PRICING INSURANCE POLICIES: THE INTERNAL RATE OF RETURN MODEL

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Financial models, which consider the time value of money, surplus commitments, and investment income, are increasingly being used in insurance rate making. This reading shows how an internal rate of return model can be used to price insurance policies. It discusses the framework of the IRR model, the various insurance, investment, and tax cash flows, the surplus commitments and equity flows, and two methods of estimating the opportunity cost of equity capital. It presents an application of the IRR model from a recent Workers' Compensation rate filing. Finally, it discusses the potential pitfalls in using IRR pricing models.

[This study note was written as educational material for the Part 10A CAS examination. The tables in Section V are reproduced with permission of the National Council on Compensation Insurance. The views expressed here do not necessarily represent the position of the Casualty Actuarial Society, the Liberty Mutual Insurance Company, the National Council on Compensation Insurance, or of any other organization. I am indebted to Robert Butsic, Richard Woll, and David Appel for numerous suggestions and comments on using IRR models, and to Charles Walter Stewart, Paul Kneuer, Jonathan Norton, John Kollar, Ira Robbin, John Coffin, Steve Lehmann, Paul Braithwaite, Leigh Halliwell, William Kahley, Len Gershun, Gerald Dolan, and Peter Murdza for extensive editing and corrections to earlier drafts of this reading. The remaining errors, of course, are my own.]

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Section I: Introduction

How should an actuary determine premium rates for insurance policies? Early rating bureau pricing procedures incorporated a fixed underwriting profit margin, such as 2.5% for Workers' Compensation and 5% for other lines of business. The simplicity of this approach led to its continued use by the actuarial profession.

During the past two decades, economists, financial analysts, and casualty actuaries have proposed alternative pricing models, sparked by the lack of theoretical justification for the traditional procedure, the high interest rates in the American economy, and the increasing competitiveness of the insurance industry. More precisely, the stimuli for more accurate pricing models fall into three categories:

(1) *The time value of money:* Insurance cash flows on a given contract occur at different times. Often, premiums are collected and expenses are paid at policy inception, whereas losses are settled months or years later. Monies exchanged at different dates have different values, which we relate to economic inflation, available interest rates, or the opportunity cost of capital. Financial insurance pricing models consider both the magnitudes and the dates of cash transactions.

(2) *Competition and expected returns:* In a free market economy, the price of a product depends on the degree of competition in the industry. If a firm prices its product above the market level, it may lose sales. If it prices its product below the market level, its profits may fall. The optimal price for products whose costs are known in advance of the sale is determined by production costs and competitive constraints. Complex insurance products, however, require an *a priori* analysis of both expected costs and achievable returns.

(3) *The rate base:* The underwriting profit margin is a return on sales. Businessmen in many industries measure profits in relation to sales, though this method is not favored by financial analysts and theoretical economists. Alternative rate bases are assets, which are used in public utility rate regulation, and equity (or net worth), which is used in most financial pricing models.

There is a wide divergence between the underwriting profit margins assumed in rate filings and actual insurance experience. Over the past 15 years, underwriting profit margins have averaged about –7%, despite the +5% or +2.5% assumed in rate filings. Much of this discrepancy stems from regulatory disapproval of requested rate revisions. In addition, some insurers do not always target a positive underwriting profit margin, since the resultant rates may not be competitive.

Actuaries have responded with new, more sophisticated pricing techniques, which consider cash flows, financial constraints, and competitive pressures. Many insurers analyze their performance with realistic profitability models. But the documentation and dissemination of these models, whether in minutes of technical rating bureau committee meetings or in academic articles, has been sparse. The practicing actuary needs a clearer exposition of the various pricing models.

This paper describes the *Internal Rate of Return* (IRR) insurance pricing model. The IRR model is used extensively by the National Council on Compensation Insurance and by various private carriers for Workers' Compensation rate filings and internal profitability analyses. Moreover, the IRR model has influenced other pricing techniques, such as the *Risk Compensated Discounted Cash Flow Model* which Fireman's Fund proposed for its California rate filings.

The expansion of "open competition" rate regulatory laws, and the replacement of the Insurance Services Office and the National Council on Compensation Insurance advisory rates by loss costs in many jurisdictions, compels company actuaries to determine appropriate profit provisions. The practicing actuary must estimate the needed provisions and justify them at rate hearings. This paper emphasizes the use of IRR pricing models for statewide rate indications, with brief comments on other applications.

IRR pricing models have numerous variations. The models change continually, in conformity with changes in tax laws, insurance regulation, and financial theories. Although this paper uses a recent NCCI Workers' Compensation rate filing as an illustration, it does not attempt to document any particular model. Rather, it shows the framework of the analysis, and discusses the assumptions and results. It clarifies the working of Internal Rate of Return models, so that you can understand their use in rate filings and actuarial analyses.[1]

Point of View

One may examine insurance transactions from two points of view:

(1) *Insurer <—> Policyholder:* The policyholder pays premiums to purchase an insurance contract, which obligates the insurer to compensate the policyholder for incurred losses. These transactions occur in the *product market*, and prices are influenced by the supply of insurance coverage and the demand for insurance services.

(2) *Equity Provider <—> Insurer:* Shareholders, or equity providers, invest funds in an insurance company. The investment provides a return, whether of capital accumulation or dividends. These transactions occur in the *financial market*, and expected returns are influenced by the risks of insurance operations.

The two views are interrelated. The supply of insurance services in the product market depends on the costs that insurers pay to obtain capital, as well as the returns achievable by investors on alternative uses of that capital. Similarly, the expected returns in the financial market,

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[1] On the development of financial pricing models, see Hanson [1970], Webb [1982], and Derrig [1990]. For examples of the major models, see Fairley [1979], Hill [1979], NAIC [1984], Urrutia [1986], Myers and Cohn [1987], Mahler [1987], Woll [1990], Butsic and Lerwick [1990], Bingham [1990], and Robbin [1991]. For analyses of these models, see Hill and Modigliani [1987], Derrig [1987], And and Lai [1987], D'Arcy and Doherty [1988], Garven [1989], D'Arcy and Garven [1990], Mahler [1991], and Cummins [1990A; 1990B; 1991].

which are influenced by the risks of insurance operations, depend on consumers' demand for insurance services.

Yet the two viewpoints may require different assumptions, use different analyses, and lead to different results. The Internal Rate of Return insurance pricing model described here uses the equity-holders' viewpoint, whereas some other financial models use the insurer-policyholder perspective. For instance, the *discounted cash flow model* used in Massachusetts insurance rate regulation assumes that the capital markets are perfectly efficient (Myers and Cohn [1987]). Were there no federal income taxes on investment income, the transactions between equity providers and an insurer would have no bearing on the "fair" price of insurance policies in the Massachusetts discounted cash flow model. Although the Myers-Cohn model uses modern portfolio theory to determine the appropriate discount rate for valuing cash flows, it largely ignores the investment activities of insurance companies or of their stockholders.

Actuarial procedures traditionally approached rate making from this first perspective: the transactions between the insurer and its policyholders. Profits were related to premiums and losses; the capital structure of the insurance company was not considered. Much economic theory, as well as several sophisticated actuarial pricing models, continues along this vein.

Financial pricing models, such as the internal rate of return model, relate profits to assets or equity. Insurance cash flows in the product market, such as premiums, losses, and expenses, are of concern only insofar as they affect the transactions between the company and its stockholders. Of course, insurance cash flows are the major determinants of stock prices and therefore of stockholder profits. The focus here is *point of view*, not *cause*: how the actuary should measure profitability, not what factors influence profitability.[2]

A Non-Insurance Illustration of the IRR Model

The internal rate of return model determines premium rates by comparing (A) the internal rate of return that sets the net present value of a project's cash flows to zero, with (B) the opportunity cost of capital, or the return demanded by investors for projects of similar risk. The decision rule of the IRR model is "Accept an investment opportunity which offers a rate of return in excess of the opportunity cost of capital."[3]

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[2] Compare Cummins [1990B], page 126: ". . . actuarial models tend to focus on supply and demand in insurance markets and typically do not give much attention to the behavior of company owners beyond the assumption that they are risk averse. Financial models tend to emphasize supply and demand in the capital markets and typically neglect the product market beyond the implicit assumption that insurance buyers are willing to pay more than the actuarial values for insurance." See also Cummins [1991].

[3] See Brealey and Myers [1988], pp. 77-85, or Weston and Copeland [1986], pp. 111-120, for introductory expositions of the Internal Rate of Return model, and Sweeney and Mantripragada [1987] and Dorfman [1981] for additional treatment. Although criticized by some financial analysts, IRR models abound: ". . . the most commonly used discounted cash flow method among practitioners is the internal rate of return method" (McDaniel, McCarty , and Jessell [1988], page 369). Gitman and Forrester [1977], page 68, find that the "internal rate of return is the dominant technique" for capital budgeting analyses, with 54% of companies using it as their primary tool, compared to 10% for the net present value method. Internal rate of return models are the dominant financial pricing technique used in life insurance, though earnings, or "statutory book profits," are generally used in place of cash flows (Anderson [1959]; Sondergeld [1982]). In a recent survey of 32 insurers, internal rate of return was the most

The importance of "point of view" can be illustrated by comparing an IRR analysis of capital budgeting with an insurance pricing analysis. In capital budgeting decisions, internal rate of return analyses are often used to value investments that require an initial outlay of capital but promise increased revenues in subsequent time periods. In property/liability insurance operations, the issuance of a policy provides an immediate inflow of cash (the premium) to the insurer, but it obligates the insurer for future loss expenditures. From the viewpoint of the equityholders, though, the insurance operations are similar to other capital budgeting decisions.

Consider first a non-insurance investment decision: Using old production machinery, a firm has $250,000 of annual revenues from a particular product and $50,000 of annual expenses. For $100,000, It can buy new equipment with a two year life span and no salvage value, which would increase annual revenues to $300,000 and reduce annual expenses to $35,000. Should it purchase the new equipment? For simplicity, assume that the purchase costs are incurred at the beginning of the year, the increases in revenues and the decreases in expenses occur at the end of each year, and there are no federal income taxes.

Exhibit 1: Equipment Purchase Decision - Revenues and Expenses

Date	Purchase Cost	Old Equipment		New Equipment		Difference
		Revenues	Expenses	Revenues	Expense	
01/01/92	–$100,000					–$100,000
12/31/92		$ 250,000	$ 50,000	$ 300,000	$ 35,000	65,000
12/31/93		250,000	50,000	300,000	35,000	65,000

The table above shows the annual revenues and expenses with and without purchase of the new equipment. The right-most column summarizes the cash flow difference: the firm pays $100,000 on January 1, 1992, to purchase the equipment, and it gains $65,000 on 12/31/92 and 12/31/93 from increased revenues and lower expenses. The internal rate of return is the value of R which satisfies the equation

$$\$100,000 = (1+R)^{-1}(\$65,000) + (1+R)^{-2}(\$65,000),$$

or R = 19.5%.

Should the firm purchase the new equipment? The answer depends on the opportunity cost of capital: How much does it cost to raise the $100,000? If the cost is 15% per annum, then purchase the equipment. If the cost is 25% per annum, then continue with the old equipment.

Insurance IRR Models

Note the initial cash outflow in the example above: the firm invests money before it realizes

common measure of profitability, slighty exceeding "present value of profits as a percentage of premium" methods (Britton, Campbell, Davlin, and Hoch [1985], page 100; see also Exhibit 1, item III.A.I, on page 120). In life insurance terminology, return on investment (ROI) compares statutory income with statutory surplus, and return on equity (ROE) compares GAAP income with GAAP equity; see Smith [1987].

future revenues. Property/Liability insurance operations seem to show the opposite pattern: the insurer collects premiums before paying losses. But this ignores the equity commitments that support the insurance operations. From the viewpoint of the equityholders, there is indeed a "cash outflow" at the inception of the policy and "cash inflows" as the policy expires and losses are paid.[4]

Two aspects of insurance operations that reflect the equity holders' perspective are incorporated in IRR pricing models:

1. When an insurer writes a policy, part of the premium is used to pay acquisition, underwriting, and administrative expenses. The remaining premium dollars are invested in financial securities, such as stocks and bonds, to support the unearned premium reserve and the loss reserve.

2. Insurance companies "commit surplus" to support their insurance writings: that is, to assure that the company has sufficient capital to withstand unexpected losses.[5]

The cash transactions provide an inflow of funds to the insurer at policy inception, and an outflow as losses are paid. But the owners of the insurer must provide funds to allow the firm to write the policy, so there is a net cash outflow at policy inception from investors. Their return, as in the illustration of the new equipment purchase, occurs in future years, as the policy expires, losses are paid, and surplus is "freed."

Equity Flows

The Internal Rate of Return pricing model takes the viewpoint of the equityholders, who commit capital to support the insurance operations. Insurance transactions are of concern only insofar as they influence the surplus funding required. But how might one determine this influence? In other words, what equity is needed to support both the surplus account and other insurance operations?

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[4] See Cummins [1990A]: "An 'off-the-shelf' approach to insurance pricing, suggested in some rate hearings, views the problem from the company perspective, considering premiums as inflows and losses as outflows. While this is not necessarily incorrect, it can be misleading in an IRR context because the signs of the flows are opposite to those in the usual capital budgeting problem; the flows are positive initially and negative later on" (page 86), and "In the NCCI application of the IRR model, it is assumed that the insurer must make an equity commitment equal to the underwriting loss early in the policy period. Under these circumstances, the early flows are likely to be negative, paralleling the usual capital budgeting example" (footnote 13). [Actually, the equity commitment in the NCCI model is both to fund the underwriting loss and to support the risk of the insurance policy.]

Benjamin [1976] models insurer profitability by means of the commitment of surplus that supports the new business strain caused by a conservative valuation basis and then the release of surplus as losses are paid. Life insurers have a first year cash outflow (surplus strain) caused by high commissions and acquisition expenses, followed by net earnings (cash inflows net of required reserves) in subsequent years.

[5] There is no explicit obligation to commit surplus, but the practice is "enforced" by the NAIC IRIS tests and by the ratings issued by the A. M. Best Corp., Moody's, and Standard and Poor's. An insurer fails the first IRIS test if its ratio of premiums written to policyholders' surplus exceeds 300% (NAIC [1989]; Bailey [1988]). The ratio of loss reserves to surplus influences the Best's rating (Best's [1991], pages xiii-xiv). The Risk Based Capital formula being developed by the NAIC will strengthen the statutory surplus requirements (Hartman, et al. [1992]; Kaufman and Liebers [1992]).

Consider utility companies. Utilities build plants and procure equipment to provide their services. The money needed for this is termed "used and useful" capital (Hanson [1970]). But in insurance operations, there is no intrinsic relationship between policyholders' surplus and premium writings. Stockholders do not continually provide funds to support new policies, and they do not continually receive the monies back, with a return on their investment, as the losses are paid. The regulatory constraints set minimum capital levels, but they do not tell us what the appropriate surplus commitment is.

To determine appropriate surplus levels, some financial analysts examine the actual surplus held by insurer carriers, presuming an overall efficiency of capital markets (Griffin, Jones, and Smith [1983], page 383). Were the insurance industry overcapitalized, investors would withdraw their funds.[6] Conversely, they would invest additional funds if the insurance industry were undercapitalized. Capital market efficiency implies that the current industry surplus levels are necessary and sufficient for insurance operations. Note, however, that the use of IRR pricing models is not dependent on any particular assumptions about capital market efficiency, since appropriate surplus levels may be determined in other ways (Hofflander [1969]; Daykin, et al. [1987]; Pentikäinen, et al. [1989]).

Even if the amount of needed surplus is estimated from industry aggregates, the timing of the surplus commitment and of its release is an assumption in the IRR model. Both the amount of surplus and the timing of its commitment affect the equity flows and the internal rate of return.

To see the importance of equity flows, consider first the association of surplus with lines of business. (On the propriety of allocating surplus to line, see Section III below.) Often, surplus is allocated in proportion to loss reserves or premium writings, or a combination of the two. The procedure used is important, since the average lag between premium collection and loss payment is greater for the Commercial lines than for the Personal lines of business, and greater for the liability lines than for the property lines. An association of surplus with reserves attributes more surplus to the long-tailed lines of business than an association of surplus with premium does.

Both the allocation of surplus to line of business and the internal rate of return depend on the pattern of equity flows. If surplus is committed when the policy is written and is no longer needed when the policy expires, then a $1,000 Homeowners' policy requires the same surplus as a $1,000 Workers' Compensation policy does. If the surplus is committed when the unearned premium reserve is set up, and the required surplus declines as losses are paid and the unearned premium plus loss reserves decrease, then the Workers' Compensation policy needs more surplus. In most instances, the more surplus that is allocated to a policy, the lower will be that policy's internal rate of return.

[6] Joskow [1973] argues that the efficient capital market hypothesis applies only if insurance policies are competitively priced. An overpricing of premiums may cause an overcapitalization of the industry, as investors strive for the higher returns. Danzon [1983], however, contests Joskow's analysis of rating bureau cartelization and overpriced insurance policies.

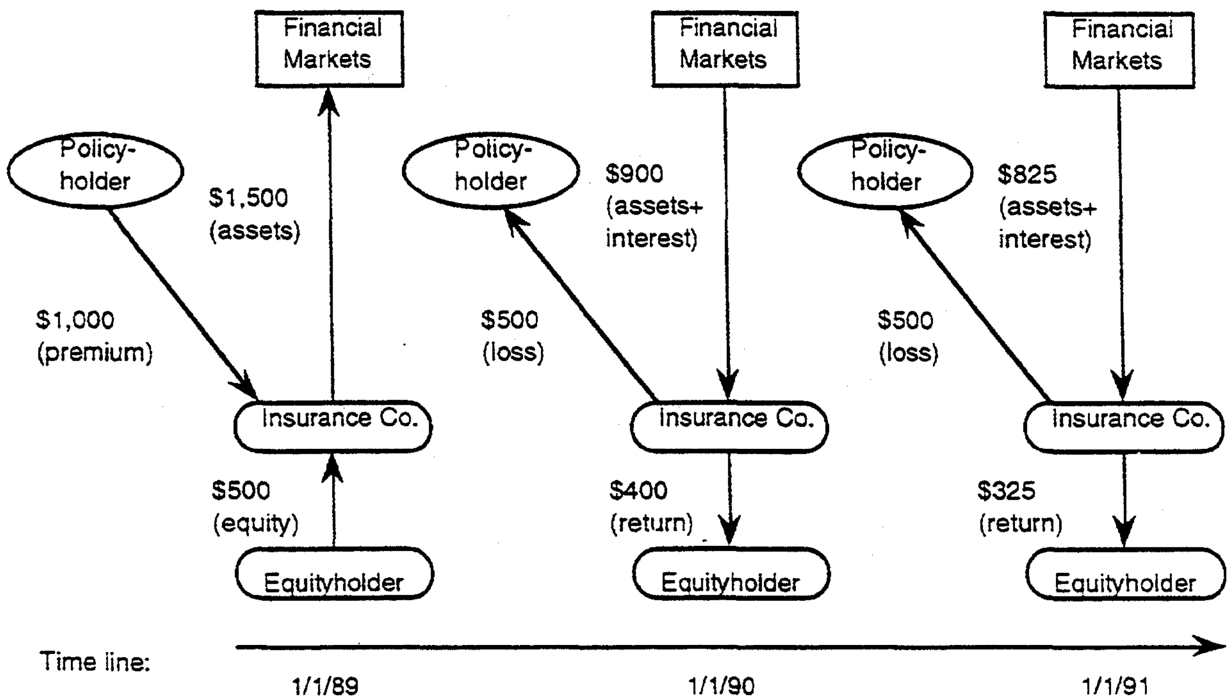
An Equity Flow Illustration

A simplified illustration of an insurance internal rate of return model should clarify the relationships between premium, loss, investment, and equity flows. There are no taxes or expenses in this heuristic example. Actual Internal Rate of Return models, of course, must realistically mirror all cash flows.

