"Fundamentals of Individual Risk Rating," Part II

PART II

Retrospective Rating

Preface on the Workers Compensation Expense Program

In the simplest case, the Standard Premium for an insured is the manual premium adjusted by its experience rating modification. This is the best prospective estimate of the correct individual risk premium, but with expenses at a flat proportion of premium, appropriate for a small risk. As risk premium sizes increase, there is gradation of expenses, so that expense becomes a lower proportion of the larger risk premiums. This is reflected in manual rules by a series of premium discount rates which increase with increasing layers of premium. If the risk is not retro rated, it will pay the Standard Premium less the discount, which is called Guaranteed Cost Premium. If retro rating is selected, the discount will be realized as a reduction to expenses in the basic premium. What the risk finally pays, in either case, is called the net premium.

The manual rules effective January 1, 1986 give the following discounts for Stock Carriers:

First	$ 5,000	0.0%
Next	95,000	10.9
Next	400,000	12.6
Over	500,000	14.4

Premium size ranges which lead to discounts calibrated in tenths of a percent are tabulated in the manual. For example, the discount on a standard premium of size $125,000 would be

$$(\$5{,}000 \times 0.0) + (\$95{,}000 \times 0.109) + (\$25{,}000 \times 0.126) = \$13{,}505,$$

which is 10.8% of the $125,000. The Guaranteed Cost Premium would then be $111,495.

A. Specifications of the Plan

1) The Formula for Retrospective Premium[6]

The basic formula for retrospective rating can be stated in words:

Subject to minimum and maximum premium constraints,

Retrospective Premium = (Basic Premium+Converted Losses)x(Tax Multiplier)

$$= \left[\text{Basic Premium Factor} \times \text{Standard Premium} + \text{Actual Losses Incurred} \times \text{Loss Conversion Factor} \right] \times \text{Tax Multiplier}$$

This may be expressed mathematically as

$$\boxed{H \le R = (b + cL)T \le G} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

where

R = The retrospective premium,

b = The basic premium: Basic premium factor times standard premium (In most of the following, it will be convenient to think of standard premium as unity. Then the basic premium and the basic premium factor are the same.),

c = The loss conversion factor for expenses which vary with loss,

L = Actual losses incurred subject to any applicable limitation, and

T = The Tax Multiplier.

H = The minimum retrospective premium

G = The maximum retrospective premium

A General Convention

As mentioned with respect to b, a great many of the symbols (G, H, L, R and L_G, E, $\hat{E}$, F, e, S, etc. mentioned below) may be considered to be ratios of the standard premium. Thus, E is not only the amount of expected losses corresponding to the standard premium for a particular risk, or group of risks, but it also can represent the expected losses per dollar of standard premium.

2) Entry Ratios

Let

E = Expected Losses of the particular risk.

Define the entry ratio, r, as the ratio of actual to expected losses:

$$r = \frac{L}{E}$$

It is called the entry ratio because ratios of actual to expected losses serve as entry values for Table M. Table M is a table of excess pure premium ratios and it is used to determine the insurance charge.

Formula (1) may be rewritten:

$$R = (b + crE)T$$

Formula (1) can also be used to form expressions for the maximum and minimum premiums.

$$\boxed{\begin{array}{l} G = (b + cL_G)T \\ G = (b + cr_G E)T \end{array}} \quad \text{(2)}$$

where G = The maximum premium,

L_G = The incurred losses that will result in the maximum premium, and

r_G = $\frac{L_G}{E}$ = The entry ratio for G.

Also,

$$\boxed{\begin{array}{l} H = (b + cL_H)T \\ H = (b + cr_H E)T \end{array}} \quad \text{(3)}$$

where H = The minimum premium.

L_H = The incurred losses that will result in the minimum premium,

r_H = $\frac{L_H}{E}$ = The entry ratio for H.

For example, assume expected losses equal 120, losses that produce maximum premium equal 144, and losses that produce minimum premium equal 48. Then

$$r_G = \frac{L_G}{E} = \frac{144}{120} = 1.2 \quad \text{and} \quad r_H = \frac{L_H}{E} = \frac{48}{120} = 0.4.$$

3) The Basic Premium

The basic premium can be defined by

$$\boxed{b = e - (c-1)\,E + cI} \quad \text{. . . . .} \quad (4)$$

where e = The provision in the Guaranteed Cost Premium for total expenses and profit exclusive of taxes, expressed as a ratio to Standard Premium.

c = The Loss Conversion Factor,

cI = The "converted" insurance charge. (The term *converted* is usually omitted.)

4) The Insurance Charge

The insurance charge is given by

$$cI = c(X_G - S_H)E$$

where

X_G = The Table M charge at entry ratio r_G (expected losses above L_G, as a percentage of expected losses),

S_H = The Table M savings at entry ratio r_H (expected savings below L_H, as a percentage of expected losses).

