THE UNDERWRITING PROFIT PROVISION

by

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OUTLINE

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THE UNDERWRITING PROFIT PROVISION

I. INTRODUCTION

A. Underwriting Profit Provisions and Actual Underwriting Profits

This paper is a presentation of methods used for computing the provision for underwriting profit in property and casualty insurance rates. The provision for underwriting profit is one component of an actuarially derived premium rate. Adding the provision for underwriting profit to the sum of provisions for losses and expenses yields the total premium rate. Here the loss provision for property lines is assumed to include an adequate load for the long-run expectation of catastrophe losses. If the premium rate is greater than the sum of the loss and expense provisions, then the underwriting profit provision is positive. The underwriting profit provision could also be negative.

Actual underwriting profit is the difference between the premium and the sum of losses and expenses. Since the rating provisions for losses and expenses may differ from the actual losses and expenses, the underwriting profit provision may differ from the actual underwriting profit.

B. Historical Overview

There are a variety of methods in use for determining profit provisions in state rate filings and for calculating profit targets. The methods are often based on different fundamental approaches and employ different sorts of data. It is instructive to review the history to see how such a situation developed. From 1921 until the 1960's, a 5% underwriting profit provision was accepted as appropriate for most lines of insurance, with the exception of Workers' Compensation which had a 2.5% load. Though they had no strong theoretical justification, these loads were used for many years without serious challenge. However, this changed in the 70's and early 80's. The impact of investment income and income taxes on the bottom line became increasingly important. Some companies apparently prospered even though actual underwriting profits were less than those provided for in the manual rates. Others even operated at an underwriting loss.

Meanwhile the rising price of insurance made the affordability of insurance a contentious issue in state politics. In some jurisdictions, the controversy led to changes in the calculation of the underwriting profit provision. The changes were often intended to simply lower the price of insurance by lowering the rates. Despite the evidence of the early 80's that some insurance companies would compete on price to the point of insolvency, the tide of increased profit provision regulation continued unabated.

As a result, the ratemaking actuary now faces a variety of algorithms for computing the underwriting profit provision. In some states, an investment income percentage is subtracted from a more traditional provision to yield the final profit figure. In others, the provision is set so that it would theoretically give the insurance company a total rate of return deemed proper by regulators.

C. Underwriting Profit and Total Profit

To understand the debate about the underwriting profit provision, one needs a clear understanding of the distinction between total profit and underwriting profit. The total profit for an insurance company is roughly the sum of underwriting gains plus investment gains less income taxes. The investment gains include the sum of interest, dividends, and real estate income plus realized capital gains. If the investment income is large enough, the company can make an overall profit, even if underwriting profits are negative. Negative underwriting profits can also reduce income taxes by offsetting otherwise taxable investment income.

In principle, one might want to define an underwriting profit provision as adequate if it would lead to an adequate total profit net of investment income and income taxes. Though this total return concept sounds straightforward, it is not obvious how to apply it in ratemaking. It also raises the question of what is an adequate total return.

Perhaps the major problem in applying a total return approach to ratemaking is that ratemaking is done on a <u>prospective policy year</u> basis, while total return is most commonly measured on a <u>calendar year</u> basis. The two perspectives are not synonymous. Generally Accepted Accounting Principles Return on Equity (GAAP ROE) is the usual measure of return for a stock insurance company. GAAP ROE is a <u>calendar year</u> return defined as the ratio of calendar year GAAP income over GAAP equity. A key point about calendar year return is that it is partly dependent on the past; the return in any calendar year includes income attributable to prior writings. However, rates are made looking to the future and rates made today will influence earnings in many subsequent calendar years. How should one define a policy year return and how should that return relate to calendar year return on equity ? What is the income and equity for a policy year (or a policy) and how does one account for the time value of money in figuring out the return? Even if such a return could be defined, how does one select an appropriate target return?

A different but related approach to selecting an appropriate underwriting profit provision is to set rates with an explicit offset for investment income on policyholder-supplied funds. Policyholder-supplied funds could be conceptually viewed as the accumulated balance of paid premiums less the sum of loss and expense payments. (One should also subtract any declared profits, as these have presumably been paid out as stockholder dividends or put into surplus used to back further writings.) The relative amount of policyholder-supplied funds by line is related to the time lag between the receipt of premiums and the payout of losses and expenses. The investment income offset due to policyholder-supplied funds could be estimated by projecting underwriting cash flows prospectively <u>or</u> by looking at calendar year data. When taking a calendar year approach, one must estimate what portion of the total investment income is generated by policyholder-supplied funds. The remaining portion is earned on stockholder-supplied funds, and some argue it therefore should not be credited to the policyholders.

The stockholder-supplied funds include the insurance company's surplus plus amounts needed to offset the portion of the loss and expense reserves not covered by premiums. The stockholders may have also supplied some cash to cover pre-paid expenses, but these sums, being already paid out, are not available for investment.

The conceptual split between policyholder and stockholder supplied funds also helps in gauging the adequacy of a given total return. As previously mentioned, it is possible for a company to have a positive total profit even if underwriting profits are negative. However, a positive total profit does not necessarily mean a stock company has generated a fair return, or even a return above breakeven, for its stockholders. For example, if the company must use some of the investment income on stockholder-supplied funds to cover underwriting losses, the total profit might still be positive but the return would be below breakeven.

D. Five Types of Underwriting Profit

To clarify matters, distinctions should be made between various types of underwriting profit.

(1) There are **underwriting profit provisions** included in **manual rates** and in rate filings to change manual rates.

(2) There are **corporate target underwriting profit provisions** which insurance company management may request from the actuary as information when deciding how much to charge. For a stock company, these are presumably sufficient to yield an expected return to stockholders comparable to the market yield on alternative investments of similar risk.

(3) Related to target profit provisions are **breakeven underwriting profit provisions**, defined to generate an expected return to stockholders equal to the rate of return on risk-free investments such as U.S government bonds. A breakeven profit provision provides the stockholder no compensation for putting his/her funds at risk of having to cover underwriting losses. This ignores the risk-return trade-off (higher return for higher risk) central to modern financial theory.

(4) The **charged underwriting profit provision** may differ from the provision in the manual rate as well as from the target and the breakeven provisions. The charged rate is obtained by applying experience and schedule rating modifications and other adjustments (eg. premium discount in Workers' Compensation) to the manual rate.

(5) **Actual underwriting profits** may differ from the provisions charged because the provisions for losses and expenses will seldom be exactly accurate and because actual catastrophe losses rarely match catastrophe provisions in any one year. If actual underwriting profits are consistently below the provisions charged, then the methods used to estimate losses and expenses should be examined for bias.

The focus of this paper will be on methods used for determining profit provisions in state rate filings or for calculating profit targets and breakeven profit provisions. However, it is beyond the scope of this paper to give any advice on what should be charged or to provide detailed analysis of actual profits.

E. Regulation

Underlying the controversy about the profit load provision is a more profound disagreement about the theory under which rates and profit provisions should be regulated. Under a "rate of return regulation" approach, the state ought to stringently regulate premium rates so that companies should be able to anticipate adequate, but not excessive, total returns. The definition of an adequate target return presumably reflects relative risk by line of business. This approach has been justified by arguing that insurance company rates should be regulated in a fashion similar to the way that utility company rates are regulated. Utility companies enjoy a natural monopoly position and their rates are rather strictly controlled. Under the Hope Decision, utility rates are supposed to be set so that the utility company should be able to achieve a rate of return commensurate with the risk involved.

Under a constrained free market theory, if the manual rate is reasonably adequate and if there is some limited flexibility to price insurance at a level different from the manual rate, then the competitive market should force the price of insurance to the optimal level. Within this context, the manual rates can be thought of as a starting point against which adjustments can be made by individual companies responding to the dynamics of the market. If the manual rate is set too low and overly stringent state regulation of prices prevents competitive adjustments, market disruptions could theoretically result.

The analogy between utilities and insurance companies has also been criticized. The insurance business is not a natural monopoly; nor is it a highly concentrated oligopoly. The large number of firms and the relatively low market share of even the biggest insurers are cited as evidence that a competitive market model is the appropriate one.

This brief and incomplete discussion of profit provision regulation theories is only intended to make one aware that there are fundamental differences in approaches to the issue.

F. Different Models

No matter what side of the issue one takes, the actuary should still be versed in the calculation of underwriting profit provisions. In a rate filing context, the calculation of the profit provision should conform with applicable state regulation. In a competitive environment, the actuary should be able to provide company management with the target profit provision needed to achieve an adequate total return.

After some preliminaries, seven models will be presented. Variants of some of these models have been used or mandated for use in state rate filings, but the specifics of state regulation will not be discussed. Simple examples will be worked out with each of these models, and some limited comparisons will be made. However, since not all of the models use the same type of data, and since the choice of parameters is often crucial, one should take extreme care in generalizing from these examples.

The first algorithm is a **Calendar Year Investment Offset Procedure**. In this procedure, the traditional profit load is reduced by subtracting an investment figure derived from calendar year data. The second algorithm, the **Present Value Offset Procedure**, also features an offset to the traditional profit load. Here the offset reflects the difference in the present values of the loss payment patterns of the line of business under review and some short-tailed reference line. The third algorithm, the **Calendar Year Return Method**, does not use the traditional profit load. Instead, the profit provision is set so as to achieve a selected target return, where return is calculated with investment income figures derived from calendar year data.

In the fourth model, the **Present Value of Income over the Present Value of Equity** method, a present value return is defined as the ratio of the present value of accounting income over the annualized present value of equity. The underwriting profit is adjusted until the present value return is equal to a target return.

In the fifth algorithm, the **Present Value Return on Cash Flow** model, the underwriting profit provision is set so that the present value of the underwriting cash flows and investment income on investible equity less income tax payments is equal to the present value of the changes in equity. In this algorithm, the underwriting cash flows, investment income on investible equity, and tax payments are discounted using the rate of return on investments, while the changes in equity are discounted at the target rate of return.

The sixth algorithm, the **Risk-Adjusted Discounted Cash Flow Method**, does not use the traditional profit load nor a target return. Rather, the premium is calculated so that the present value of premium payments will equal the present value of paid losses, expenses, and income taxes. However, since the loss payments are not known with certainty, they are discounted at a new money risk-adjusted rate which is lower than the new money risk-free rate.

Finally, an **Internal Rate of Return on Equity Flows** algorithm will be presented. The strategy in this algorithm is to estimate flows of money between a hypothetical stockholder and a hypothetical company formed to write one policy. These "equity flows" are related to the projected stream of total income and the assumed surplus requirements associated with the writing of the policy. The profit provision is found by adjusting the premium until the yield on the equity flows is equal to a target return.

Which, if any, of the models is "right" is not a question that can be easily decided. Each method has its adherents. Implicit in the use of any model in a rate regulatory context is a theory on the role of the state in controlling insurance prices and assumptions about the competitiveness of the market. Once beyond such overall contextual issues, there are a multitude of modelling questions to consider. Should surplus should be in the model, and if so, how should the surplus requirement be determined? Should risk be reflected in the model, and if so, how ? Is it better to use cash flows or income flows? How does one reflect income taxes? After questions of model construction are resolved, there are parameter selection issues. What is the appropriate rate for discounting the flows? Should new money or embedded yields be used? What is the right target return?

Even if one is only considering profit targets so that the role of the state can be momentarily ignored, there is no simple empirical test of which model is better. While one can compare profit targets with actual profits, a judgement on the adequacy of actual profits depends on some underlying economic theory. In other words, a model that accurately estimates what underwriting profits <u>were</u> does not necessarily say what they <u>should have been</u>. This is conceptually different from the situation encountered when evaluating the performance of algorithms for estimating losses and expenses. With loss and expense estimation procedures, one could, in principle, compare the differing prior estimates against subsequent results. No external theory would be required.

