurance can reduce the need for capital, and an insurer who effectively uses reinsurance can provide catastrophe insurance for a lower cost. However reinsurance has its own costs. This section examines how insurers and reinsurers may work together to provide coverage for the least cost.

Case 1—"Local" Reinsurance

By "local" reinsurance, we mean that the primary insurers and the reinsurers are operating in the same market. Since all reinsurers are competing for the same insurance contract, we assume that each of them uses the same risk load multiplier.

Let

$$Y = Y_1 + \cdots + Y_g$$

where Y_k is the amount paid by the kth reinsurance contract.

As a matter of convenience, we will only consider contracts for which $\mathrm{Cov}[Y_k, Y_j] \geq 0$. This is true for quota share and excess of loss reinsurance contracts where an increase in Y_k is never associated with a decrease in Y_j.

We have

$$\begin{aligned} \mathrm{Var}[Y] &= \sum_{k=1}^{g} \mathrm{Var}[Y_k] + 2 \cdot \sum_{k=2}^{g} \sum_{j=1}^{k-1} \mathrm{Cov}[Y_k, Y_j] \\ &\geq \sum_{k=1}^{g} \mathrm{Var}[Y_k]. \end{aligned} \tag{11.1}$$

Thus the variance part of the risk load,

$$\overline{\lambda} \cdot \mathrm{Var}[Y], \tag{11.2}$$

is reduced when the loss Y is distributed among the g insurers.

We also have

$$\mathrm{Cov}[\overline{X}_i, Y] = \sum_{k=1}^{g} \mathrm{Cov}[\overline{X}_i, Y_k] \tag{11.3}$$

for all i.

Thus, the covariance part of the risk load,

$$2 \cdot \overline{\lambda} \cdot \sum_{i=1}^{n} \mathrm{Cov}[\overline{X}_i, Y] \tag{11.4}$$

is *not* reduced when the loss Y is distributed among the g insurers.

Now examine how much the risk load can be reduced by sharing the loss among g insurers. Suppose an insured faces a random loss Y. If the loss Y is split equally among g insurers instead of kept exclusively with a single insurer, the total risk load is reduced by

$$\overline{\lambda}\cdot\left(\mathrm{Var}[Y]-g\cdot\mathrm{Var}\left[\frac{Y}{g}\right]\right)=\overline{\lambda}\cdot\mathrm{Var}[Y]\cdot\left(1-\frac{1}{g}\right). \tag{11.5}$$

Equation 11.5 represents the theoretical maximum that the variance part of the risk load can be reduced by sharing the loss among g insurers. Consider the case of $g = 2$. We have

$$\begin{aligned}\mathrm{Var}[Y] &= \mathrm{Var}[Y_1]+2\cdot\mathrm{Cov}[Y_1,Y_2]+\mathrm{Var}[Y_2]\\ &= \mathrm{Var}[Y_1]+2\cdot\rho\cdot\sqrt{\mathrm{Var}[Y_1]\cdot\mathrm{Var}[Y_2]}+\mathrm{Var}[Y_2],\end{aligned} \tag{11.6}$$

where ρ is the coefficient of correlation between Y_1 and Y_2. Let $g = \sqrt{\mathrm{Var}[Y_1]/\mathrm{Var}[Y]}$, $Y_1' = g\cdot Y$, and $Y_2' = (1-g)\cdot Y$. We have $\mathrm{Var}[Y_1'] = \mathrm{Var}[Y_1]$, and the coefficient of correlation between Y_1' and Y_2' is 1. Since Equation 11.6 must hold for Y_1' and Y_2', we must have that $\mathrm{Var}[Y_2'] \leq \mathrm{Var}[Y_2]$. Thus we can replace any shared contract by a proportional contract with a total risk load at least as small.