Charges and savings X_r and S_r are listed in Table M for entry ratio(s) r. The formal definitions of these functions may be found in Section G. It should be noted that X_G and S_H are shorthand notation. For instance, X_G is an abbreviation for X_{r_G}. The problem, which will unfold below, is that charges can only be listed by entry ratio r. Finding r_G from G is the principle calculation problem in the application of retrospective rating.

Formula (4) can now be restated:

$$\boxed{b = e - (c - 1)E + c(X_G - S_B)\,E} \quad \text{. . . . .} \quad (5)$$

5) Expenses

If D is the discount appropriate for a risk of given size, T is the Tax Multiplier, and E is the expected losses (as a percent of standard premium), then the expense ratio (to standard premium) is

$$\boxed{e = \frac{(1 - D)}{T} - E} \quad \text{. . . . .} \quad (6)$$

Note that $T(e + E) = (1 - D)$ is the Guaranteed Cost Premium.

6) The Tax Multiplier

The Tax Multiplier is designed to recover certain costs of writing compensation coverage, one of which is of course premium tax. In any particular state, there are frequently several levies which are clearly not proportional to Standard Premium. These include charges for Insolvency Funds, Second Injury Funds, rehabilitation programs or other worthwhile projects. Any of these may be proportional to actual (net) premium or to loss, as the state sees fit. These are referred to as "taxes" and "assessments," respectively, although the former may include many things that are not actually taxes.

The Tax Multiplier has recently been revised to include a "tax" component for assigned risk subsidy. The calculation of and rationale for this are outside the scope of this monograph.

If the various taxes (premium-based levies) are $\tau = t_1 + t_2 + \ldots$ and the assessments are $\mu = u_1 + u_2 + \ldots$, then the Tax Multiplier is

$$\boxed{T = \frac{0.2 + E(1 + \mu)}{0.2 + E} \cdot \frac{1}{1 - \tau}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

where E is the permissible loss ratio in the plan. The factor of 0.2 accounts for the expense portion of the retro premium, which is not subject to assessments. The provision for assessments is on an average basis and is somewhat conservative.

7) The Balance Equations

Most insurance companies sell their plans by using maximum and minimum premium factors G and H. Insurance charges could be derived directly from Table M if maximum and minimum loss ratios r_G and r_H were selected instead. This section shows how r_G and r_H are derived from G and H.

The use of Table M is facilitated by two important relationships: The Table M entry difference and the corresponding Table M value difference. The Table M entry ratio difference is easily obtained by the subtraction of formulas (2) and (3).

$$G - H = (b + cr_G E - b - cr_H E)T$$

$$G - H = (r_G - r_H)cET$$

$$\boxed{r_G - r_H = \frac{(G - H)}{cET}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

The difference between losses associated with the minimum premium and losses associated with the maximum premium, loaded for loss adjustment expenses, must equal the difference between the minimum and maximum premiums (excluding taxes).

To obtain the Table M value difference it is necessary to use formulas (3) and (5) and the following relationship between X and S:

For any entry ratio r,

$$S_r = X_r + r - 1,$$

but the subscripts are usually left understood; thus,

$$\boxed{S = X + r - 1} \quad \text{(9)}$$

The proof of this relationship is outlined below. We use it in the following analysis:

$$\begin{aligned} H/T &= b + cr_H E \\ &= e - (c - 1) E + c (X_G - S_H)E + cr_H E \\ &= e - cE + E + c (X_G - X_H - r_H + 1)E + cr_H E \\ &= e - cE + E + c (X_G - X_H)E - cr_H E + cE + cr_H E \\ &= e + E + c (X_G - X_H) E \end{aligned}$$

$$\boxed{X_H - X_G = \frac{e + E - H/T}{cE}} \quad \text{(10)}$$

The difference between the Guaranteed Cost Premium and the Minimum Premium, excluding taxes, is the converted expected losses between L_H and L_G.

The use of maximum and minimum premium factors introduces a complexity into the insurance charge calculation. This is due to the fact that both the minimum and maximum premiums contain the same basic premium: expense, profit and insurance charge, excluding taxes. Since the insurance charge depends on the entry ratio to produce the maximum premium, and this insurance charge is a variable portion of the basic premium (which is a component of the maximum premium), a trial and error procedure is necessary to determine the correct entry ratios.

Two entry ratios, r_G and r_H, must be found which have a difference equal to the difference between the maximum and minimum premium factors, divided by the product of the converted expected loss ratio and the tax multiplier (formula (8)). In addition, the selected entry ratios must have corresponding excess pure premium ratios with a difference equal to the factor for loss and expense in the guaranteed cost premium less the minimum premium factor, both excluding the tax provision, all divided by the converted expected loss ratio (formula (10)).

Equations (8) and (10) are called the balance equations. A search in Table M must be used to solve them simultaneously. Their satisfaction assures that expected retrospective premium will equal guaranteed cost, i.e., the aggregate premiums should balance.

