SOCIETY OF ACTUARIES

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B5-Call for Papers: Advanced Risk Modeling Concepts

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Local knowledge. Global power.

A Risk Managment Tool for Long Liabilities: the Static Control Model

May 1, 2009

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Group Risk

LIFE INSURANCE | PENSIONS | INVESTMENTS

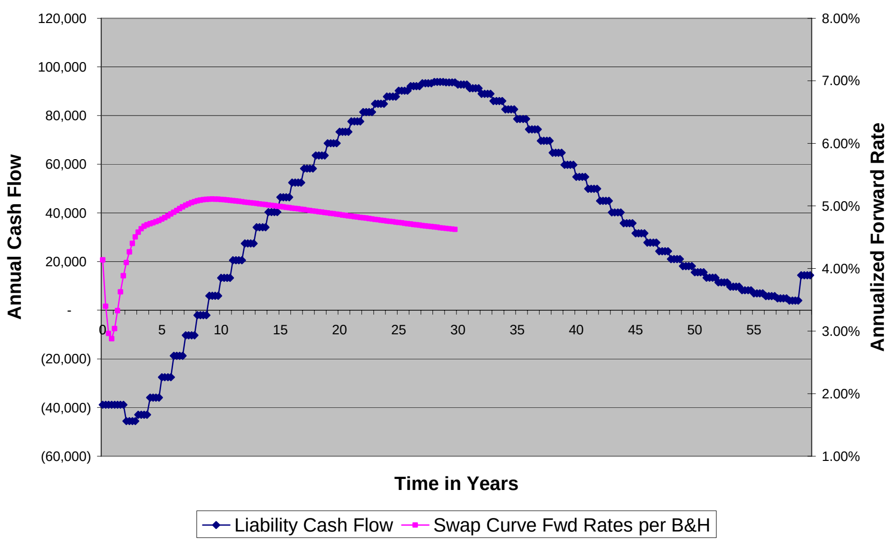
Agenda

- Introduction
  - Hedgeable vs Non Hedgeable Risk
  - Examples of Long Liabilities
- Critique of Yield Curve Extension Approach
  - Monopole model – use a fixed long forward rate
  - Dipole model - extend yield curve using last fwd rate
  - Formal "Vasicek" Extension model
- Static Replication Approach
  - Simple Total Return model & Marginal Cost Yield Curve
  - Static Control Model
  - Formal theoretical properties (3 good, 2 not so good)
  - Technical Caveats
- A Risk Management Example – two approaches produce similar results in practice

Introduction & Examples

- Canada
  - Term to 100 life insurance has been important since 1980
  - No cash value, low lapse rates => long liability
- USA
  - Long Term Care health Ins. Low lapse rates => long liability
  - Pension Plans
- Taiwan
  - Local debt market limited relative to North America
  - Even “normal” insurance products long relative to available fixed income assets
- Problem Definition: Must be able to do three things
  - Put a value on a long risk (extend the yield curve)
  - Decompose risk into hedgeable and non-hedgeable parts
  - Deal appropriately with non-hedgeable risk (economic capital and margins)

Model Inputs: Long Liability at 9/08



Simple Model #1

- Simplifying Assumption: All forward rates after 30 years are constant at f = 5.27% .
- Value in terms of known zero coupon bonds $Z_1,\ldots,Z_{30}$

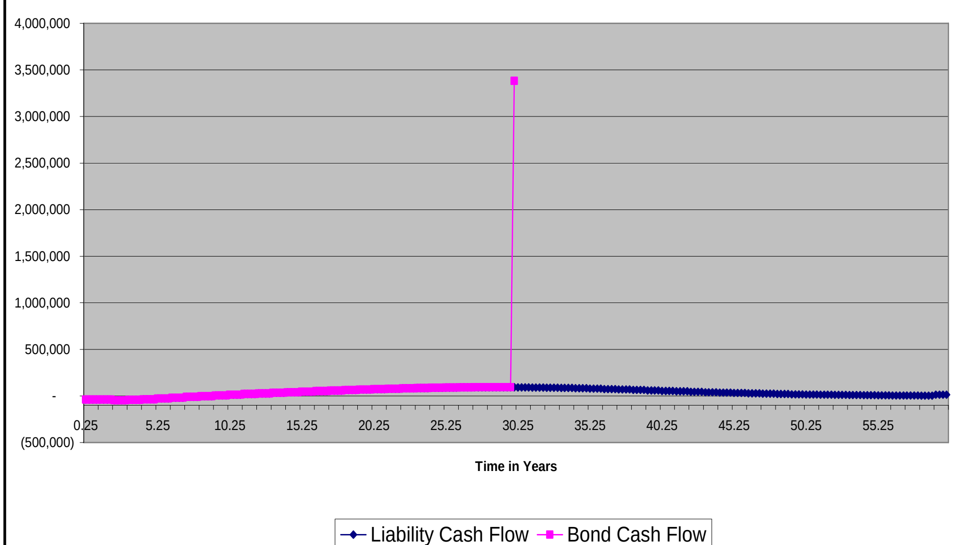
$$V(Z_1,\ldots,Z_{30}) = \sum_{j=1}^{30} a_j Z_j + Z_{30}\sum_{j=31}^{60} a_j (1+f)^{-(j-30)}$$