Thus, the maximum reduction of risk load will occur with a proportional sharing contract of the form $Y_1 = g\cdot Y$ and $Y_2 = (1-g)\cdot Y$. In this case the reduction is

$$2\cdot p\cdot(1-p)\cdot\mathrm{Var}[Y]. \tag{11.7}$$

This expression is maximized when $g = 1/2$. Thus the maximum reduction in the risk load is:

$$\frac{\mathrm{Var}[Y]}{2}. \tag{11.8}$$

If $g > 2$, any two insurers with different liabilities can get together and reduce their joint share by each taking 1/2 of their joint liability. If each insurer takes $1/g$ of the total liability, no

TABLE 2

REINSURANCE PRICES FOR SAMPLE BOOKS OF BUSINESS IN THE STATE OF EQUILIBRIUM

Book	Exposure Distribution	Expected Loss (000)	Total Risk Load (000)	Percentage Risk Load	Variance Risk Load	Covariance Risk Load
1	Industry	2,500	4,696	187.8%	16.5%	171.3%
2	Territory 25	2,500	8,741	349.6	93.4	256.3
3	Uniform	2,500	3,717	148.7	11.9	136.8
4	Industry	5,000	10,219	204.4	33.1	171.3
5	Industry	1,250	2,245	179.6	8.3	171.3

reduction in the total risk load can occur. Thus Equation 11.5 gives the theoretical maximum reduction in the risk load by g insurers.[9]

In theory, the variance part of the risk load can be eliminated entirely by increasing g indefinitely. In practice, g will not be increased indefinitely because of the transaction costs involved in reinsuring. If the transaction costs of adding a reinsurer exceed the corresponding reduction in the risk load, it will not be economical to add that reinsurer to the contract. The expense of reducing the risk load will exceed the cost of capital needed to bear the risk.

We now continue the illustrative example started in Section 9. Suppose an insurer wants to reinsure all its property insurance in the State of Equilibrium. Table 2 gives the expected losses and the risk loads for various books of business when a single reinsurer takes all the business.

The first book of business consists of 6,250,000 units of exposure, distributed among the territories in proportion to the entire

[9]The variance part of the risk load is the same as the variance principle for calculating premiums. The analogous result for the variance principle is well known. See Daykin, Pentikainen and Pesonen [5, Chapter 6] for a standard reference on this subject.

industry. The total risk load for reinsuring the entire book of business equals 187.8% of the expected loss. The variance part of the risk load equals 16.5% of the expected loss. The second book consists of 3,549,523 units of exposure concentrated in Territory 25. The third book consists of 6,398,443 units of exposure, uniformly spread over the 50 territories. We chose these exposure levels so that the expected loss is the same for the first three cases.

Books 4 and 5 illustrate the effect of changing the overall exposure level while maintaining the same relative concentration as Book 1. The covariance risk load is a constant percentage of the expected loss. However, the variance risk load, expressed as a percentage of the expected loss, increases directly with the overall exposure level.[10] Thus, an insurer may expand more efficiently by moving into other geographic regions or to other lines of business. Such a decision will depend upon the other costs of doing business.

The single (or direct) reinsurer arrangement described in Table 2 may not be the most efficient one available. In fact, most catastrophe reinsurance is done through the brokerage market. To continue our example, assume that the reinsurance broker charges an additional commission (above that of the direct reinsurer) equal to 10% of the expected loss. Assume also that each reinsurer involved in the contract incurs an additional expense equal to 0.5% of the expected loss. Then the minimum risk load plus transaction cost occurs when

$$\text{Broker's Commission }\% + \frac{\text{Variance Risk Load }\%}{g} + 0.5\% \cdot g \tag{11.9}$$

is a minimum.

[10]Part of this effect may be an artifact of this example. Here we assume that each hurricane inflicts damages on all properties in a territory in a constant, non-random manner. A more detailed model might include some random effects of hurricanes on the property in a given territory.