8) Table M

Current Table M lists insurance charges for entry ratios varying from 0.01 to 6.00. There are sets of charges, listed in columns, indexed by the charge at unity, X_1, in percent. The column to use is determined by the size of insured, and there are about seventy-five useful size groups.

The smallest eligible insureds have a high charge at r = 1, usually about 0.80, thus enter column 80. The larger insureds have low charges X_1 at unity, and may qualify for 0.05 in column 5.

B. Insurance Charge Reflecting Loss Limitations

The Retrospective Rating Plan includes the optional provision of limiting ratable losses to amounts of $25,000, $50,000, etc. per accident.

If a loss limit is selected, the Retrospective Rating formula changes. In this case,

$$\boxed{H \le R = (\hat{b} + cF + c\hat{L})T \le G} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

where

$\hat{b}$ is the basic premium (factor) calculated as described below, and

F is the ELF, a tabular factor applicable to standard premium to generate expected loss excess of the selected retention. F = E • LER, where LER is the (pure) loss elimination ratio for losses excess of the retention.

$\hat{L}$ is the actual losses as limited by the selected per occurrence loss limitation

The expected limited losses we denote $\hat{E}$

$$\boxed{\hat{E} = E - F} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

where E is the expected unlimited losses.

It should be observed that the actual subject losses of this plan are *limited* and, as such, have a different aggregate distribution than the unlimited losses.

Excess pure premium ratios for this distribution should come from a limited loss Table M. We denote such charges and savings with a hat:

$\hat{X}_G$ = charge for maximum

$\hat{S}_H$ = savings for minimum

The limited loss table should be indexed by entry ratios of actual *limited* losses to expected *limited* losses. (We will omit hats on r_G and r_H)

Now

$$G = (\hat{b} + cr_G \hat{E} + cF)\, T$$

$$H = (\hat{b} + cr_H \hat{E} + cF)\, T$$

$$G - H = (r_G - r_H)\, c\hat{E}T$$

Thus

$$\boxed{r_G - r_H = \frac{G - H}{c\hat{E}T}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

is the first balance equation.

For the second balance equation, use $\hat{b} = e - (c\text{-}1)E + c(\hat{X}_G - \hat{S}_H)\hat{E}$ in

$$\frac{H}{T} = \hat{b} + cr_H \hat{E} + cF$$

Then $\frac{H}{T}$

$$= e - (c\text{-}1)E + c(\hat{X}_G - \hat{S}_H)\, \hat{E} + cr_H\hat{E} + cF$$

$$= e - cE + E + c(\hat{X}_G - \hat{X}_H - r_H + 1)\, \hat{E} + cr_H\hat{E} + cF$$

$$= e - cE + E + c(\hat{X}_G - \hat{X}_H)\, \hat{E} - cr_H\hat{E} + c\hat{E} + cr_H \hat{E} + cF$$

$$= e + E + c(\hat{X}_G - \hat{X}_H)\, \hat{E} - cr_H\hat{E} + cr_H \hat{E} + c\hat{E} + cF - cE$$

these cancel these cancel

Thus,

$$\frac{H}{T} = e + E + c\,(\hat{X}_G - \hat{X}_H)\,\hat{E}$$

and

$$\boxed{\hat{X}_H - \hat{X}_G = \frac{e + E - H/T}{c\hat{E}}} \qquad (14)$$

is the second balance equation. These differ from equations (8) and (10) in the substitution of expected limited for expected total losses in the denominators, but not the numerator of (14).

The filed plan approximates Limited Loss Table M by shifting columns to one with lower charges. For a selected loss limit, we adjust expected losses by multiplying expected losses E by an adjustment factor

$$E \rightarrow E \cdot \frac{1 + 0.8\ LER}{1 - LER}$$

$$= E \cdot \frac{1 + 0.8\ (F/E)}{1 - (F/E)}$$

to assign the curve of a larger size group. The standard premium used in the retro formula is unchanged.

C. Simplified Construction of Table M[a]

1) Construction

TABLE 1						
(1) LOSS RATIO	(2) ENTRY RATIO	(3) NO. OF RISKS AT LOSS RATIO	(4) NO. OF RISKS OVER GIVEN LOSS RATIO	(5) LOSSES OVER GIVEN LOSS RATIO	(6) (5)/60 EXCESS RATIO	(7) (6)+(2)-1 SAVINGS
0%	0	0	10	60	1.000	0
10	.167	1	9	50	.833	0
20	.333	1	8	41	.683	.016
30	.500	0	8	33	.550	.050
40	.667	1	7	25	.417	.084
50	.833	0	7	18	.300	.133
60	1.000	4	3	11	.183	.183
70	1.167	0	3	8	.133	.300
80	1.333	1	2	5	.083	.416
90	1.500	0	2	3	.050	.550
100	1.667	1	1	1	.017	.684
110	1.833	1	0	0	.000	.833
120	2.000	0	0	0	.000	1.000
		10	60			

Table 1 shows the development of a hypothetical Table M using the experience of ten risks with equal standard premium. The number of risks at each loss ratio is given in Column (3). Column (4) is simply an upward summation of Column (3), and should be interpreted as the number of risks over a given loss ratio. For example, since there are only ten risks in the distribution, Column (4) should be read as ten risks with loss ratios in excess of 0%, nine risks with loss ratios in excess of 10%, eight risks with loss ratios in excess of 20%, etc.