- Do we have a Static Hedge Portfolio ? Yes – the monopole

$$b_k = \frac{\partial V}{\partial Z_k} \Rightarrow b_{30} = a_{30} + \sum_{j=31}^{60} a_j (1+f)^{-(j-30)}$$

- Is Static Hedge self financing as time moves forward? No

Long Liability Monopole Static Hedge



Simple Model #1 Roll Forward

- Assume yield curve evolves from $Z_1,\ldots,Z_{30}$ to $Z'_1,\ldots,Z'_{30}$ over one year

Table 1

$K = \sum_{j=30}^{60} a_j (1+f)^{-(j-30)}$	**Assets**	**Liabilities**	**Difference**
t=0	KZ_{30}	KZ_{30}	0
t=1	$KZ'_{29} = (1+f'_{30})KZ'_{30}$	$(1+f)KZ'_{30}$	$(f'_{30}-f)KZ'_{30}$

- Conclusion: Take gains when $f'_{30} > f$ and losses when $f'_{30} < f$
- This is the unhedged risk for this approach
- Suggests a model where forward rate grades from f_{30} to f over some reasonable time frame (see Model #3)

Simple Model #2

- Simplifying Assumption: All forward rates after 30 years are constant at f_{30}.
- Value in terms of known zero coupon bonds $Z_1,\ldots,Z_{30}$

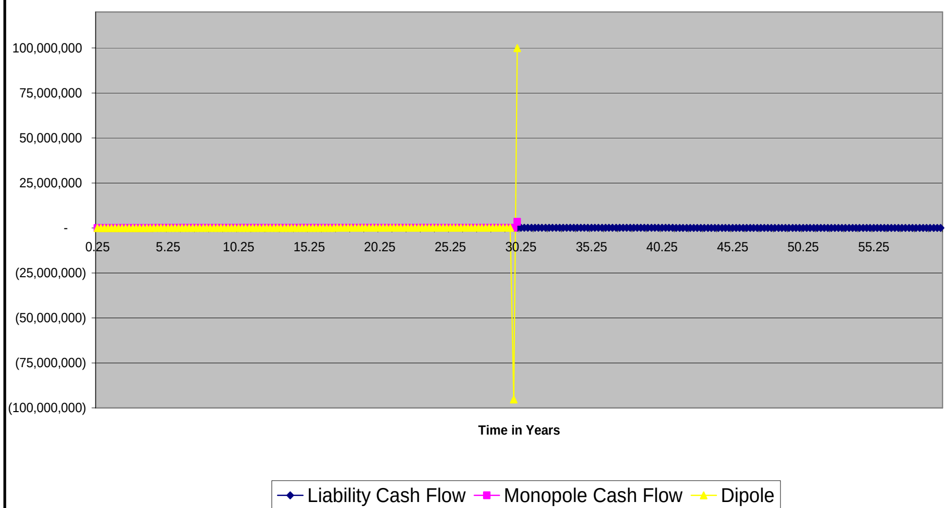
$$V(Z_1,\ldots,Z_{30}) = \sum_{j=1}^{30} a_j Z_j + Z_{30} \sum_{j=31}^{60} a_j \left(\frac{Z_{29}}{Z_{30}}\right)^{-(j-30)}$$

- Do we have a Static Hedge Portfolio ? Yes – the dipole

$$b_k = \frac{\partial V}{\partial Z_k} \Rightarrow \begin{cases} b_{30} = a_{30} + \sum_{j=31}^{60} a_j \left(\frac{Z_{29}}{Z_{30}}\right)^{-(j-30)} + \frac{1}{Z_{30}} \sum_{j=31}^{60} a_j (j-30) \left(\frac{Z_{29}}{Z_{30}}\right)^{-(j-30)} \\ b_{29} = a_{29} - \frac{1}{Z_{29}} \sum_{j=31}^{60} a_j (j-30) \left(\frac{Z_{29}}{Z_{30}}\right)^{-(j-30)} \end{cases}$$

- Is Static Hedge self financing as time moves forward? Still no.