Suppose an insurer

- collects $1,000 of premium on January 1, 1989,
- pays two claims of $500 each on January 1, 1990 and January 1, 1991,
- wants a 2:1 ratio of undiscounted reserves to surplus, and
- earns 10% on its financial investments.

These cash flows are diagrammed below.



The internal rate of return analysis models the cash flows to and from investors. The cash transactions among the insurer, its policyholders, claimants, financial markets, and taxing authorities are relevant only in so far as they affect the cash flows to and from investors.

Reviewing each of these transactions should clarify the equity flows. On January 1, 1989, the insurer collects $1,000 in premium and sets up a $1,000 reserve, first as an unearned premium reserve and then as a loss reserve. Since the insurer desires a 2:1 reserves to surplus ratio, equityholders must supply $500 of surplus. The combined $1,500 is invested in the capital markets (e.g., stocks or bonds).

At 10% per annum interest, the $1,500 in financial assets earns $150 during 1989, for a total of $1,650 on December 31, 1989. On January 1, 1990, the insurer pays $500 in losses, reducing the loss reserve from $1,000 to $500, so the required surplus is now $250.

The $500 paid loss reduces the assets from $1,650 to $1,150. Assets of $500 must be kept for the second anticipated loss payment, and $250 must be held as surplus. This leaves $400 that can be returned to the equityholders. Similar analysis leads to the $325 cash flow to the equityholders on January 1, 1991.

Thus, the investors supplied $500 on 1/1/89, and received $400 on 1/1/90 and $325 on 1/1/91. Solving the following equation for "v"

$$\$500 = (\$400)(v) + (\$325)(v^2)$$

yields "v" = 0.769, or "r" = 30%. ["V" is the discount factor and "r" is the annual interest rate, so $v = 1/(1+r)$.]

The internal rate of return to investors is 30%. If the cost of equity capital is less than 30%, the insurer has a financial incentive to write the policy. [The insurer may have other reasons for writing or not writing the policy, such a desire for market share growth, expectations about the future, or concerns about policyholder relationships; see Smith [1983] for a discussion of internal rate of return versus marketing objectives for writing insurance contracts.] Since we are analyzing these transactions from the stockholder's point of view, we compare the internal rate of return with the cost of equity capital.[7]

Actual IRR models are more complex. The following sections (i) describe the insurance cash flows, (ii) explain the surplus commitment and equity flow assumptions, (iii) show how to determine the cost of equity capital, (ii) provide an illustration from a recent Workers' Compensation filing, and (ii) discuss potential pitfalls in using IRR models.

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[7] Textbook presentations of the IRR model for other industries use the firm's weighted cost of capital, which is a combination of the cost of equity capital and the cost of debt capital. Since we are considering the equity holders' perspective, only the cost of equity capital is relevant (see Modigliani and Miller [1958] for further analysis).

Section II: Cash Flows

A. Premium Cash Flows

Insurance pricing models often assume that premium is collected at the inception of the policy. Although once true, this assumption is no longer valid. Large Commercial Lines risks may pay monthly premiums, may spread their premium payments over the first three quarters of the policy year, or may pay a deposit premium at inception and the remainder over the final three quarters of the policy year. A portion of the premium on policies subject to audit may not be collected until after the policy expires. Retrospectively rated policies, particularly for Workers' Compensation, may show return premiums at first adjustment, but additional premiums at subsequent adjustments. Insureds on "cash flow" premium payment plans may not pay the premium until shortly before the insurer expects to pay the loss. The resultant premium cash flow pattern on many retrospectively policies is from policyholder to insurers at inception, from insurer to policyholder at first adjustment (approximately 18 to 21 months after policy inception), and from policyholder to insurer at subsequent adjustments. Combining retrospectively rated policies with special payment plans, audited exposure bases, or cash flow plans produces intricate premium transactions.

An insurance pricing model must examine both premium collection and loss payment patterns. We commence with premiums, after two caveats: (1) Industry-wide *loss* payment patterns by line of business may be determined from Schedule P of the Annual Statement. Most insurers carefully analyze their own loss payment patterns, in greater detail and for more finely segmented blocks of business, to judge reserve adequacy. Industry-wide *premium* collection patterns are not available, and few insurers even track their own experience. (2) Loss payment patterns are stable from year to year, since they depend more on the external insurance environment, such as statutory compensation systems, court delays, and claim emergence, than on internal insurer operations, such as claim settlement philosophy. Premium payment patterns vary widely, since they depend on the payment plans offered by the insurer or chosen by the policyholder.

The pricing actuary can determine premium payment patterns for each block of business only after discussions with the underwriting, audit, and billing personnel at his company. The following discussion of Workers' Compensation premium collection patterns shows some of the factors that must be considered in an internal rate of return pricing model.

Premium Collection Patterns

Exhibit 2 displays sample Workers' Compensation premium cash flow patterns by quarter since policy issue.[8]

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[8] The pattern was developed by the Workers' Compensation Rating and Inspection Bureau of Massachusetts from a study of 350 risks with effective dates in 1986; see its *Filing for 1/1/91 Rates*, Section VIII, "Underwriting Profit Provision," pages 369-536. Exhibit 2 shows the "trimmed flow" from page 383 and patterns for two policy types from pages 386-389. See also MARB [1981], Section VIII, Subsection C, for an earlier study.

Exhibit 2: Workers' Compensation Premium Payment Patterns

Quarter	1	2	3	4	5	6	7	8	9	10	11	12+
All Policies	22%	21%	23%	21%	4%	3%	3%	−1%	0%	3%	1%	0%
Small GC	31	18	18	17	5	6	4	−1	1	0	0	0
Large RR	5	20	24	20	3	2	4	−9	4	22	5	1

Small GC – Guaranteed cost policies issued by stock companies; standard premium less than $5,000.
Large RR – Retrospectively rated policies issued by mutuals; standard premium greater than $500,000.

This payment pattern is for an annual policy written at the beginning of the first quarter. Since the premium is earned evenly over the year, it will be matched with an *accident year* loss payment pattern. Other applications of the IRR model, such as that used by the NCCI (see Section V), use a policy year model: that is, policies written evenly over the course of a year. This premium collection pattern must be matched with a *policy year* loss payment pattern.

Workers' Compensation premium is collected rather evenly over the policy term, since many large policyholders are billed as the premium is earned (NAIC [1990]). During the subsequent year, premium flows depend on standard premium audits (cash inflows) and first adjustments on retrospectively rated policies (cash outflows). For all insureds combined, the premium audits are generally larger than the first retrospective adjustments, and they precede them, on average, by one or two quarters. Second adjustments on retrospectively rated policies provide additional cash inflows in the tenth and eleventh quarters.

The premium cash flow pattern differs between small guaranteed cost policies, where much premium is paid up-front, and large retrospectively rated policies, where much premium is collected at the second adjustment (10th quarter). Small policyholders with guaranteed cost policies pay 50% of the premium within the first 6 months and the balance within the next 15 months. Large policyholders with retrospectively rated policies pay only 25% of the premium within the first 6 months; over 30% remains unpaid after 2 years. Note the return premiums at the first retrospective adjustment (-9% in the eighth quarter) and the additional premiums from the second adjustment (22% in the tenth quarter) for these policies. [The 9% is the net of the retrospective returns with the additional audit premiums.]

Much Commercial Lines insurance is distributed by independent agents, who sometimes hold the premiums received for a month or two before forwarding them to the insurer, at least in the first year that the policy is issued. In 1990, industry-wide agents' balances in course of collection were 7.4% of written premiums. (The percentage is greater for Commercial Casualty insurers. Premium balances, which include both agents' balances and accrued retrospective premiums, are about 10% of premium collected for Personal Lines writers but 30% of premium collected for Commercial Casualty writers; see Best's [1991A].) In other words, a portion of the premium paid by policyholders is not received by the insurer for several weeks. The magnitude of this delay, which depends on the distribution system used by the insurer, affects the premium collection pattern.

B. Loss Cash Flows

In many lines of business, losses are not paid until long after the accident has occurred. The average time lag ranges from a few months for Homeowners' and Automobile Physical Damage, to several years for Products Liability or Workers' Compensation. The investment income earned on the "float" between premium collection and loss payment is one stimulus for insurance pricing models that account for the time value of money.

Payment Lags

Several factors cause the lag between the occurrence and payment of claims:

- Loss adjustment procedures, such as claim filing and investigation, take several weeks or months, particularly when the insurer must first determine the liability of its insured.

- In Workers' Compensation and no-fault Personal Automobile, payments for lost income are paid periodically, as the income loss accrues. [Under tort liability systems, payments for anticipated future lost income are usually made as lump sum settlements.]

- An injury may not be reported by the victim until years after the accident or exposure that caused it, particularly for latent diseases, such as black lung or asbestosis, and for Professional Liability claims.

- Litigated claims often remain unpaid for long periods, since court backlogs, discovery procedures, and legal negotiations delay settlement.

Workers' Compensation claims show all these effects. Moreover, the lag between occurrence and payment is long and stable in Workers' Compensation. Annual Statement data shows an average lag of four to five years, with little fluctuation from year to year (Woll [1987]; see also Kahane [1978]; Noris [1985]).

Most insurers collect data on loss payment patterns, particularly if paid loss development methods are used for reserve indications. Alternatively, one can use industry patterns, as compiled by the A. M. Best Co. from Annual Statement Schedule P data.

The chart below shows a hypothetical 20 year countrywide loss payment pattern for Workers' Compensation.[9] Note the long tail: over 12% of losses remain unpaid after 10 years.

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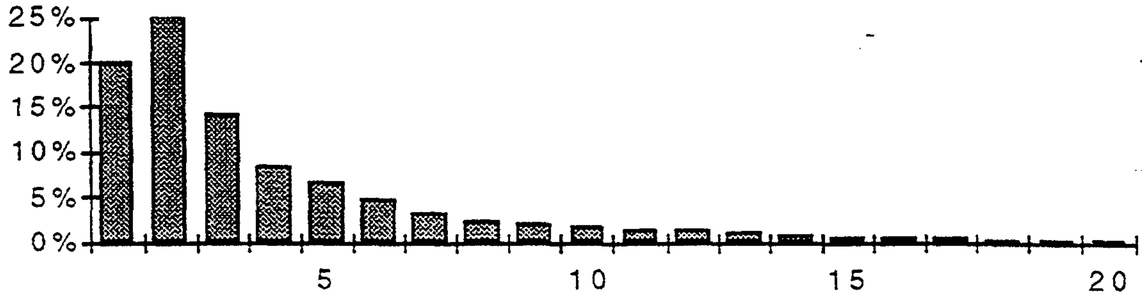
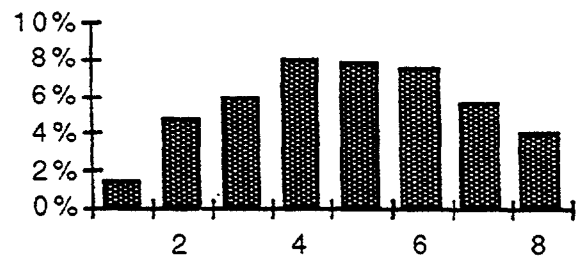
[9] The countrywide pattern is not appropriate for all states, since statutory benefit provisions and limitations on the size and duration of recoveries affect the expected payment patterns; see NCCI [1991], Section 5, pages 219-391, for paid loss patterns by state and Chamber of Commerce [1991], Part 2, pages 17-29 (or NCCI [1991], Section 4, pages 134-191) for benefit provisions by state.

WC Accident Year Loss Payment Pattern



The cash flow pattern for loss payments during the first two years is important for the IRR model, since the payments are not evenly distributed by quarter. Loss payments increase rapidly during the first four quarters since the inception of the policy, remain level from the fourth through the sixth quarters, and then slowly decline, as shown in the accompanying chart. The figures are shown as percentages of ultimate losses. 45% of losses are paid during the first two years: 1.5% in the first quarter, 5% in the second quarter, and so forth.

WC Quarterly Payment Pattern



Loss Adjustment Expenses

Loss adjustment expenses are a large component of insurance costs in several lines, particularly General Liability. Allocated loss adjustment expenses (ALAE), such as defense counsel fees, relate to specific claims. Unallocated loss adjustment expenses (ULAE), such as Claims Department salaries and overhead costs, can not be related to specific claims.

The cash flow patterns for losses, ALAE, and ULAE are different. ALAE is paid more slowly than losses, since defense counsel costs are greatest for the heavily litigated, slowly settling cases. ULAE is incurred primarily when the claim is first reported, since setting up files and investigating the cases consume a large portion of Claims Department salaries. The statutory distribution formula for paid ULAE prescribed in Schedule P uses two assumptions:[10] (1) Half of the ULAE is expended when the claim is reported, and half is expended when the claim is settled. (2) 90% of claims are reported during the calendar year during which the loss occurred, and 10% are reported in the subsequent year. The actual percentages vary by line of business. For instance, fewer than 75% of General Liability claims are reported during the calendar year in which the loss occurred, though 90% for Workers' Compensation are indeed reported by then.

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[10] See Salzmann [1988]; Feldblum [1991]. Alternatively, one may assume that ULAE is expended while a claim is open, with additional amounts paid when it is reported or settled (Kittel [1981]; Johnson [1989]).

For example, the average ULAE to loss ratio is 6% for Workers' Compensation. Many WC claims have periodic indemnity payments, with little Claims Department involvement, in contrast with GL bodily injury claims, where the insurer may negotiate with the claimant until the settlement date. Thus, most ULAE on long-duration disability cases is expended when the claim is first reported and the insurer investigates the injury. We assume that 60% of total Workers' Compensation ULAE is incurred when the claim is reported, and we distribute this figure in proportion to the claims emergence pattern: 90% in the first year and 10% in the next half year. The remaining 40% of ULAE is distributed in proportion to the WC loss payout pattern.

C. Expense Cash Flows

Traditional rate making procedures place little emphasis on expense costs. Historical loss ratios are developed and trended to project anticipated loss ratios during the future policy period. A target loss ratio is set as the complement of the expense ratio, which is determined from (i) Insurance Expense Exhibit data, adjusted for trends in expense levels, (ii) agency contracts, and (iii) company budgets. Pricing actuaries have refined the analysis of expenses, with emphasis on fixed versus variable expenses, expense flattening procedures, expenses by size of risk, and the relationship of expenses to policyholder persistency (McConnell [1952], Wade [1973]; Hunt [1978]; Childs and Currie [1980]; Feldblum [1990A]).

In theory, we should analyze expenses as rigorously as we analyze premiums and losses. In practice, several problems hinder this analysis:

1. *Data:* Few companies monitor their expense payment patterns. Loss payment patterns are shown in the Annual Statement, Schedule P, and are used to estimate bulk reserves. But most companies keep expense data only on a calendar year basis. The pricing actuary can find out the "other acquisition expense" in the month of January but he cannot always determine when that expense is paid in relation to the policy inception dates.

2. *Company:* Expense levels vary widely by company. Loss payment patterns depend more on external influences than on internal company operations. But expense levels and payment patterns depend on the company's distribution system, underwriting philosophy, and the services it provides.

3. *Risk Size:* Traditional rate making procedures assume that insurance costs vary directly with premium. For instance, premium taxes and some commissions vary with gross premium; losses and loss adjustment expenses vary with net premium. The assumption is not necessarily true for overhead expenses, such as policy issuance and billing costs, advertising, Home Office rent, and employee salaries, or for expenses that vary by policy size, such as Workers' Compensation commissions. In general, overhead expenses are a lower percentage of premium for larger risks. To set more equitable rates, actuaries use premium discounts in Workers' Compensation and expense flattening procedures in Personal Automobile.

4. *Policy Year:* Underwriting expenses, other acquisitions costs, and commissions for direct writers are higher in the first policy year than in subsequent years. Long-term expense costs depend on policyholder persistency: insureds who persist for several renewals have lower average annual expense costs that those who terminate in the first few years (Feldblum [1992B; 1993]).

In sum, the expense levels and cash flow patterns should depend on company characteristics, size of the risk, and new versus renewal underwriting. The following paragraphs discuss the types of expenses, following the Insurance Expense Exhibit categories, but they do not incorporate all the refinements mentioned above. Industry average Workers' Compensation expense levels are used as illustrations.

Types of Expenses

1. *State premium taxes* are paid quarterly as a percentage of the previous year's tax liability, with an adjustment in the first quarter of the following year to reflect the actual liability calculated on the prior year's written premium. If the volume of business is not changing, the premium tax cash flows should reflect the written premium pattern. Most Personal Lines premium is booked at policy inception, so the premium tax is paid in the first quarter. For large Commercial Lines risks, premium often is booked as it is earned, so premium taxes are incurred monthly throughout the policy year, as well as at audit dates.

Premium taxes and special assessments (such as for guarantee funds) vary by state. Workers' Compensation rates range from 2.5% to 8%, with a countrywide average of 3.5 to 4% (NCCI [1991], Section 3, pages 129-133). Certain assessments, such as those for second-injury funds and administrative costs, are sometimes based on incurred benefits, not only premium writings. Other taxes, licenses, and fees, which average 0.5% to 1% of written premium, appear with premium taxes on line 8 of the Insurance Expense Exhibit and may be included with premium taxes and assessments in the cash flow patterns.

2. The *commission* payment pattern follows the premium collection pattern. The overall commission level depends on several factors. For Personal Lines, the commission level varies by policy year: first year commissions may be 25% of premium and renewal commissions may be 5% of premium. For Workers' Compensation, commissions vary by size of risk, ranging from 2% to 15%. Contingent commissions depend on the profitability or persistency of an agent's book of business and vary by company. The industry average direct commission level for Workers' Compensation is 5.1 of adjusted direct written premium (Best's [1991A], page 119).

3. *Other acquisition expenses*, such as overhead expenses for agents' offices or for advertising, are difficult to model. First, the expenses are related to blocks of business, not to specific policies. Second, these expenses are incurred and paid several months before policies are issued. The industry average expense level for Workers' Compensation is 3.4% (Best's [1991A], page 119). A sample cash flow pattern might be 20% in the first quarter after policy inception, 50% in the quarter before policy inception, and 30% in the next preceding quarter.

4. *Underwriting and administrative expenses*, such as salaries of Home Office personnel or rent and utilities on company owned buildings, have a mixed payment pattern. Many of these expenses are incurred primarily when the policy is first written, so they vary by year since the original inception date.