TABLE 3

SAMPLE REINSURANCE ARRANGEMENT FOR BOOK 1—INDUSTRY EXPOSURE DISTRIBUTION PRIMARY INSURER RETAINS 10% OF ALL LOSSES

Layer	Expected Loss	Total Risk Load	Percentage Risk Load	Variance Risk Load	Covariance Risk Load
0 } 2,000,000	755,870	706,169	93.4%	1.8%	91.7%
2,000,000 } 6,000,000	723,195	1,154,388	159.6	4.8	154.8
6,000,000 } 12,000,000	489,581	1,181,366	241.3	8.4	232.9
12,000,000 } 20,000,000	247,524	797,542	322.2	11.1	311.1
20,000,000 } 30,000,000	33,830	133,824	395.6	7.7	387.9
Total	2,250,000	3,973,288	176.6%	5.3%	171.3%

For the least concentrated example, Book 2 of Table 2, the minimum variance risk load plus brokerage expense is $10 + 11.9/5 + 0.5 \cdot 5 = 14.9\%$. This does not compare favorably with the 11.9% original reducible risk load and so the contract will stay with the direct reinsurer.

In Book 1, the insurer follows the industry concentration. The minimum reducible risk load plus brokerage expense is $10 + 16.5/6 + 0.5 \cdot 6 = 15.8\%$. This is slightly lower than the 16.5% original reducible risk load, and so further investigation is called for. In practice, reinsurers rarely use this optimal contract. (Could it be that reinsurance underwriters don't believe actuarial theory?) Reinsurers usually require the primary insurer to retain a certain proportion of the loss, to assure diligence in adjusting claims. The remaining losses are parceled out in various layers.

Suppose that the broker comes up with the agreement described in Table 3. With this agreement, the total reducible

TABLE 4

SAMPLE REINSURANCE ARRANGEMENT FOR BOOK 2—
ALL EXPOSURE IN TERRITORY 25 PRIMARY INSURER
RETAINS 10% OF ALL LOSSES

Layer	Expected Loss	Total Risk Load	Percentage Risk Load	Variance Risk Load	Covariance Risk Load
0 – 4,000,000	227,184	474,078	208.7%	6.7%	201.9%
4,000,000 – 12,000,000	454,369	978,807	215.4	13.5	201.9
12,000,000 – 24,000,000	546,325	1,391,386	254.7	19.3	235.4
24,000,000 – 40,000,000	552,566	1,655,379	299.6	25.5	274.1
40,000,000 – 60,000,000	390,499	1,393,837	356.9	30.1	326.8
60,000,000 – 84,000,000	79,057	331,920	419.9	24.3	395.6
Total	2,250,000	6,225,408	276.7%	20.4%	256.3%

risk load plus brokerage expense is $10 + 5.3 + 0.5 \cdot 5 = 17.8\%$. This does not compare favorably with the original 16.5% reducible risk load, so the contract will stay with the direct reinsurer.

In Book 2 of Table 2, all the primary insurer's business was in Territory 25. The minimum variance risk load plus brokerage expense is $10 + 93.4/14 + 0.5 \cdot 14 = 23.7\%$. This compares favorably with the 93.4% original reducible risk load, so further investigation is necessary.

Suppose that the broker comes up with the agreement described in Table 4. With this arrangement, the total variance risk load plus brokerage expense is $10 + 20.4 + 0.5 \cdot 6 = 33.4\%$. This compares very favorably with the original 93.4% variance risk load, so the brokered contract is sold. Note that the cost of the

TABLE 5

SAMPLE REINSURANCE ARRANGEMENT FOR BOOK 1—INDUSTRY EXPOSURE DISTRIBUTION PRIMARY INSURER RETAINS 10% OF ALL LOSSES

Layer	Expected Loss	Total Risk Load	Percentage Risk Load	Variance Risk Load	Covariance Risk Load
0 – 2,000,000	755,870	151,866	20.1%	1.8%	18.3%
2,000,000 – 6,000,000	723,195	258,660	35.8	4.8	31.0
6,000,000 – 12,000,000	489,581	269,020	54.9	8.4	46.6
12,000,000 – 20,000,000	247,524	181,511	73.3	11.1	62.2
20,000,000 – 30,000,000	33,830	28,851	85.3	7.7	77.6
Total	2,250,000	889,909	39.6%	5.3%	34.3%

brokered contract differs from that of the optimal contract. The broker may be able to come up with a better contract.