Long Liability Dipole Static Hedge



Simple Model #2 Roll Forward

Table 2

$b_{30} = \frac{DK}{Z_{30}},$ $b_{29} = -\frac{DK}{Z_{29}}$	**Assets**	**Liabilities** $K = \sum_{j=30}^{60} a_j (\frac{Z_{29}}{Z_{30}})^{-(j-30)}$	**Difference**
t=0	$KZ_{30} + \{b_{30}Z_{30} + b_{29}Z_{29}\}$	$V = KZ_{30}$	0
t=1	$(1+f'_{30})(K+b_{30})Z'_{30}$ $(1+f'_{29})b_{29}Z'_{29}$ $+b_{30}dZ_{30} + b_{29}dZ_{29}$	$(1+f'_{30})KZ'_{30}$ $+b_{30}dZ_{30} + b_{20}dZ_{29}$ $+\frac{1}{2}(C+D)Vdf_{30}^2$ $+\ldots$	$(f'_{30} - f'_{29})DKZ'_{30}$ $-\frac{1}{2}(C+D)KZ'_{30}df_{30}^2$ $+\ldots$

- Still have a theoretical bias – method not acceptable
- Is the dipole strategy realistic?

Formal Solution – the “Vasicek” extension

- Want a method that eliminates bias in Model #1
  - Result is a mixture of issues in Models #1 and #2
- Assume we can hedge the monopole risk
- Assume we go naked on the “dipole” risk and hold economic capital for that
- Two risk variable model
  - Z = n year 0 coupon bond, f = n year forward
- Extrapolate yield curve given f and discount remaining cash flows to time n. $V(t,Z,f) = Z\,K(t,f)$
- Invest $V(t,Z,f)$ in n year zero coupon bond
- Engineer $K(t,f)$ so that roll forward bias is equal to cost of capital

Formal Solution – the “Vasicek” extension

- Make enough simplifying assumptions to allow closed form solution
- Mean reverting forward rate $df = \alpha(\bar{f} - f)dt + \sigma dw$
- Constant volatility long bond
  $$dZ = -nZ[\mu dt + s dw'],\ dw dw' = \rho dt$$
- Asset increment $dA = fKZdt + KdZ$
- Liability Increment $dV = KdZ + ZdK + dKdZ$
- Incremental Gain (“Z” risk hedged, “f” risk is not)

$$\begin{aligned} dA - dV &= fKZdt + KdZ - \{KdZ + ZdK + dKdZ\} \\ &= fKZdt - \{ZdK + dKdZ\} \end{aligned}$$

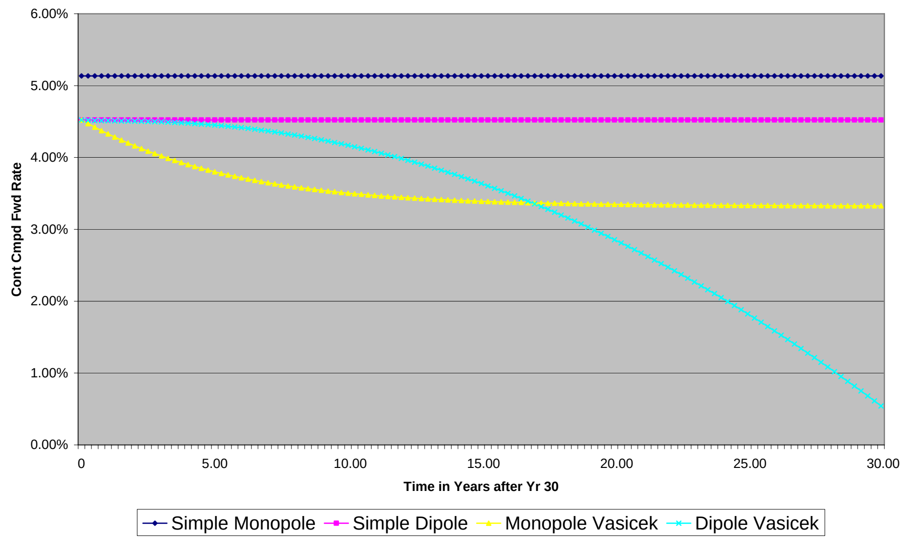
Formal Solution – the "Vasicek" extension

- Formal unhedged loss is $-\frac{\partial K}{\partial f} Z\sigma dw$
- Suggests economic capital should be
$$EC = DZ[K(t, f-\lambda\sigma) - K(t,f)] \approx DZ[-\lambda\sigma\frac{\partial K}{\partial f} + \frac{1}{2}\lambda^2\sigma^2\frac{\partial^2 K}{\partial f^2}]$$
- Formal valuation equation to get desired bias
$$\frac{\partial K}{\partial t} + \alpha(\bar{f} - \sigma\frac{ns\rho + \pi\lambda}{\alpha} - f)\frac{\partial K}{\partial f} + \frac{(1+\pi\lambda^2)\sigma^2}{2}\frac{\partial^2 K}{\partial f^2} = fK$$
- Formal solution per Vasicek for extrapolated fwd rates
$$\delta_f(s-n) = fe^{-\alpha(s-n)} + (\bar{f} - \sigma\frac{ns\rho+\pi\lambda}{\alpha})(1-e^{-\alpha(s-n)}) - \frac{(1+\pi\lambda^2)\sigma^2}{\alpha^2}(1-e^{-\alpha(s-n)})^2$$