General expenses incurred for Workers' Compensation average 4.9% of earned premium. Actuaries often assume that about half of these are underwriting expenses, which are incurred on or prior to the inception date. A sample cash flow pattern would be 65% in the first quarter (underwriting costs), 10% each in the next three quarters (other Home Office costs), and 5% during the second year (continuing actuarial, accounting, and systems costs for open cases).

These expense levels and sample cash flow patterns are summarized in the exhibit below. The cash flow pattern for "total expenses" is a weighted average (by expense level) of the patterns for each expense item.

Exhibit 3: Workers' Compensation Expense Levels and Cash Flow Patterns

Expense Type	Expense Level	Cash Flow Pattern by Quarter −1	0	1	2	3	4	5	6	7	8	9+
Taxes, licenses, fees	4.5%	0%	0%	50%	12%	12%	12%	4%	4%	4%	2%	0%
Commissions	5.1	0	0	25	21	20	20	4	3	2	3	2
Other acquisition	3.4	30	50	20	0	0	0	0	0	0	0	0
General expenses	4.9	10	25	25	10	10	10	6	2	1	1	0
Total expenses	17.9%	8%	16%	30%	12%	11%	11%	4%	3%	2%	2%	1%

D. Investment Cash Flows

The investment income earned by insurance companies has been the stimulus for financial pricing models. Yet the *investment* rate of return is different from the *internal* rate of return. The former relates to the insurer's transactions, whereas the latter relates to those of the equity holders in the insurance company (Smith [1987]; Griffin, Jones, and Smith [1983]).

IRR models for some other industries consider the cash flows from operations: sales (premiums), production costs (losses), and expenses. What the firm does with its extra cash, whether it invests the monies in financial securities, uses them to finance other activities, or pays larger dividends to stockholders, is not necessarily relevant to its pricing decisions. The firm's financial holdings are distinct from its business operations.

Insurance operations are different. Financial holdings corresponding to the premium, loss, and loss adjustment expense reserves are needed to pay claims covered by the policies. The remaining assets, corresponding to surplus, support the insurance operations. The financial holdings can not necessarily be used to pay stockholder dividends or to finance other activities.

Investment Yield and Internal Rate of Return

Thus, there are two "rates of return" in the IRR insurance pricing model. The internal rate of return is earned by equity holders for supplying funds to the insurance company. The investment rate of return is earned by the insurance company for supplying funds to the stock or bond markets. The investment returns are a part of the insurer's business operations from which the internal rate of return is determined.

Although the internal rate of return and investment rate of return are distinct, there are close links between the two. If the returns are expressed in nominal dollars, both will vary with economic inflation. If the returns are expressed in real terms, the connection is more subtle. The internal rate of return varies with the riskiness of the insurance operations. If the insurer invests in higher risk securities, such as common stocks and junk bonds, rather than in low risk securities, such as Treasury notes, the insurer's investment return will be high, but so will the cost of equity capital, or the internal rate of return demanded by equity holders.[11]

To illustrate the derivation of the investment rate of return and the opportunity cost of equity capital, Exhibit 4 shows the 1989 insurance industry financial portfolio and average rates of return, using data from the A. M. Best Corp., DRI, and Value Line. The investment yields are "new money rates of return," not imbedded yields or portfolio returns. The distribution of securities is taken from the actual industry portfolio, which consists predominantly of municipal bonds, Treasury securities, investment grade corporate bonds, and common stocks, with smaller proportions of mortgage loans, short term investments, and other holdings (such as real estate).[12]

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[11] When considering the inclusion of investment income in the rate making procedures, some observers have wondered: "Should policyholders receive the benefits of a successful investment strategy? And should they incur the costs of a poor strategy?" To avoid shifting investment risk to policyholders, some pricing models use "risk free" interest rates (Cooper [1974], Wohl [1987], Bingham [1990]). Other models, such as that described in this reading, use expected returns determined from the insurer's financial portfolio and base the "target" rate of return on the cost of equity capital. An insurer with an aggressive investment strategy may earn higher investment yields. The additional investment risk, though, raises the opportunity cost of equity capital, so there is an offsetting effect on premium rates.

[12] The 1989 financial holdings by type of security are taken from Best's [1990]. Bonds and stocks are from the Annual Statement Schedule D, "Summary by Country," with amortized values for bonds, market values for common stocks, and statement values for preferred stocks. The remaining invested assets are taken from the Annual Statement Balance Sheet, page 2, "Assets," lines 3 through 7.

Yields on bonds are available from such sources as DRI (= Standard & Poor's), the Wall Street Journal, or Moody's. For bonds, the most recent new money yield is a good proxy for the expected new money yield; that is, the analyst may have no better data than the current rates to forecast future rates. (On the rationale for using new money yields versus portofio returns, see Cummins and Chang [1983].) As Robert Butsic has pointed out to me, an adjustment for default risk should be subtracted from the reported returns, though it is hard to quantify the amount (Vanderhoof, et al. [1989]; Altmann [1989]). For common stocks, expected yields must be estimated, using a CAPM or DGM model (see Section IV). Average yields in 1989 for the other securities are taken from Best's [1990], page 2, except for "Other invested assets," whose high reported return (18.2% in 1989 and 17.7% in 1990) seems unusual.

Exhibit 4: Property/Liability Insurance Company Investment Portfolios and Average Yields

	1989 Holdings ($000,000)	Percent of Invested Assets	Expected Yield	Tax Rate	Expected After-Tax Yield
Treasury Bonds	81,107	18.2%	8.5%	34.0%	5.6%
Municipal Bonds	151,407	33.9	7.0	5.1	6.6
Corporate and Foreign Gov't Bonds	78,958	17.7	9.2	34.0	6.1
Preferred Stocks	10,956	2.5	8.8	13.8	7.6
Common Stocks	73,049	16.3	15.0	26.9	11.0
Mortgage Loans	6,461	1.4	9.9	34.0	6.5
Cash and Short-Term Investments	33,853	7.6	8.9	34.0	5.9
Other Invested Assets	10,748	2.4	10.0	34.0	6.6
Total:	446,539	100%	9.3%	22.5%	7.0%

Taxes on Investment Income

The Federal Income Tax rates reflect the provisions of the 1986 tax amendments. All investment returns except stock dividends and municipal bond yields are taxed at 34%. The "Dividends Received Deduction" provides a tax exemption of 70% of most common and preferred stock dividends received by corporations; the remaining 30% of dividends is taxed at 34%. The "Proration" provision subjects 15% of tax-exempt municipal bond income and the dividends received deduction to the 34% tax rate, unless the securities were owned before Aug 8, 1986.

Thus, interest on municipal bonds is taxed at (15%)(34%), or 5.1%. Stock dividends are taxed as follows:

Dividends received deduction:	(.70)	x	(0.15)	x	(.34)	= 3.6%
Remaining dividends:	(.30)	x	(1.00)	x	(.34)	= 10.2%
Total:						13.8%

The dividends received deduction does not affect capital gains and losses, which are taxed at 34%. In 1989, insurers earned $2,509 million in common stock dividends and $12,169 million in realized plus unrealized capital gains on common stocks (Bests's [1990], page 62). This high proportion of 1989 common stock returns coming from capital gains (83%) reflects the bull market of the 1980's, not a predominance of growth stocks. Using 65% as the expected percentage of common stock returns coming from capital gains, the combined tax rate is

$$(65\%)(34\%) \quad + \quad (35\%)(13.8\%) \quad = \quad 26.9\%.$$

For the overall financial portfolio of the Property/Casualty insurance industry, the 1989 pre-tax return was 9.3%, the average tax rate was 22.5%, and the after-tax return was 7%.

Adjustments for expenses, depreciation on real estate, and other "write-in deductions" reduce the expected investment return. Investment expenses do not relate to the writing or servicing of insurance policies, so they are not included in the expense cash flows above. Rather, investment expenses are deducted from the gross investment returns to determine net investment returns. Similarly, depreciation on real estate and other "write-in deductions from investment income" must be subtracted from the gross totals above.

In 1989, the industry reported $2,176 million of such expenses, depreciation, and other deductions, or 0.5% of the $447 billion in invested assets shown above (Best's [1990],p. 62). The full 34% tax rate is applicable to these expenses and deductions, so the after-tax yield is reduced by 0.3%. The expected net investment yields are 8.8% pre-tax and 6.7% after-tax.

E. Federal Income Taxes

Federal income taxes depend both on the magnitude and incidence of the firm's earnings and on the accounting methods of the company. For all but the smallest firms, the tax rate on underwriting income is 34%. The incidence of taxes is determined by statutory accounting, federal statute, and IRS regulation.

Until 1987, taxable income was generally based upon statutory income as reported in the NAIC accounting blank, with adjustments for tax exempt income, depreciation and amortization of assets expensed on the Annual Statement, and other miscellaneous adjustments. Two provisions of the 1986 Tax Reform Act changed this relationship, by accelerating the timing of federal income taxes. First, "revenue offset" includes in current income 20% of the change in the unearned premium reserve, as a proxy for deferred acquisition expenses (DAC). Second, insurers must discount loss reserves, using a prescribed interest rate and either an industry wide or a company payment pattern.[13]

Several relationships between tax provisions and other actuarial assumptions should be noted:

1. *Revenue Offset:* The IRS assumes a 20% deferred acquisition cost expense ratio for all lines. The actual expense ratio differs by company and by line of business, ranging from 13% in Workers' Compensation to 37% in Fire Insurance (Best's [1991A]; Feldblum [1992B]).

2. *Discount Rates:* The prescribed interest rate is a 60 month moving average of yields on fixed income Treasury securities with maturities between 3 and 9 years. If interest rates are stable, this is similar to new money rates earned by insurers.

3. *Surplus Commitments:* Investment income on surplus funds is taxed. Were equity holders provided no return from insurance operations, they would prefer to invest their funds

[13] See NCCI [1987]; Gleeson and Lenrow [1987]; Almagro and Ghezzi [1988]; and Friedman and Heitz [1988]. The 1986 Act also includes a transition provision whereby 20% of the December 31, 1986 unearned premium reserve is brought into income ratably over six years commencing in 1987. In addition, insurers are subject to an Alternative Minimum Tax, if this tax is greater than the normal income tax. The AMT may be viewed as an additional cost of business, since it restricts otherwise optimal investment and underwriting strategy (see the references cited above). The complexities of the federal income tax regulations prevent their full treatment in this reading.

directly in the capital markets, rather than provide it to insurers and face double taxation.[14] To induce equity holders to invest in the company, insurers must use earnings from insurance operations to compensate equity holders for the tax on capital and surplus funds.

The incidence of this tax depends on the assumed surplus commitment pattern. If surplus is committed in proportion to premiums earned, this tax is incurred during the policy year. If surplus is committed in proportion to loss plus LAE reserves, much of this tax is incurred in subsequent years.

Federal income taxes are a complex but essential aspect of financial pricing models. The pricing actuary must consider the relationship between the tax effects of the insurance transactions and the insurer's overall tax situation. Further analysis of federal income taxes is provided in the illustration in Section V.

[14] The "double taxation" stems from the corporate income tax paid by the insurer and the personal income tax paid by its equity holders. Suppose an investor provides $1 million to an insurer, which uses this money to buy stocks or bonds, providing a 10% return. Of the $100,000 investment income, $34,000 is paid in corporate income taxes, and $66,000 is disbursed as dividends to equity holders. Of this $66,000, an additional amount, say $22,000, is paid in personal income taxes, and the equity holders remain with $44,000. The equity holders would prefer to invest directly in stocks and bonds, and have federal income taxes deducted only once; see Myers and Cohn [1987], pages 57-58; Cummins [1990A], page 84; Garven [1987].

Section III: Surplus

Financial pricing models calculate a return on the capital used by a firm to furnish goods or services. Manufacturers use fixed assets, such as plants and equipment, to produce their output. The return on investment relates the firm's output to the capital required for its production.

The insurance contract is a promise: a commitment to compensate policyholders if certain contingent events occur. The worth of the promise depends on the integrity and financial strength of the insurer. The surplus held by an insurer backs its promises. An insurer could not fulfill its obligations if it lacked the financial assets that support them, even as a manufacturer could provide no output if it lacked the fixed assets to produce it.

There are important differences between the manufacturer's fixed assets and the insurer's surplus. The fixed assets needed for production can be objectively measured, for the firm as a whole and often for each product line which it produces. The surplus "needed" by an insurer, and its allocation to lines of business and time periods, is a theoretical construct. The pricing actuary using an IRR model may assume a relationship between surplus commitment and insurance transactions. He can examine the assumption for reasonableness, but he can not always test it empirically.

The Individual Firm and the Industry

This difference between (i) insurance pricing and (ii) the uses of internal rate of return models in other industries has several implications:

- IRR models used for capital budgeting decisions examine the funds invested by a particular firm and the returns expected by that firm. A firm may invest more funds if it anticipates larger returns. Similarly, if it invests less funds, it may generate less revenue.

  For the insurance pricing model, the assumption of capital markets efficiency implies that the industry is neither over- nor under-capitalized. But any given insurer, because of favorable or adverse operating results in the past, may have more or less surplus than it needs. Whereas manufacturers of similar products often have similar levels of operating leverage (that is, the ratio of sales to fixed assets), insurers writing identical lines of business have diverse levels of insurance leverage (that is, the ratio of premium to surplus). The actual surplus held by an insurer may reflect historical happenstance, not deliberate strategy.

- The internal rate of return model concentrates on the *capital market*: the returns earned by providers of equity to insurance firms and the opportunity cost of this capital. The prices of insurance contracts are determined in the *product market*, by the supply of and demand for insurance policies.

  At the industry level, the capital and product markets are intertwined. If marketplace prices are inadequate, and industry returns are below the opportunity cost of capital, firms will leave the industry and prices will rise. This relationship does not necessarily hold for the individual firm. Industry rates may be adequate, but the firm's internal rate of return may be depressed by operating inefficiencies.

In sum, the required surplus is a theoretical construct. Even if the industry as a whole has an appropriate surplus level, individual firms may be over- or under-capitalized. Moreover, the relationship between the product market and the capital market is strong for the industry as a whole but weak for any individual firm.

Some analysts therefore determine the surplus commitments in the IRR model from industry-wide practice (Griffin, Jones, and Smith [1983], page 383). The surplus actually held by the individual insurer is ignored, and replaced with the required surplus posited by the actuary.

Surplus Allocation

Many financial pricing models examine returns on an insurer's surplus or equity. Surplus exists for the company as a whole, not for each particular line of business. Whether surplus can be allocated to line of business is a disputed issue.[15] Even when an allocation is needed for regulatory purposes or for a financial pricing model, many allocation methods are possible.

This paper does not contend that an allocation of surplus by line of business is theoretically "correct." But an allocation of surplus is needed for the internal rate of return model, and the pricing actuary must posit surplus allocation assumptions. The following pages highlight two aspects of these assumptions:

1. How is surplus allocated? Actuaries generally allocate surplus in proportion to another base, such as premium writings, statutory reserves, or the present value of future loss

[15] In testimony regarding California's Proposition 103, Bass [1990] says: "By its fundamental nature, surplus is not allocatable, whether to line of business, to jurisdiction, or to any other segment of an insurer's operation" (page 231). Similarly, McClenahan [1990] says: "The fact is that the entire surplus of an insurer stands behind each and every risk" (page 152). After reviewing several allocation methods, Kneuer [1987] concludes that none "addresses the philosophical questions that underlie any attempt to allocate surplus" (page 224). See also Roth [1992], who argues against surplus allocation and proposes an alternative measure of return.

These quotations relate primarily to solvency, which differs from pricing. As Peter Murdza has pointed out to me, "allocation for ratemaking purposes only does not mean that surplus is actually allocated for solvency or other purposes." Similarly, Callaghan and Derrig [1982] say: "A company's surplus is not in fact or in law allocated by line and state. A company's entire surplus is available to meet the losses on any line in any state. . . .The fact that surplus is not actually allocated by line and state does not, however, mean that it need not be allocated for purposes of determining an appropriate underwriting profit provision for each line. As noted above . . . Massachusetts law requires the determination of rates by line. Thus it is not only appropriate but required that the ratemaker . . . consider surplus by line, just as other elements of the rate-making methodology must be considered by line. . . .Such consideration requires that surplus be allocated by line and state for purposes of rate-making, even though it is not allocated by line and state by law. Indeed, such allocation is unavoidable. Any profit methodology which purports to determine profit provisions by line assumes an allocation of surplus by line and state." In a paper arguing against surplus allocation for regulatory purposes, Bass and Khury [1992], page 563, note: ". . . nothing we say here should be construed as challenging the idea of allocating surplus for the purposes of deriving estimates of branch office profitability, deriving estimates of of business profitability, etc., for purposes of *internal* management of an insurer's operation."

payments. The *timing* of the surplus allocation is equally important: when is the surplus "committed," and when is it "freed"?

2. Should the surplus allocation depend on the type of policy? For instance, does a retrospectively rated Workers' Compensation policy require less surplus than a guaranteed cost policy? Does a claims-made policy require less surplus than an occurrence policy?

Premiums and Reserves

The ratio of written premium to policyholders' surplus is a common test of surplus adequacy (NAIC [1989]; Best's [1991B]; Ludwig and McAuley [1988]). The test indicates when surplus is so low that the insurer's solidity should be examined. This surplus adequacy test may be extended to the surplus allocation issue, which implies that

- Required surplus varies directly with premium. If one line has twice the premium of a second line, the first line needs twice the surplus commitment.

- Surplus is committed when the premium is written, and it is released when the policy expires.

An alternative base is the reserves held for the block of business, or the anticipated future loss and expense payments. The corresponding implications are

1. Required surplus varies directly with reserves. If one line has twice the reserves of a second line, the first line needs twice the surplus commitment.

2. If the allocation base is loss and expense reserves, then surplus is committed when the losses occur, and it is released when the losses are paid. If the allocation base includes the loss portion of the unearned premium reserves in addition to loss reserves, then surplus is committed when the policy is written, and it is released when the losses are paid.[16]

These assumptions affect the internal rate of return for each line of business. Increasing the surplus allocated to a line moves the internal rate of return closer to the after-tax investment yield. The insurance industry's cost of equity capital, as well as its total return on capital, generally exceeds the after-tax investment yield, so increasing the surplus allocation decreases

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[16] The NCCI uses undiscounted reserves in its IRR model (see Section V). Cummins [1990A] criticizes this: ". . . the present market value, not the book value, of liabilities must be used in the NCCI approach. Thus, the surplus-to-reserves ratio should be based on the estimated market value of liabilities, not the book value." Butsic [1988] also uses the present value of reserves.

Mixtures of the premium and reserves bases are also possible. For instance, Myers and Cohn [1987] use the statutory premium to surplus ratio to allocate surplus by line of business, but they use loss payments to model the timing of surplus flows between the equity providers and the insurer. Daykin, *et al.* [1987] use technical provisions (loss reserves) to determine the required asset margin for existing business and net written premium to determine the required asset margin for future business.

the internal rate of return.[17]

Long- and -Short-Tailed Lines

Slow paying lines with large loss reserves, such as Workers' Compensation and General Liability, receive a greater allocation of surplus if reserves are used as the base rather than premiums. This result may also be viewed as a timing phenomenon: when reserves are the allocation base, the surplus is committed for longer periods in the slow paying lines than in the quick paying lines.