As these examples show, "local" reinsurance helps very little when the insureds are geographically diversified, but it can help when the insureds are geographically concentrated. But does it help enough? We move on to the next case.

Case 2—"Global" Reinsurance

By "global" reinsurance, we mean that the reinsurer's market covers a much larger area than the primary insurer's market. This case is certainly closer to the norm for catastrophe reinsurance.

As Section 6 shows, the risk load depends upon how the business of competitors is related to, or covaries with, the contract

TABLE 6

SAMPLE REINSURANCE ARRANGEMENT FOR BOOK 2—ALL EXPOSURE IN TERRITORY 25 PRIMARY INSURER RETAINS 10% OF ALL LOSSES

Layer	Expected Loss	Total Risk Load	Percentage Risk Load	Variance Risk Load	Covariance Risk Load
0 – 4,000,000	227,184	107,076	47.1%	6.7%	40.4%
4,000,000 – 12,000,000	454,369	244,801	53.9	13.5	40.4
12,000,000 – 24,000,000	546,325	362,621	66.4	19.3	47.1
24,000,000 – 40,000,000	552,566	443,874	80.3	25.5	54.8
40,000,000 – 60,000,000	390,499	372,777	95.5	30.1	65.4
60,000,000 – 84,000,000	79,057	81,734	103.4	24.3	79.1
Total	2,250,000	1,612,883	71.7%	20.4%	51.3%

under consideration. Global reinsurers should have a very diversified book of business. A fairly large portion of the business should be independent of the primary insurer's business. We now illustrate this effect with the examples described in Tables 3 and 4, with one change. The average exposure in the State of Equilibrium of the competing reinsurers is lower by a factor of five. The remaining exposures of the competing reinsurers have losses independent of the losses in the State of Equilibrium. Assume no change in the capital requirements or the average size of the competing reinsurers. Thus the risk load multiplier remains the same (see Table 5). Here we see that "global" reinsurance can have a dramatic effect on the overall risk load. By comparing Tables 2 through 4 with Tables 5 through 7, it would appear that an insurer could compete far more effectively with the aid of a "global" reinsurer.

TABLE 7

SAMPLE REINSURANCE ARRANGEMENT FOR BOOK 3—UNIFORM EXPOSURE DISTRIBUTION PRIMARY INSURER RETAINS 10% OF ALL LOSSES

Layer	Expected Loss	Total Risk Load	Percentage Risk Load	Variance Risk Load	Covariance Risk Load
0 – 2,000,000	823,024	153,419	18.6%	1.7%	16.9%
2,000,000 – 6,000,000	776,838	248,637	32.0	4.4	27.6
6,000,000 – 12,000,000	578,988	274,577	47.4	8.2	39.2
12,000,000 – 20,000,000	71,151	37,899	53.3	4.7	48.5
Total	2,250,000	714,532	31.8%	4.4%	27.4%

12. THE COMPOUNDING EFFECT OF BUILDING CODES

So far, we have only discussed the insurance side of risk management. This section discusses the effects of loss mitigation efforts.

Assume the existence of a loss mitigation technology that can reduce the expected loss to each insured by a factor of v. If Y is the loss random variable for the insured, the expected loss after loss mitigation is $v \cdot E[Y]$. Since loss mitigation is intended to reduce losses, $v < 1$.

Under normal conditions,[11] an insurer will reduce its rate by a factor of v when there is convincing evidence that the insured's expected losses are reduced by a factor of v. However, as we shall argue, the positive effects of loss mitigation are compounded when a catastrophe exposure is present.

[11] Here we ignore considerations such as fixed expenses which figure into pricing deductibles.

In the discussion that follows, R will be the risk load that applies before any loss mitigation measures take place.