Formal Solution – the "Vasicek" extension

- Summary: If our model is correct then
$$dA - dV = \pi ECdt + Z\frac{\partial K}{\partial f}\sigma dw, \quad EC = -\lambda\sigma Z\frac{\partial K}{\partial f} + Z\frac{\lambda^2\sigma^2}{2}\frac{\partial^2 K}{\partial f^2}$$
- Extrapolated forward rates
$$\delta_f(s-n) = fe^{-\alpha(s-n)} + (\bar{f} - \sigma\frac{ns\rho+\pi\lambda}{\alpha})(1-e^{-\alpha(s-n)}) - \frac{(1+\pi\lambda^2)\sigma^2}{2\alpha^2}(1-e^{-\alpha(s-n)})^2$$
- Limiting fwd rate
$$f_\infty = [(\bar{f} - \sigma\frac{ns\rho}{\alpha}) - \frac{\sigma^2}{2\alpha^2}] - \frac{\pi\sigma\lambda}{\alpha}(1+\frac{\sigma\lambda}{2\alpha})$$
$$= [.0515 - .01\frac{30(.01)(.75)}{.2} - \frac{(.01)^2}{2(.2)^2}] - \frac{.04(.01)2.8}{.2}(1+\frac{(.01)2.8}{2(.2)})$$
$$= .0515 - .0113 - .0013 - .0060 = .0330$$

Yield Curve Extension Models: Fwd Rates



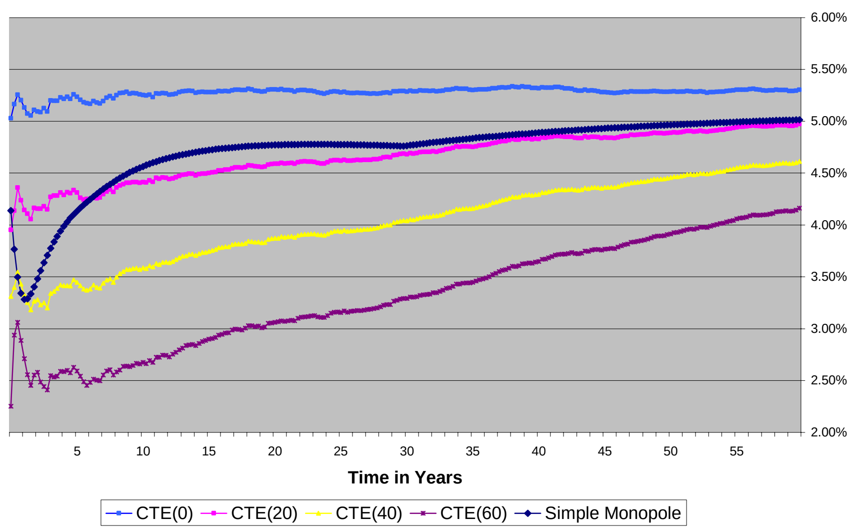
Simple Model #3 – Pure Total Return

- Imagine we have only one asset class
  - Total Return Vehicle with known statistics for all years
    - e.g. Log Normal Equity Return factors $A(z) = exp(\mu + \sigma z)$
    - Expected Return $E\,[A(z)] = exp(\mu + ½\ \sigma^2) = 1.08$
    - Volatility $\sigma = 16\%$
    - Expected Discount $E\,[1/A(z)] = exp(-\mu + ½\ \sigma^2) = 1/(1.0527)$
    - Generate 5000 random return scenarios and look at distribution of PV of cash flow
- PV means amount of initial asset required to mature obligation assuming Total Return Vehicle is the sole asset class.

Simple Model #3 – Marginal Cost Yield Curve

- Use a coherent risk measure (e.g. CTE(x%)) to assign a value to cash flow
- Define a function $L(\boldsymbol{a}) = L(a_1, a_2, \ldots) = CTE\ PV\ (a_1, a_2, \ldots)$
- What if we shock the cash flows $\boldsymbol{a} \rightarrow \boldsymbol{a} + \Delta\boldsymbol{a}$?
- If the shock is small enough it will not change the ordering of the scenario results
- Conclusion1: The partial derivatives $\partial L / \partial a_i$ are discount factors!
- $\partial L / \partial a_i$ is the average PV at duration i over the set of scenarios included in the CTE calculation
- Conclusion 2 : $L = \sum_i a_i\, \partial L / \partial a_i$, $\Delta L \approx \sum_i \Delta a_i\, \partial L / \partial a_i$
- Leads to concept of the *Marginal Cost Yield Curve* (*MCYC*)
- Impact of adding a new (small) block of business to the portfolio is the same as valuing the new block on the *MCYC*