In summary, each actuary must make his or her own decision about which underwriting profit provision model best captures the situation and is most appropriate in a given context.

II. UNDERWRITING PROFIT PROVISION IN THE PREMIUM FORMULA

Let L be the prospective estimate of loss and let c denote the ratio against loss of expenses proportional to loss. Let FX denote the provision for fixed expenses. Assume unallocated and allocated loss adjustment expenses are contained in some fashion in the loss, the expenses proportional to loss, or the fixed expenses. Write VR to stand for the variable expense ratio where variable expense encompasses all expenses such as commissions and premium taxes that are proportional to premium. Write U for the underwriting profit provision and let CR stand for the combined ratio. To summarize the notation:

(II.1)

Symbol	Description
L	**Loss**
c	**Loss Proportional Expense Ratio**
FX	**Fixed Expense**
VR	**Variable Expense Ratio**
CR	**Combined Ratio**
U	**Underwriting Profit Provision**

The premium, P, is given as:

(II.2) $P = (1+c)L + FX + P*VR + P*U$, which can be rewritten as:

(II.3) $P = ((1+c)L + FX)/(1-(VR+U))$

The combined ratio is the ratio of losses and expenses to premium:

(II.4) $CR = VR + ((1+c)L+FX)/P$

The combined ratio is related to the underwriting profit provision as follows:

(II.5) $U = (1-CR)$

Note that a profit provision of -100% does not correspond to a $0 premium, but rather a combined ratio of 200%. In the following chapters, the expenses proportional to loss will be omitted in the interest of simplifying matters.

III. CALENDAR YEAR INVESTMENT INCOME OFFSET PROCEDURE

Under the procedure to be discussed in this section, an offset is made against the traditional underwriting profit provision in order to arrive at a final number. The offset is an investment income figure which is expressed as a percentage of premium. It is obtained by looking at calendar year investment returns and allocating them by line. Adjustments are made to reflect income taxes and investment income earned from stockholder-supplied funds.

A defining feature of this method is its reliance on calendar year figures. Since calendar year results are reported by companies in their Annual Statements, the data supporting the calculations is easily obtained and verified. This is a distinct advantage. Use of reported figures can dispel criticism that the insurance company is making unduly pessimistic investment projections in its rate filings while earning record investment gains. Finally, in most cases, calendar year investment portfolio yields tend to be relatively stable.

However, since calendar year results are an inherently retrospective summary of contributions from current and prior policy years, their applicability in prospective ratemaking could be challenged. In particular, the prior growth history and loss experience of the line could distort the answers. However, unless the line has experienced rapid growth or decline, the figures, which might not be exactly correct for any specific year, will tend to balance out over several years of reviews. Also, certain adjustments can be made to partly correct the problem. For instance, the portion of the investment income offset due to loss reserves can be stated relative to incurred loss and then be put at a level consistent with the permissible loss ratio in the rates.

Next, a particular Calendar Year Investment Offset Procedure will be described. It is a streamlined version of procedures, such as the Insurance Services Office (ISO) State "X" Method, which have been actually used in rate filings. The simplified version is shown in Exhibit 2.

The first step in the algorithm is to find the ratio of pre-tax investment income relative to average invested assets. By applying appropriate tax rates to the various classes of investment income, one can calculate an after-tax portfolio yield. One should exercise care in making sure the right tax rates are used in this step of the calculation. The interest on taxable bonds is fully taxable at a 34% rate, while stock dividends are subject to an 80% exclusion. Under the proration provisions of the 1986 Tax Reform Act, the interest on "tax exempt" bonds acquired after August 1986 is subject to tax after an 85% exclusion. The actuary should review current tax law to ensure that investment taxes are correctly reflected.

One issue to face in this phase of the calculation is how to treat capital gains, both realized and unrealized. Some argue that capital gains should not be included. Others include only realized capital gains, perhaps using a multi-year rolling average of realized capital gains to invested assets to enhance stability. Others reflect all capital gains in computing the portfolio yield, again with some averaging over several years to avoid excessive volatility of results.

Once the after-tax portfolio yield is computed, the next goal of the calculation is to relate invested assets to premiums. Since policyholder premiums only generate part of the invested assets, a algorithm is needed to estimate the policyholder-supplied funds. The estimate is done indirectly by looking at the unearned premium reserve and loss reserve. These liabilities are offset by assets, only some of which are investible, and only some of which were supplied by the policyholder. A somewhat detailed series of computations is needed to estimate the investible balance due to policyholder supplied funds.

With respect to the unearned premium reserve, the first step is to obtain the ratio of average direct unearned premiums to direct earned premiums from the Annual Statement. Then this ratio is reduced to account for prepaid expenses and premium balances owed to the company.

The prepaid expenses include all or part of the commissions, premium taxes, other acquisitions expenses, and company overhead. The point here is that the company has already paid out these expenses, so the premiums paid to cover them are not generating investment income. For example, if the prepaid expense ratio against premiums was 20%, then one would reduce the unearned premium to 80% of its original value.

The premium balances cover all premiums owed (and less than 90 days late) including those deferred and not yet due. Again, the point is that the company does not have the cash to invest; it merely has an IOU. The appropriate unpaid premium balances are obtained from the Annual Statement, ratioed against direct earned premium, and then the ratio is subtracted from the previously calculated unearned premium ratio (to direct earned premium) net of pre-paid expense.

With respect to the calculation of the contribution associated with the loss reserve, one first computes the ratio of calendar year loss reserves to calendar year incurred losses. To obtain more stable results , one might calculate a multi-year average for this factor. This reserves-to-incurred loss factor is then multiplied by the permissible loss ratio. Thus we arrive at a ratio of reserves to premiums.

Note that actual ratios of reserves-to-premiums are not used in the calculation. Instead, a reserves-to-premium ratio consistent with the permissible loss ratio in the prospective rate is calculated as the product of the permissible loss ratio times the historic average reserves-to-incurred loss ratio. While the reserves-to-incurred ratio could be distorted by rapid growth or decline and changes in reserve adequacy, a reserves-to-premium ratio would also be subject to distortion from changes in premium adequacy. Using the reserves-to-incurred ratio along with the permissible loss ratio provides the policyholder credit for the investment income associated with the loss provision of the premium.

To summarize, the equation for policyholder-supplied funds is:

(III.1)

$$PHSF = \left(\frac{UEPR}{PREM}\cdot(1-PPACQ) - \frac{RECV}{PREM}\right) + PLR\cdot\frac{LRES}{INCL}$$

where:

PHSF = Policyholder-Supplied Funds (as a ratio to premium)

UEPR = Unearned Premium Reserves

PPACQ = Prepaid Acquisition Expense Ratio

RECV = Premiums Receivable

LRES = Loss Reserves

INCL = Incurred Loss

PLR = Permissible Loss Ratio

The final underwriting profit provision is then given as:

(III.2) $$\mathbf{U = U^0 - i_{AFIT} \cdot PHSF}$$

where U^0 is the traditional underwriting profit provision and i_{AFIT} is the after-tax portfolio yield.

There is a slight problem in determining the permissible loss ratio. Note the permissible loss ratio is used to calculate the profit provision which, in turn, is used to calculate the permissible loss ratio. To be technically correct, the calculation should be done iteratively until the permissible loss ratio is consistent with the offset and vice versa.

The key advantage of this method is its practicality. Figures come from documents already filed with statutory authorities, and the numbers are generally stable, especially if capital gains are excluded. The calculation is fairly short and the logic behind it is not too difficult. The key disadvantage is the lack of an underlying general economic theory to support the calculation. Other problems include the possibility of distortion when there is rapid growth or decline in loss volume or when there are significant changes in loss reserve adequacy. However, overall, this algorithm does account for investment income in a fairly straightforward manner. In stable growth scenarios with stable patterns of reserving, it seems appropriate for use in state rate filings.

IV. PRESENT VALUE OFFSET METHOD

In this method, as with the Calendar Year Investment Offset Procedure, an offset is applied against the traditional underwriting profit provision to get a final number. The offset is based on the difference between the present value of losses for a short-tailed reference line and the present value of losses for the line under review. The traditional provision is assumed to be correct for the reference line. Suppose $\vec{x} = (x(1), x(2), \ldots, x(n))$ is a loss payout pattern, where x(j) is the fraction of ultimate loss paid out at the end of the jth year. Thus, the sum of the x(j) is unity. If the ultimate loss is L, then the amount paid out at the end of the jth year is L(j), where $L(j) = L \cdot x(j)$. Given an interest rate, i, let $v = 1/(1+i)$. The present value of losses would be given as:

(IV.1)

$$PV(\vec{L};i) = L \cdot PV(\vec{x};i) = L \cdot \sum_{j=1}^{n} x(j) \cdot v^{j}$$

Suppose $\vec{x}^0$ is a reference loss payout pattern and $\vec{x}$ is the loss payout pattern for the line under review. The present value loss differential is defined as the difference in present values of loss under these two patterns. Letting the total amount of loss be given by a permissible loss ratio and expressing the present value loss differential as a ratio to premium, one obtains:

(IV.2)

$$DELPVLR = PLR \cdot (PV(\vec{x}^0;i) - PV(\vec{x};i))$$

The underwriting profit provision is then given as:

(IV.3) $$U = U^0 - DELPVLR$$

where U^0 is the traditional underwriting profit provision.

One version of this method is demonstrated in Exhibit 3. It is similar, though not identical, to the algorithm used in Florida filings.

The intent of the calculation is to adjust for the differences in investment income potential between lines of business. To understand how this is accomplished, one could ignore loss proportional expenses and rewrite the premium formula as:

(IV.4) $$P = PVL + FX + VR \cdot P + U^0 \cdot P + L(1 - PVx^0)$$

where:

PVL = present value of losses for the line

L = full value of losses for the line

PVx^0 = the present value loss pattern for the reference line.

From this perspective, the loss provision is now being counted only at its present value, while the profit provision is effectively the sum of the traditional underwriting profit provision plus the scant investment income potential of the reference line. The thought is that if the present value loss provision were collected at policy inception, it would, on average, generate enough investment income so that the (full value) loss payouts could be covered.

The selection of the interest rate for discounting losses is critical in this method. One could use the portfolio yield from a recent year, the current embedded portfolio yield, an estimate of the portfolio yield for the year the rates will be in effect, or a new money yield. In keeping with the prospective nature of ratemaking, new money yields are theoretically preferable. However, portfolio yields are more stable and more easily verifiable. Also, their use may eliminate concern that the company is using low yields in its rate filings, while reporting high yields in its financial statements.

One way to partially account for income taxes within the context of this method is to use an after-tax interest rate for discounting the loss payout patterns. If the present value loss provision is computed with the after-tax interest rate, the after-tax accumulation of the present value loss provision will be sufficient to cover the expected full value loss payments. Note that the reflection of income taxes is only accounting for the income tax on investment income associated with the present value loss provision. Since this is only part of overall income tax, the tax adjustment is, in some sense, incomplete. A more elaborate model could be constructed within the same general framework by computing projected income taxes for the reference line and the line under review. An adjustment could then be made for the difference in the present values of the projected income tax payments. This will not be done here.

One could take a prospective or retrospective approach in putting the interest rate on an after-tax basis. Under the prospective approach, one applies the appropriate prospective tax rates to the pre-tax yields of each type of investment to get after-tax yields. The after-tax yield of the whole portfolio is calculated by weighting the after-tax yields by the assumed mix of assets.

Under the retrospective approach, the tax rate is derived from a company's actual income tax rate on all income for the prior calendar year. The implicit assumption is that the previous calendar year's actual income tax on all income for all lines and all states is a good indicator of the projected income tax on investment income associated with the line and state under review. This assumption is debateable. For instance, the tax paid in the past reflects past underwriting profits which may or may not be consistent with the underwriting profit provision in the prospective rates. As well, one could raise questions about implicit interstate and interline rate subsidies. Finally, one could argue that use of the prior year's actual tax rate operates perversely in that it effectively penalizes companies that lost money and rewards those that were profitable.