Column (5) is an upward summation of Column (4) and is needed to develop the values in Column (6) which are the Table M charges or excess pure premium ratios. Column (6) provides the percentage of total losses in excess of a given loss ratio. This column should be read in the following manner: 100% of the losses are in excess of the 0% loss ratio, 83.3% of the losses are in excess of the 10% loss ratio, 68.3% of the losses are in excess of the 20% loss ratio, etc.

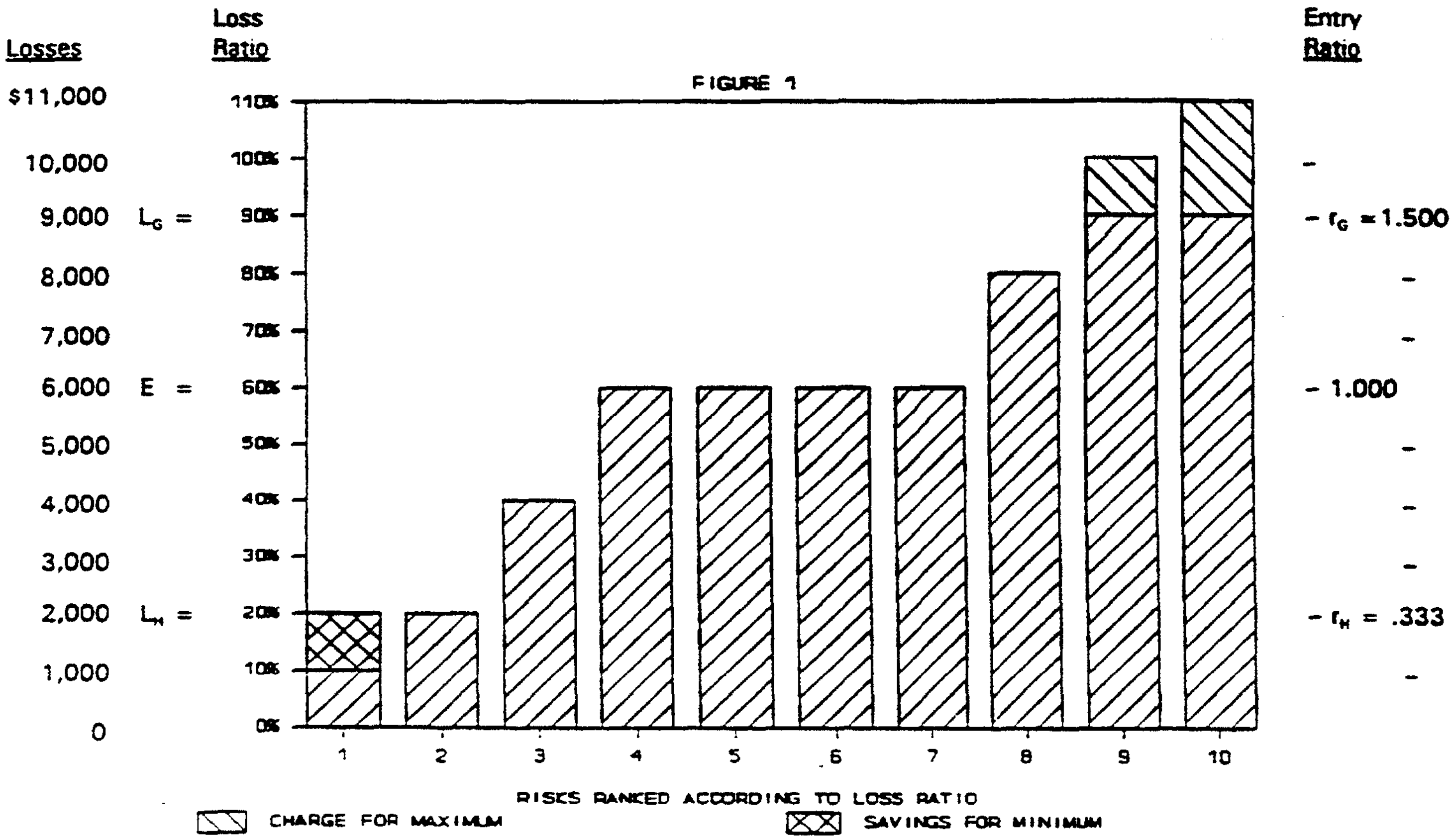
2) Insurance Charge and Savings

Figure 1 is constructed from the same ten risks with loss ratios as indicated in Table 1. On the graph, these risks are arrayed in order of increasing loss ratio from left to right. It is assumed that each risk has a standard premium of $10,000. The vertical axis of the graph shows the dollar loss and the loss ratio for each risk.