Simple Bonds: Annual MCYC Spot Yields



Simple Model #3 – Total Return Hurdle

Table 3: Long Liability using Sept. 2008 $US Swap Curve

Amounts in $millions — 01/19/09

Quarterly Time Step

Strategy	CTE	Static Hedge	Duration	Total Return	Total Return Hurdle	EC	Total Liability	Level Val'n %	VaR Level	Std Err
No Bonds	0%	-		1,522	5.23%		1,522	5.30%	64%	28
	20%	-		1,911	5.21%		1,911	4.66%	72%	34
	40%	-		2,394	5.06%		2,394	4.01%	80%	43
	60%	-		3,092	4.77%		3,092	3.25%	87%	60
	80%	-		4,366	4.33%		4,366	2.21%	94%	102
	90%	-		5,829	3.89%		5,829	1.33%	97%	173

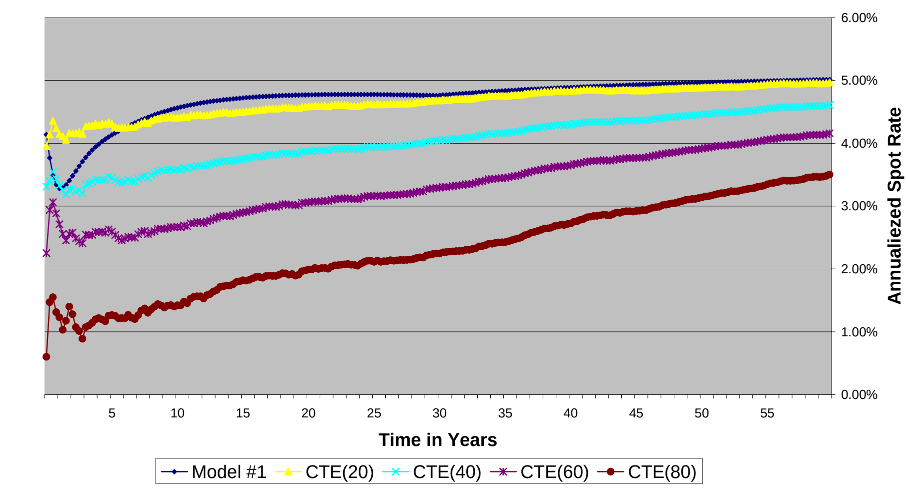
- Question: If we use one of these valuation models, what rate would our equities have to earn over a (short) time frame in order to avoid recognizing a loss over that time frame?
- Answer: Total Return Hurdle
- How do we choose the CTE level?
- One Answer: Choose the level that gives us a reasonable total return target e.g. 4.00% => between CTE(80%) & CTE(90%)

Model #3 – Simple Bond Strategy

- Comparing the MCYC to the swap curve we draw a few conclusions
- Where liability cash flows are positive then, on the margin, we are better off backing them with bonds rather than equity
- Try a strategy of buying bonds to match first 30 years of liability cash flow
- Calculate the PV of the net cash flow $a(t)$-$b(t)$ over each scenario using the simulated returns
- Assign a value W to the liability as

$$W(a,b,Z) = \sum_t b(t)Z(t) + L(\boldsymbol{a}\text{-}\boldsymbol{b})$$

Marginal Cost Spot Rates - Simple Bonds



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Amounts in $millions

01/22/09

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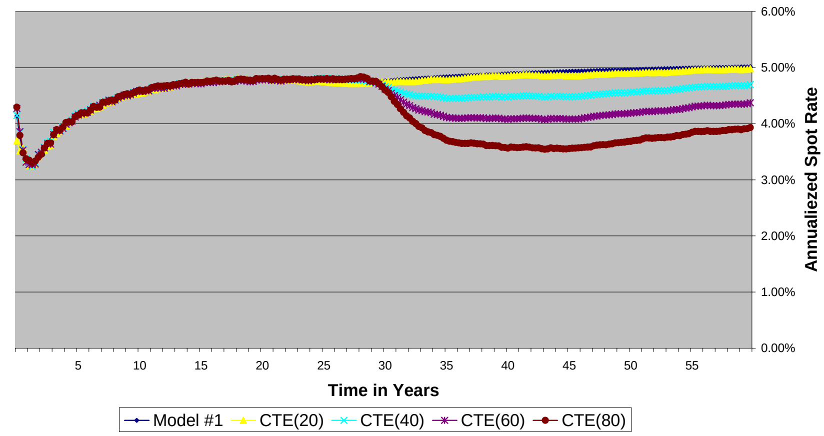
Simple Bonds	0%	981	37.6	696	5.21%		1,677	5.03%	67%	12
(match first	20%	981	37.6	836	5.08%		1,817	4.81%	74%	15
30 years)	40%	981	37.6	1,020	4.90%		2,001	4.53%	81%	19
	60%	981	37.6	1,298	4.60%		2,279	4.15%	87%	27
	80%	981	37.6	1,833	4.12%		2,814	3.53%	94%	47
	90%	981	37.6	2,469	4.41%		3,450	2.92%	97%	83
Monopole Vasicek	N/A	1,873	34.0	-	N/A	74	1,873	4.80%		
Dipole Vasicek	N/A	1,847	37.5	-	N/A	0	1,847	4.78%		