However the tax issue is settled, this method does account for investment income in a direct and simple fashion. While calendar year investment returns may be used in arriving at an appropriate discount rate, the method is not distorted by rapid growth or decline as was the Calendar Year Investment Offset procedure of Chapter III. Also, there is no need to select a target return or estimate a surplus requirement as is needed in some of the methods that follow.

V. CALENDAR YEAR RETURN ON EQUITY (ROE) METHOD

This method utilizes calendar year figures to calculate a rate of return on equity. The underwriting profit provision is chosen so that the rate of return is equal to a specified target return.

Return on equity is related to underwriting profit by the general formula:

(V.1)

$$ROE = \frac{INC}{EQ} = \frac{U \cdot P + II - FIT}{EQ}$$

where:

ROE	=	Return on Equity
INC	=	Total Income
U	=	Underwriting Profit Provision
P	=	Premium
II	=	Investment Income
FIT	=	Federal Income Tax
EQ	=	Equity

Equity is often set by first choosing a Statutory Surplus requirement. This is usually done by selecting a premium-to-surplus ratio that may vary by line of insurance. GAAP Equity can then be estimated by applying an historic equity-to-surplus ratio. A more complicated alternative is to add in the deferred acquisition expense balance and make other GAAP adjustments.

Taxes apply to both underwriting income and investment income. Whereas one uniform tax rate generally applies to all underwriting income, the investment portfolio may contain a variety of securities with effective tax rates that differ from each other and from the rate applicable to underwriting income. For this reason, it may be useful to split the tax into its two components: tax on underwriting income, and tax on investment income. Then, the total income numerator may be rewritten as the sum of after-tax investment income and after-tax underwriting income. Now suppose the tax on underwriting income is simply computed by applying a tax rate. This, of course, is incorrect as it ignores the 1986 Tax Reform Act, but under this assumption, total income may be conveniently expressed as:

(V.2)

$$INC = (1 - t_u) \cdot U \cdot P + II_{AFIT}$$

where:

t_u = Income tax rate on underwriting income

II_{AFIT} = After-tax investment income

After-tax investment income is computed using an after-tax yield calculated in the same fashion as the after-tax yield obtained in the Calendar Year Investment Offset Method (See Chapter III). Recall that in that calculation each class of investment was subject to the appropriate prospective tax rate. Alternatively one could use a tax rate based on the company's recent tax returns. Counterarguments made in Chapter IV still apply here.

The after-tax yield should be multiplied against all investible funds no matter whether they were provided by policyholders or stockholders. Thus, the after-tax investment income may be expressed as:

(V.3)

$$II_{AFIT} = i_{AFIT} \cdot (PHSF \cdot P + S)$$

where:

i_{AFIT} = After-tax return on invested assets

PHSF = Policyholder-supplied funds (as a ratio to premium)

S = Surplus

The method is illustrated in the Exhibit 4. Note that the policyholder-supplied funds and investment yields from the Calendar Year Investment Offset example were used in this Calendar Year ROE example. The Calendar Year ROE Method shown here is roughly similar to methods promulgated in California under Proposition 103 regulation.

If the target return and necessary leverage ratios are given then one can solve for the corresponding underwriting profit provision to obtain the formula:

(V.4)

$$U=\frac{1}{1-t_u}\left[\ r\cdot\left(\frac{QSR}{PSR}\right)\ -\ i_{AFIT}\cdot\left(PHSF+\frac{1}{PSR}\right)\right]$$

where

r	=	Target Return on Equity
PSR	=	Premium-to-Surplus Ratio
QSR	=	Equity-to-Surplus Ratio

Using a calendar year return method raises several issues. First, as a calendar year method, it is subject to biases due to rapid growth or changing reserve adequacy similar to those discussed earlier. Second, there is the question of how to select and defend a target return. Related to this is the issue of whether one should divide by GAAP Equity instead of Statutory Surplus. If one uses a Statutory Surplus denominator, the resulting return will not be comparable to GAAP ROE. How to pick an appropriate leverage ratio (premium-to-surplus ratio) is another question. Should it balance against a company's actual surplus or should a target leverage ratio be assumed? Should the leverage ratios vary by line?

However, if one can address these issues, a calendar year return methodology does have some positive features. Figures used in the calculation are published in the Insurance Expense Exhibit and the Annual Statement, making verification relatively simple. Moreover, the key attraction of the method is that it produces a return on equity which is in some sense comparable to the GAAP ROE commonly used to measure profitability in many industries.

VI. PRESENT VALUE OF INCOME OVER PRESENT VALUE OF EQUITY (PVI/PVE) MODEL

Under this method, the underwriting profit provision is set so as to achieve a target "present value return" called the "PVI/PVE". As the name suggests, the PVI/PVE is the ratio of the present value of accounting income over the present value of equity. It is calculated with a single policy model constructed so that income is earned and equity is evaluated in accord with accounting (usually GAAP) conventions. The income is the sum of underwriting income plus investment income less income taxes.

The fundamental equation is:

(VI.1)

$$r = \frac{PV(INC)}{PV(EQ)}$$

where:

r = target return

PV(INC) = present value of income

PV(EQ) = present value of equity

The PVI/PVE is calculated by first constructing a model to produce Statutory and GAAP balance sheets and income statements for a company hypothetically writing a single policy. In the model, premium is earned evenly over the policy term and the unearned premium reserve is calculated as the difference between written premium less premium earned to date. Premiums receivable are the difference between written premiums less premiums paid to date. Loss and loss adjustment expense is incurred uniformly over the policy term, and the loss and loss adjustment expense reserve is equal to the difference between amounts incurred to date less amounts paid to date. Thus, the reserve in the model is always exactly adequate. Implicitly, it includes claims incurred but not reported (IBNR) as well as case reserves on known claims. Following Statutory accounting, expenses such as premium tax, commission, and acquisition expense are incurred up-front when the policy is written. General expenses, on the other hand, need not all be incurred at policy inception. Statutory expense reserves are posted as the difference between expenses incurred to date less expenses paid to date. Under GAAP, the premium tax and commission are incurred uniformly over the policy term instead of up-front. The incurral of some other acquisition expenses may also be deferred. The difference between Statutory incurred expenses to date less GAAP incurred expenses to date is sometimes called the Deferred Acquisition Balance and is counted as a GAAP asset. It is also sometimes referred to as the "equity in the unearned premium reserve" since it may be calculated by multiplying the unearned premium reserve by the ratio of deferrable expense relative to premium.

In the model, investment income for each accounting period is approximated by applying the pre-tax investment yield to the average balance of invested assets during the period. The invested assets can be computed by taking total assets and subtracting non-investible assets such as premiums receivable. The assets for the hypothetical company are equal to the sum of Statutory Reserves plus Statutory Surplus. This follows because any balance sheet must be in balance.

Income taxes should be based on relevant provisions of the tax code as applied to the hypothetical company. A detailed calculation of taxable income would require consideration of reserve discounting, the 20% disallowance for unearned premium, and other provisions of the Tax Reform Act of 1986. An issue that arises in modelling taxes is what to do if tax accounting income is negative. The usual practice is to allow a negative tax (i.e. a payment from the government to the company) when taxable income is negative. Another option is to allow a negative tax in any period only if it offsets positive taxes paid in previous periods.

The equity in the model is often set to be level for one year, and the amount of equity is determined by selecting a premium-to-equity ratio. A ratio of expected loss-to-equity can also be used for this purpose. More general assumptions are often made. For example, some equity may be held over several years in proportion to loss reserves. Also, in many models, the amount of equity may vary during a calendar period. For instance, if one starts with a "block" surplus assumption that fixes the level of Statutory Surplus, the corresponding level of GAAP Equity for a single policy will generally vary over the year. Note that some of the GAAP Equity is non-investible; in particular, the "equity in the unearned premium reserve" which is associated with pre-paid expense.

Since present values are commonly taken of a series of flows rather than a series of balances, the calculation of the present value of equity is a somewhat unique feature of the model. Further, the present value of equity must be defined with some care since one needs a definition that yields sensible results when equity is held over several years or when equity is evaluated more frequently than at the end of each year. One way to fashion such a definition is to first define <u>average equity balances during</u> each accounting period based on the <u>equity</u>

balances at the end of each time period. Let EQ_j be the equity balance as of the end of the j^{th} time period where each time period lasts an m^{th} of a year. Write EQ_0 for the balance at policy inception. Next, let EQB_j denote the average equity balance during the j^{th} time period. The average equity balance during the j^{th} period may usually be approximated as the numerical average of consecutive quarter ending equity values. However, when equity is based on a "block" surplus which is taken down immediately after the end of a year, this rule is violated and one must take more care in defining the average equity balance during the period. In a quarterly model with a "block" equity requirement, the balances would be :

(VI.4)

Quarter	Qtr Ending Equity Balance (EQ_j)	Average Equity Balance During Qtr (EQB_j)
0	EQ_0	-----
1	EQ_1	$EQB_1 = (EQ_0+EQ_1)/2$
2	EQ_2	$EQB_2 = (EQ_1+EQ_2)/2$
3	EQ_3	$EQB_3 = (EQ_2+EQ_3)/2$
4	EQ_4	$EQB_4 = (EQ_3+EQ_4)/2$
5	$EQ_5 = 0$	$EQB_5 = 0$

Consider how the use of the average equity balances during each quarter eliminates the problem of having too many quarter ending balances and the problem of how to handle the takedown of equity just after the end of the fourth quarter.

One way of defining the PVI/PVE is to calculate an m^{th}ly effective PVI/PVE return as follows:

(VI.5)

$$PVI/PVE = \frac{\sum_{j=0} INC_j \cdot v_{(m)}^j}{\sum_{j=1} EQB_j \cdot v_{(m)}^j}$$

Here $v_{(m)} = (1+i)^{-1/m}$ where i is the interest rate used for discounting.

Now consider this definition as applied to a simple example in which a balance of $100 is maintained in a bank account for n quarters and $2 of interest is received at the end of each quarter. The present value of the income is $\$2 \cdot (v_{(4)} + v_{(4)}^2 + \ldots + v_{(4)}^n)$, while the present value of equity is $\$100 \cdot (v_{(4)} + v_{(4)}^2 + \ldots + v_{(4)}^n)$. Thus, the PVI/PVE is 2% effective quarterly. More generally, if one has a venture yielding interest of $y_{(m)} \cdot B_j$ paid at the end of the j^{th} period on a balance of B_j, the PVI/PVE will be $y_{(m)}$ effective m^{th}ly. This will be true irrespective of the interest rate used in discounting the income and the equity, as long as the same rate is used in discounting both.

In more general cases, when income in each period is not a fixed multiple of the equity balances, the PVI/PVE as defined by VI.5 will depend on the interest rate (or rates) used in discounting the income and the equity. This leads to some disagreeable results even in some very simple scenarios. For example, if $8 is paid at the end of the year on a fixed balance of $100 and one uses a quarterly model with a 10% rate for discounting, the effective annual return would be calculated as 7.942% ($\{1+ .08*1.1^{-1}/(1.1^{-.25}+1.1^{-.50}+1.1^{-.75}+1.1^{-1})\}^4$) rather than 8%. A related problem with VI.5 is that the present value of equity undergoes drastic changes if one shifts from an annual model to, say, a quarterly model. For instance, if a balance of $100 is held constant for one year, its present value is $100 in an annual model. Using a 10% rate for discounting, the present value balance becomes $377 in a quarterly model.