If only one insured takes the loss mitigation measure, the risk load, R_M, for that insured becomes

$$\begin{aligned} R_M &= \overline{\lambda} \cdot \left(\mathrm{Var}[v \cdot Y] + 2 \cdot \sum_{i=1}^{n} \mathrm{Cov}[\overline{X}_i, v \cdot Y] \right) \\ &= v \cdot \overline{\lambda} \cdot \left(v \cdot \mathrm{Var}[Y] + 2 \cdot \sum_{i=1}^{n} \mathrm{Cov}[\overline{X}_i, Y] \right) \\ &\approx v \cdot R. \end{aligned} \tag{12.1}$$

This last approximation is good for individual properties which are part of a catastrophe exposure. In this case, as discussed in Section 9, the covariance risk load is much larger than the variance risk load.

If all insureds take the loss mitigation measure, the risk load, R_M, for an insured becomes

$$\begin{aligned} R_M &= \overline{\lambda} \cdot \left(\mathrm{Var}[v \cdot Y] + 2 \cdot \sum_{i=1}^{n} \mathrm{Cov}[v \cdot \overline{X}_i, v \cdot Y] \right) \\ &= v^2 \cdot \overline{\lambda} \cdot \left(\mathrm{Var}[Y] + 2 \cdot \sum_{i=1}^{n} \mathrm{Cov}[\overline{X}_i, Y] \right) \\ &= v^2 \cdot R. \end{aligned} \tag{12.2}$$

As argued above, the risk load can be a significant part of the overall property rate. Thus the message contained in Equations 12.1 and 12.2 is that the premium for an individual insured can be significantly reduced if its neighbors also take steps to mitigate losses. All insureds have an interest in community-wide loss mitigation. Effective building codes are one way to express this interest.

13. CONCLUDING REMARKS

This paper has derived the Competitive Market Equilibrium risk load formula from standard competitive market economic assumptions, as they apply to the business of insurance. The paper applies the risk load formula to lines of business with a significant catastrophe exposure. The formula uses output from newly developed catastrophe models. The key idea is as follows:

> The marginal capital needed to support an insurance contract increases with the concentration of exposure.

We define the risk load as the cost of marginal capital needed to support the insurance contract. The Competitive Market Equilibrium (CME) risk load is the risk load that matches the supply and demand for insurance.

Through examples, the possibility that the risk load can be very high relative to the expected loss is raised. Rather than pass this risk load on to the insured, cooperative risk management arrangements can result in significantly lower risk loads.

This paper provides a way to balance price, concentration, and the transaction costs of reinsurance.

Market equilibrium is a rare phenomenon in real economic behavior. Shocks to the system happen too often for an equilibrium to develop. However, the examples in this paper show that the CME risk load formula can provide guidance for pricing and managing the catastrophe risk in an evolving insurance market.

REFERENCES

[1] Albrecht, Peter, "Premium Calculation without Arbitrage? —A Note on a Contribution by G. Venter," *ASTIN Bulletin* 22, 1992, pp. 247–254.

[2] Ang, James S., and Tsong-Yue Lai, *Insurance Premium Pricing and Ratemaking in Competitive Insurance and Capital Asset Markets*, 1987.

[3] American Academy of Actuaries Property/Casualty Risk Based Capital Task Force, "Report on Reserve and Underwriting Risk Factors," *Casualty Actuarial Society Forum*, Summer 1993, pp. 105–171.

[4] J. F. Weston and T. E. Copeland, *Financial Theory and Corporate Policy*, Addison-Wesley, 1979, Appendix to Chapter 7.

[5] Daykin, C., T. Pentikainen, and M. Pesonen, *Practical Risk Theory for Actuaries*, Chapman and Hall, London and New York, 1994.

[6] Heckman, Philip E., "Some Unifying Remarks about Risk Loads," *Casualty Actuarial Society Forum*, Spring 1992, pp. 31–44.

[7] Insurance Services Office, *The Impact of Catastrophes on Property Insurance*, 1994.

[8] Kreps, Rodney, "Reinsurer Risk Loads from Marginal Surplus Requirements," *PCAS* LXXVII, 1990, pp. 196–203.