For instance, suppose an insurer writes $100 million of annual business in each of two lines, Homeowners' and Workers' Compensation, and it holds $100 million of surplus. The expected loss ratio is 50% for Homeowners' and 75% for Workers' Compensation. The average duration between occurrence and payment of a claim is half a year for Homeowners' and four years for Workers' Compensation.

If premium is used as the allocation base, the $100 million of surplus is split equally, half to Homeowners' and half to Workers' Compensation. If reserves are used as the allocation base, the split is different. The anticipated Homeowners' losses of $50 million a year, using the expected 50% loss ratio, spend half a year as reserves. In a steady state, there are $25 million of reserves at any point in time. The anticipated Workers' Compensation losses of $75 million a year, using the expected 75% loss ratio, spend four years as reserves. In a steady state, there are $300 million of reserves at any point in time. Twelve times as much surplus would be allocated to Workers' Compensation as to Homeowners'.[18]

Insurance Risks

Surplus protects the insurer against several types of risk. Asset risk is the risk that financial assets will depreciate (e.g., bonds will default or stock prices will drop). Pricing risk is the risk that at policy expiration, incurred losses and expenses will be greater than expected. Reserving risk is the risk that loss reserves will not cover ultimate loss payments. Asset-liability mismatch risk is the risk that changes in interest rates will affect the market value of certain assets, such as bonds, differently than that of liabilities. Catastrophe risk is the risk that unforeseen losses, such as hurricanes or earthquakes, will depress the return realized by the insurer. Reinsurance risk is the risk that reinsurance recoverables will not be collected. Credit risk is the risk that agents will not remit premium balances or that insureds will not

[17] However, if the premium rates are so inadequate that the net present value of the insurance cash flows discounted at the after-tax investment yield on surplus funds is negative, then increasing the surplus commitment increases the internal rate of return. See Section VI for further discussion of this issue.

[18] Using the present value of future loss payments instead of undiscounted statutory reserves reduces the spread between slow paying and quick paying lines of business. An alternative adjustment for the illustration in the text is to substitute a discounted loss ratio for the 75% undiscounted WC loss ratio. The higher expected loss ratio in Workers' Compensation than in Homeowners' reflects the greater investment income in the former line.

remit accrued retrospective premiums.[19]

Pricing and catastrophe risk occur during the policy period; other risks continue until all losses are paid. For instance, an insurer may write a General Liability or Workers' Compensation policy on January 1, but it may have paid only 20% of the incurred losses by December 31. Many financial pricing models therefore allocate surplus based on unpaid losses, in addition to or instead of premium writings.

If reserves are used as the surplus allocation base, what reserves should be used? Loss and loss adjustment expense reserves support claims that have already occurred, whether or not they have been reported to the insurer. Unearned premium reserves, minus the "equity" reflecting acquisition and underwriting expenses, support the claims anticipated over the policy term.

Legally, the insurer is not always "at risk" for the unexpired portion of the policy, since in many jurisdictions it may unilaterally cancel contracts, particularly in the Commercial Lines. [In Personal Lines, several states prohibit mid-term cancellations, or even non-renewals, except for non-payment of premium.] In practice, an insurer rarely cancel contracts in mid-term, so pricing risk continues through the expiration date. Moreover, the pricing risk during the policy term is generally greater than the reserving risk that remains after the policy expires (Butsic [1988]).[20]

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[19] See Hofflander [1969], pages 72-74, Pentikäinen, et al. [1989], Section 3, Kaufman and Liebers [1992], and Feldblum [1992A]. Life actuaries divide insurance risk into three parts: asset risks (C-1), pricing risk (C-2), and the financial effects caused by changes in interest rates (C-3). For the SOA classification system, see Hickman, Cody, Maynard, Trowbridge, and Turner [1979] and CAS [1991]. On asset risks, see Sega [1986], Cody [1988], and Vanderhoof, Albert, Tenenbein, and Verni [1989]; on C-3 risk, see SOA [1987], Geyer [1989], and Mereu [1989]. For a recent treatment of all risks, see Steinig, et al. [1991].

[20] The NCCI's IRR model does not consider pricing risk, explaining that "for workers compensation, surplus exists solely to support the capacity to pay *incurred* loss claims; there is almost no danger that unearned premium reserves will be affected by adverse exposure, as in lines subject to catastrophe" (NCCI [n.d.], page 5); but note that the NCCI illustration in Section V uses unearned premium reserves in addition to loss reserves for the surplus commitment assumption. The NCCI also uses surplus to fund the underwriting loss on the insurance contract, since it uses undiscounted loss reserves. Cummins [1990A] suggests that this is not appropriate: "The NCCI model assumes that the company fully funds the underwriting loss at policy inception. The NCCI maintains that its approach is consistent with the realities of a regulated insurance market where loss liabilities must be fully funded at nominal values. . . . In a regulated industry, there may be a justification for departing from financial theoretic principles provided that the departures realistically reflect the impact of regulation on the firm's market value. . . . The issue is whether or not the requirement that the company set up nominally valued reserves actually affects the market value of the firm; and, if so, whether its effect is captured accurately by the NCCI model. . . . it is not unknown for insurers to be significantly underreserved for sustained periods of time without incurring regulatory intervention. It seems unlikely that the statutory reserve constraint is as stringent as the NCCI model assumes" (page 95); and: "The NCCI approach is correct only if this treatment of surplus is an economic reality, i.e., if writing a block of policies requires the firm to completely forego the use of surplus equal to the underwriting loss early in the policy period rather than funding the loss more gradually out of investment income as losses are paid . . . This depends upon the stringency of regulatory reserving and premiums-to-surplus constraints" (page 101).

Arguments can be made for both the NCCI and Cummins positions, as in the following simulated dialogue. (NCCI:) The NCCI model is used for rate filings, where adherence to statutory accounting principles is required; the fact that

Policy Form

Surplus is committed to guard against the risks to the insurance company. In most insurance transactions, the insured pays a fixed premium and the insurer is liable for random loss occurrences. Surplus protects policyholders from adverse fluctuations in loss payments that might threaten the insurer's solvency.

Occurence contracts subject the insurer to liability for accidents that occur during the policy term. Claims made forms impose this liability only for claims reported to the insurer, or accidents reported to the insured, during the policy term. The elimination of much of the IBNR liability reduces the loss uncertainty and – in theory – the surplus needed to support the risk.

Service contracts, where the insurer handles claims but does not incur loss liabilities, involve no insurance risk (though the risk remains that expense charges will not cover actual expenses incurred). In fact, since the insurer receives expense fees but no premium, no surplus is required for statutory financial statements. Capital is needed to hire the requisite personnel, but no surplus is needed for insurance "risk."

Retrospective rating is a cross between an insurance policy and a service contract. There is little insurance risk in the primary layer, where losses are fully reimbursed by the policyholder, though there is the "credit risk" of the insured's failure to pay the premiums (Livingston [1982]; Greene [1988]; Brown [1992]). There is substantial risk where the loss limit or the maximum premium curtails the reimbursement. Overall, the retrospectively rated policy is more risky than a service contract but less risky than a prospectively rated insurance policy.

Although risk varies by policy type, there is no simple procedure to account for this. Some applications of the IRR pricing model make no distinction among retrospectively rated policies, excess coverage, large deductible policies, and first dollar contracts. Other applications count only premium and reserves for which true insurance protection is provided. For retrospectively rated policies, this would be the insurance charge and the reserves for claims in excess of the loss limit, or claim payments that would not be reimbursed by the policyholder because the ultimate premium exceeds the maximum premium (see Simon [1965]; Snader [1990]).

Equal treatment of all policy types overstates the risk on retrospectively rated policies and understates the risk on excess coverage. Considering only premium and reserves which cover true insurance risk understates the risk for retrospectively rated polices, excess coverage, and large deductible policies, since loss fluctuations are greater on higher layers of coverage. The ideal procedure lies somewhere in the middle, though the continual changes in policy forms and insurance services make this terrain too slippery for fixed techniques.

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many insurers are underreserved in practice is not necessarily germane. (Cummins:) If so, one should use reserves to suplus ratios derived from grossed up reserves and restated surplus. This would raise the ratios and lower the underwriting profit provision needed to obtain the desired internal rate of return. (NCCI:) The NAIC requires the low leverage ratios, in addition to full value loss reserves; the IRR model must comport with statutory requirements.

Section IV: The Cost of Equity Capital

The magnitude and timing of the insurance cash flows (premiums, losses, expenses, investment income, taxes) and the surplus commitment assumptions allow us to determine the internal rate of return on investor supplied equity. The insurer has a financial incentive to write the policy if the internal rate of return exceeds the opportunity cost of this equity capital. But what return do investors demand for the use of their money?

If the capital market is efficient, then the returns achieved by investors is an indicator of the returns needed to elicit equity capital, assuming that all other factors remain steady. If the returns that investors achieved were less than what they demanded, they would withdraw some capital (Bailey [1967]; but cf. Matison [1987]). With lower industry capacity, premium rates would rise, underwriting standards would tighten, and overall returns would increase. Conversely, if the returns that investors achieved were greater than what they demanded, they would commit additional capital. The combination of greater industry capacity, more firms, and increased competition in the insurance market would cause premium rates to fall, underwriting standards to loosen, and overall returns to decline.

If other factors are changing, the returns demanded by investors may not be equal to the returns achieved in the past. For instance, if inflation accelerates, investors demand higher returns to compensate them for the reduced value of money. Similarly, an increase in the perceived risk of holding insurance stocks, due perhaps to more stringent rate regulation, lower profits, and an increased number of insolvencies, may cause investors to demand higher returns.

This section presents two models for estimating the return on stockholder supplied equity, or the opportunity cost of equity capital. It begins with the Dividend Growth Model (DGM), which directly estimates the cost of equity capital but relies on uncertain projections about future dividend payments. It proceeds to the Capital Asset Pricing Model (CAPM), which relies on observed historical data, but whose theoretical foundations are often questioned.[21]

The Dividend Growth Model

What determines the prices of stocks? The stock certificate is a piece of paper, with no intrinsic worth. In a free market, of course, its value is determined by the forces of supply and demand: what others are willing to pay for it. But this only begs the question: What determines how much others are willing to pay for the stock certificate?

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21 Returns on either statutory surplus or GAAP equity may be derived from financial statements. The historical returns on surplus derived from Annual Statement figures are often used by regulators for estimating "equitable" returns. Growth rates, payment lags, and accounting conventions cause the financial and accounting estimates of return on equity (or surplus) to diverge (Anderson [1972]; Feldblum [1992B]; Butsic [1990]; Bingham [1992]; see also Beaver, Kettler, and Scholes [1970]). In general, financial models are better indicators of the returns demanded by investors. On the cost of equity capital for insurers, see Haugen and Kronke [1971], Quirin and Waters [1975], Lee and Forbes [1980], and Cummins [1992]. On the application of financial methods to insurance, see D'Arcy [1989]).

A stock certificate is a financial asset, like a bond. The worth of a bond is determined by the cash payments to the owner: semiannual coupons and the par value at maturity. At any time, the worth of a bond is the present value of these future cash payments.

A stock has three differences from a bond.

- First, the stock never matures: there are periodic dividends, but no "repayment of principal at maturity."
- Second, the dividend payments are less certain. If the firm faces financial difficulties, it may eliminate or reduce a dividend payment. If it earns unusually large profits one year, it may provide a larger dividend.
- Third, bond coupons have fixed amounts. Stock dividends are not fixed in nominal terms, but generally grow with monetary inflation and with the earnings of the firm.

If we knew the amounts of all future dividend payments, we could estimate the price of the stock as the present value of the future cash flows. The actual future dividends are uncertain, but we can use historical experience to forecast them. To determine present values, we must know the appropriate discount rate, which is the opportunity cost of equity capital. So if we know the current price, and we forecast future dividends, we can solve for the discount rate.[22]

Forecasting future dividends is a difficult task. To simplify, assume that the firm's earnings, assets, dividends, and stock price are all increasing at a constant rate. This growth rate, in combination with the dividend to price ratio, determines the cost of equity capital.

For example, suppose a firm is growing 10% per annum, its stock price increases at the same rate, and it pays an annual dividend at the end of each year equal to 5% of its stock price. What is the return to the equity holders in this firm?

Imagine an investor who buys a share of common stock for $100 on January 1, receives the dividend on December 31, and then sells the stock. (The $100 price is arbitrary; any price gives the same result.) On December 31, the stock price is $110 (10% per annum capital appreciation), and the dividend is $5.50. The annual return to the investor, or the cost of equity capital, is ($10 + $5.50) / $100, or 15.5% (Butters, et al. [1981], page 140).

Derivation of the DGM

In mathematical terms, let

K be the cost of equity capital,
D be the stockholder dividend at the end of the previous year,
P be the stock price at the beginning of the year, and
G be the anticipated (uniform) growth rate of stockholder dividends.

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22 On the Dividend Growth Model, see Gordon and Shapiro [1956], Sharpe and Alexander [1990], chapter 16, Weston and Copeland [1986].

We assumed above that all financial characteristics of the firm, such as earnings, assets, stock price, and dividends, are growing at the same rate. This is the common situation, since dividends can not grow indefinitely if earnings do not keep pace. The mathematical derivation, though, only needs the growth rate of dividends (hence the name Dividend Growth Model).

On January 1, the investor pays P for the stock. If the firm grows 100G% per annum, he can sell the stock on December 31 for (P)(1 + G). In addition, he receives the stockholder dividend on December 31. The dividend the previous year was D, so this year it will be (D)(1 + G). The return to the investor, or the cost of equity capital, is

$$\{ (P)(1 + G) + (D)(1 + G) - P \} / P, \text{ or}$$

$$K = (D/P)\ (1 + G)\ +\ G.$$

A more rigorous derivation examines only future cash flows, the stockholder dividends. The price of the stock equals the present value of future returns. If dividends are growing at 100G% per annum, the future returns are D(1+G) in one year's time, $D(1+G)^2$ another year later, and so forth. Discounting these at the cost of equity capital ("K"), we obtain

$$P = D(1+G)/(1+K) + D(1+G)^2/(1+K)^2 + D(1+G)^3/(1+K)^3 + \ldots$$

Now $(x + x^2 + x^3 + \ldots) = x/(1-x)$ for positive $x < 1$. If dividends are positive, $K > G$, so

$$P = D\ \{ (1+G)/(1+K) \} / \{ 1 - [(1+G)/(1+K)] \}.$$

Simplifying this expression gives

$$P = D\ (1 + G) / (K - G), \text{ or}$$
$$K = (D/P)\ (1 + G) + G.$$

Both parameters of the dividend growth model, the ratio of stockholder dividend to stock price (or "dividend yield") and the anticipated dividend growth rate, are calculated or projected by investment firms for the major publicly traded stock companies. The dividend yield is generally stable from year to year, and averaged between 4% and 4.5% for Property/Liability insurers in 1989.

Changes in Dividend Growth Rates

The anticipated dividend growth rate is a subjective estimate, for which investment firms provide differing forecasts. Moreover, the growth rate of the firm is often inversely related to the dividend yield: "growth stocks" pay low dividends, whereas "income stocks" pay higher dividends but grow more slowly. These phenomena complicate the Dividend Growth Model.

Suppose an aggressive stock company perceives an opportunity for rapid and profitable growth. To finance its expansion, it retains most of its earnings, keeping its ratio of dividends to price at

2%. During its expansion, its stock price and its dividends both grow at 20% per annum.[23]

During this period, the return on equity to stockholders is 22.4% [= 20% + (2%)(1.2)]. An investment analyst estimating the future return on equity may reason that the insurer can not continue growing rapidly. He may reduce the growth rate to 10% per annum, and calculate the anticipated return on equity as 12.24% [= 10% + (2%)(1.12)].

This estimate is not correct, since a sustained reduction in growth may lead to an increased dividend yield. Since it no longer need finance a rapid expansion, the insurer can retain less of its earnings and pay greater stockholder dividends. Investment analysts who foresee a reduction in the growth rate often anticipate an increase in the dividend yield.

Stock versus Mutual Insurers

Cost of capital estimates derived from stock prices and dividends can be done only for publicly traded companies. Mutual companies and privately owned companies can not be included in a dividend growth model analysis. Moreover, if insurance risk differs by industry segment (life, health, Property/Casualty) and line of business, then the opportunity cost of capital may differ as well. Some insurers sell predominantly Property/Liability coverages; others have substantial life or health business as well.

For its cost of equity capital analyses, the National Council on Compensation Insurance examines 14 publicly traded stock insurers specializing in Property/Liability coverages, as well as 7 insurers who write both life/health and Property/Casualty coverages. In 1989, the NCCI

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[23] The inverse relationship of business expansion and free surplus is particularly strong for insurance companies, for several reasons:

(a) Statutory accounting disallows the capitalization of acquisition expenses, and prohibits the recognition of the "equity" in the unearned premium reserve. A growing insurer has an increasing "equity." Since all acquisition costs are expensed when they are incurred and are also held as unearned premium liabilities (pro-rata over the term of the policy), the increase in the equity is double counted as charges to the statutory income statement (Morgan [1988]). See NAIC [1984], page 131, table 8-5, and Feldblum [1992B] for the magnitude of this effect.

(b) In both statutory and GAAP financial statements, loss and loss expense reserves are shown at nominal values, not discounted values. New business in a long-tailed line generally shows an underwriting loss during the policy year and investment income gains in subsequent years (Lowe [1989]; Woll [1987]). For an insurer that is not growing, the investment income gains from previous blocks of policies often exceed the underwriting loss from the new business (if the coverage has been reasonably priced). For a rapidly growing insurer, the new business underwriting loss may outweigh the investment income gains from previous business, resulting in an apparent accounting loss.

(c) New business shows worse loss ratios and higher average loss costs than renewal business does. Reunderwriting of the existing book, the "transient" nature of many poor risks, and the attraction of "marginal" risks by rapidly growing insurers contribute to this phenomenon (Feldblum [1990B; 1993]; D'Arcy and Doherty [1989; 1990]; Conning and Co. [1988]). Rapidly growing insurers show poor accounting results but build up a potentially profitable renewal book of business.

Because of these phenomena, an insurer that successfully implements a profitable growth strategy will show lower gains or greater losses in its financial statements during its expansion as compared with "true economic" results. It will be forced to reduce its dividend yield even more than growth firms in other industries do.

estimated a dividend yield of 4.5% and a growth rate of 11%, for a cost of equity capital of 16%.

The IRR pricing model seeks the cost of *surplus* funds, which are needed by all insurers. The cost of equity capital is needed for the financial management of publicly traded stock companies, and it is a proxy for the cost of surplus funds of other insurers. The IRR model assumes that the "cost of surplus funds" for mutual and privately owned insurers is similar to the cost of equity capital of stock insurers (Launie [1971], page 265; Cummins [1992]; but see Roth [1992] who disputes this).[24]

Capital Asset Pricing Model

The Dividend Growth Model works best in an unchanging environment: inflation remains level, the firm grows steadily, and the economy expands slowly. If inflation accelerates suddenly, the economy enters a recession, or the firm's book of business changes rapidly, the Dividend Growth Model may not provide reasonable forecasts.