To address these concerns, one can use an alternate definition of PVI/PVE in which all income is evaluated as of the end of the first year and in which the present value of equity is put on an "annualized" basis as follows:

(VI.6)

$$PVE_{Ann} = \frac{\sum_{j=1} EQB_j \cdot v_{(m)}^{j-1}}{\sum_{j=1}^{m} v_{(m)}^{j-1}}$$

Note that when equity is held fixed for one year, its annualized present value equals the "block" amount under any m^{th}ly model irrespective of "m" or the interest rate used in discounting.

Under this approach, the annual effective PVI/PVE is given as:

(VI.7)

$$PVI/PVE_{Ann} = \frac{(1+i) \cdot \sum_{j=0} INC_j \cdot v_{(m)}^{j}}{PVE_{Ann}}$$

The PVI/PVE model is demonstrated in the Exhibit 5 using the annualized definitions. In the interest of simplicity, the only GAAP adjustment is for deferred acquisition costs, and income taxes are incorrectly modelled as a percent of GAAP income. Actual tax law should be followed in any business applications.

The rate used in discounting the income numerator is often selected to be the pre-tax, risk-free, new money yield on taxable investments. Usually, this same rate is used to discount the equity denominator, though sometimes the target return is also employed for this purpose. The rationale for using the pre-tax yield is that income taxes are explicitly included in the income. The target return is set a few points above the pre-tax, risk-free rate on taxables. The spread would be dependent on the risk involved. While measurement of risk is problematic, if the rate is risk-free, there is no need to worry about the investment default risk. Another justification for using the risk-free rate is that the resulting underwriting profit provision will not depend on the particular investment strategy of a company.

An alternative philosophy in selecting investment yields, discount rates, and target returns is to use actual portfolio yields in conjunction with discount rates and target returns comparable to historically acceptable GAAP ROE targets.

While one common rate is generally used in discounting both income and equity, one could conceive of using different rates. In particular, an argument could be made for discounting the equity using the target rate of return. The general dependence of results on the interest rate selection can be deduced as follows. In practice, since income is usually negative for the first few periods and then subsequently positive, it follows that an increase in the rate for discounting income will typically result in a decrease of the PVI/PVE. With respect to the denominator, a boost in the discount rate will always reduce the "PVE". This will enhance "leverage" so that positive PVI/PVE grow more positive and negative PVI/PVE become more negative. When using PVI/PVE to find an underwriting profit provision consistent with a selected (and presumably positive) target return, the leverage effect will boost PVI/PVE return. This will lead to a reduction in the indicated underwriting profit provision, assuming the target return stays fixed.

The key advantage of this method is that it is based on a measure of return that is both comparable to GAAP ROE and a generalization of the standard definition of the rate of interest. However, it does require selection of rates for calculating investment income and taking present values. Also, one must choose a target return. As with other methods that have a target return and an explicit surplus requirement, one may encounter some debate on the choice of these parameters. For instance, a regulator may lean toward a relatively low target return and a relatively small amount of surplus. In such a situation, a company actuary should be ready to support his or her selections with data on the returns achieved by the insurance industry, financial services industries, and other industries. The actuary can also buttress his or her case by citing surplus benchmarks accepted by the industry or promulgated by regulators.

VII. PRESENT VALUE CASH FLOW RETURN MODEL

With this method the underwriting profit provision is set so that the present value of total cash flow equals the present value of the changes in equity. Here an investment rate of return is used to calculate the present value of the total cash flow while a target rate of return is used in computing the present value of the changes in equity. The total cash flow is defined to be the sum of the underwriting cash flow plus the investment income on investible equity less income taxes. Underwriting cash flow consists of paid premiums less paid losses and paid expenses. The flows are derived from a single policy (or policy year) model based on assumptions about the ratios and payment patterns for losses and expenses. Note the present value of total cash flow has no distinct term for investment income on policyholder-supplied funds, while there is a term for the present value of investment income on assets that offset equity. This disparate treatment of investment income arises because the present value of investment income on policyholder-supplied funds is already taken into account in computing the present value of the underwriting cash flows.

The basic relation may be expressed in mathematical terms as:

(VII.1)

$$PV(\Delta EQ; r) = PV(TCF; i)$$

where

EQ	=	Equity
r	=	Target Rate of Return
TCF	=	Total Cash Flow
i	=	Investment rate used for discounting cash flows

The idea behind this method is that the target rate of return on equity can be used along with assumptions about the level of equity to arrive at a target value for the present value of the total cash flow. To see how this works, consider a two year venture in which $100 of equity is needed for the first year and $40 for the second. The equity changes are +100, -60, and -40. With a 20% target return, the target present value "profit" (evaluated at the end of the first year) comes to $26.67 (100·1.2 + -60 + -40/1.2). Note that the investment of $100 for the first year and $40 for the second year requires the same present value profit as a one year investment of $133 (20%·$133 = $26.67). It may also prove insightful to observe that $133 = 100 + 40/1.2.

The equation for total cash flow is:

(VII.2)

$$TCF = UWCF + INVIEQ - FIT$$

where:

UWCF = Underwriting Cash Flow

INVIEQ = Investment Income on Investible Equity

FIT = Federal Income Tax

"Investible Equity" is somewhat of a misnomer as equity is merely the difference between assets and liabilities. The term is used here to refer to that portion of the equity which can be associated with investible assets. Usually this consists of roughly the Statutory Surplus. Various ways of modelling Statutory Surplus were mentioned in VI.

Income taxes can be modelled in a simple, though inaccurate, fashion by applying the appropriate tax rate to the present value of the underwriting cash flows and to the investment yield on investible equity. The tax rate would not be applied to the interest rate used in discounting flows to present value. To get a more accurate treatment of taxes, one would have to compute income according to tax accounting conventions.

A simple example of the Present Value Cash Flow Return is given in Exhibit 6. It is similar to the fifth model presented by Mahler in his paper.

Perhaps the biggest criticism of the method is that it is not exactly clear just what sort of profit is being measured. To get a return comparable to GAAP ROE, then the present value of GAAP income should equal the present value of the total cash flow. Yet, the present value of total cash flow cannot be easily reconciled with GAAP accounting since the timing of underwriting cash flows is not generally equal to the timing of GAAP underwriting income. Nonetheless, the method has great appeal, perhaps because the present value of underwriting cash flows is precisely what most people first think of when trying to measure the underwriting profit net of associated investment income.

VIII. RISK-ADJUSTED DISCOUNTED CASH FLOW MODEL

Under this method, the strategy is to compute a "fair" premium directly. The underwriting profit provision is then calculated after the fact by comparing the fair premium against the sum of loss and expense provisions. Of course, the key question is: what is the fair premium? The answer is that the fair premium is the sum of the risk-adjusted present value of the underwriting cash flows plus an amount to cover the present value of income taxes. The risk adjustment is accomplished by discounting flows at a risk-adjusted rate that is usually less than the risk-free rate. Though one could, in principle, calculate a separate risk-adjusted discount rate for each flow, in what follows, only the paid losses will be subject to risk-adjusted discounting. The premium, expense, and income tax flows will be discounted at the risk-free rate. While surplus plays no critical role in this model, one must not forget to reflect income taxes on the investment income associated with the surplus.

The basic equation is:

(VIII.1)

$$PV(P;i_f) = PV(L;i_r) + PV(FX+VX;i_f) + PV(FIT;i_f)$$

where:

P = Premium

L = Loss

FX = Fixed Expense

VX = Variable Expense

FIT = Federal Income Tax

i_f = Risk-free rate

i_r = Risk-adjusted rate

The method is demonstrated in Exhibit 7. The demonstration is a simplified version of the Myers-Cohn model that has been used in Massachusetts rates filings.

The risk-adjusted rate is key to the method. It is calculated with the following formula:

(VIII.2)

$$i_r = i_f + \beta \cdot (i_m - i_f)$$

This formula quantifies the risk-return trade-off and is a simplified expression of the concepts underlying the Capital Asset Pricing Model (CAPM). CAPM was originally formulated to describe market prices of stocks and bonds. The idea is that the market demands higher expected returns for riskier investments. The term, $(i_m - i_f)$, is the market "premium" for an investment of average risk, or, in other words, i_m is the return on an average market portfolio.

The "beta" coefficient relates to the relative systematic risk of the particular investment under consideration. It is the ratio of the covariance between the market return and the return of the particular asset, divided by the variance of the market return. Under CAPM, a critical distinction is made between non-diversifiable, systematic risk and diversifiable, non-systematic risk. A higher return is warranted only for the systematic risk component. To get a stronger intuition, consider a stock that moves with the market, plus or minus some random fluctuations. In this case, the beta is unity. If a stock moves in the same direction as the market but with swings always twice as large, then the beta is two. The same beta results if some random fluctuations are thrown on top of the systematic double amplitude, market-following swings.

In principle, it is possible to have a negative beta, even for an asset. For example, suppose the return on a particular stock deviated from the risk-free return in the same magnitude but opposite direction as the market return. Then, an investor could theoretically achieve the risk-free rate with a portfolio half in that stock and half in a diversified market basket of stocks. Since the expected market return is always above the risk-free rate, the expected return on the hypothetical stock should be below the risk-free rate. Thus beta in this pathological case would be negative. As far as the author knows, the real world provides no good example of a stock with a negative beta, though some suggest one might be found among the stocks of gold mining companies.

Applying the CAPM concept to liabilities is a bit tricky. With stocks and bonds, the theory leads to predictions that can be checked against actual market prices. With liabilities such as loss reserves, there is no open market with publicly disclosed prices to provide empirical validation for the theory. Anecdotal evidence suggests that the price for loss portfolio transfers is usually greater than the present value of the losses transferred when present values are computed at the risk-free rate. This implies that the beta for liabilities is negative, but supplies no data for estimating "beta" magnitudes or how beta varies between lines of business.

An indirect approach to estimating a liability beta is to first calculate the beta for stocks of insurance companies and then the beta for the investment portfolios held by these companies. The difference between the market value of insurance stocks and the market value of insurer asset portfolios must be due to an implicit market valuation of insurer liabilities. With sophisticated techniques that will not be presented here, a beta for liabilities can be derived. The trouble is that most insurance companies are multi-line companies so that this method has not, in practice, been able to distinguish betas by line of business.

Because the method has great intuitive appeal and is directly grounded in modern financial theory, one might be able to accept some uncertainty about the appropriate beta. By defining a fair premium without resort to a total rate of return measure, the method neatly sidesteps the question of what is the right target rate of return. Also, since there no return relative to equity, the selection of a surplus requirement is not as critical as with other models.

IX. INTERNAL RATE OF RETURN ON EQUITY FLOW MODEL

Under this method, the underwriting profit provision is set so as to achieve a selected target return. The return is calculated by modelling a single policy. As with the PVI/PVE (Present Value of Income over Present Value of Equity) return discussed in Chapter VI, one constructs a model to calculate income statements and balance sheets for a fictitious company writing the single policy. Then, using basic accounting relations, one calculates flows of money between this company and its hypothetical stockholders. These flows are called equity flows. The return on the policy is defined to be the internal rate of return (IRR) on the equity flows. If some typical restrictions prevail, the IRR can be interpreted as the interest rate paid to stockholders on a series of equity "loans" made to the insurance company.

In general, the IRR on a sequence of flows is the rate, y, if it exists and is unique, for which the present value of the flows is zero:

(IX.1)

$$0 = PV(\vec{x};y) = \sum_{j=0}^{n} x_j \cdot (1+y)^{-j}$$

Thus, the IRR is the solution to an n^{th} degree polynomial equation. If, for example, the flows are (-200, +110, +121), then the IRR is 10% (assuming the flows are at annual intervals), since $-200 + 110*(1.1^{-1}) + 121*(1.1^{-2}) = 0$. IRR is a generalization of the interest rate on a loan. It can be shown that if the IRR exists, then the flows can be expressed as a sum of overlaid simple loans all carrying an interest rate equal to the IRR. For instance, in the previous example, the simple loans are:

(-100, +110, 0) and (-100, 0, +121),

both of which are simple loans at a 10% rate. Adopting the convention that negatives represent cash outflows and positives denote cash inflows, one can interpret the flows from the point of view of a lender obtaining 10% on the loans made. However, if all signs of the flows were reversed, then one would still obtain the same IRR. Only in this case, one would be viewing matters as a borrower paying 10% on a series of loans. More generally, one should take care with IRR to be sure to identify what side of the table one is sitting on. The IRR only measures the interest rate on the loans; not whether one is doing the borrowing or the lending.