Assume a hypothetical retrospective rating contract with a maximum premium produced by a 90% loss ratio and a minimum premium produced by a 20% loss ratio. The selection of maximum and minimum loss ratios is intentional and serves to substantially simplify the calculation. In practice, most operating companies do not select maximum and minimum loss ratios. Instead selected maximum and minimum premiums are used.

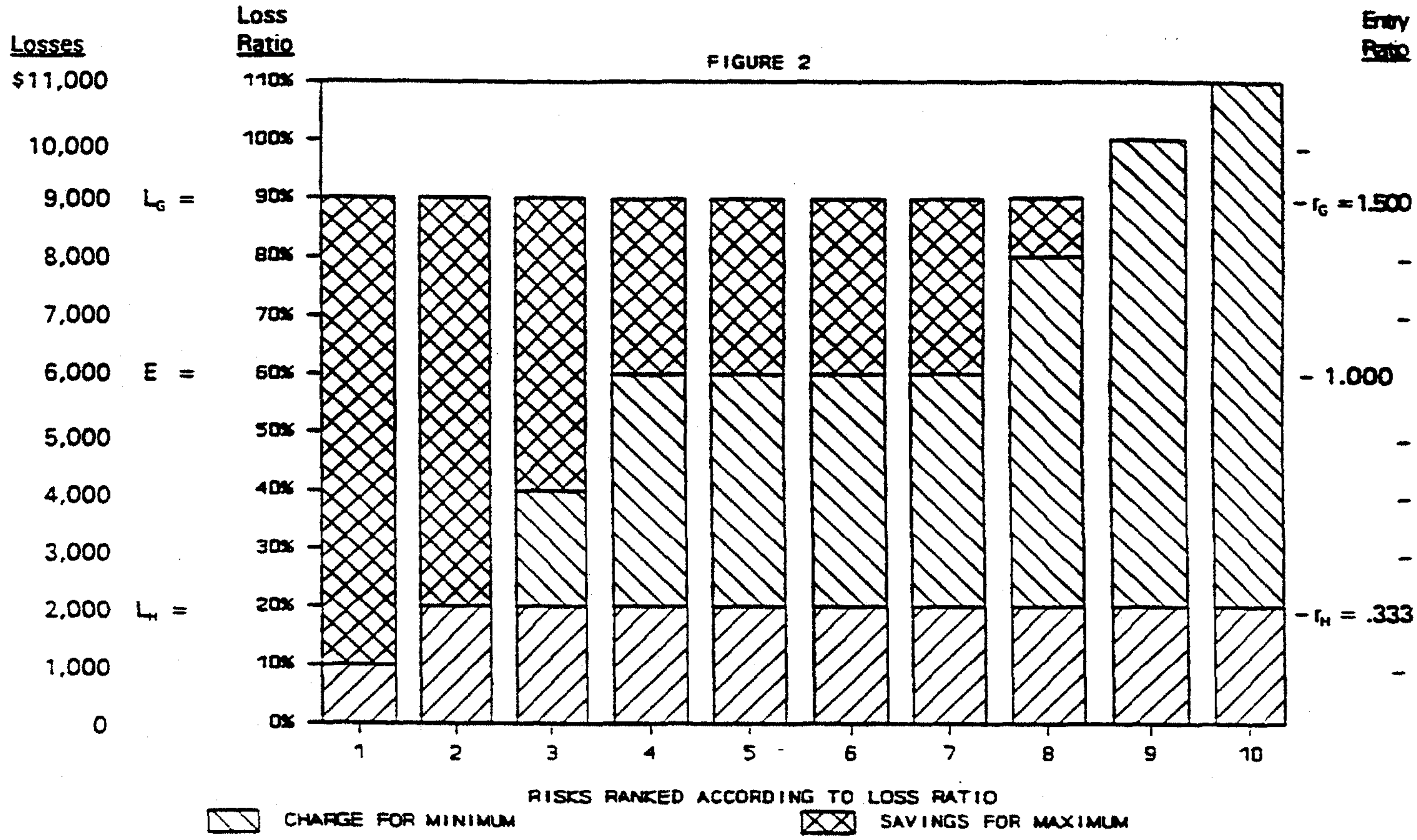
The symbol L_G represents the maximum loss ratio, and the symbol L_H represents the minimum loss ratio. In addition, another symbol, r, is introduced on the vertical axis. This is known as the entry ratio or the ratio of rated (actual) to expected losses. The expected loss ratio in this example is 60%. The 90% maximum loss ratio is equivalent to an entry ratio of 1.50 (r_G = 1.50). In other words, the maximum loss ratio is 150% of the expected loss ratio. The entry ratio at the minimum is .333 (r_H = .333), which means that the minimum loss ratio is 33.3% of the expected loss ratio.