[9] Ludwig, Stephen J., "An Exposure Rating Approach to Pricing Property Excess-of-Loss Reinsurance," *PCAS* LXXVIII, 1991, pp. 110–145.

[10] Meyers, Glenn G., "The Competitive Market Equilibrium Risk Load Formula for Increased Limits Ratemaking," *PCAS* LXXVIII, 1991, pp. 163–200.

[11] Meyers, Glenn G., "Author's Reply to a Discussion by I. Robbin of: The Competitive Market Equilibrium Risk Load Formula for Increased Limits Ratemaking," *PCAS* LXXX, 1993, pp. 396–415.

[12] Robbin, I., “A Discussion of: The Competitive Market Equilibrium Risk Load Formula for Increased Limits Ratemaking,” *PCAS* LXXIX, 1992, pp. 367–384.

[13] Schnieper, Rene, “The Insurance of Catastrophe Risks,” *International Prize in Actuarial Science 1992: Catastrophe Risks*, SCOR Reinsurance, 1992.

[14] Venter, Gary G., “Premium Calculation Implications of Reinsurance Without Arbitrage,” *ASTIN Bulletin* 21, 1991, pp. 223–230.

EXHIBIT 1

MODELING THE EFFECTS OF EARTHQUAKES (SHAKE DAMAGE) ON INSURED LOSSES

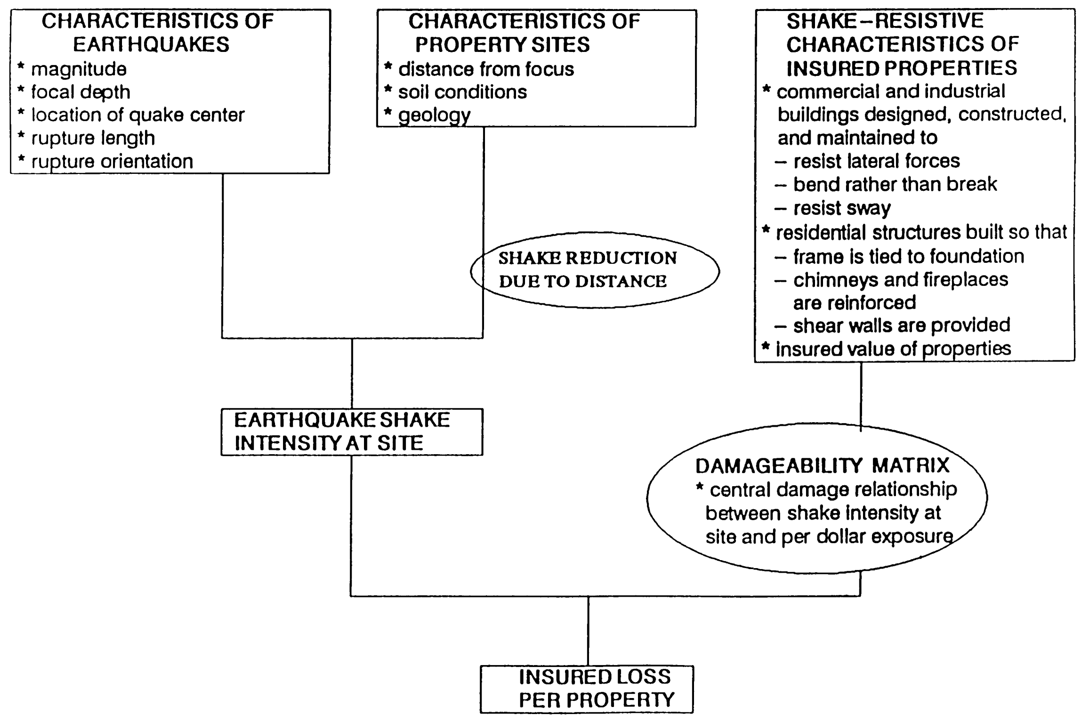

EXHIBIT 2

MODELING THE EFFECTS OF HURRICANE WINDS ON INSURED LOSSES

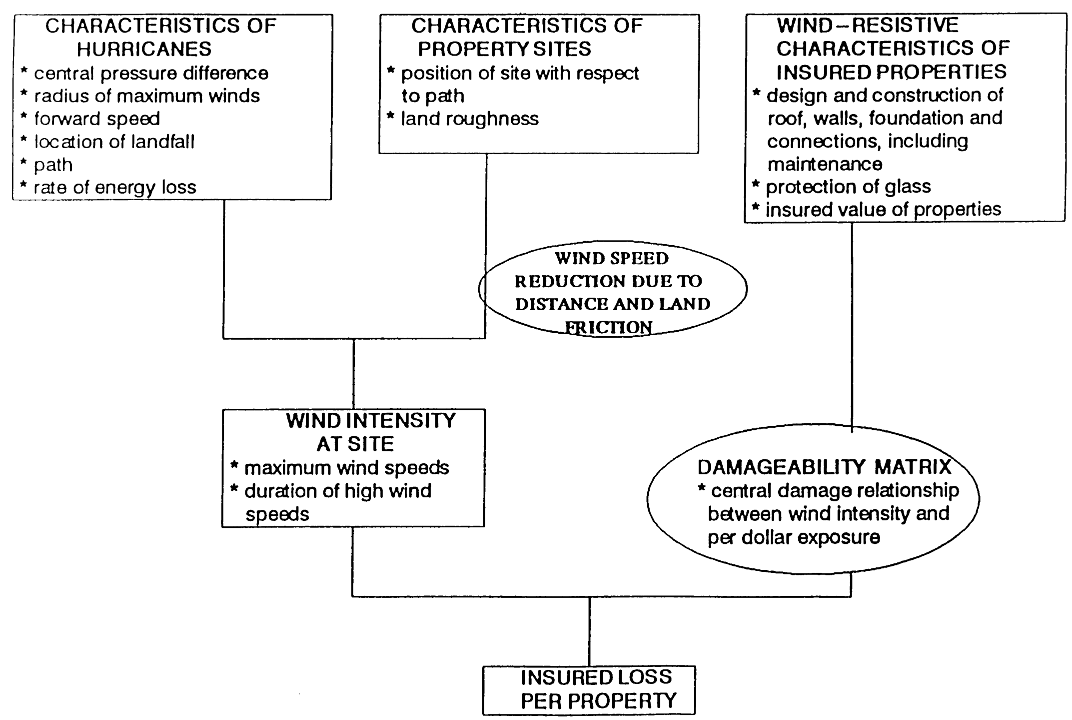

EXHIBIT 3

MAP OF THE STATE OF EQUILIBRIUM

Key

WW XXXXXX YYY ZZZ.ZZ%	WW = Territory	XXXXXX = $\frac{\text{Total Number of Exposure Units}}{\text{Number of Insurers}}$
	YYY = Expected Loss for 100 Exposure Units	ZZZ.ZZ% = $\frac{\text{Risk load}}{\text{Expected Loss}}$ (%)