Of course, an n^{th} degree polynomial could have as many as n real roots. If the defining equation has more than one real root, then the IRR fails to exist. In such cases, the flows can be decomposed in more than one way into a sum of simple loans such that, for each different decomposition, the simple loans carry a different interest rate. For example, the flows:

(-100, +230, -132)

have a zero present value at rates of both +10% and +20%. These flows can be expressed either as:

(-100, +110, 0) + (0, +120, -132) or

(-100, +120, 0) + (0, +110, -132) .

The first decomposition is the sum of 10% simple loans and the second is a sum of 20% loans. Not coincidentally, in each of these decompositions, one of the simple loans is a borrowing transaction and one is a lending transaction.

While the problem of multiple roots could in principle be fatal, in practice it almost never arises in modelling property and casualty insurance ventures. Thus, the previous discussion is merely a caution about the uncritical use of IRR in all situations and not a telling objection to the use of IRR for the purpose at hand.

The equation for equity flow is:

(IX.2)

$$EF_j = INC_j - (SCHNG)_j$$
$$for\ j = 0,1,2,\ldots,n$$

where:

EF = Equity Flow

INC = Total Statutory Income

SCHNG = Change in Statutory Surplus

Here a variable with a subscript, j, connotes a flow occurring or income accruing as of the end of the j^{th} time period and a balance sheet evaluated as of the end of the j^{th} time period. The "0" subscript refers to the start of the first time period. A possible confusion arises under these conventions with respect to the Change in Surplus variable. The Change in Statutory Surplus is usually equal to the difference between consecutive Statutory Surplus balances. However, an exception occurs when surplus is taken down at the start of an accounting period. For example, in a quarterly model run with a one year, block surplus of $100, the Statutory Surplus account will be $100 for indices zero through four and then become zero at index five. The Change in Statutory Surplus will be $100 at index zero and then be -$100 at index four (and not five).

An simple example of the method is shown in Exhibit 8. It is similar to the procedure used by the National Council of Compensation Insurance in several states.

The accounting principle behind Equation IX.2 is that the surplus of the company can change only if it declares income (either a gain or loss) or if it receives or distributes funds to stockholders. If one ignores unrealized capital gains, changes in non-admitted assets, and other sundry adjustments, this is exactly true. Note that equity flows are defined here so that a positive equity flow denotes a flow of money to stockholders.

Inherent in the equation is a critical distinction between the surplus of the company and the stockholder investment in the company. From the equity flow perspective, what is most important is what the stockholders will put into the company and what they will get back. Statutory and GAAP accounting rules and requirements are important, not because Statutory Surplus or GAAP Equity is in the denominator of an ROE measure, but because they can affect the equity flows. Statutory Surplus requirements and the conservative bias of Statutory accounting rules tend to increase the amount of money required from stockholders and delay or decrease the flow of funds to them. GAAP accounting rules can also impact the flow of equity, because, in a few cases, GAAP is more conservative than Statutory accounting. One example of such a "concept violation" is in the accrual of a GAAP policyholder dividend reserve liability not recognized under Statutory accounting.

While the defining equity flow equation has been given using Statutory Income and Statutory Surplus, one should more properly determine each equity flow so that the company maintains a sound balance sheet under both accounting systems. Further, once each equity flow is calculated as the difference between Statutory Income and the change in the Statutory Surplus (during period) balance, one could restate it as the difference between GAAP Income and the change in the (during period) GAAP Equity balance. Though the company may have different balance sheets and income statements under GAAP, the equity flows stay the same. Thus the IRR on Equity Flows is not a GAAP return or a Statutory return; rather it is the return to stockholders reflecting the constraints of both accounting systems.

Following the rules of Statutory accounting, expenses such as commissions, premium taxes, and acquisitions expenses are incurred up-front at policy inception. Other expenses are incurred as paid or incurred uniformly over the policy term. Expense reserves are held so that incurred equals paid plus the change in reserves. Premiums are earned and loss and loss expenses are incurred evenly over the policy term. Earned premium equals written less the change in the unearned premium reserve. Appropriate loss reserves are posted so that incurred loss is equal to paid plus the change in loss reserves.

One can now see why the "multiple roots" problem seldom arises in property and casualty insurance models. Consider that the up-front expense declaration rules of Statutory accounting and the need to fund the initial required surplus usually force the first equity flow to be negative. If underwriting losses are sufficiently large, equity flows could remain negative for the policy term. This happens, since, in the model, the stockholders must fund reserves when premiums are inadequate to do so. After the first year, a property casualty venture usually generates investment income and hence positive equity flows. The repatriation of surplus also leads to positive equity flows. Note that in the model the stockholders are a "bottomless well" providing funds as needed to maintain the pre-set surplus requirement. For its part, the company immediately returns excess funds to stockholders. It does not build up profits or retain surplus above the predetermined surplus requirement level.

The main virtue of this model is that it calculates a return to stockholders that is directly analogous to the interest rate on a loan. It is clear what return is being measured. Another possible strength of the model is that it reflects the rules of accounting insofar as they impact the flow of funds to stockholders. In other words, the cost of conservatism in Statutory accounting is included. The unified treatment of all income is an aesthetic plus.

Drawbacks include the need to select a target return and a surplus requirement. This again forces one to address the knotty issues of how to "price" the elements of risk in terms of increasing the required surplus, increasing the target return, or both.

X. CONCLUSION

The reader has been introduced to a profusion of models and may find the whole subject a bit overwhelming. Indeed, perhaps the major thesis of this paper is that the methods differ not just in their details but, more importantly, in their basic foundations. It is thus time to step back, survey the field, and pick out key characteristics. What makes one method different from another?

The first distinction is whether the traditional underwriting profit provision is used as a starting point to make adjustments or whether an attempt is made to define a "correct" profit provision from first principles. The Calendar Year Investment Offset and the Present Value Loss Offset Methods are procedures of the former type. The other methods are independent of the traditional load. Except for the Risk-Adjusted Discounted Cash Flow method in which a "fair" premium is obtained directly, a notion of total return is defined in these other methods and the profit provision is set to hit a selected return target.

The methods can also be classified by their degree of reliance on calendar year data. Calendar year investment figures play a key role in the Calendar Year Investment Offset and Calendar Year Return algorithms. At the other extreme, calendar year numbers play no role at all in the Risk-Adjusted Discounting method. With the Present Value Offset, PVI/PVE, Present Value Cash Flow Return, and IRR on Equity Flows, calendar year figures might be used in selecting interest rates or rates used for discounting, but market risk-free rates are preferred.

Another defining characteristic of a model is how "risk" is reflected. In the Risk-Adjusted

Discounted Cash Flow method, pricing for risk is central to the model. With all the total return models, risk is reflected in the selection of the surplus requirement and the target return. Risk was not considered in the models that use the traditional profit load as a starting point, though one might be able to put "risk" in after the fact.

When looking at a model, one should also understand the role of accounting conventions. Accounting constraints are central to the IRR on Equity Flows method because these constraints impact the modelled flows of stockholder equity. Whether one should be measuring GAAP ROE or Statutory ROS (Return on Surplus) is a major issue in the Calendar Year Return Procedure.

There are a host of other concerns. How are income taxes handled, if at all ? How stable is the data and the resulting profit provision? How defensible is the method if senior corporate managers or state regulators question it?

To conclude, no "one true" method has been espoused in this paper. While each actuary may prefer one method above all others or feel that some are outright nonsense, it was not the purpose here to spark vituperative debate. Rather, the intent was to impart sufficient enlightenment so the reader could judge what models are appropriate in a given situation and to make some sense of models encountered in practice. The reader should also have enough information to construct simple versions of various models. The underwriting profit provision will likely be the subject of contention for years to come, and it is hoped that the reader now has sufficient background to understand the confusion.

ACCOUNTING GLOSSARY

GAAP (Generally Accepted Accounting Principles) Vs. Statutory Accounting Principles

All insurance companies must report their results to the regulators in states in which they are licensed using the prescribed format of the NAIC Annual Statement. The rules for preparing the NAIC Annual Statement constitute Statutory accounting principles. They are balance sheet oriented and emphasize the valuation of assets and liabilities on a "liquidation basis" rather than on the "going-concern basis" used for GAAP financial statements. Generally, Statutory financial statements display a more conservative financial position and results of operations than the results reported under GAAP. The Internal Revenue Service Form which Property-Casualty insurance companies use to report their taxable income follows neither GAAP nor Statutory insurance accounting principles entirely.

Capital Gains

The sale of a capital asset at more (or less) than its cost basis for tax purposes gives rise to a realized gain (or loss). Unrealized gains reflect the excess of Market Value over Book Value. Unrealized gains and losses in the investment portfolio are not included in investment income.

GAAP Equity Vs. Statutory Surplus

Surplus is a Statutory accounting term representing the excess of assets over liabilities. Statutory accounting conservative valuation rules exclude some assets from the balances sheet (non-admitted assets). Another area of distinction is in the treatment of prepaid acquisition expenses for which Statutory accounting requires immediate expensing and for which GAAP allows amortization. Both of these differences tend to make the GAAP equity larger that the Statutory Surplus.

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GLOSSARY OF EXHIBITS

EXHIBIT	TITLE
1A	Loss and Expense Parameters Used for All Methods
1B	Payment Patterns
1C	Investment Portfolio Yield Calculation
1D	Financial Assumptions by Model
2	Calendar Year Investment Offset Method
3	Present Value Offset Method
4	Calendar Year ROE Method: Summary
5A	PVI/PVE Model: Summary
5B	PVI/PVE Model: Inputs and Assumptions
5C	PVI/PVE Model: Underwriting Cash Flows
5D	PVI/PVE Model: Underwriting Income
5E	PVI/PVE Model: Balance Sheet
5F	PVI/PVE Model: Income
6A	Present Value Cash Flow Return Model: Summary
6B	Present Value Cash Flow Return Model: Inputs and Assumptions
6C	Present Value Cash Flow Return Model: Investment Income on 'Investible Equity'
6D	Present Value Cash Flow Return Model: Underwriting Cash Flows
6E	Present Value Cash Flow Return Model: Changes in Equity
6F	Present Value Cash Flow Return Model: Discount Factors
7A	Risk–Adjusted Discounted Cash Flow: Summary
7B	Risk–Adjusted Discounted Cash Flow: Inputs and Assumptions
7C	Risk–Adjusted Discounted Cash Flow: Underwriting Cash Flow Patterns
7D	Risk–Adjusted Discounted Cash Flow: Tax on Investment Income on Surplus
7E	Risk–Adjusted Discounted Cash Flow: Discount Factors
8A	IRR Model: Summary
8B	IRR Model: Inputs and Assumptions
8C	IRR Model: SAP Underwriting Income
8D	IRR Model: SAP Balance Sheeet
8E	IRR Model: SAP Income
8F	IRR Model: GAAP Income