Consider the effects of inflation. If inflation accelerates, and investors seek the same return in inflation-adjusted dollars, then the *nominal* cost of equity capital will rise. But so will the nominal costs of other financial instruments, such as the coupon rate on bonds, or the mortgage rate on home loans.

Few pricing actuaries try forecast future inflation or economic conditions. Instead, they seek a relationship between the cost of equity capital and some steady and accessible index. The Capital Asset Pricing Model (CAPM) provides such a relationship.

Price Fluctuation

The Capital Asset Pricing Model presumes that there are two influences on common stock price fluctuations. Some price changes are peculiar to the specific firm. For instance, the stock price for an oil company may increase if the company discovers an untapped oil source. Similarly, the stock price of an auto manufacturer may drop if its employees declare a strike.

A second influence on the prices of individual stocks is the movement in the stock market as a whole. During a "bull market," the prices of most stocks increase. The prices of some stocks are highly responsive to market movements: if the market as a whole goes up 12%, the prices of these stocks may increase 15%. The prices of other stocks are less responsive, and may increase only 10% during this period.

Price fluctuations that are peculiar to individual firms are referred to as *firm-specific*, *unsystematic*, or *diversifiable* risk. Price movements that reflect overall market returns are

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[24] Modigliani and Miller [1958] argue that the cost of capital depends on the riskiness of the firm, not on its capital structure (Proposition I); see also Fama [1978]. Thus, the appropriate discount rate should not necessarily be higher for stock companies. In fact, since mutual and privately owned insurers have grown more rapidly than publicly traded stock companies during the second half of the twentieth century, the cost of capital estimates derived here may underestimate the true cost of surplus funds. The Dividend Growth Model will show low annual growth rates, providing a low estimate of the cost of capital. Rapidly expanding insurers have a higher cost of capital, but they are under-represented among publicly traded stock companies.

termed *systematic* or *undiversifiable* risk. The Capital Asset Pricing Model hypothesizes that

- The *expected* return from a common stock is related only to the stock's systematic risk;
- The difference between the *expected* return from a common stock and return on a risk-free security is proportional to the firm's systematic risk; and
- The systematic risk and the factor of proportionality are relatively constant over time.[25]

Formally, the Capital Asset Pricing Model posits the following relationship:

$$R = R_f + \beta\ (R_m - R_f),$$

where R is the expected return on a given stock,
R_f is the risk free rate, such as the rate on Treasury bills,
R_m is the overall market return, and
β quantifies the undiversifiable or systematic risk associated with this stock.

The "market risk spread," or $(R_m - R_f)$, has averaged about 8.6 percentage points over the past 60 years, if R_f is the return on short term Treasury bill.[26] The β parameters, which reflect systematic risk, are estimated from historical data, and have averaged about unity for most Property/Liability insurers.

In sum, the Capital Assets Pricing Model estimates that the cost of equity capital for Property/Liability insurers is about 8.6 percentage points higher than the return on Treasury bills. The Treasury bill returns are readily available, and they closely track monetary inflation, economic prosperity, and other external conditions that affect the cost of capital.

In 1989, the return on Treasury bills was between 7.5 and 8%, implying a cost of equity capital between 16 and 16.5%. (The returns on Treasury bills dropped sharply in 1990 and 1991; see Section V.) This is consistent with the Dividend Growth Model estimate derived above. The similarity is not unexpected: although the formulas in these two model are different, the financial data used and many of the underlying assumptions are the same. As noted above, other methods of estimating the opportunity cost of capital have been proposed, and the range of results is wider than this section implies.

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[25] See Sharpe [1970] and Lintner [1965]. Good introductions to the CAPM are Weston and Copeland [1986], chapters 16 and 17, Brealey and Myers [1988], chapter 9, or Cohen, Zinberg, and Zeikel [1982], pp. 143-241. For application of these concepts to insurance returns, see Williams [1983] and Cooper [1974]. CAPM estimates of the cost of capital have been used in public utility regulation; see the testimony of Stewart C. Myers in the 1971 AT&T rate case (Butters, et al. [1981], page 131, note 22).

[26] This figure uses the arithmetic average of the difference between stock returns and the return on Treasury bills. The averages from 1926 to 1986 are 12.12% for stock returns and 3.51% for T-Bills, for a difference of 8.61% (Sharpe and Alexander [1990], pages 5-6). Other analysts, such as Cox and Griepengrog [1988] and Quirin and Waters [1975], use geometric averages, not arithmetic averages. The geometric averages are 9.98% for stock returns and 3.45% for T-Bills, for a difference of 6.53%. See Ibbotson and Sinquefield [1982], pages 57-61, for further discussion of when to use each type of average.

Section V: A Rate Filing Illustration

The illustrations in previous sections were heuristic, using simplified cash flows to emphasize concepts, not details. This section provides selected exhibits from a 1991 NCCI Workers' Compensation rate filing, to show an actual application of the IRR model. Note several characteristics of this illustration:

- Policy year cash flows are used, since the filing sets rates for all policies issued during a future policy year. The previous sections used accident year cash flows, to show the expected internal rate of return on an individual policy.

- Quarterly cash flows are used for the first 6 years, beginning one year prior to policy inception, followed by annual cash flows for the next 19 years.

- The IRR model is used to justify the underwriting profit provision specified in the rate filing, not to independently determine the proper profit provision.

- The NCCI model incorporates some elements, such as tax credits, not discussed previously.

These are differences of technique and format. The same principles underlie the model in this illustration as those discussed throughout the paper.

Assumptions – Table I

The tables in this section are from a 1991 NCCI Workers' Compensation rate filing that uses a 0% underwriting profit and contingencies provision (Table I, row 5). To justify this factor, the NCCI shows that the implied equity cash flows generate a 10.42% internal rate of return. This rate of return must be compared with the cost of equity capital: If investors demand a return greater than 10.42%, a positive underwriting profit provision should be used; if investors would be satisfied with a lower return, a negative provision may be used.

Workers' Compensation rates may be stated on three bases. *Manual* premiums are the prices determined from bureau or company rate manuals. *Standard* premiums are the manual premium after adjustment for experience rating plan modifications. *Net* premiums are the standard premiums after adjustment for premium discounts and retrospective rating (NCCI [1984]).

Table I shows loss and expense provisions relative to *NCCI net* premiums. The experience rating plan modifications and premium discounts are incorporated in net premiums, so no adjustment for these items is shown. Member company rate deviations and schedule rating adjustments, which are not incorporated in the NCCI net premiums, average 3.2% of net premium in the state under review (row 6). Thus, if net written premium at bureau rates is $1 million, the collected premium is $968,000 (rows 10 and 11).

The differences between ratios to standard vs. net premium may be seen from rows 2 and 3. Commissions are 15% of the first $1,000 of premium, grading down at higher layers, for an average of 6.61% to *net* premium (row 2). Similarly, the average ratio of other expenses to net premiums is 9.99% (row 3).

Premium taxes are state specific. The tax and assessment provisions shown on rows 4A, 4B, and 4C are (i) state premium tax (0.63%), (ii) Uninsured Employers' Fund (0.25%), and (iii) The Workers' Compensation Account of the Insurance Guaranty Association (0.17%). These provisions have different payment patterns, so they are modeled separately.

The expense provisions amount to 17.65% (= 6.61% for commissions + 9.99% for other expenses + 1.05% for state taxes and assessments + 0% for profit and contingencies). The loss ratio of 82.35% (row 1) is the complement of the expense ratio.

Row 8B shows the after-tax yield on investments, which equals the pre-tax yield (row 8A) times the complement of the tax rate (row 8C). The pre-tax yield is determined from the distribution of assets held by Commercial Lines insurers and the current yields by asset class (see the discussion below). For Treasury securities and bonds, the NCCI used average yields in July 1991, when interest rates were low, so the yields are below those shown in Section II.D.

The reserves to surplus ratio (row 9) is the ratio of loss, loss adjustment expense, and unearned premium reserves to policyholders' surplus for Commercial Casualty predominating companies for 1985 through 1989 (see Best's [1991A]). The statewide average policyholder dividend ratio (row 7) is taken from industry Page 14 experience.

The caption at the bottom of Table 1 notes that the implied internal rate of return is 10.42%. The following exhibits show the derivation of this rate of return.

Cash Flows – Table II

Table II shows premium, loss, and expense cash flow patterns. The premium cash flow pattern (column 1) is derived from the Massachusetts study discussed in Section II.A, converted to a policy year pattern. [The Massachusetts pattern assumes one policy written at time "zero"; the NCCI pattern assumes that policies are written evenly through the year.]

The first nine years of the loss payout pattern (column 2) are derived from a policy year call for experience by state. Subsequent years are modeled in a manner consistent with the payout pattern in the sixth through ninth years (cf. McClenahan [1975] and Woll [1987]).

The commission cash flow pattern is the same as the premium collection pattern, so no separate column is needed. The cash flow pattern for other expenses (column 3) is derived from the 1977 Massachusetts study converted to a policy year basis (see Section II.C above).

Tax and assessment cash flows (columns 4, 5, 6) are determined from state regulations. The premium tax and the Uninsured Employers Fund follow the premium collection pattern, but they are paid annually one quarter subsequent to the year of premium collection. Thus, the 0.19% in row 5 of columns 4 and 5 equals the sum of rows 1 through 4 in column 1, and the 50.83% in row 9 of columns 4 and 5 equals the sum of rows 5 through 8 in column 1. The guaranty fund assessment (column 6) is paid in the first quarter subsequent to the policy year.

Policyholder dividends (column 7) are paid in the second quarter after expiration of the policy. Since policies are issued evenly over the first four quarters, dividends are paid evenly in the sixth through ninth quarters.

Investment Yield – Tables III-A and III-B

Financial pricing models incorporate investment income by projecting future yields on invested assets. Some models use risk free investments and marginal tax rates, arguing that the gains or losses from more aggressive investment strategies should be allocated to stockholders, not policyholders. The IRR model illustrated here compares the internal rate of return with the cost of equity capital. Since the cost of capital reflects the investment strategy, the internal rate of return should do so as well. Tables III-A and III-B therefore show actual investment yields and the associated federal income taxes on insurers' financial portfolios.

The top half of Table III-A separates realized capital gains and losses (row 1) from net investment income earned (row 2), since different tax rates are applied to each.[27] Rows 4 and 5 show the proportion of total (statutory) investment return derived from each part. Rows 6 through 8 determine investment expenses as a percent of total assets.

Realized capital gains are taxed at 34%, and investment expenses provide a tax credit of 34%. Other investment income, such as yields on tax exempt bonds and dividends from common and preferred stocks, incur lower tax rates (see Section II.D above). The appropriate post-tax yield on these investments is shown in Table III-B.

Rows 9 and 10 shows the pre-tax and post-tax yields derived in Table III-B. Row 11 shows the pre-tax yield with an offset for post-tax investment expenses. The pre-tax yield is 7.731%, investment expenses are 0.466%, post-tax investment expenses are (1 – 34%)(0.466%) = 0.308%, so the weighted pre-tax yield is 7.424%. Row 13 shows the post-tax yield:

- *Realized capital gains and losses:* Composite pre-tax yield (7.731%) times the percentage derived from realized capital gains and losses (15.6%) times the complement of the marginal tax rate (1 – 34%) = 0.796%.
- *Net investment income earned:* Composite post-tax yield (6.160%) times the percentage derived from investment income (84.4%) = 5.199%.
- *Investment expense credit:* Investment expense ratio (0.466%) times the complement of the marginal tax rate (1 – 34%) = 0.308%.

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[27] The statutory income statement shows net investment income earned and realized capital gains and losses; unrealized capital gains and losses are a direct credit or charge to surplus. Table III-B, which shows a pre-tax yield on common stock 730 basis points above that on Treasury securities, implicitly includes unrealized capital gains and losses. Table III-B calculates the expected pre-tax and pre-expense investment yield, which is a total return; Table III-A applies the appropriate tax rates to each part of the yield. Statutory accounting ignores taxes on unrealized capital gains and losses; GAAP sets up a deferred tax liability (AICPA [1990]). This IRR model supports a state rate filing, so it follows the statutory treatment.

The estimated post-tax yield is therefore 0.796% + 5.199% − 0.308% = 5.687% (row 13). Row 12 shows the implied investment income tax rate, or the complement of the ratio of the post-tax yield to the pre-tax yield: (1 − 5.687/7.424) = 23.39%.

Table III-B shows pre- and post-tax yields on various investments. The calculations are similar to those in Section II.D; see particularly the discussion there on investment tax rates. This illustration uses July 1991 yields on fixed income investments (see the footnotes to column 2 in Table III-B), which are lower than the 1989 yields.

Cash Flows Supporting Reserves – Table IV

The illustration assumes that the carrier holds full value statutory reserves and has a 3.5 to 1 reserves to surplus ratio. Statutory reserves are greater than economically adequate reserves, since (i) the loss reserves are undiscounted, (ii) the unearned premium reserves have no offset for deferred acquisition costs, and (iii) some assets, such as overdue agents' balances, are not admitted. Statutory reserves at early durations may exceed the premium, net of expenses paid, losses paid, and non-admitted assets, received by the insurer.

Column 1 of Table IV shows the cumulative premium *collected*, derived from column 1 of Table II. For example, Table IV, column 1, row 5, shows $45,399.20 collected by the end of the first quarter. This equals the sum of rows 4 and 5 in column 1 of Table II, or 4.69%, times the total premium collected, or $968,000.

The illustration assumes that agents' balances are not overdue on individual policies unless the aggregate balances are overdue. Agents' balances are the difference between written and collected premium. For example, since policies are issued evenly through the year, first quarter writings are $968,000/4, or $242,000. Collected premium is $45,399.20 (column 1), so the difference, or $196,600.80, is being held by agents (column 2).

No agents' balances are overdue during the first two years, since much of the premium is "deferred and not yet due." [One may think of the "agents' balances" column as either funds not yet remitted by the agent to the insurer, or funds not yet remitted by the insured to the agent.] All premium is due by the end of the second year, so the difference between written and collected premium at time 2.25, or $968,000.00 − $958,900.80 = $9,099.20, is overdue (column 3). The "admitted agents' balances" (column 4) is the difference between columns 2 and 3.[28]

Losses incurred (column 5) presume that policies are written evenly through the year and losses are incurred evenly through the policy term. The full policy year incurred losses are $823,500, or 82.35% *x* $1 million. During the first quarter, 25% of the policies are written. A policy written on January 1 has been in effect for three months by the end of the quarter, so 25% of its losses are incurred by March 31. A policy written on March 31 has had no exposure during the first quarter. The average policy written during the first quarter has

[28] In practice, audit and retrospective premium are not booked until after the policy expires, so the $9,099.20 "overdue agents' balances" in column 3 may be overstated. Conversely, the zeros in the preceding four rows may be understated, since premium on some individual policies may be overdue. [This is the implication of positive figures in column for the third through fifth years, or time 2.25 through 5.00.] These adjustments are minor.

been in effect for 6 weeks by March 31, or 12.5% of the year. Combining, we have

$$25\% \times 12.5\% \times \$823{,}500 = \$25{,}734.38 \text{ (column 5).}$$

Unearned premium (column 6) is the difference between written and earned premium. For example, $242,000 (= $968,000/4) is written during the first quarter. On average, 12.5% of the policy is earned by March 31, or 87.5% is unearned. Combining, we have

$$87.5\% \times \$242{,}000 = \$211{,}750.00 \text{ (column 6).}$$

"Total premium net of reserves" is admitted premium written (columns 1 plus 4) minus premium reserves and incurred losses (column 5 plus 6). Continuing with row 5, we have

$$\$45{,}399.20 + \$196{,}600.80 - \$25{,}734.38 - \$211{,}750.00 = \$4{,}515.63 \text{ (column 7)}$$

Column 8 shows the incremental premium net of reserves, or the first differences of column 7. For example, the $13,546.88 in row 6 of column 8 equals $18,062.50 – $4,515.63 from column 7. The column 8 figures are the excess of (admitted) booked premium over statutory reserves. The figures are positive for the eight quarters during which premiums are in force, since the "zero" underwriting profit provision and the positive return on the policy implies that the premium suffices to cover the losses. During the quarter following the last policy expiration ("2.00 to 2.25"), there is no new written premium or incurred losses, but $9,099.20 of agents' balances become non-admitted, so this amount appears in column 8. Similarly, column 8 in all subsequent quarters is the change in column 3 (overdue agents' balances).[29]

Tax Credits – Table V-A

Expositions of concepts may ignore federal income taxes; practical exhibits must include them. Income tax details obscure the principles of the pricing model, but tax flows may differentiate profitable from unprofitable contracts.

Several characteristics of tax liabilities complicate the treatment:

- The statutory loss reserves in the NCCI's IRR model are undiscounted, but IRS taxable income uses discounted reserves. [In the illustration, federal income taxes exceed the insurer's statutory income during the first year, which show negative cash flows from underwriting in column 5 of Table V-B; see below for further discussion.]

- The discount rate and loss payout pattern are prescribed by the IRS. The discount rate does not necessarily equal the internal rate of return or the current investment yield in the

[29] Since statutory accounting does not recognize the equity in the unearned premium reserves (that is, no acquisition costs may be deferred and amortized over the policy term), one might expect negative entries in column 8 for the four quarters in which policies are issued and large positive amounts in the following four quarters, instead of the mirror images shown in Table IV. This does not occur because expense charges are not included until Table V-A, column 3 and Table V-B, column 3. Column 5 of Table V-B, "net cash flow from underwriting," is indeed negative during each quarter of policy issuance.

model, and the payout pattern does not necessarily equal the loss payment cash flow in column 2 of Table II.

- The NCCI model uses policy year cash flow patterns; IRS discounting procedures prescribe accident year cash flow patterns. Table V-A therefore separates the policy year's paid losses and loss reserves into "accident year 1" and "accident year 2" components.

Because federal income taxes are based on discounted reserves but statutory income is based on undiscounted reserves, taxes are paid before the net income is booked. Consider a policy issued on January 1, with income taxes determined from discounted reserves as of December 31. If there were no investment income received in subsequent years, the insurer would receive a tax refund as discounted reserves are converted into undiscounted paid losses. Since this IRR model uses an *after-tax* investment yield and statutory financial statements, part of the taxes paid during the first two years form "tax credits."[30]

Table V-A calculates the tax credit. Column 1 shows premium written, or $968,000 during the policy year. Since policies are written evenly throughout the year, half of this premium is unearned, so $484,000 appears in column 2. The "revenue offset" provision of the 1986 Federal Income Tax Amendments allows only 80% of the unearned premium reserve as a reduction of taxable income, or $387,200 in this case. [The remaining 20% represent prepaid expenses, which are included in column 3.]

Total expenses paid during the policy year are $95,622.71 (column 3). Policyholder dividends are not paid until three months after policy expiration, so column 4 contains a zero in row 2.