The entry ratios are used to allow Table M to be applied in various states for compensation and other lines of insurance. Since we are concerned only with ratios of rated to expected losses, differences in underlying expected loss ratios are not significant.



Losses
Loss Ratio
Entry Ratio
$11,000
10,000
9,000
8,000
7,000
6,000
5,000
4,000
3,000
2,000
1,000
0
L_G =
E =
L_H =
110%
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
r_G = 1.500
1.000
r_H = .333
1 2 3 4 5 6 7 8 9 10
RISKS RANKED ACCORDING TO LOSS RATIO
CHARGE FOR MAXIMUM
SAVINGS FOR MINIMUM

FIGURE 1



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The total standard premium for all risks is $100,000, and the total losses are $60,000. In the example, the maximum premium is produced by a 90% loss ratio or an entry ratio of 1.50. The excess pure premium ratio corresponding to the maximum loss ratio is displayed in Figure 1. Symbolically, the insurance charge or excess pure premium ratio at the maximum is described as X_G. The upper shaded area in Figure 1 is 5% of the total area representing all losses for the ten risks. In other words, risk number nine, which had a 100% loss ratio, had losses which were $1,000 in excess of the maximum loss ratio; and risk number ten, with a 110% loss ratio, had losses $2,000 above the maximum loss ratio. This $3,000 of excess losses represents 5% of the total losses for all risks; and, therefore, the excess pure premium ratio, or charge for the maximum premium, is 5% of the total losses.

A quick reference to Table 1 will show that the excess pure premium ratio for a 90% loss ratio is 5% as developed by the double summation method. The purpose of Figure 1 is simply to illustrate the true nature of an excess pure premium ratio.

Under a retrospective plan in which there is no specified minimum premium, 5% would be the final insurance charge. Many risks, however, desire minimum premiums, and it is necessary to subtract the savings arising from the minimum. The savings is denoted by S_H and is represented by the cross-hatched area in Figure 1. The area of that region is one-sixtieth (1.7%) of the area representing the total losses and this constitutes the savings credit.

The savings can be more easily understood if it is recognized that only risk number one, with a 10% loss ratio, would actually have been subject to the minimum premium in the example given. This risk's losses were $1,000 less than the losses that would have produced the minimum premium. One thousand dollars is 1.7% of the total losses. The net insurance charge is 5.0% minus 1.7%, or 3.3%.

It should also be noted that there exists a charge for the minimum, X_H, and savings for the maximum, S_G; these are displayed graphically in Figure 2.

3) Summary

Total Premium	$100,000	Average Premium	$10,000
Total Losses	$ 60,000	Average Loss Ratio	60%

Description	Symbol	Value
Charge for Maximum	$= X_G$	$\frac{1,000 + 2,000}{60,000} = .050$
Charge for Minimum	$= X_H$	$\frac{60,000 - 9(2,000) - 1,000}{60,000} = .683$
Savings for Minimum	$= S_H$	$\frac{1,000}{60,000} = .017$
Savings for Maximum	$= S_G$	$\frac{8,000+7,000+5,000+4(3,000)+1,000}{60,000} = .55$

It should be mentioned that this hypothetical Table M is inappropriate for a $10,000 risk in the 1990's. Such risks have a far more skewed loss ratio distribution, and consequent larger excess ratios.

D. Illustrations

1) Selected Loss Ratio Example

This example is a practical demonstration using the Table M previously constructed. A $10,000 risk insured with a mutual company is placed on a retrospective plan in which the maximum premium is reached at a 90% loss ratio and the minimum premium is reached at a 20% loss ratio. Expected losses are $6,000 and the factor for total losses, expenses and profit exclusive of taxes is 0.958. Values of the provisions for expenses for various risk sizes have been compiled by the National Council in tables captioned "Table of Compensation Expense Ratios - Excluding Taxes." A loss conversion factor of 1.30 and a premium tax rate of 3%, leading to a tax multiplier of 1.0/0.97 = 1.031, are assumed for the example. It can be seen from formula (6) that 1 - D = .988, and so the premium discount on this size insured is 1.2%.

	Values	Factors
Standard Premium	10,000	1.000
Guaranteed Cost Premium	9,880	.988 = $1 - D$
Provision for Losses and Expenses exclusive of Taxes	9,580	.958 = $E+e$
Expected Losses	6,000	.600 = E
Loss Conversion Factor	1.30	1.30 = c
Tax Multiplier	1.031	1.031 = T
Selected Maximum Loss Ratio	L_G = 90%	r_G = 1.50
Selected Minimum Loss Ratio	L_H = 20%	r_H = .333

Formula (10) is used to derive the minimum premium ratio. The total expenses, the permissible loss ratio, the excess pure premium ratios at the minimum and the maximum, the converted expected loss ratio, and the tax rate are known values; and the minimum premium ratio can therefore easily be found to equal 47.8% of the standard premium.

$$H/T = e + E - c(X_H - X_G)E \qquad \text{(from (10))}$$

$$.970\,H = .958 - (1.30)\,(.683 - .050)\,(.600)$$

$$= .464$$

$$H = .478$$

Formula (3) is used to calculate the basic premium factor, which is equal to 20.4% of the standard premium.

$$H/T = b + cL_H$$

$$.464 = b + 1.30\ (.20)$$

$$b = .204$$

In addition, formula (2) is used to calculate the maximum premium factor, which is equal to 141.8% of the standard premium.

$$G/T = b + cL_G$$

$$.970\ G = .204 + 1.30\ (.90)$$

$$= 1.374$$

$$G = 1.418$$

The unusual values for the minimum and maximum premium factors occur because of the selection of given minimum and maximum loss ratios.