LOSS AND EXPENSE PARAMETERS USED FOR ALL METHODS

Loss	$65
Fixed Expense	$15
Variable Expense Ratio	25%

Payment Patterns

PREMIUM				LOSS				EXPENSE		
Qtr	Percent of Ultimate	Percentage Increments	Qtr	Dollars by Quarter *	Percent of Ultimate	Percentage Increments	Qtr	Percent of Ultimate	Percentage Increments
0	40.0%	40.0%	0	$0.00	0.0%	0.0%	0	30.0%	30.0%
1	55.0%	15.0%	1	$2.00	3.1%	3.1%	1	47.5%	17.5%
2	70.0%	15.0%	2	$4.00	9.2%	6.2%	2	65.0%	17.5%
3	85.0%	15.0%	3	$7.00	20.0%	10.8%	3	82.5%	17.5%
4	100.0%	15.0%	4	$8.00	32.3%	12.3%	4	100.0%	17.5%
5	100.0%	0.0%	5	$8.50	45.4%	13.1%	5	100.0%	0.0%
6	100.0%	0.0%	6	$8.00	57.7%	12.3%	6	100.0%	0.0%
7	100.0%	0.0%	7	$6.00	66.9%	9.2%	7	100.0%	0.0%
8	100.0%	0.0%	8	$5.00	74.6%	7.7%	8	100.0%	0.0%
9	100.0%	0.0%	9	$4.00	80.8%	6.2%	9	100.0%	0.0%
10	100.0%	0.0%	10	$3.00	85.4%	4.6%	10	100.0%	0.0%
11	100.0%	0.0%	11	$2.00	88.5%	3.1%	11	100.0%	0.0%
12	100.0%	0.0%	12	$2.00	91.5%	3.1%	12	100.0%	0.0%
13	100.0%	0.0%	13	$1.00	93.1%	1.5%	13	100.0%	0.0%
14	100.0%	0.0%	14	$1.00	94.6%	1.5%	14	100.0%	0.0%
15	100.0%	0.0%	15	$1.00	96.2%	1.5%	15	100.0%	0.0%
16	100.0%	0.0%	16	$1.00	97.7%	1.5%	16	100.0%	0.0%
17	100.0%	0.0%	17	$0.50	98.5%	0.8%	17	100.0%	0.0%
18	100.0%	0.0%	18	$0.50	99.2%	0.8%	18	100.0%	0.0%
19	100.0%	0.0%	19	$0.50	100.0%	0.8%	19	100.0%	0.0%
20	100.0%	0.0%	20	$0.00	100.0%	0.0%	20	100.0%	0.0%
Total	100.0%	100.0%	Total	$65.00	100.0%	100.0%	Total	100.0%	100.0%

* Total losses fixed at $65.00 .

Used in the following models: Present Value Offset, Present Value Cash Flow, PVI/PVE, Risk–Adjusted Discounted Cash Flow, IRR

INVESTMENT PORTFOLIO YIELD CALCULATION

INVESTMENT	Average Asset Value	Interest, Dividends, and Rents Earned	Pre Tax Yield	Average Federal Tax Rate	Post Tax Yield	Net Realized Capital Gains	Pre Tax Yield	Average Federal Tax Rate	Post Tax Yield	Total Pre Tax Yield	Total Post Tax Yield
BONDS											
Taxable	565,000	54,000	9.56%	34.00%	6.31%	0	0.00%	34.00%	0.00%	9.56%	6.31%
Tax–Exempt	155,000	10,000	6.45%	0.62%	6.41%	0	0.00%	34.00%	0.00%	6.45%	6.41%
Total	720,000	64,000	8.89%	28.78%	6.33%	0	0.00%	34.00%	0.00%	8.89%	6.33%
STOCKS (Unaffiliated)											
Common	90,000	8,000	8.89%	12.67%	7.76%	870	0.97%	34.00%	0.64%	9.86%	8.40%
Preferred	25,000	3,000	12.00%	12.67%	10.48%	0	0.00%	34.00%	0.00%	12.00%	10.48%
Total	115,000	11,000	9.57%	12.67%	8.35%	870	0.76%	34.00%	0.50%	10.32%	8.85%
MORTGAGES	86,500	8,600	9.94%	34.00%	6.56%	0	0.00%	34.00%	0.00%	9.94%	6.56%
REAL ESTATE	5,300	1,300	24.53%	34.00%	16.19%	340	6.42%	34.00%	4.23%	30.94%	20.42%
SHORT TERM	70,100	6,900	9.84%	34.00%	6.50%	0	0.00%	34.00%	0.00%	9.84%	6.50%
CASH	19,200	1,200	6.25%	34.00%	4.13%	0	0.00%	34.00%	0.00%	6.25%	4.13%
TOTAL	1,016,100	93,000	9.15%	27.89%	6.60%	1,210	0.12%	34.00%	0.08%	9.27%	6.68%

Portfolio yields are used in : **Calendar Year Investment Offset Method**
Present Value Offset Procedure
Calendar Year ROE Method

FINANCIAL ASSUMPTIONS BY MODEL

Assumption Type	Calendar Year Investment Income Offset Method	Present Value Offset Procedure	Calendar Year ROE Method	Present Value Cash Flow Return Method	Present Value of Income over Present Value of Equity Model	Risk-Adjusted Discounted Cash Flow Model	Internal Rate of Return on Equity Flow Model
Surplus & Equity	N/A	N/A	"Block" held one yr	"Block" held one yr	"Block" held one yr	"Block" held one yr	"Block" held one yr
Premium to Surplus Ratio	N/A	N/A	3.0	3.0	3.0	3.0	3.0
Premium to Equity Ratio	N/A	N/A	2.5	2.5	N/A	N/A	N/A
Investment Yield, Pre-Tax Post-Tax	Portfolio Yield 9.27% 6.68%	N/A	Portfolio Yield 9.27% 6.68%	New Money 8.00%	New Money 8.00%	New Money 8.00%	New Money 8.00%
Discount Rate	N/A	5.28% (Post-Tax)	N/A	8.00%	8.00%	Losses: Risk-Adj. Rate Other: Risk-Free Rate	8.00%
Income Tax Rate	N/A	34%	34.0%	34.0%	34.0%	34.0%	34.0%
Target Return	N/A	N/A	15.0%	15.0%	15.0%	N/A	15.0%

CALENDAR YEAR INVESTMENT OFFSET METHOD

Policyholder–supplied funds on Unearned Premiums		
Average Direct Unearned Premium	$50,000	(1)
Pre–paid Expense Ratio	18%	(2)
Average Premiums Receivable	$28,000	(3)
Average Portfolio Balance due to UEPR	$13,000	(4)=(1)x[1.0–(2)]–(3)
Direct Earned Premium	$160,000	(5)
Balance Relative to Earned Premium	8.13%	(6)=(4)/(5)
Policyholder–supplied funds on Loss Reserves		
Loss Reserves to Loss Incurred (average ratio over three years)	1.20	(7)
Permissible Loss Ratio	60%	(8)
Reserves Relative to Premiums	72%	(9)=(7)x(8)
Investment Offset and Final U/W Profit		
Policyholder–supplied Funds (as a percent of premiums)	80.13%	(10)=(6)+(9)
After–tax Portfolio Yield	6.68%	(11) (see Exh. 1D)
Investment Income Offset	5.35%	(12)=(10)x(11)
Traditional U/W Profit Provision	5.00%	(13)
Final U/W Profit Provision	–0.35%	(14)=(13)–(12)

PRESENT VALUE OFFSET METHOD
LOSS PAYOUT PATTERNS

Qtr.	Discount Factor	Reference Line Pattern	Present Values at Year Zero	Evaluated Line Pattern	Present Values at Year Zero
1	0.987	10.0%	9.9%	3.1%	3.0%
2	0.975	15.0%	14.6%	6.2%	6.0%
3	0.962	20.0%	19.2%	10.8%	10.4%
4	0.950	25.0%	23.7%	12.3%	11.7%
5	0.938	15.0%	14.1%	13.1%	12.3%
6	0.926	10.0%	9.3%	12.3%	11.4%
7	0.914	5.0%	4.6%	9.2%	8.4%
8	0.902	0.0%	0.0%	7.7%	6.9%
9	0.891	0.0%	0.0%	6.2%	5.5%
10	0.879	0.0%	0.0%	4.6%	4.1%
11	0.868	0.0%	0.0%	3.1%	2.7%
12	0.857	0.0%	0.0%	3.1%	2.6%
13	0.846	0.0%	0.0%	1.5%	1.3%
14	0.835	0.0%	0.0%	1.5%	1.3%
15	0.825	0.0%	0.0%	1.5%	1.3%
16	0.814	0.0%	0.0%	1.5%	1.3%
17	0.804	0.0%	0.0%	0.8%	0.6%
18	0.793	0.0%	0.0%	0.8%	0.6%
19	0.783	0.0%	0.0%	0.8%	0.6%
20	0.773	0.0%	0.0%	0.0%	0.0%
Totals		100.0%	95.4%	100.0%	91.9%

Difference in Present Value Loss Patterns =	3.5%	(1)=[95.4%−91.9%]
Projected Loss Ratio =	65.0%	(2)
Underwriting Profit Prov. Offset =	2.3%	(3)=(1)x(2)
Standard U/W Profit Provision =	5.0%	(4)
U/W Profit Provision with Offset =	2.7%	(5)=(4)−(3)

Rate Used for Discounting = 5.28%

CALENDAR YEAR ROE METHOD
SUMMARY

	$	% of Prem	
Premium	$103.35	100.00%	(1)
Loss	$65.00	62.89%	(2)
Fixed Expense	$15.00	14.51%	(3)
Variable Expense	$25.84	25.00%	(4)=(1) x 25%
U/W Gain	($2.49)	−2.41%	(5)=(1) − [(2)+(3)+(4)]
U/W Gain After−Tax	($1.64)	−1.59%	(6)=(5) x (1−34%)
Policyholder Supplied Funds	$82.81	80.13%	(7) (See Exh.2 Line 10)
Surplus	$34.45	33.33%	(8)=(1)/(P/S Ratio) = (1)/3.0
Investible Funds	$117.26	113.46%	(9)=(7)+(8)
Investment Income	$10.87	10.52%	(10)=(9)x(Pre−Tax Yld)=(9)x9.27%
Investment Income AFIT	$7.83	7.58%	(11)=(9)x(After−Tax Yld)=(9)x6.68%
Total Net Income	$6.19	5.99%	(12)=(6)+(11)
Equity	$41.34	40.00%	(13)=(1)/(P/E Ratio)=(1)/2.5

Return on Equity (ROE)	15.0%	(14)=(12)/(13)

PVI/PVE MODEL
SUMMARY

	PRESENT VALUE *	FULL VALUE $	FULL VALUE % of Prem	
Earned Premium	$111.07	$107.89	100.00%	(1)
Incurred Loss	$66.92	$65.00	60.25%	(2)
Incurred Expense	$43.74	$41.97	38.90%	(3)
Underwriting Income	$0.41	$0.92	0.85%	(4)=(1)−(2)−(3)
Investment Income	$10.77	$10.80	10.01%	(5) (see Exh. 6F)
Income Tax	$3.80	$3.98	3.69%	(6) (See Exh. 6F)
Total Income	$7.38	$7.73	7.17%	(7)=(4)+(5)−(6)
Annualized Present Value Equity**	$49.21	----	----	(8) (see Exh. 6E)
Annual Return	15.0%	----	----	(9)=(7)/(8)
Combined	----	$106.97	99.15%	(10)=(2)+(3)
Underwriting Profit Provision	----	----	0.85%	(11)=1−(10)

* Evaluated at the end of the first year
** Evaluated at the beginning of the first year

PVI/PVE MODEL
INPUTS AND ASSUMPTIONS

Loss	$65
Fixed Expense	$15
Variable Expense Ratio	25%
Total Expense Incurral Assumption:	
GAAP	25% up front; 75% evenly for next 4 qtrs.
SAP	75% up front; 25% evenly for next 4 qtrs.
Surplus	"Block" held one year
Premium–to–Surplus Ratio	3.0
Yield on Investments	8.00%
Rate Used for Discounting	8.00%
Income Tax Rate	34%
Target Return	15.0%