N
W← • →E
S

Inland #4	Inland #3	Inland #2	Inland #1	Landfall	Ocean
1 25000 169 85.74%	2 75000 242 85.74%	3 75000 345 85.75%	4 25000 493 85.75%	5 25000 704 85.76%	\/\/\/ \/\/\/
6 25000 169 101.10%	7 75000 242 101.10%	8 75000 345 101.11%	9 25000 493 101.11%	10 25000 704 101.12%	\/\/\/ \/\/\/
11 25000 169 78.15%	12 25000 242 78.16%	13 25000 345 78.16%	14 25000 493 78.17%	15 25000 704 78.17%	\/\/\/ \/\/\/
16 25000 169 144.26%	17 25000 242 144.26%	18 25000 345 144.26%	19 25000 493 144.27%	20 25000 704 144.28%	\/\/\/ \/\/\/
21 25000 169 256.26%	22 25000 242 256.26%	23 25000 345 256.26%	24 225000 493 256.27%	25 225000 704 256.28%	\/\/\/ \/\/\/
26 25000 169 144.26%	27 25000 242 144.26%	28 25000 345 144.26%	29 25000 493 144.27%	30 25000 704 144.28%	\/\/\/ \/\/\/
31 25000 169 100.61%	32 25000 242 100.61%	33 25000 345 100.62%	34 25000 493 100.62%	35 25000 704 100.63%	\/\/\/ \/\/\/
36 125000 169 179.41%	37 25000 242 179.41%	38 125000 345 179.41%	39 125000 493 179.42%	40 25000 704 179.43%	\/\/\/ \/\/\/
41 125000 169 183.21%	42 25000 242 183.21%	43 125000 345 183.21%	44 125000 493 183.22%	45 25000 704 183.23%	\/\/\/ \/\/\/
46 25000 169 94.70%	47 75000 242 94.70%	48 25000 345 94.71%	49 25000 493 94.71%	50 25000 704 94.72%	\/\/\/ \/\/\/

APPENDIX

PARAMETERS FOR SAMPLE HURRICANES

The sample hurricanes used in this paper travel from east to west. As a hurricane moves inland, the damage per exposure unit, d_{hi}, is multiplied by 0.7 as it crosses each territory.

Hurricane Number h	Landfall Territory i	Average Damage Per Exposure Unit d_{hi}	Annual Probability p_h
1	5	41.46	0.01618123
2	5	82.91	0.01294498
3	5	124.37	0.00485437
4	10	41.46	0.01618123
5	10	82.91	0.01294498
6	10	124.37	0.00485437
7	15	41.46	0.01618123
8	15	82.91	0.01294498
9	15	124.37	0.00485437
10	20	41.46	0.01618123
11	20	82.91	0.01294498
12	20	124.37	0.00485437
13	25	41.46	0.01618123
14	25	82.91	0.01294498
15	25	124.37	0.00485437
16	30	41.46	0.01618123
17	30	82.91	0.01294498
18	30	124.37	0.00485437
19	35	41.46	0.01618123
20	35	82.91	0.01294498
21	35	124.37	0.00485437
22	40	41.46	0.01618123
23	40	82.91	0.01294498
24	40	124.37	0.00485437
25	45	41.46	0.01618123
26	45	82.91	0.01294498
27	45	124.37	0.00485437
28	50	41.46	0.01618123
29	50	82.91	0.01294498
30	50	124.37	0.00485437
31	5 10	124.37	0.00485437

PARAMETERS FOR SAMPLE HURRICANES

Continued

Hurricane Number h	Landfall Territory i		Average Damage Per Exposure Unit d_{hi}	Annual Probability p_h
32	5	10	165.82	0.00647249
33	5	10	207.28	0.00323625
34	10	15	124.37	0.00485437
35	10	15	165.82	0.00647249
36	10	15	207.28	0.00323625
37	15	20	124.37	0.00485437
38	15	20	165.82	0.00647249
39	15	20	207.28	0.00323625
40	20	25	124.37	0.00485437
41	20	25	165.82	0.00647249
42	20	25	207.28	0.00323625
43	25	30	124.37	0.00485437
44	25	30	165.82	0.00647249
45	25	30	207.28	0.00323625
46	30	35	124.37	0.00485437
47	30	35	165.82	0.00647249
48	30	35	207.28	0.00323625
49	35	40	124.37	0.00485437
50	35	40	165.82	0.00647249
51	35	40	207.28	0.00323625
52	40	45	124.37	0.00485437
53	40	45	165.82	0.00647249
54	40	45	207.28	0.00323625
55	45	50	124.37	0.00485437
56	45	50	165.82	0.00647249
57	45	50	207.28	0.00323625
58		5	124.37	0.00485437
59		5	165.82	0.00647249
60		5	207.28	0.00323625
61		50	124.37	0.00485437
62		50	165.82	0.00647249
63		50	207.28	0.00323625