In order to check the basic premium derived from formula (3), the factor of .204 can be separated into its components, the expense in the basic and the insurance charge. If these two components are calculated separately, their sum should equal the value of b independently derived from formula (3).

Expense in Basic $= e + E - cE = .178$

Savings at Minimum $= S_H$

$$= X_H + r_H - 1$$

$$= .683 + .333 - 1$$

$$= .017$$

Insurance Charge $= c(X_G - S_H)E$

$$= (1.30)(.050 - .017)(.600) = .026$$

Total Basic = Expense in Basic + Insurance Charge

= .178 + .026 = .204

This example demonstrates the simplicity of calculating a retrospective plan if given maximum and minimum loss ratios are selected.

2) A Special Case, Minimum equals Basic

Retrospective rating calculations are substantially simplified if a plan is elected in which there is no specified minimum premium. Consider formula (2).

$$G/T = b + cL_G$$

$$= b + cr_G E$$

$$= e - (c - 1)\,E + c\,(X_G - S_H)\,E + cr_G E$$

$$= e - (c - 1)\,E + c\,(S_G - r_G + 1 - S_H)\,E + cr_G E$$

$$= e - cE + E + c\,(S_G - S_H)\,E - cr_G E + cE + cr_G E$$

$$\boxed{G/T = e + E - c(S_H - S_G)\,E} \qquad (15).$$

But when the minimum premium equals the basic times the tax multiplier, $S_H = 0$.

$$G/T = e + E + cS_G E$$

$$cS_G E = G/T - e - E$$

$$\boxed{S_G = \frac{G/T - (e + E)}{cE}} \qquad (16).$$

Formula (16) is the key to this special case. Since all values on the right-hand side of the equation are known, the savings at the maximum can be quickly calculated. That savings ratio has a corresponding entry ratio (for the maximum) which can be substituted into formula (2) to determine the basic ratio. If, for further analysis, it is desirable to divide the basic factor into components, this is done simply by determining the amount of expense in the basic; and the residual insurance charge is equal to the total basic minus the expense provision.

As an example, consider a risk with a standard premium of $10,000 insured by a stock company. Using our hypothetical Table M, the expected losses are $6,000.

$e + E = .917$ ($E = .600$; e is taken from the Table of Compensation Expense Ratios—Excluding Taxes, based on the Stock Premium Discounts in the Preface)

$c = 1.10$

Selecting a maximum of 1 times the tax multiplier,

$$G/T = 1.00$$

From formula (16),

$$S_G = \frac{1.00 - .917}{(1.10)\ (.600)} = .126$$

Using linear interpolation in Table 1, $r_G = 0.81$

From formula (2),

$$1 = b + (1.10)(0.81)(0.60)$$

$$\rightarrow b = .465$$

Expense in Basic $= .917 - (1.10)(.600) = .257$

Insurance Charge $= .465 - .257 = .208$

Notice that the Table M charge for $r_G = 0.81$ is about .316. With a minimum entry ratio of 0, the net insurance charge is then $(1.10)(0.60)(.316) = .209$, which agrees with our backdoor calculation.

This example illustrates the simplicity of the calculation. The rating values are applicable to stock companies, but the general principles would apply to mutual companies.

This type of calculation is used extensively in the insurance business to determine various sliding scale dividend plans used in certain sections of the country.

E. **Aggregate Balance**

The same ten risks which produce the loss ratios used in constructing the simplified Table M were run through retrospective rating calculations using the values developed in the first example. Each risk was retrospectively rated based on the losses actually incurred subject to the maximum and minimum premium factors. The sum of the retrospective premiums for the ten risks is $95,800, which is equal to the total premium before tax which would have been collected on a guaranteed cost basis.

(1) Risk Number	(2) L Losses	(3) cL Converted Losses (2)x1.30	(4) $cL + b$ Converted Losses Plus Basic (3) + $2,040	(5) Retrospective Premium (before Tax): (4) Subject to Max. or Min.
1	$1,000	1,300	$3,340	$4,640
2	2,000	2,600	4,640	4,640
3	4,000	5,200	7,240	7,240
4	6,000	7,800	9,840	9,840
5	6,000	7,800	9,840	9,840
6	6,000	7,800	9,840	9,840
7	6,000	7,800	9,840	9,840
8	8,000	10,400	12,440	12,440
9	10,000	13,000	15,040	13,740
10	11,000	14,300	16,340	13,740
		$78,000	$98,400	$95,800

Total on Guaranteed Cost Basis (exclusive of Taxes)
= $10,000 (.958) = $9,580 x 10 risks = $95,800.