Since the second accident year of this policy year has not yet commenced by December 31, all losses paid and loss reserves are related to "accident year 1" (columns 5 and 7). In subsequent years, both losses paid and loss reserves must be allocated to accident year to compute the federal income tax liability. Since policies are written evenly throughout the year, the "accident year 2" figures in column 6 and 8 equal the "accident year 1" figures in column 5 and 7 in the previous row.[31]

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[30] If the financial statements used discounted reserves, or "present value accounting," or if before-tax investment yields were used, no tax credits would be available; see FASB [1990] and Woll [1987].

[31] The actual lag is two thirds of a year: The average accident date of claims occurring in the first accident year of a given policy year is September 1, or two thirds of the way through the year. The average accident date of claims occurring in the second accident year is May 1, or one third of the way through the year. The time span between these two average accident dates is two thirds of a year.

The NCCI uses an "accident year loss payout pattern" to determine the paid losses in columns 5 and 6. The computations are unclear to me. The total losses paid in the second row of columns 5 and 6 of Table V-A (the two accident year pieces of the first calendar year) should equal the total losses paid determined from the loss payout pattern in Table II, column 2, for the four quarters of the first calendar year (time 0 through time 1). Column 5 of Table V-A shows $100,611.11 for year 0. The latter calculation gives

$$(0.675\% + 2.025\% + 3.375\% + 4.725\%) \times 82.35\% \text{ (loss ratio)} \times \$1 \text{ million (premium)} = \$88{,}938.$$

Columns 7 and 8 show the change in discounted loss reserves, where the discount is computed using IRS discount factors and payout patterns (not shown in the tables). Assuming a tax rate of 34% on underwriting income, income taxes are

$968,000 – (80%)($484,000) – $95,622.71 – $100,611.11 – $245,099.97 = $139,466.21

and (34%)($139,466.21) = $47,418.51

The inverse of the tax liability is the "tax credit" shown in column 9.

Cash Flow from Underwriting – Table V-B

Table V-B shows the net cash flow from underwriting. The premium flow net of reserves (column 1), taken from Table IV, column 8, equals collected premium + admitted agents' balances – losses incurred – unearned premium. The tax credits (column 2) are determined on an annual basis in Table V-A, column 9; they are spread evenly to quarter for Table V-B.

Expenses (column 3) are the sum of commission, other expenses, and premium taxes. Cash flow patterns are shown in Table II, column 1, 3, 4, 5, and 6. [The cash flow pattern for commissions is the same as the cash flow pattern for premiums (Table II, column 1).] Expense levels are shown in Table I. The expense amounts in Table V-B, column 3, are derived from the net premium, the expense levels, and the expense cash flow patterns.

For instance, the fifth row of column 3 ($17,346.94) is derived as follows:

- Commissions: Net premium of $968,000 (Table I, row 11) *x* commission rate of 6.61% (Table I, row 2) *x* the entry from premium collection pattern of 4.5% (Table II, column 1, row 5) = $2,879.

- Other expenses: Net premium of $968,000 (Table I, row 11) *x* other expense rate of 9.99% (Table I, row 3) *x* the entry from other expense payment pattern of 14.9579% (Table II, column 3, row 5) = $14,464.77.

- Tax & assess: [Note that the premium tax and the Uninsured Employer's Fund have the same payment pattern, and there is no Insurance Guaranty Fund assessment in this quarter.] Net premium of $968,000 (Table I, row 11) *x* combined premium tax and Uninsured Employer's Fund rate of 0.88% (Table I, rows 4A + 4B) *x* the entry from the premium tax payment pattern of 0.19 (Table II, column 4 , row 5) = $16.18.

Dividends (column 4) are derived from the average dividend level in Table I, row 7 (4.9%) and the dividend cash flow pattern in Table II, column 7. For example, the $11,858 is column 5, rows 10 through 13, equals

$968,000 (net premium) x 4.9% (dividend level) x 25% (cash flow pattern) = $11,858

The net cash flow from underwriting (column 5) equals the premium flow net of reserves (column 1) + the tax credit (column 2) – expenses (column 3) – dividends (column 4).

Surplus Funds and Invested Assets – Table VI

Assets support two items on the liability side of the balance sheet: (i) premium and loss reserves and (ii) policyholders' surplus. Assets may be either invested assets or admitted but not invested assets, such as agents' balances. Table VI shows how much invested assets is needed.

Loss and premium reserves are shown in columns 1 and 2, and admitted agents' balances are shown in column 3. Premium reserves are taken from Table IV, column 6. Loss reserves are incurred losses (Table IV, column 5) minus paid losses (derived from the loss payout pattern in Table II, column 2, and the expected loss ratio in Table I, row 1). Admitted agents' balances are taken from Table IV, column 4.

The "cash level" in Table VI, column 4, is the amount of invested assets needed to support the reserves, or loss and premium reserves minus admitted agents' balances. For example, row 5 gives

$20,175.75 + $211,750.00 – $196,600.80 = $35,324.95

Required policyholders' surplus equals the premium and loss reserves divided by 3.5 (Table I, row 9). For Table VI, column 5, row 5, we have

($20,175.75 + $211,750.00) / 3.5 = $66,264.50

Since admitted agents' balances have all be used to support the reserves, invested assets must support policyholders' surplus.

Equity Flows – Table VII

Table VII shows the net cash flows to and from equity holders, derived as the after tax cash flows from underwriting and investments and the required contributions or reductions to surplus.

The net cash flow from underwriting (column 1), taken from Table V-B, column 5, is already an after-tax figure.

The pre-tax income from invested cash (column 2) is derived from the cash level (Table VI, column 4) and the pre-tax investment yield (Table I, row 8A). The cash level of Table VI, column 4, is the end-of-quarter cash level. We need the average cash level during the quarter to determine investment income, so we use the mean cash levels of this and the preceding quarter. The pre-tax investment yield of 7.424% in Table III, row 11, is the annual yield; the fourth root of this figure is the quarterly yield. Thus, the $335.68 in Table VII, column 2, row 5, is derived as follows:

- Average cash level during quarter = ($1,839.20 + $35,324.95) / 2 = $18,582.08, using end-of-quarter cash levels from Table VI, column 4, rows 4 and 5.
- Quarterly pre-tax investment yield = $1.074240^{0.25} - 1 = 0.0180646$.
- $18.582.08 *x* 0.018055 = $335.68.

The tax rate on investment income is 23.391% (Table III-A, row 12). $335.68 *x* 23.391% = $78.52 (Table VII, column 3; the 2 cent discrepancy is a rounding error).

The net cash flow from the surplus account (column 4) is the first difference from Table VI, column 5, "funds in surplus account." For instance,

$126,772.29 – $66,264.50 = –$60,507.79 (Table VII, column 4, row 6).

The pre-tax income from invested surplus (column 5) and the tax on income from invested surplus (column 6) are determined in the same manner as used for columns 2 and 3.

The net cash flow to investors (column 7) equals the sum of the entries in columns 1 through 6. The internal rate of return, or 10.42%, is the interest rate that sets the present value of these cash flows to zero. Although there is no closed form equation to calculate the IRR, many microcomputer spreadsheets, such as Excel or Lotus 1-2-3, have built-in functions that perform the needed calculations.

INTERNAL RATE OF RETURN ANALYSIS
STATE OF XXXXXXXX

TABLE I: ASSUMPTIONS

	ITEMS		PERCENT
(1)	LOSS RATIO		82.35
(2)	COMMISSIONS		6.61
(3)	OTHER EXPENSES		9.99
(4)	STATE PREMIUM TAXES		
	(A)	TAX1	0.63
	(B)	TAX2	0.25
	(C)	TAX3	0.17
(5)	PROFIT AND CONTINGENCY		0.00
(6)	DEVIATIONS AND SCHEDULE RATING		3.20
(7)	DIVIDENDS TO POLICYHOLDERS		4.90
(8)	INVESTMENT INCOME		
	(A)	PRE-TAX RETURN ON ASSETS	7.42
	(B)	POST-TAX RETURN ON ASSETS	5.69
	(C)	INVESTMENT INCOME TAX RATE	23.4%
(9)	RESERVE TO SURPLUS RATIO		3.50
(10)	PREMIUMS WRITTEN		1,000,000
(11)	COLLECTED PREMIUM		968,000

INTERNAL RATE OF RETURN: 10.42 %

NATIONAL COUNC COMPENSATION INSURANCE
INTERNAL R OF RETURN ANALYSIS
STATE OF XXXXXXXX

TABLE II: CASH FLOW PATTERNS (%)

TIME INTERVAL FROM	:	TO	(1) PREMIUM COLLECTION	(2) LOSS PAYOUT	(3) OTHER EXPENSES	(4) TAX 1	(5) TAX 2	(6) TAX 3 *	(7) DIVIDENDS PAID
-1.00	:	-0.75	0.00000	0.00000	0.1664	0	0	0	0.00
-0.75	:	-0.50	0.00000	0.00000	0.6747	0	0	0	0.00
-0.50	:	-0.25	0.00000	0.00000	2.7575	0	0	0	0.00
-0.25	:	0.00	0.19000	0.00000	8.3026	0	0	0	0.00
0.00	:	0.25	4.50000	0.67500	14.9579	0.19	0.19	0	0.00
0.25	:	0.50	10.60000	2.02500	18.3660	0	0	0	0.00
0.50	:	0.75	15.59000	3.37500	17.9559	0	0	0	0.00
0.75	:	1.00	20.14000	4.72500	13.9586	0	0	0	0.00
1.00	:	1.25	19.13000	9.60000	8.8096	50.83	50.83	100	0.00
1.25	:	1.50	14.13000	8.25000	5.8757	0	0	0	25.00
1.50	:	1.75	9.39000	6.90000	4.2865	0	0	0	25.00
1.75	:	2.00	4.39000	5.55000	2.7388	0	0	0	25.00
2.00	:	2.25	1.00000	4.35000	1.0661	47.04	47.04	0	25.00
2.25	:	2.50	0.06000	4.35000	0.0836	0	0	0	0.00
2.50	:	2.75	(0.12000)	4.35000	0.0000	0	0	0	0.00
2.75	:	3.00	0.10000	4.35000	0.0000	0	0	0	0.00
3.00	:	3.25	0.13000	2.60000	0.0000	1.04	1.04	0	0.00
3.25	:	3.50	(0.02000)	2.60000	0.0000	0	0	0	0.00
3.50	:	3.75	(0.03000)	2.60000	0.0000	0	0	0	0.00
3.75	:	4.00	0.11000	2.60000	0.0000	0	0	0	0.00
4.00	:	4.25	0.23000	1.65000	0.0000	0.19	0.19	0	0.00
4.25	:	4.50	0.22000	1.65000	0.0000	0	0	0	0.00
4.50	:	4.75	0.17000	1.65000	0.0000	0	0	0	0.00
4.75	:	5.00	0.08000	1.65000	0.0000	0	0	0	0.00
5.00	:	6.00	0.01000	4.40000	0.0000	0.7	0.7	0	0.00
6.00	:	7.00	0.00000	3.40000	0.0000	0.01	0.01	0	0.00
7.00	:	8.00	0.00000	2.60000	0.0000	0	0	0	0.00
8.00	:	9.00	0.00000	2.30000	0.0000	0	0	0	0.00
9.00	:	10.00	0.00000	2.06324	0.0000	0	0	0	0.00
10.00	:	11.00	0.00000	1.72765	0.0000	0	0	0	0.00
11.00	:	12.00	0.00000	1.44664	0.0000	0	0	0	0.00
12.00	:	13.00	0.00000	1.21134	0.0000	0	0	0	0.00
13.00	:	14.00	0.00000	1.01431	0.0000	0	0	0	0.00
14.00	:	15.00	0.00000	0.84932	0.0000	0	0	0	0.00
15.00	:	16.00	0.00000	0.71118	0.0000	0	0	0	0.00
16.00	:	17.00	0.00000	0.59550	0.0000	0	0	0	0.00
17.00	:	18.00	0.00000	0.49864	0.0000	0	0	0	0.00
18.00	:	19.00	0.00000	0.41753	0.0000	0	0	0	0.00
19.00	:	20.00	0.00000	0.34962	0.0000	0	0	0	0.00
20.00	:	21.00	0.00000	0.29275	0.0000	0	0	0	0.00
21.00	:	22.00	0.00000	0.24514	0.0000	0	0	0	0.00
22.00	:	23.00	0.00000	0.20526	0.0000	0	0	0	0.00
23.00	:	24.00	0.00000	0.17188	0.0000	0	0	0	0.00

* EXCLUDES POLICYHOLDER DIVIDENDS. REFUND INCLUDED IN TABLE V-B, COLUMN (3).

NATIONAL COUNCIL ON COMPENSATION INSURANCE

TOTAL ESTIMATED YIELD FOR THE COMPOSITE PORTFOLIO

STATE OF XXXXXXXX

COUNTRY-WIDE DATA:

1.	REALIZED CAPITAL GAINS, 1985-89:	23,066,973
2.	NET INVESTMENT INCOME EARNED, 1985-89:	124,412,729
3.	TOTAL REALIZED CAPITAL GAINS AND NET INVESTMENT INCOME EARNED, 1985-89:	147,479,702
4.	REALIZED CAPITAL GAINS AS A PROPORTION OF TOTAL REALIZED CAPITAL GAINS AND NET INVESTMENT INCOME EARNED:	0.156
5.	NET INVESTMENT INCOME EARNED AS A PROPORTION OF TOTAL REALIZED CAPITAL GAINS AND NET INVESTMENT INCOME EARNED:	0.844
6.	MEAN TOTAL INVESTED ASSETS, 1985-1989:	356,137,393
7.	MEAN TOTAL DEDUCTIONS FROM INVESTMENT INCOME 1985-1989:	1,660,844
8.	INVESTMENT EXPENSES AS PERCENT OF MEAN TOTAL ASSETS:	0.466

CALCULATION OF STATE SPECIFIC PRE AND POST TAX PORTFOLIO YIELDS:

9.	WEIGHTED PRE-TAX YIELD FOR THE COMPOSITE PORTFOLIO:	7.731
10.	WEIGHTED POST TAX YIELD FOR THE COMPOSITE PORTFOLIO:	6.160
11.	WEIGHTED PRE-TAX YIELD FOR THE COMPOSITE PORTFOLIO AFTER INVESTMENT EXPENSES AND EXPENSE TAX CREDITS:	7.424
12.	INVESTMENT INCOME TAX RATE:	23.391
13.	TOTAL ESTIMATED POST-TAX YIELD FOR COMPOSITE PORTFOLIO:	5.687

SOURCES- 1, 2: BEST'S AGGREGATES AND AVERAGES, EDITIONS 1986-90
6, 7: BEST'S AGGREGATES AND AVERAGES, EDITIONS 1986-90
(3) = (1) + (2)
(4) = (1) / (3)
(5) = (2) / (3)
(8) = (7) / (6)
(9), (10): TABLE III-B
(11) = (9) - {(8) x .66}
(12) = 1 - (13) / (11)
(13) = {(9) x (4) x .66} + {(10) x (5)} - {(8) x .66}

NOTES: (1) ALL DOLLAR AMOUNTS ARE IN THOUSANDS OF DOLLARS.

NATIONAL COUNCIL ON COMPENSATION INSURANCE

WEIGHTED PRE-TAX AND POST-TAX ESTIMATED YIELDS ON INVESTMENT INCOME
FOR A COMPOSITE PORTFOLIO

STATE OF XXXXXXXX

	(1) PORTFOLIO COMPOSITION	(2) PRE-TAX YIELDS	(3) TAX RATE	(4) POST-TAX YIELDS
U.S. TREASURY SECURITIES	18.7 %	7.149 %	34.0 %	4.718 %
BONDS EXEMPT FROM U.S. TAX	35.4	5.109	5.1	4.848
BONDS NOT EXEMPT FROM U.S. TAX	17.6	7.498	34.0	4.949
PREFERRED STOCKS	2.6	8.850	13.8	7.631
COMMON STOCKS	16.7	14.463	13.8	12.471
REAL ESTATE	1.3	9.803	34.0	6.470
SHORT TERM INVESTMENTS	7.8	6.375	34.0	4.208
	100.0 %	7.731 %		6.160 %

SOURCES:

COLUMN (1): BEST'S AGGREGATES AND AVERAGES, 1990 EDITION

COLUMN (2):
- FEDERAL RESERVE STATISTICAL BULLETIN: YIELDS AVAILABLE 7/7/91.
  - U.S. TREASURY BILLS AND BONDS
  - COMMERCIAL PAPER
  - MOODY'S 'AAA' SEASONED CORPORATE BONDS
  - STATE AND LOCAL BONDS
- MOODY'S BOND RECORD, 7/91 EDITION.
  - 'A' RATED PREFERRED UTILITY STOCKS
- NEW YORK TIMES, FIRST SUNDAY EDITION OF 7/91.
  - 7 DAY YIELDS ON TAX-EXEMPT MONEY MARKET FUNDS
- STOCKS, BONDS, BILLS, AND INFLATION 1989 YEARBOOK (IBBOTSON ASSOCIATES INC., CHICAGO, ILLINOIS 1989)
  - EXHIBIT 28, PAGE 86
- IBBOTSON AND SIEGEL, "REAL ESTATE RETURNS: A COMPARISON WITH
  - OTHER INVESTMENTS", AREUEA JOURNAL, VOL. 12, NO.3, 1984

COLUMN (3): THE TAX REFORM ACT OF 1986.

COLUMN (4): COLUMN(2) x (1 - COLUMN(3)).