- A big improvement over no bonds
- Still a long way from the answers given by yield curve extension models
- Which bond strategy minimizes the value?
- Answer: the Static Control Model

Marginal Cost Spot Rates - Static Control



Looks like the monopole model is the CTE(0) limit.

Table 3: Long Liability using Sept. 2008 $US Swap Curve

Quarterly Time Step — Amounts in $millions — 01/22/09

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Monopole Vasicek	N/A	1,873	34.0	-	N/A	74	1,873	4.80%		
Dipole Vasicek	N/A	1,847	37.5	-	N/A	0	1,847	4.78%		
Static Control	20%	1,016	40.7	792	4.78%		1,807	4.82%	75%	13
	40%	1,509	35.3	389	2.35%		1,898	4.68%	82%	10
	60%	1,683	34.5	321	-2.17%		2,004	4.53%	88%	10
	80%	1,883	34.1	276	-10.90%		2,159	4.31%	94%	12
	90%	2,108	32.5	193	-27.62%		2,301	4.13%	97%	16

Equity Model: Log Normal — Yield Curve: $US Swaps at Sept. 30, 2008

Volatility 15.99%

Long Fwd Rate 5.27%

Static Control Model – 3 Theoretical Results

- Market Consistency – first order optimality condition is equivalent to statement of market consistency

$$W(a,b,Z) = b \cdot Z + L(a-b), \quad \frac{\partial W}{\partial b} = 0 \Rightarrow Z = \left.\frac{\partial L}{\partial a}\right|_{a-b}$$

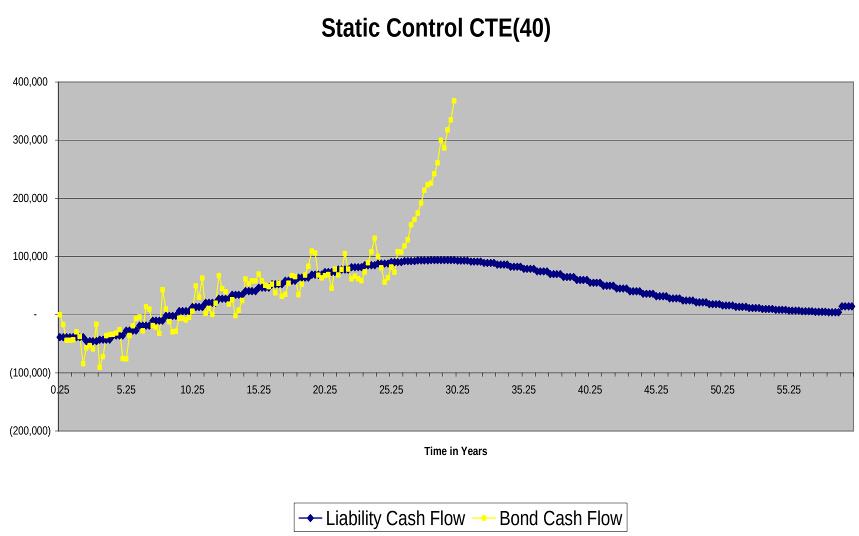
- Static Hedge – the optimal bond position b^* is the static hedge

$$V(a,Z) = W(a,b^*,Z) \Rightarrow \frac{\partial V}{\partial Z} = b^*$$

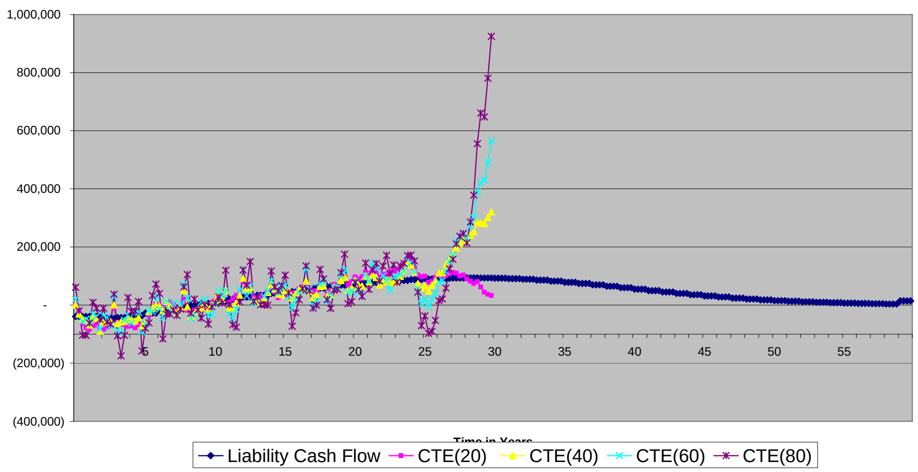
- Convexity Margin – because we are optimizing over static investment strategies (not dynamic) there is an element of conservatism. The static hedge portfolio is always more convex than the liability.