PVI/PVE MODEL
UNDERWRITING CASH FLOWS

Qtr	Paid Premium	Paid Loss	Paid Expense
0	43.2	0.0	12.6
1	16.2	2.0	7.3
2	16.2	4.0	7.3
3	16.2	7.0	7.3
4	16.2	8.0	7.3
5	0.0	8.5	0.0
6	0.0	8.0	0.0
7	0.0	6.0	0.0
8	0.0	5.0	0.0
9	0.0	4.0	0.0
10	0.0	3.0	0.0
11	0.0	2.0	0.0
12	0.0	2.0	0.0
13	0.0	1.0	0.0
14	0.0	1.0	0.0
15	0.0	1.0	0.0
16	0.0	1.0	0.0
17	0.0	0.5	0.0
18	0.0	0.5	0.0
19	0.0	0.5	0.0
20	0.0	0.0	0.0
Totals	107.9	65.0	42.0

PVI/PVE MODEL
UNDERWRITING INCOME

Qtr	Earned Premium	Incurred Loss	Statutory Incurred Expense	GAAP Incurred Expense	Statutory U/W Income	GAAP U/W Income
0	0.0	0.0	31.5	10.5	−31.5	−10.5
1	27.0	16.3	2.6	7.9	8.1	2.9
2	27.0	16.3	2.6	7.9	8.1	2.9
3	27.0	16.3	2.6	7.9	8.1	2.9
4	27.0	16.3	2.6	7.9	8.1	2.9
5	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0
Totals	107.9	65.0	42.0	42.0	0.9	0.9

PVI/PVE MODEL
BALANCE SHEET

Qtr	Unearned Premium Reserve	Statutory Expense Reserve	Loss Reserve	End of Qtr Surplus	During Qtr Surplus	Assets	Premium Receivable	Investable Assets	Average Investable Assets	Deferred Acquisition	End–Qtr GAAP Equity	During–Qtr GAAP Equity	Present Value GAAP Equity
0	107.9	18.9	0.0	36.0	0.0	162.7	64.7	98.0	0	21.0	56.9	0.0	0.0
1	80.9	14.2	14.3	36.0	36.0	145.3	48.6	96.7	97.4	15.7	51.7	56.9	55.9
2	53.9	9.4	26.5	36.0	36.0	125.9	32.4	93.5	95.1	10.5	46.5	51.7	49.8
3	27.0	4.7	35.8	36.0	36.0	103.4	16.2	87.2	90.4	5.2	41.2	46.5	43.9
4	0.0	0.0	44.0	36.0	36.0	80.0	0.0	80.0	83.6	0.0	36.0	41.2	38.2
5	0.0	0.0	35.5	0.0	0.0	35.5	0.0	35.5	57.7	0.0	0.0	0.0	0.0
6	0.0	0.0	27.5	0.0	0.0	27.5	0.0	27.5	31.5	0.0	0.0	0.0	0.0
7	0.0	0.0	21.5	0.0	0.0	21.5	0.0	21.5	24.5	0.0	0.0	0.0	0.0
8	0.0	0.0	16.5	0.0	0.0	16.5	0.0	16.5	19.0	0.0	0.0	0.0	0.0
9	0.0	0.0	12.5	0.0	0.0	12.5	0.0	12.5	14.5	0.0	0.0	0.0	0.0
10	0.0	0.0	9.5	0.0	0.0	9.5	0.0	9.5	11.0	0.0	0.0	0.0	0.0
11	0.0	0.0	7.5	0.0	0.0	7.5	0.0	7.5	8.5	0.0	0.0	0.0	0.0
12	0.0	0.0	5.5	0.0	0.0	5.5	0.0	5.5	6.5	0.0	0.0	0.0	0.0
13	0.0	0.0	4.5	0.0	0.0	4.5	0.0	4.5	5.0	0.0	0.0	0.0	0.0
14	0.0	0.0	3.5	0.0	0.0	3.5	0.0	3.5	4.0	0.0	0.0	0.0	0.0
15	0.0	0.0	2.5	0.0	0.0	2.5	0.0	2.5	3.0	0.0	0.0	0.0	0.0
16	0.0	0.0	1.5	0.0	0.0	1.5	0.0	1.5	2.0	0.0	0.0	0.0	0.0
17	0.0	0.0	1.0	0.0	0.0	1.0	0.0	1.0	1.3	0.0	0.0	0.0	0.0
18	0.0	0.0	0.5	0.0	0.0	0.5	0.0	0.5	0.8	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.3	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
													49.2 (Annualized)*

* Present Value GAAP Equity is annualized via dividing the sum of the discounted quarterly equity values by the sum of the discount factors.

PVI/PVE MODEL
INCOME

Qtr	Investment Income	Statutory U/W Income	Statutory Pre-tax Income	GAAP U/W Income	GAAP Pre-tax Income	Income Tax*	Statutory Income	GAAP Income
0	0.0	-31.5	-31.5	-10.5	-10.5	-3.6	-27.9	-6.9
1	1.9	8.1	10.0	2.9	4.7	1.6	8.4	3.1
2	1.8	8.1	9.9	2.9	4.7	1.6	8.3	3.1
3	1.8	8.1	9.9	2.9	4.6	1.6	8.3	3.0
4	1.6	8.1	9.7	2.9	4.5	1.5	8.2	3.0
5	1.1	0.0	1.1	0.0	1.1	0.4	0.7	0.7
6	0.6	0.0	0.6	0.0	0.6	0.2	0.4	0.4
7	0.5	0.0	0.5	0.0	0.5	0.2	0.3	0.3
8	0.4	0.0	0.4	0.0	0.4	0.1	0.2	0.2
9	0.3	0.0	0.3	0.0	0.3	0.1	0.2	0.2
10	0.2	0.0	0.2	0.0	0.2	0.1	0.1	0.1
11	0.2	0.0	0.2	0.0	0.2	0.1	0.1	0.1
12	0.1	0.0	0.1	0.0	0.1	0.0	0.1	0.1
13	0.1	0.0	0.1	0.0	0.1	0.0	0.1	0.1
14	0.1	0.0	0.1	0.0	0.1	0.0	0.1	0.1
15	0.1	0.0	0.1	0.0	0.1	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Totals	10.8	0.9	11.7	0.9	11.7	4.0	7.7	7.7

* Income Tax is obtained by applying tax rate to GAAP Pre-Tax Income. This simplified treatment is not in accord with current tax code provisions.

PRESENT VALUE CASH FLOW RETURN MODEL
SUMMARY

	PRESENT VALUE *	FULL VALUE $	FULL VALUE % of Prem	
Premium	$103.22	$106.20	100.00%	(1)
Loss	$57.34	$65.00	61.21%	(2)
Expense	$40.19	$41.55	39.12%	(3)
Underwriting Cash Flow	$5.69	($0.35)	−0.33%	(4)=(1)−(2)−(3)
After−tax U/W Cash Flow	$3.76	($0.23)	−0.22%	(5)=(4)x(1−34%)
Investment Income on Surplus	$2.70	$2.83	2.67%	(6)
After−tax Inv Income on Surplus	$1.78	$1.87	1.76%	(7)=(6)x(1−34%)
Total Cash Flow	$5.54	$1.64	1.54%	(8)=(5)+(7)
Changes in Equity	$5.54 **	$0.00	----	(9)
Combined	----	----	100.33%	(10)=(2)+(3)
Underwriting Profit Provision	----	----	−0.33%	(11)=(1)−(2)−(3)

* Evaluated at the start of the first year

** Evaluated at the start of the first year at target rate of return

PRESENT VALUE CASH FLOW RETURN MODEL
INPUTS AND ASSUMPTIONS

Loss	$65
Fixed Expense	$15
Variable Expense Ratio	25%
Surplus	"Block" held one year
Premium–to–Surplus Ratio	3.0
Equity	"Block" held one year
Equity–to–Surplus Ratio	1.2
Yield on Investments	8.00%
Rate Used for Discounting	8.00%
Income Tax Rate	34%
Target Return	15.0%

EXHIBIT 6C

PRESENT VALUE CASH FLOW RETURN MODEL
INVESTMENT INCOME ON 'INVESTIBLE EQUITY' **

Qtr	Surplus	Investment Income on Surplus
0	$35.40 *	$0.00
1	$35.40	$0.71
2	$35.40	$0.71
3	$35.40	$0.71
4	$35.40	$0.71
5	$0.00	$0.00
6	$0.00	$0.00
7	$0.00	$0.00
8	$0.00	$0.00
9	$0.00	$0.00
10	$0.00	$0.00
11	$0.00	$0.00
12	$0.00	$0.00
13	$0.00	$0.00
14	$0.00	$0.00
15	$0.00	$0.00
16	$0.00	$0.00
17	$0.00	$0.00
18	$0.00	$0.00
19	$0.00	$0.00
20	$0.00	$0.00
Total		$2.83
Present Value Factor (discounting to start of 1st quarter)		0.953

* $106.20 /3.00= $35.40

** Assumes Surplus equals Investible Equity

PRESENT VALUE CASH FLOW RETURN MODEL
UNDERWRITING CASH FLOWS

Qtr	Paid Premium	Paid Loss	Paid Expense	Net U/W Cash Flow
0	42.5	0.0	12.5	30.0
1	15.9	2.0	7.3	6.7
2	15.9	4.0	7.3	4.7
3	15.9	7.0	7.3	1.7
4	15.9	8.0	7.3	0.7
5	0.0	8.5	0.0	−8.5
6	0.0	8.0	0.0	−8.0
7	0.0	6.0	0.0	−6.0
8	0.0	5.0	0.0	−5.0
9	0.0	4.0	0.0	−4.0
10	0.0	3.0	0.0	−3.0
11	0.0	2.0	0.0	−2.0
12	0.0	2.0	0.0	−2.0
13	0.0	1.0	0.0	−1.0
14	0.0	1.0	0.0	−1.0
15	0.0	1.0	0.0	−1.0
16	0.0	1.0	0.0	−1.0
17	0.0	0.5	0.0	−0.5
18	0.0	0.5	0.0	−0.5
19	0.0	0.5	0.0	−0.5
20	0.0	0.0	0.0	0.0
Totals	106.2	65.0	41.5	−0.3
Present Value Factors (discounting to start of 1st quarter)	0.972	0.882	0.967	

EXHIBIT 6E

PRESENT VALUE CASH FLOW RETURN MODEL
CHANGES IN EQUITY

Qtr	Equity	Change in Equity	PV Change in Equity
0	42.5	42.5	42.5
1	42.5	0.0	0.0
2	42.5	0.0	0.0
3	42.5	0.0	0.0
4	42.5	−42.5	−36.9
5	0.0	0.0	0.0
6	0.0	0.0	0.0
7	0.0	0.0	0.0
8	0.0	0.0	0.0
9	0.0	0.0	0.0
10	0.0	0.0	0.0
11	0.0	0.0	0.0
12	0.0	0.0	0.0
13	0.0	0.0	0.0
14	0.0	0.0	0.0
15	0.0	0.0	0.0
16	0.0	0.0	0.0
17	0.0	0.0	0.0
18	0.0	0.0	0.0
19	0.0	0.0	0.0
20	0.0	0.0	0.0
Totals		0.0	5.5

PRESENT VALUE CASH FLOW RETURN MODEL

Discount Factors (discounting to start of first year)

Item	Rate Used in Discounting	Discount Factor
Premium	8.00%	0.972
Losses	8.00%	0.882
Expenses	8.00%	0.967
Inv Income on Inv Equity	8.00%	0.953
Changes in Equity	15.00%	