NATIONAL COUNCIL ON COMPENSATION INSURANCE
INTERNAL RATE OF RETURN ANALYSIS
STATE OF XXXXXXXX

TABLE IV: CASH FLOWS FOR LOSS AND UNEARNED PREMIUM RESERVES

TIME INTERVAL FROM	TO	(1) PREMIUM COLLECTED	(2) AGENTS BALANCES	(3) OVERDUE AGENTS BALANCES	(4) ADMITTED AGENTS BALANCES	(5) LOSSES INCURRED	(6) UNEARNED PREMIUM	(7) TOTAL PREMIUM NET OF RESERVES	(8) PREMIUM NET OF RESERVES
-1.00 :	-0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-0.75 :	-0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-0.50 :	-0.25	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
-0.25 :	0.00	1,839.20	(1,839.20)	0.00	(1,839.20)	0.00	0.00	0.00	0.00
0.00 :	0.25	45,399.20	196,600.80	0.00	196,600.80	25,734.38	211,750.00	4,515.63	4,515.63
0.25 :	0.50	148,007.20	335,992.80	0.00	335,992.80	102,937.50	363,000.00	18,062.50	13,546.88
0.50 :	0.75	298,918.40	427,081.60	0.00	427,081.60	231,609.38	453,750.00	40,640.63	22,578.13
0.75 :	1.00	493,873.60	474,126.40	0.00	474,126.40	411,750.00	484,000.00	72,250.00	31,609.37
1.00 :	1.25	679,052.00	288,948.00	0.00	288,948.00	591,890.63	272,250.00	103,859.37	31,609.38
1.25 :	1.50	815,830.40	152,169.60	0.00	152,169.60	720,562.50	121,000.00	126,437.50	22,578.13
1.50 :	1.75	906,725.60	61,274.40	0.00	61,274.40	797,765.63	30,250.00	139,984.37	13,546.88
1.75 :	2.00	949,220.80	18,779.20	0.00	18,779.20	823,500.00	0.00	144,500.00	4,515.63
2.00 :	2.25	958,900.80	9,099.20	9,099.20	0.00	823,500.00	0.00	135,400.80	(9,099.20)
2.25 :	2.50	959,481.60	8,518.40	8,518.40	0.00	823,500.00	0.00	135,981.60	580.80
2.50 :	2.75	958,320.00	9,680.00	9,680.00	0.00	823,500.00	0.00	134,820.00	(1,161.60)
2.75 :	3.00	959,288.00	8,712.00	8,712.00	0.00	823,500.00	0.00	135,788.00	968.00
3.00 :	3.25	960,546.40	7,453.60	7,453.60	0.00	823,500.00	0.00	137,046.40	1,258.40
3.25 :	3.50	960,352.80	7,647.20	7,647.20	0.00	823,500.00	0.00	136,852.80	(193.60)
3.50 :	3.75	960,062.40	7,937.60	7,937.60	0.00	823,500.00	0.00	136,562.40	(290.40)
3.75 :	4.00	961,127.20	6,872.80	6,872.80	0.00	823,500.00	0.00	137,627.20	1,064.80
4.00 :	4.25	963,353.60	4,646.40	4,646.40	0.00	823,500.00	0.00	139,853.60	2,226.40
4.25 :	4.50	965,483.20	2,516.80	2,516.80	0.00	823,500.00	0.00	141,983.20	2,129.60
4.50 :	4.75	967,128.80	871.20	871.20	0.00	823,500.00	0.00	143,628.80	1,645.60
4.75 :	5.00	967,[illegible]	96.80	96.80	0.00	823,500.00	0.00	144,403.20	774.40
5.00 :	6.00	968,[illegible]	0.00	0.00	0.00	823,500.00	0.00	144,500.00	96.80
6.00 :	7.00	968,[illegible]	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
7.00 :	8.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
8.00 :	9.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
9.00 :	10.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
10.00 :	11.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
11.00 :	12.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
12.00 :	13.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
13.00 :	14.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
14.00 :	15.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
15.00 :	16.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
16.00 :	17.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
17.00 :	18.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
18.00 :	19.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
19.00 :	20.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
20.00 :	21.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
21.00 :	22.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
22.00 :	23.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00
23.00 :	24.00	968,000.00	0.00	0.00	0.00	823,500.00	0.00	144,500.00	0.00

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TABLE V-A: TAX CREDITS AVAILABLE FROM UNDERWRITING OPERATIONS

YEAR	(1) PREMIUM WRITTEN (POST DEVIATION)	(2) CHANGE IN UNEARNED PREMIUM RESERVE	(3) EXPENSES	(4) DIVIDENDS	(5) LOSSES PAID: ACCIDENT YEAR 1	(6) LOSSES PAID: ACCIDENT YEAR 2	(7) CHANGE IN DISCOUNTED LOSS RESERVE: ACCIDENT YEAR 1	(8) CHANGE IN DISCOUNTED LOSS RESERVE: ACCIDENT YEAR 2	(9) TAX CREDIT
-1	0.00	0.00	11,626.64	0.00	0.00	0.00	0.00	0.00	3,953.06
0	968,000.00	484,000.00	95,622.71	0.00	100,611.11	0.00	245,099.97	0.00	(47,418.51)
1	0.00	(484,000.00)	57,016.51	35,574.00	100,858.16	100,611.11	(90,261.65)	245,099.97	20,977.35
2	0.00	0.00	5,764.10	11,858.00	58,674.38	100,858.16	(49,078.03)	(90,261.65)	12,857.08
3	0.00	0.00	210.22	0.00	35,781.08	58,674.38	(28,627.83)	(49,078.03)	5,766.33
4	0.00	0.00	464.30	0.00	23,099.18	35,781.08	(16,830.54)	(28,627.83)	4,721.30
5	0.00	0.00	66.03	0.00	16,264.13	23,099.18	(11,873.58)	(16,830.54)	3,646.57
6	0.00	0.00	0.85	0.00	12,517.20	16,264.13	(7,455.98)	(11,873.58)	3,213.89
7	0.00	0.00	0.00	0.00	10,149.64	12,517.20	(6,113.06)	(7,455.98)	3,093.25
8	0.00	0.00	0.00	0.00	9,031.57	10,149.64	(5,422.29)	(6,113.06)	2,599.59
9	0.00	0.00	0.00	0.00	7,873.58	9,031.57	(3,840.49)	(5,422.29)	2,598.41
10	0.00	0.00	0.00	0.00	6,592.91	7,873.58	(3,346.02)	(3,840.49)	2,475.19
11	0.00	0.00	0.00	0.00	5,520.54	6,592.91	(2,922.43)	(3,346.02)	1,987.30
12	0.00	0.00	0.00	0.00	4,622.60	5,520.54	(2,560.33)	(2,922.43)	1,584.53
13	0.00	0.00	0.00	0.00	3,870.72	4,622.60	(2,251.83)	(2,560.33)	1,251.59
14	0.00	0.00	0.00	0.00	3,241.13	3,870.72	(1,990.49)	(2,251.83)	975.64
15	0.00	0.00	0.00	0.00	2,713.94	3,241.13	(2,606.43)	(1,990.49)	461.77
16	0.00	0.00	0.00	0.00	2,272.51	2,713.94	(2,182.48)	(2,606.43)	67.16
17	0.00	0.00	0.00	0.00	1,902.87	2,272.51	(1,827.49)	(2,182.48)	56.24
18	0.00	0.00	0.00	0.00	1,593.36	1,902.87	(1,530.24)	(1,827.49)	47.09
19	0.00	0.00	0.00	0.00	1,334.20	1,593.36	(1,281.34)	(1,530.24)	39.43
20	0.00	0.00	0.00	0.00	1,117.18	1,334.20	(1,072.93)	(1,281.34)	33.02
21	0.00	0.00	0.00	0.00	935.47	1,117.18	(898.41)	(1,072.93)	27.65
22	0.00	0.00	0.00	0.00	783.31	935.47	(752.28)	(898.41)	23.15
23	0.00	0.00	0.00	0.00	389.24	1,172.55	(373.82)	(1,126.10)	21.04

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TABLE V-B: NET CASH FLOW FROM UNDERWRITING

TIME INTERVAL FROM	:	TO	(1) PREMIUM FLOWS NET OF RESERVES	(2) TAX CREDITS	(3) EXPENSES	(4) DIVIDENDS	(5) NET CASH FLOW FROM UNDERWRITING
-1.00	:	-0.75	0.00	988.26	160.85	0.00	827.41
-0.75	:	-0.50	0.00	988.26	652.27	0.00	335.99
-0.50	:	-0.25	0.00	988.26	2,665.71	0.00	(1,677.45)
-0.25	:	0.00	0.00	988.26	8,147.80	0.00	(7,159.54)
0.00	:	0.25	4,515.63	(11,854.63)	17,356.94	0.00	(24,695.94)
0.25	:	0.50	13,546.88	(11,854.63)	24,540.38	0.00	(22,848.13)
0.50	:	0.75	22,578.13	(11,854.63)	27,338.44	0.00	(16,614.94)
0.75	:	1.00	31,609.37	(11,854.63)	26,386.96	0.00	(6,632.21)
1.00	:	1.25	31,609.38	5,244.34	26,738.27	0.00	10,115.44
1.25	:	1.50	22,578.13	5,244.34	14,705.52	11,858.00	1,258.95
1.50	:	1.75	13,546.88	5,244.34	10,134.89	11,858.00	(3,201.68)
1.75	:	2.00	4,515.63	5,244.34	5,437.83	11,858.00	(7,535.87)
2.00	:	2.25	(9,099.20)	3,214.27	5,657.66	11,858.00	(23,400.59)
2.25	:	2.50	580.80	3,214.27	119.24	0.00	3,675.83
2.50	:	2.75	(1,161.60)	3,214.27	(76.82)	0.00	2,129.49
2.75	:	3.00	968.00	3,214.27	64.02	0.00	4,118.25
3.00	:	3.25	1,258.40	1,441.58	171.81	0.00	2,528.17
3.25	:	3.50	(193.60)	1,441.58	(12.80)	0.00	1,260.79
3.50	:	3.75	(290.40)	1,441.58	(19.21)	0.00	1,170.39
3.75	:	4.00	1,064.80	1,441.58	70.42	0.00	2,435.96
4.00	:	4.25	2,226.40	1,180.33	163.42	0.00	3,243.30
4.25	:	4.50	2,129.60	1,180.33	140.84	0.00	3,169.09
4.50	:	4.75	1,645.60	1,180.33	108.83	0.00	2,717.10
4.75	:	5.00	774.40	1,180.33	51.21	0.00	1,903.51
5.00	:	6.00	96.80	3,646.57	66.03	0.00	3,677.34
6.00	:	7.00	0.00	3,213.89	0.85	0.00	3,213.04
7.00	:	8.00	0.00	3,093.25	0.00	0.00	3,093.25
8.00	:	9.00	0.00	2,599.59	0.00	0.00	2,599.59
9.00	:	10.00	0.00	2,598.41	0.00	0.00	2,598.41
10.00	:	11.00	0.00	2,475.19	0.00	0.00	2,475.19
11.00	:	12.00	0.00	1,987.30	0.00	0.00	1,987.30
12.00	:	13.00	0.00	1,584.53	0.00	0.00	1,584.53
13.00	:	14.00	0.00	1,251.59	0.00	0.00	1,251.59
14.00	:	15.00	0.00	975.64	0.00	0.00	975.64
15.00	:	16.00	0.00	461.77	0.00	0.00	461.77
16.00	:	17.00	0.00	67.16	0.00	0.00	67.16
17.00	:	18.00	0.00	56.24	0.00	0.00	56.24
18.00	:	19.00	0.00	47.09	0.00	0.00	47.09
19.00	:	20.00	0.00	39.43	0.00	0.00	39.43
20.00	:	21.00	0.00	33.02	0.00	0.00	33.02
21.00	:	22.00	0.00	27.65	0.00	0.00	27.65
22.00	:	23.00	0.00	23.15	0.00	0.00	23.15
23.00	:	24.00	0.00	21.04	0.00	0.00	21.04

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TABLE VI: DERIVATION OF CASH LEVEL AND FUNDS IN SURPLUS ACCOUNT

TIME INTERVAL FROM	:	TO	(1) LOSS ADJUSTMENT AND LOSS RESERVES	(2) UNEARNED PREMIUM RESERVES	(3) ADMITTED AGENTS BALANCES	(4) CASH LEVEL	(5) FUNDS IN SURPLUS ACCOUNT
-1.00	:	-0.75	0.00	0.00	0.00	0.00	0.00
-0.75	:	-0.50	0.00	0.00	0.00	0.00	0.00
-0.50	:	-0.25	0.00	0.00	0.00	0.00	0.00
-0.25	:	0.00	0.00	0.00	(1,839.20)	1,839.20	0.00
0.00	:	0.25	20,175.75	211,750.00	196,600.80	35,324.95	66,264.50
0.25	:	0.50	80,703.00	363,000.00	335,992.80	107,710.20	126,772.29
0.50	:	0.75	181,581.75	453,750.00	427,081.60	208,250.15	181,523.36
0.75	:	1.00	322,812.00	484,000.00	474,126.40	332,685.60	230,517.71
1.00	:	1.25	423,896.63	272,250.00	288,948.00	407,198.63	198,899.04
1.25	:	1.50	484,629.75	121,000.00	152,169.60	453,460.15	173,037.07
1.50	:	1.75	505,011.38	30,250.00	61,274.40	473,986.97	152,931.82
1.75	:	2.00	485,041.50	0.00	18,779.20	466,262.30	138,583.29
2.00	:	2.25	449,219.25	0.00	0.00	449,219.25	128,348.36
2.25	:	2.50	413,397.00	0.00	0.00	413,397.00	118,113.43
2.50	:	2.75	377,574.75	0.00	0.00	377,574.75	107,878.50
2.75	:	3.00	341,752.50	0.00	0.00	341,752.50	97,643.57
3.00	:	3.25	320,341.50	0.00	0.00	320,341.50	91,526.14
3.25	:	3.50	298,930.50	0.00	0.00	298,930.50	85,408.71
3.50	:	3.75	277,519.50	0.00	0.00	277,519.50	79,291.29
3.75	:	4.00	256,108.50	0.00	0.00	256,108.50	73,173.86
4.00	:	4.25	242,520.75	0.00	0.00	242,520.75	69,291.64
4.25	:	4.50	228,933.00	0.00	0.00	228,933.00	65,409.43
4.50	:	4.75	215,345.25	0.00	0.00	215,345.25	61,527.21
4.75	:	5.00	201,757.50	0.00	0.00	201,757.50	57,645.00
5.00	:	6.00	165,523.50	0.00	0.00	165,523.50	47,292.43
6.00	:	7.00	137,524.50	0.00	0.00	137,524.50	39,292.71
7.00	:	8.00	116,113.50	0.00	0.00	116,113.50	33,175.29
8.00	:	9.00	97,173.00	0.00	0.00	97,173.00	27,763.71
9.00	:	10.00	80,182.20	0.00	0.00	80,182.20	22,909.20
10.00	:	11.00	65,955.02	0.00	0.00	65,955.02	18,844.29
11.00	:	12.00	54,041.96	0.00	0.00	54,041.96	15,440.56
12.00	:	13.00	44,066.62	0.00	0.00	44,066.62	12,590.46
13.00	:	14.00	35,713.80	0.00	0.00	35,713.80	10,203.94
14.00	:	15.00	28,719.61	0.00	0.00	28,719.61	8,205.60
15.00	:	16.00	22,863.06	0.00	0.00	22,863.06	6,532.30
16.00	:	17.00	17,959.10	0.00	0.00	17,959.10	5,131.17
17.00	:	18.00	13,852.79	0.00	0.00	13,852.79	3,957.94
18.00	:	19.00	10,414.39	0.00	0.00	10,414.39	2,975.54
19.00	:	20.00	7,535.26	0.00	0.00	7,535.26	2,152.93
20.00	:	21.00	5,124.44	0.00	0.00	5,124.44	1,464.13
21.00	:	22.00	3,105.75	0.00	0.00	3,105.75	887.36
22.00	:	23.00	1,415.40	0.00	0.00	1,415.40	404.40
23.00	:	24.00	(0.00)	0.00	0.00	(0.00)	(0.00)

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TABLE VII: NOMINAL CASH FLOW TO INVESTORS

TIME INTERVAL FROM : TO	(1) NET CASH FLOW FROM UNDERWRITING	(2) PRE-TAX INCOME FROM INVESTED CASH	(3) TAX ON INCOME FROM INVESTED CASH	(4) NET FLOW FROM SURPLUS ACCOUNT	(5) PRE-TAX INCOME FROM INVESTED SURPLUS	(6) TAX ON INCOME FROM INVESTED SURPLUS	(7) NET CASH FLOW TO INVESTORS
-1.00 : -0.75	827.41	0.00	0.00	0.00	0.00	0.00	827.41
-0.75 : -0.50	335.99	0.00	0.00	0.00	0.00	0.00	335.99
-0.50 : -0.25	(1,677.45)	0.00	0.00	0.00	0.00	0.00	(1,677.45)
-0.25 : 0.00	(7,159.54)	16.61	(3.89)	0.00	0.00	0.00	(7,146.82)
0.00 : 0.25	(24,695.94)	335.68	(78.54)	(66,264.50)	598.52	(140.04)	(90,244.82)
0.25 : 0.50	(22,848.13)	1,291.94	(302.28)	(60,507.79)	1,743.56	(407.94)	(81,030.64)
0.50 : 0.75	(16,614.94)	2,853.85	(667.72)	(54,751.07)	2,784.62	(651.52)	(67,046.78)
0.75 : 1.00	(6,632.21)	4,885.89	(1,143.16)	(48,994.36)	3,721.68	(870.76)	(49,032.92)
1.00 : 1.25	10,115.44	6,682.85	(1,563.59)	31,618.68	3,878.62	(907.48)	49,824.51
1.25 : 1.50	1,258.95	7,773.72	(1,818.82)	25,861.96	3,359.44	(786.01)	35,649.23
1.50 : 1.75	(3,201.68)	8,376.97	(1,959.97)	20,105.25	2,944.25	(688.87)	25,575.96
1.75 : 2.00	(7,535.87)	8,492.61	(1,987.02)	14,348.54	2,633.05	(616.06)	15,335.24
2.00 : 2.25	(23,400.59)	8,268.90	(1,934.68)	10,234.93	2,411.00	(564.10)	(4,984.55)
2.25 : 2.50	3,675.83	7,791.40	(1,822.96)	10,234.93	2,226.11	(520.85)	21,584.47
2.50 : 2.75	2,129.49	7,144.29	(1,671.56)	10,234.93	2,041.23	(477.59)	19,400.79
2.75 : 3.00	4,118.25	6,497.17	(1,520.15)	10,234.93	1,856.34	(434.33)	20,752.21
3.00 : 3.25	2,528.17	5,980.23	(1,399.20)	6,117.43	1,708.64	(399.77)	14,535.49
3.25 : 3.50	1,260.79	5,593.45	(1,308.70)	6,117.43	1,598.13	(373.92)	12,887.17
3.50 : 3.75	1,170.39	5,206.66	(1,218.21)	6,117.43	1,487.62	(348.06)	12,415.83
3.75 : 4.00	2,435.96	4,819.88	(1,127.71)	6,117.43	1,377.11	(322.20)	13,300.47
4.00 : 4.25	3,243.30	4,503.77	(1,053.75)	3,882.21	1,286.79	(301.07)	11,561.25
4.25 : 4.50	3,169.09	4,258.31	(996.32)	3,882.21	1,216.66	(284.66)	11,245.29
4.50 : 4.75	2,717.10	4,012.85	(938.89)	3,882.21	1,146.53	(268.25)	10,551.55
4.75 : 5.00	1,903.51	3,767.39	(881.46)	3,882.21	1,076.40	(251.85)	9,496.21
5.00 : 6.00	3,677.34	13,633.47	(3,189.84)	10,352.57	3,895.28	(911.38)	27,457.45
6.00 : 7.00	3,213.04	11,249.14	(2,631.97)	7,999.71	3,214.04	(751.99)	22,291.97
7.00 : 8.00	3,093.25	9,415.04	(2,202.85)	6,117.43	2,690.01	(629.38)	18,483.51
8.00 : 9.00	2,599.59	7,917.19	(1,852.39)	5,411.57	2,262.06	(529.26)	15,808.77
9.00 : 10.00	2,598.41	6,583.42	(1,540.33)	4,854.52	1,880.98	(440.09)	13,936.90
10.00 : 11.00	2,475.19	5,424.61	(1,269.20)	4,064.91	1,549.89	(362.63)	11,882.77
11.00 : 12.00	1,987.30	4,454.29	(1,042.17)	3,403.73	1,272.65	(297.76)	9,778.04
12.00 : 13.00	1,584.53	3,641.79	(852.07)	2,850.10	1,040.51	(243.45)	8,021.41
13.00 : 14.00	1,251.59	2,961.45	(692.89)	2,386.52	846.13	(197.97)	6,554.83
14.00 : 15.00	975.64	2,391.77	(559.60)	1,998.34	683.36	(159.89)	5,329.62
15.00 : 16.00	461.77	1,914.75	(448.00)	1,673.30	547.07	(128.00)	4,020.90
16.00 : 17.00	67.16	1,515.32	(354.54)	1,401.13	432.95	(101.30)	2,960.72
17.00 : 18.00	56.24	1,180.86	(276.29)	1,173.23	337.39	(78.94)	2,392.49
18.00 : 19.00	47.09	900.80	(210.76)	982.40	257.37	(60.22)	1,916.68
19.00 : 20.00	39.43	666.29	(155.89)	822.61	190.37	(44.54)	1,518.27
20.00 : 21.00	33.02	469.93	(109.95)	688.81	134.27	(31.41)	1,184.65
21.00 : 22.00	27.65	305.50	(71.48)	576.77	87.29	(20.42)	905.31
22.00 : 23.00	23.15	167.83	(39.27)	482.96	47.95	(11.22)	671.40
23.00 : 24.00	21.04	52.54	(12.29)	404.40	15.01	(3.51)	477.18

INTERNAL RATE OF RETURN: 10.42 %

Section VI: Potential Pitfalls in IRR Analyses

Internal rate of return capital budgeting techniques have been criticized on theoretical grounds, and IRR insurance pricing models have been criticized on practical grounds. Advocates of IRR analyses respond that the criticisms are not material, particularly compared to the benefits of the models. This section reviews the arguments on both sides.