$$\frac{\partial^2 V}{\partial Z_i \partial Z_j} = -\left.\left(\frac{\partial^2 W}{\partial b_i \partial b_j}\right)^{-1}\right|_{b=b^*} \Leftarrow \textit{negative definite}$$

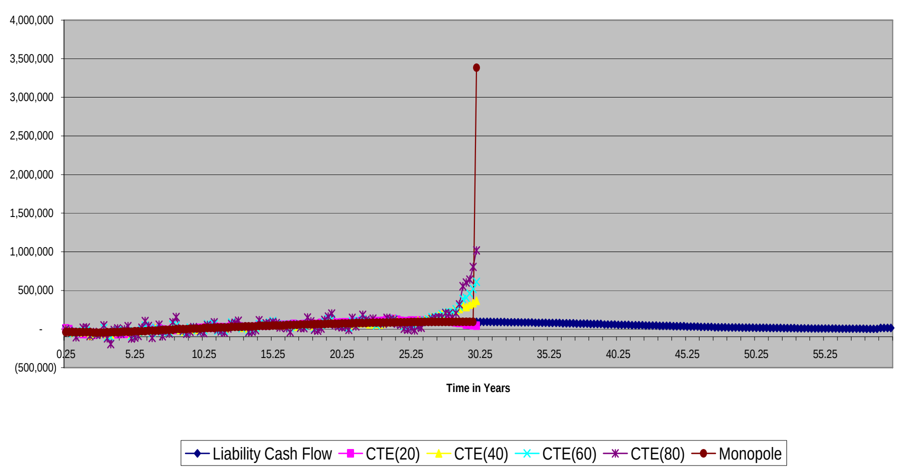
Static Control Model – Replicating Bond Example



Static Control Optimal Bonds



Static Hedge Bond Cash Flows



Static Control – Summary so far

- Good News
  - Method assigns a value very similar to yield curve extension models
  - Much more reasonable static hedge strategy due to use of total return asset class (no big monopole or dipole)
  - Market Consistent – model will properly price all instruments built into the optimization step
  - Convexity margin for risks which are not a static linear combination of available instruments
- Bad News:
  - Technical Complexity : This approach will require an investment in the proper tools. They do exist today.
  - Method is sub-additive when aggregating risks
  - Extrapolated Forward Rate curve is NOT continuous, but that is how we avoid the "dipole" problem

Static Control – The Unhedged Risk

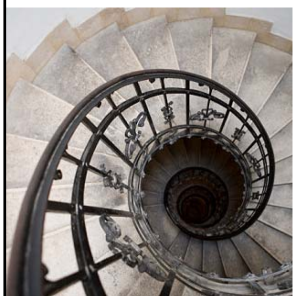
- Consider the following example

Table 4 : Yield Curve Shock Analysis

	CTE	Static Hedge	Duration	Total Return	Total Return Hurdle	Total Liability	Level Val'n %	VaR Level	Std Err
Base +50 bp	30%	1,288	37.8	312	1.33%	1,600	5.16%	80%	7
Base	30%	1,383	36.3	471	3.88%	1,854	4.75%	79%	10
Base -50 bp	30%	805	43.4	1,282	4.94%	2,087	4.41%	77%	20
				Simple $ Dur'n Liab		48,713			
				Static Hedge $ Dur'n		50,200			
Yield Curve - 50 bp									
	30%	805	43.4	1,282	4.94%	2,087	4.41%	77%	20
	40%	1,477	34.7	697	4.47%	2,174	4.29%	82%	14
	60%	1,864	33.1	446	1.31%	2,310	4.11%	88%	13
	80%	2,168	32.2	334	-7.00%	2,502	3.88%	94%	16
		Estimated Economic Capital				128			

Static Control – Unhedged Risk

- Can't hedge the risk that we have to change the CTE level to keep the total return target manageable
- Just like the yield curve extension models
  - We take a gain if the yield curve rises
  - We take a loss if the yield curve drops
- Must hold Economic Capital for this risk
- As an offset we have the convexity gain



Stochastic Trend Models in Casualty and Life Insurance

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Introduction

- We focus on trends that are not only stochastic, but dynamic
  - Meaning the parameters may change over time.
  - What are the implications for forecast errors?
- In casualty, we will focus on trends on the diagonal of the triangle, i.e. the calendar year of payment.
- In life, we will focus on mortality trends by calendar year of death.