This demonstration is useful in understanding the actuarial balance between retrospective and prospective rating. If a group of risks with a distribution of loss ratios similar to the distribution underlying the Table M values are rated retrospectively, the total retrospective premium will be approximately equal to the total premium collected on a guaranteed cost basis.

It should be recognized, however, that if the variation of loss ratios of the retrospectively rated business is different than anticipated by Table M, or if the aggregate loss ratio on the retrospectively rated business differs from the expected loss ratio, the total retrospective premium may not balance to guaranteed cost on that particular book of business.

F. Summary

The foregoing explanations are intended to remove some of the mystery from retrospective rating. The insurance charge concept is essentially a simple one. The key statistic is the percentage of total losses in excess of a given loss ratio or entry ratio.

The algebraic development is relatively straightforward. In general, the selection of maximum and minimum premium factors makes it necessary to find two entry ratios with a given difference which have excess pure premium ratios with another given difference. As has been observed, this necessitates a trial-and-error exploration of Table M.

Retrospective rating is designed to be in balance with prospective rating. Given the same loss ratio distribution pattern as contemplated by Table M and an emerged overall loss ratio equal to the expected, there should be no difference in earned premiums. However, if the total block of retrospectively rated business suffers from a higher-than-average loss ratio, it is an indication that insurance charges may be inadequate.

In this development, no special attention was paid to dividend provisions. The formulas apply equally if dividend considerations are introduced, since both sides of each equation are multiplied or divided by a given factor.

G. <u>Excess Pure Premium Ratio, Formal Definition</u>

1) Definition

The excess pure premium ratio as defined by Dorweiler[9] is, for an individual risk, the ratio of the risk's losses in excess of a specific selected loss ratio to the total losses for the risk. For a group of risks, it is the ratio of the aggregate of the losses in excess of the selected loss ratio for each risk to the aggregate total losses of the group. The validity of the definition is not changed if the words "entry ratio" are substituted for the words "loss ratio."

2) The Continuous Case[10]

The Table M functions can easily be stated in the continuous form. For entry ratios r, there is a distribution F with density f such that:

$$\int_0^\infty f(r)\, dr = 1$$

$$\int_0^\infty r \cdot f(r) dr = 1$$

We now define the excess pure premium ratio $X(r_0)$:

$$\boxed{X(r_0) = \frac{\int_{r_0}^\infty (r - r_0) f(r)\, dr}{\int_0^\infty r\, f(r)\, dr}} \qquad (17)$$

The derivatives of $X(r_0)$ can be obtained if the following rule of calculus is recalled:

1. If $F(x) = \int_a^x f(y)\, dy$, then $\frac{dF(x)}{dx} = f(x)$

2. If $F(x) = \int_x^b f(y)\, dy$, then $\frac{dF(x)}{dx} = - f(x)$

Applying this rule to $X(r_0)$,

$$X(r_0) = \int_{r_0}^{\infty} (r - r_0)\, f(r)dr$$

$$X'(r_0) = \frac{dX(r_0)}{dr_0} = \frac{d}{dr_0} \int_{r_0}^{\infty} (r - r_0)\, f(r)dr$$

$$= \frac{d}{dr_0} \int_{r_0}^{\infty} rf(r)dr - \frac{d}{dr_0} \left[r_0 \int_{r_0}^{\infty} f(r)dr\right]$$

$$= - r_0 f(r_0) - r_0 \cdot \frac{d}{dr_0} \int_{r_0}^{\infty} f(r)dr - \int_{r_0}^{\infty} f(r)dr \cdot \frac{d(r_0)}{dr_0}$$

$$= - r_0 f(r_0) + r_0 f(r_0) - \int_{r_0}^{\infty} f(r)dr$$

$$\boxed{X'(r_0) = - \int_{r_0}^{\infty} f(r)dr} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (18).$$

$$X''(r_0) = \frac{d}{dr_0} \left(- \int_{r_0}^{\infty} f(r)dr\right)$$

$$= - \frac{d}{dr_0} \int_{r_0}^{\infty} f(r)dr$$

$$= - (- f(r_0))$$

$$\boxed{X''(r_0) = f(r_0)} \qquad \text{(19).}$$

Thus, the underlying risk distribution can be determined from the second derivative of the excess pure premium ratios (or the second difference from the table).

The savings is defined as

$$\boxed{S(r_0) = \frac{\int_0^{r_0} (r_0 - r)\, f(r)dr}{\int_0^{\infty} r\, f(r)dr}} \qquad \text{(20),}$$

The student should show

$$S(r_0) = X(r_0) + r_0 - 1.$$

This can be done easily by subtracting (17) from (20).