General Criticisms

Two widely used capital budgeting techniques consider the time value of money:

- Internal rate of return analyses determine the interest rate that sets the present value of cash inflows equal to the present value of cash outflows. [When used for pricing insurance policies, as in this reading, the IRR model generally considers "equity capital" inflows and outflows, not cash inflows and outflows.[32]] If this interest rate exceeds the cost of capital, the project may be accepted; if it does not, the project should be rejected.

- Net present value analyses use the cost of capital to discount all cash flows to the same time. If the sum of the discounted values is positive, the project may be accepted; if it is negative, the project should be rejected.

The IRR model solves for the discount rate such that the net present value is zero – that is, such that the present value of outflows equals the present value of inflows. The NPV model determines the net present value of all transactions at a given discount rate.

The IRR and NPV models may be thought of as inverse functions:

- IRR: The implied discount rate is a continuous function of the net present value.
- NPV: The net present value is a continuous function of the discount rate.

In other words, the same relationship between net present value and discount rate underlies both the IRR and NPV models.

For most "accept or reject" decision with no constraints, the two methods give the same answer. When there are budget constraints, the projects are mutually exclusive, or there are unusual cash flows, the two methods may give different answers. Often, however, the different answers stem from different decision rules, not from the mathematics involved.

One goal of capital budgeting is to optimize the net worth of the corporation. Some academic theoreticians argue that net present value analyses achieve this objective, whereas internal rate of return analyses may not. Many corporate analysts find the academic arguments of little practical concern and consider the IRR analyses useful and clear.

[32] Cf. Sondergeld [1982], page 425: "The internal rate of return is the yield rate at which the present value of the surplus transfers equals zero." (Sondergeld uses an IRR model with implied "surplus transfers" between benchmark surplus, insurance surplus, and corporate surplus.)

Cash Flow Patterns

The cash flow pattern affects the number of positive solutions to the internal rate of return equation. Most projects involve an initial cash outflow (or a series of cash outflows) followed by cash inflows. There is a single "sign reversal" between outflows and inflows in this pattern, and there is at most one positive real root to the IRR equation.[33]

The maximum number of positive real roots is equal to the number of sign reversals in the cash flow pattern (Descartes's "rule of signs"). If the project involves an outflow followed by an inflow followed by another outflow, there may be two positive real roots. Solomon [1956] provides an "oil pump" illustration of this, which has been repeated often in financial texts.

Suppose a company's present equipment enable it to extract oil from a well over two years, for revenues of $10,000 each year. By purchasing a more efficient pump for $1,600, the company can extract all the oil in one year, for revenues of $20,000.

To simplify the IRR analysis, assume that the cash outflows and inflows occur at discrete times separated by one year. Purchasing the more efficient equipment means a cash outflow of $1,600 in the first year, an extra cash inflow of $10,000 in the second year, and a cash outflow (or the loss of a cash inflow) of $10,000 in the third year. The IRR equation is

$$\$1,600 = (1+r)^{-1}(\$10,000) - (1+r)^{-2}(\$10,000)$$

R = 25% and r = 400% both satisfy the equation; there is no unique solution.

In truth, neither solution is realistic. Solomon [1956] notes that if the purchase price of the pump is $0, the internal rate of return is 0%; if the price is $827, the IRR becomes 10%; if it is $1,600, the IRR is 25%; if it is $2,500, the IRR is 100%. The more the pump costs, the more profitable the project seems.

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[33] This is Descartes's "rule of signs." Roach [1987], pages 189-190, provides a formal statement:

> *"If f(x) represents a polynomial with real coefficients and with its terms arranged in descending powers of x, the difference v – p between the number v of variations in signs of f(x) and the number p of positive roots of f(x) = 0 is zero or an even positive integer. In symbols,*
>
> $$v - p = 2k, \text{ } k \text{ is a positive integer or zero."}$$

For an intuitive understanding of the statement in the text, suppose there is a cash outflow at time 0 and ten cash inflows at times 1 through 10. The IRR equation is

$$(1+r)^0 out_0 = (1+r)^1 in_1 + (1+r)^1 in_1 + (1+r)^2 in_2 + \ldots + (1+r)^{10} in_{10}.$$

Since $(1+r)^0 = 1$, the left hand side of the equation is constant, whereas the right hand side is a monotonically decreasing function of "r," so there is at most one positive real solution. If there are multiple cash outflows followed by multiple cash inflows, set the time origin to a point between the outflows and inflows. The left hand side (outflows) is an increasing function of "r," and the right hand side (inflows) is a decreasing function of "r," so again there is at most one positive real solution.

Various methods have been proposed to resolve such problems. Solomon rephrases the question as "What is it worth to the investor to receive $10,000 one year earlier than he would otherwise have received it?" If the investor can obtain x% per annum on his money, the cash inflow and outflow of $10,000 apiece can be replaced by a single cash inflow of $10,000$x$. The IRR analysis can now be used to determine whether the project is profitable.[34]

Oversimplifications

In most cases, sign reversals in the projected cash flows result from inaccuracies or oversimplifications, not true reversals in the expected flows. For example, the "net cash flow to investors" (Table VII, column 7) in the illustration of Section V shows a general pattern of net outflows followed by net inflows. There are two exceptions: net inflows in the first two quarters of the year preceding policy inception ($827.41 and $335.99), and a net outflow in the quarter following termination of the last policy ($4,984.55).

Both of the exceptions result from oversimplifications, not from true reversals. The two early "inflows" stem from prepaid expenses and tax credits. The prepaid expenses occur almost entirely in the last two quarters of the year preceding policy inception ($813.12 in the first two quarters and $10,813.51 in the latter two quarters). The tax credit is determined on an annual basis and spread evenly to all four quarters ($3,953.06 for the full year, spread as $988.26 each quarter).

The inaccuracy due to this simplification is not material to the IRR analysis as a whole, but it causes a sign reversal. A more precise analysis would spread the first year's tax credit in the same proportion as the prepaid expenses. All four quarters would show cash outflows, and there would be no sign reversal.

The net outflow of equity funds of $4,984.55 in the fifth quarter following the policy year stems from the assumption that no agents' balances are overdue until this quarter, and all remaining agents' balances become overdue at this time. This simplification is unrealistic, but correcting it requires unavailable data and extensive analysis. A precise projection of the overdue portion of agents' balances would show a decrease in this quarter, not an increase. The net outflow is results from the oversimplification.

In sum, expected cash flows in insurance pricing models generally show a single sign reversal. Additional reversals indicate oversimplifications. If these are material, the projections should be reexamined. One should not tinker with the IRR equation and leave the errors untouched.

Actual cash flows may indeed show multiple sign reversals. An unanticipated IBNR loss may cause a cash outflow from investors in the middle of the stream of inflows. Net present value

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[34] Butler and Appel [1989] expand on Solomon's work, outlining an algorithm to eliminate all but the first sign reversal from the cash flow pattern. This transformation is not always easy. The IRR model implicitly assumes that returns can be reinvested at the internal rate of return. Solomon's transformation is the text uses a fixed outside return. Paquin [1987] uses separate borrowing (fixed) and lending (IRR) discount rates for insurance transactions with multiple cash flow sign changes. Since Paquin uses an IRR model from the insurer-policyholder perspective, not the equity holders' perspective, sign changes in the cash flow pattern are not unusual.

methods can be used to estimate prospectively the expected profitability of a contract and to determine retrospectively the actual results. Internal rate of return analyses on a single policy or a small group of policies are better for projecting estimates than for determining results.

The focus differs between IRR and NPV analyses. The IRR model derives the rate of return, whereas the NPV model determines the dollar return. To determine the rate of return with an NPV analysis, one must compare the present value of profits to some base, such as the present value of surplus (Robbin [1991]; Sondergeld [1982]). If the assumed, or benchmark, surplus varies with loss, some of the problems noted here with regard to IRR models apply to NPV models as well.

Mutually Exclusive Projects and Reinvestment Rates

Some projects pose "accept or reject" decisions, where net present value and internal rate of return analyses give the same answer. Other projects are mutually exclusive, particularly if there are aggregate budget constraints or if the projects accomplish the same goal. Net present value and IRR analyses do not necessarily provide the same ranking of projects.

Consider two mutually exclusive projects, each of which requires an initial capital outlay of $12,000. Project A returns $10,000 one year hence and $6,500 two years hence; project B returns $5,000 one year hence and $12,500 two years hence. The net present values of these projects at interest rates between 10% and 30% are shown below.

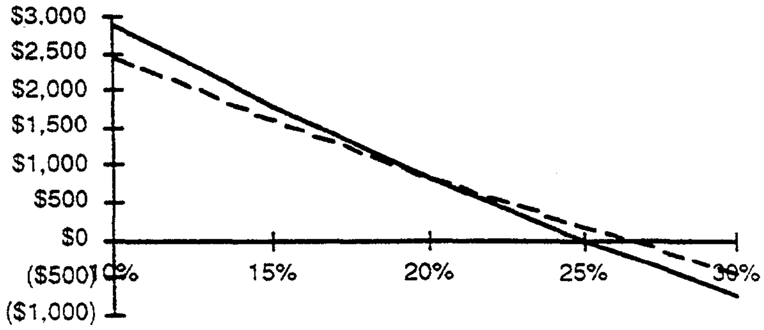
Net Present Values at Varying Interest Rates

	Cash Flows at Time			Net Present Value at Interest Rate of				
Project	0	1	2	10%	15%	20%	25%	30%
A	−12,000	10,000	6,500	2,463	1,611	847	160	−462
B	−12,000	5,000	12,500	2,876	1,800	847	0	−757

On a present value basis, if the opportunity cost of capital is less than 20%, project B is preferable; if it is more than 20%, project A is preferable. On an Internal Rate of Return basis, project A, with an IRR of 26.3%, is preferable to project B, with an IRR of 25%.

The two projects differ in the aggregate dollars of revenue and in the timing of the cash inflows. Project B has a larger nominal revenue ($17,500) than project A has ($16,500), but project A has quicker receipt of the revenue (61% in the first year) than project B has (29% in the first year). At low rates of interest, it is often wise to defer income for a larger total return; at high rates of interest, deferral of income may be expensive.

The accompanying chart graphs the net present values of the two projects at varying interest rates. The two lines cross at 20%, but they have different slopes. At interest rates below 20%, project B (the solid line) is the better investment; at interest rates above 20%, project A (the broken line) is better.



The criticism of the internal rate of return analysis runs as follows. Project A has a higher IRR (26.3%) than project B has (25%). But all this means is that project A is preferable if the revenue received in the first year ($10,000) can be invested at 26.3%. In truth, the firm's opportunity cost of capital is 15%; this is the return that the firm's owners can receive on their funds. The IRR analysis incorrectly mixes the interest rate that equates real values of inflows and outflows with the interest rate at which the firm can invest the funds it receives.[35]

This argument is valid for certain capital budgeting decisions. It is of dubious merit for many applications of the IRR insurance pricing model, for the following reasons:

1. The IRR pricing model is often used to set statewide manual rates, not to price individual policies. If the cost of capital is 15%, but the pricing model shows an IRR of 20%, the insurer can plough back the revenue it receives by writing more policies. As long as the insurer can grow at the internal rate of return and maintain the same quality of risks, the IRR assumptions are correct (Dorfman [1981]).

2. When the pricing model is used to determine the underwriting profit provision, the analyst selects a premium rate that equalizes the internal rate of return and the cost of equity capital. In such cases, there is no difference between net present value and IRR analyses.

Practical Criticisms

One IRR decision rule is: "If the internal rate of return is less than the cost of equity capital, reject the project." The internal rate of return may be positive, and may even exceed the investment yield, but if it is less than the cost of capital, then the project may be undesirable.[36]

Some regulators take another perspective. "A low rate of return may not be desired, but as long as it is positive, isn't the insurer making money? And if it exceeds the investment return, isn't

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[35] This criticism is often denoted as the "reinvestment rate assumption"; see particularly Weston and Copeland [1986], Sweeney and Mantripragada [1987], and McDaniel, McCarty, and Jessell [1988].

[36] An alternative IRR decision rule is "Between two similar projects, the one with the higher internal rate of return is preferable." The decision rule used must be appropriate for the type of IRR analysis.

it more than sufficient?"

The presentation of results is crucial in rate filings. Both the net present value and the IRR analyses may show that the project is not profitable. But the former discounts the cash flows at the cost of capital and thereby shows a negative net present value – a clear indication that the project is not profitable. The latter may show a positive, though inadequate, return – a less convincing demonstration of unprofitability.

In utility regulation, this difference is less important. As costs increase, the internal rate of return drops, soon becoming negative. In the insurance pricing model, as costs increase, the internal rate of return subsides slowly, hovering at low positive values even as premiums become severely inadequate.

The difference between utility and insurance regulation stems from the equity base against which returns are measured. The "used and useful" capital in utility regulation is a fixed amount; it does not vary with the projections in the model. But the "required surplus" in the insurance pricing model is an assumption posited by the actuary. If required surplus is determined by a "reserves to surplus" ratio, then as costs increase, so do surplus and investment income. Although the total return is inadequate, the added investment income offsets some of the underwriting loss, and the internal rate of return declines more slowly with increasing loss.

Premium Inadequacies and IRR Analyses

In such cases, the implied equity flow assumptions are not reasonable. As costs rise but premium rates are depressed, equity holders will not provide more capital, as the IRR model implies. Rational investors will provide less capital, if they provide any at all.

To clarify this problem, consider a simple IRR analysis:

- Net premium (that is, premium less expenses) of $10,000 is collected at policy inception.
- One loss will be paid four years after policy inception.
- The insurer funds the loss with a four year zero-coupon bond yielding 10% per annum.
- The equity commitment assumes a 2:1 ratio of undiscounted reserves to surplus.

The assumptions are simplified, but they are realistic for Workers' Compensation. We exclude expenses and taxes, which would complicate the analysis. The loss payment four years after policy inception means a lag of 3.5 years from occurrence to settlement. Workers' Compensation claims actually have weekly benefit payments; to simplify, we use an average payment date and a single cash transaction. Semi-annual coupons are the most common, but the zero-coupon bond makes the exposition clearer. The 2:1 reserves to surplus ratio is low (a more realistic ratio would lie between 2.5:1 and 4:1) but it highlights the problem.

The expected profitability of the contract depends on the expected losses and the cost of capital. Suppose expected losses are $12,000, and the cost of capital is 15% per annum. At policy inception, the equity holders contribute $2,000 to fund the underwriting loss ($12,000 minus $10,000) plus $6,000 as supporting surplus ($12,000 / 2). The total assets of $18,000, consisting of net premium ($10,000) plus equity contribution ($8,000) are invested in a four year 10% zero coupon bond, where they grow to $26,354. The loss of $12,000 is paid, and $14,354 is returned to the equity holders.

Only two equity flows affect the equity holders: the outflow of $8,000 at policy inception and the inflow of $14,354 four years later. The internal rate of return is 15.75%, so the contract is acceptable.

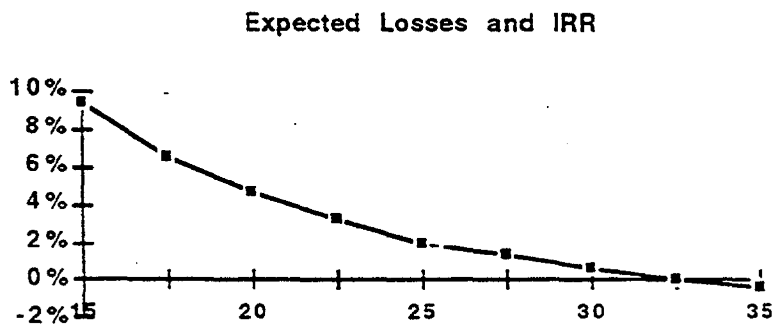
Suppose the expected loss is $15,000. The contract is clearly unprofitable, since the net premium of $10,000 accumulated at 10% interest is only $14,641 after four years. In other words, part of the loss must be funded with existing surplus, since premium plus investment income will not cover it.

The IRR analysis, when properly interpreted, says the same. The equityholders contribute $12,500 at policy inception: $5,000 to fund the underwriting loss and $7;500 as supporting surplus. The $22,500 of assets grow to $32,942 after four years, when the loss of $15,000 is paid and $17,942 is returned to the equityholders.

The internal rate of return is 9.42%. Since this is less than the cost of capital, the contract should be rejected. Since this is even less than the investment yield, the operating ratio exceeds 100%, and existing surplus must be used to fund the loss.

Let us now choose an extreme example: suppose the expected loss is $25,000. With a loss ratio above 200%, results are clearly unprofitable. But the IRR analysis says: equityholders contribute $27,500 at policy inception. Assets of $37,5000 grow to $54,904 after four years, when the $25,000 is paid and $29,904 is returned to the equityholders. The internal rate of return is 2%.

Eventually the IRR becomes negative, but expected losses rise considerably before this happens. The accompanying chart graphs the relationship between the IRR and the expected losses for this example. The horizontal axis shows expected losses in thousands of dollars, and the vertical axis shows the internal rate of return.



The problem is rate filing presentation and acceptance by regulators. When the contract is clearly unprofitable, the actuary should show the negative expected net present value and the insufficiency of net premiums plus investment income to fund the losses. For internal use, the IRR analysis is sound.

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