RISK–ADJUSTED DISCOUNTED CASH FLOW MODEL
SUMMARY

	PRESENT VALUE *	FULL VALUE $	FULL VALUE % of Prem	
Premium	$106.84	$101.78	100.00%	(1)
Loss	$62.58	$65.00	63.86%	(2)
Expense	$42.25	$40.45	39.74%	(3)
FIT on U/W Cash Flow	$0.68	($1.25)	−1.22%	(4)=[(1)−(2)−(3)]x34%
FIT on Inv Income on Surplus	$0.95	$0.92	0.91%	(5) (see Exh 7D)
Total	$106.47	----	----	(6)=(2)+(3)+(4)+(5)=(1)
Combined	----	$105.45	103.60%	(7)=(2)+(3)
Underwriting Profit Provision	----	----	−3.60%	(8)=1−(7)

* Evaluated at the end of the first year

RISK–ADJUSTED DISCOUNTED CASH FLOW MODEL
INPUTS AND ASSUMPTIONS

Loss	$65
Fixed Expense	$15
Variable Expense Ratio	25%
Surplus	"Block" held one Year
Premium–to–Surplus Ratio	3.0
Risk–Free Rate	8.00%
Average Market Return	10.50%
Beta Coefficient	–0.750
Risk–Adjusted Rate	6.13%
Income Tax Rate	34%

RISK-ADJUSTED DISCOUNTED CASH FLOW MODEL
UNDERWRITING CASH FLOW PATTERNS

Qtr	Premium Payment Pattern	Paid Loss Pattern	Paid Expense Pattern
0	0.400	0.000	0.300
1	0.150	0.031	0.175
2	0.150	0.062	0.175
3	0.150	0.108	0.175
4	0.150	0.123	0.175
5	0.000	0.131	0.000
6	0.000	0.123	0.000
7	0.000	0.092	0.000
8	0.000	0.077	0.000
9	0.000	0.062	0.000
10	0.000	0.046	0.000
11	0.000	0.031	0.000
12	0.000	0.031	0.000
13	0.000	0.015	0.000
14	0.000	0.015	0.000
15	0.000	0.015	0.000
16	0.000	0.015	0.000
17	0.000	0.008	0.000
18	0.000	0.008	0.000
19	0.000	0.008	0.000
20	0.000	0.000	0.000
Totals	1.000	1.000	1.000
Rates Used in Discounting	8.00%	6.13%	8.00%
Discount Factors (discounting to end of first year)	1.050	0.963	1.045

RISK–ADJUSTED DISCOUNTED CASH FLOW MODEL
TAX ON INVESTMENT INCOME ON SURPLUS

Qtr	Surplus	Investment Income on Surplus	Federal Income Tax
0	33.927	0.000	0.000
1	33.927	0.679	0.231
2	33.927	0.679	0.231
3	33.927	0.679	0.231
4	33.927	0.679	0.231
5	0.000	0.000	0.000
6	0.000	0.000	0.000
7	0.000	0.000	0.000
8	0.000	0.000	0.000
9	0.000	0.000	0.000
10	0.000	0.000	0.000
11	0.000	0.000	0.000
12	0.000	0.000	0.000
13	0.000	0.000	0.000
14	0.000	0.000	0.000
15	0.000	0.000	0.000
16	0.000	0.000	0.000
17	0.000	0.000	0.000
18	0.000	0.000	0.000
19	0.000	0.000	0.000
20	0.000	0.000	0.000
Totals		2.714	0.923
Discount Factor			1.030

RISK–ADJUSTED DISCOUNTED CASH FLOW MODEL
DISCOUNT FACTORS (discounting to end of first year)

Item	Rate Used in Discounting	Discount Factor
Premium	8.00%	1.050
Losses	6.13%	0.963
Expenses	8.00%	1.045
Tax on Inv Inc on Surplus	8.00%	1.030

Calculation of Risk–Adjusted Rate

Risk–Free Rate	8.00%
Average Market Rate	10.50%
Beta Coefficient	–0.750
Risk–Adjusted Rate	6.13%

IRR MODEL
SUMMARY

	FULL VALUE		
	$	**% of Prem**	
Premium*	$108.51	100.00%	(1)
Loss	$65.00	59.90%	(2)
Expense	$42.13	38.82%	(3)
Combined	$107.13	98.73%	(4)=(2)+(3)
Underwriting Profit Provision	----	1.27%	(5)=(1)−(4)

* Premium at which IRR on Equity Flows is equal to the Target Return

EXHIBIT 8B

IRR MODEL
INPUTS AND ASSUMPTIONS

Loss	$65
Fixed Expense	$15
Variable Expense Ratio	25%
Total Expense Incurral Assumption:	
GAAP	25% up front; 75% evenly for next 4 qtrs.
SAP	75% up front; 25% evenly for next 4 qtrs.
Surplus	"Block" held one year
Premium–to–Surplus Ratio	3.0
Yield on Investments	8.00%
Rate Used for Discounting	8.00%
Income Tax Rate	34%
Target Return	15.0%

IRR MODEL
SAP UNDERWRITING INCOME

Qtr	Paid Premium	Paid Loss	Paid Expense	Earned Premium	Incurred Loss	Statutory Incurred Expense	Statutory U/W Income
0	43.4	0.0	12.6	0.0	0.0	31.6	−31.6
1	16.3	2.0	7.4	27.1	16.3	2.6	8.2
2	16.3	4.0	7.4	27.1	16.3	2.6	8.2
3	16.3	7.0	7.4	27.1	16.3	2.6	8.2
4	16.3	8.0	7.4	27.1	16.3	2.6	8.2
5	0.0	8.5	0.0	0.0	0.0	0.0	0.0
6	0.0	8.0	0.0	0.0	0.0	0.0	0.0
7	0.0	6.0	0.0	0.0	0.0	0.0	0.0
8	0.0	5.0	0.0	0.0	0.0	0.0	0.0
9	0.0	4.0	0.0	0.0	0.0	0.0	0.0
10	0.0	3.0	0.0	0.0	0.0	0.0	0.0
11	0.0	2.0	0.0	0.0	0.0	0.0	0.0
12	0.0	2.0	0.0	0.0	0.0	0.0	0.0
13	0.0	1.0	0.0	0.0	0.0	0.0	0.0
14	0.0	1.0	0.0	0.0	0.0	0.0	0.0
15	0.0	1.0	0.0	0.0	0.0	0.0	0.0
16	0.0	1.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.5	0.0	0.0	0.0	0.0	0.0
18	0.0	0.5	0.0	0.0	0.0	0.0	0.0
19	0.0	0.5	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Totals	108.5	65.0	42.1	108.5	65.0	42.1	1.4

IRR MODEL
SAP BALANCE SHEET

Qtr	Unearned Premium Reserve	Statutory Expense Reserve	Loss Reserve	SAP Surplus	Assets	Premium Receivable	Average Investable Assets
0	108.5	19.0	0.0	36.2	163.6	65.1	0.0
1	81.4	14.2	14.3	36.2	146.0	48.8	97.9
2	54.3	9.5	26.5	36.2	126.4	32.6	95.5
3	27.1	4.7	35.8	36.2	103.8	16.3	90.7
4	0.0	0.0	44.0	36.2	80.2	0.0	83.8
5	0.0	0.0	35.5	0.0	35.5	0.0	57.8
6	0.0	0.0	27.5	0.0	27.5	0.0	31.5
7	0.0	0.0	21.5	0.0	21.5	0.0	24.5
8	0.0	0.0	16.5	0.0	16.5	0.0	19.0
9	0.0	0.0	12.5	0.0	12.5	0.0	14.5
10	0.0	0.0	9.5	0.0	9.5	0.0	11.0
11	0.0	0.0	7.5	0.0	7.5	0.0	8.5
12	0.0	0.0	5.5	0.0	5.5	0.0	6.5
13	0.0	0.0	4.5	0.0	4.5	0.0	5.0
14	0.0	0.0	3.5	0.0	3.5	0.0	4.0
15	0.0	0.0	2.5	0.0	2.5	0.0	3.0
16	0.0	0.0	1.5	0.0	1.5	0.0	2.0
17	0.0	0.0	1.0	0.0	1.0	0.0	1.3
18	0.0	0.0	0.5	0.0	0.5	0.0	0.8
19	0.0	0.0	0.0	0.0	0.0	0.0	0.3
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0

EXHIBIT 8E

IRR MODEL
SAP INCOME

Qtr	Investment Income	Statutory U/W Income	Statutory Pre–tax Income	Income Tax*	Statutory Income	Change in Surplus	Equity Flow
0	0.0	−31.6	−31.6	−3.6	−28.0	36.2	−64.2
1	1.9	8.2	10.1	1.7	8.5	0.0	8.5
2	1.9	8.2	10.1	1.6	8.5	0.0	8.5
3	1.8	8.2	10.0	1.6	8.4	0.0	8.4
4	1.6	8.2	9.9	1.6	8.3	−36.2	44.5
5	1.1	0.0	1.1	0.4	0.7	0.0	0.7
6	0.6	0.0	0.6	0.2	0.4	0.0	0.4
7	0.5	0.0	0.5	0.2	0.3	0.0	0.3
8	0.4	0.0	0.4	0.1	0.2	0.0	0.2
9	0.3	0.0	0.3	0.1	0.2	0.0	0.2
10	0.2	0.0	0.2	0.1	0.1	0.0	0.1
11	0.2	0.0	0.2	0.1	0.1	0.0	0.1
12	0.1	0.0	0.1	0.0	0.1	0.0	0.1
13	0.1	0.0	0.1	0.0	0.1	0.0	0.1
14	0.1	0.0	0.1	0.0	0.1	0.0	0.1
15	0.1	0.0	0.1	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Totals	10.8	1.4	12.2	4.2	8.1	0.0	8.1

* Income Tax is obtained by applying the tax rate to Statutory Pre–tax Income.
This simplified treatment is not in accord with current tax code provisions.

IRR MODEL
GAAP INCOME

Qtr	Statutory Incurred Expense	GAAP Incurred Expense	Deferred Acquisition	GAAP U/W Income	GAAP Pre–tax Income	GAAP Income	Change in Surplus	Change in Deferred Acquisition	Equity Change	Equity Flow
0	31.6	10.5	21.1	–10.5	–10.5	–7.0	36.2	21.1	57.2	–64.2
1	2.6	7.9	15.8	3.0	4.9	3.2	0.0	–5.3	–5.3	8.5
2	2.6	7.9	10.5	3.0	4.8	3.2	0.0	–5.3	–5.3	8.5
3	2.6	7.9	5.3	3.0	4.7	3.1	0.0	–5.3	–5.3	8.4
4	2.6	7.9	0.0	3.0	4.6	3.0	–36.2	–5.3	–41.4	44.5
5	0.0	0.0	0.0	0.0	1.1	0.7	0.0	0.0	0.0	0.7
6	0.0	0.0	0.0	0.0	0.6	0.4	0.0	0.0	0.0	0.4
7	0.0	0.0	0.0	0.0	0.5	0.3	0.0	0.0	0.0	0.3
8	0.0	0.0	0.0	0.0	0.4	0.2	0.0	0.0	0.0	0.2
9	0.0	0.0	0.0	0.0	0.3	0.2	0.0	0.0	0.0	0.2
10	0.0	0.0	0.0	0.0	0.2	0.1	0.0	0.0	0.0	0.1
11	0.0	0.0	0.0	0.0	0.2	0.1	0.0	0.0	0.0	0.1
12	0.0	0.0	0.0	0.0	0.1	0.1	0.0	0.0	0.0	0.1
13	0.0	0.0	0.0	0.0	0.1	0.1	0.0	0.0	0.0	0.1
14	0.0	0.0	0.0	0.0	0.1	0.1	0.0	0.0	0.0	0.1
15	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Totals	42.1	42.1		1.4	12.2	8.1				8.1