The Data Triangle

	0	1	2	3	4	5	6	7	8	9
1998	4,645	4,927	3,016	1,485	1,172	806	594	438	316	316
1999	4,205	5,412	3,114	1,865	1,018	584	532	447	356	
2000	4,543	5,800	3,335	1,867	1,145	641	596	471		
2001	4,546	5,773	3,414	1,858	738	443	488			
2002	4,253	5,258	3,002	1,650	1,106	614				
2003	4,273	5,177	2,938	1,748	1,145					
2004	4,624	5,174	2,675	1,661						
2005	4,865	5,082	2,843							
2006	5,130	5,594								
2007	5,212									

Accident Year (AY) 2002

Calendar Year of Payment (CY) 2007

(CY = AY + DY)

Development Year (DY) 1

(payments made in the year after the accident year)

The Casualty Data

- U.S Industry Workers Compensation payments
- Accident Years (AY) 1979 – 2007
- Development Years (DY) 0 – 9
- Net, derived by published data (Schedule P)
  - Adjusted to remove effects of aggregate reinsurance
  - Companies with problematic data excluded

The Trapezoid -- US WC Industry Loss Payments

	0	1	2	3	4	5	6	7	8	9
1979	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx
1980	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx
1981	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx
1982	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx
1983	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx
1984	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx
1985	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx
1986	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx
1987	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx
1988	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx
1989	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx
1990	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx
1991	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx
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1993	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx
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2004	xxxx	xxxx	xxxx	xxxx						
2005	xxxx	xxxx	xxxx							
2006	xxxx	xxxx								
2007	xxxx									

The Life Data

- French Mortality
- Deaths in years 1947 – 2004
- Age at death from 50 – 99
- We arrayed it as a casualty-like triangle.

The Life Data as a Triangle

	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99
1897	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx	xxxx
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Casualty Model - Background
2 – way multiplicative fixed effects (MFE)

$q_{w,d}$ is incremental paid losses

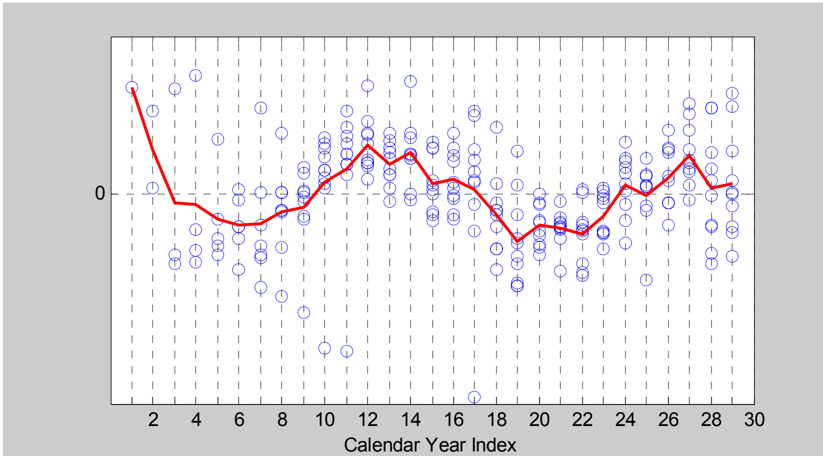
- ***w*** for rows (AY)
- ***d*** for columns (DY)

$$q_{w,d} = U_w * g_d + e_{w,d}$$

- If $q_{w,d}$ is Poisson (or over-dispersed Poisson), then model is "chain-ladder consistent."
- Possible shortcomings:
  – No calendar year (may not fit real data)
  – May be too many parameters

The Fitted MFE2/ODP

- Deviance residuals vs. calendar year of payment:



Chain Ladder – consistent models dramatically overstate the accuracy of the chain-ladder forecast

Using the chain-ladder to predict payments for the next 5 CY's:

- Predicted standard error of 3% versus observed mean square error of 8% – 11%**
- Only ***10%*** of the observed variance is predicted
- ***75%*** of observed errors exceed 2 predicted standard deviations.

** Predicted CV of forecast error (next 5 CY's of payments, no tail) from bootstrapping the MFE2/ODP.

3 – way multiplicative fixed effects (MFE)

Calendar year parameters are added

$$q_{w,d} = U_w * g_d * h_{w+d} + e_{w,d}$$

- Now even more parameters
- To reduce parameters:
  - Losses per exposure, so $y_{w,d} = q_{w,d} / E_w$
  - Convert ***g***'s and ***h***'s to from levels to trends
  - Consider parameters in each direction as parameter *types*, not necessarily all different.

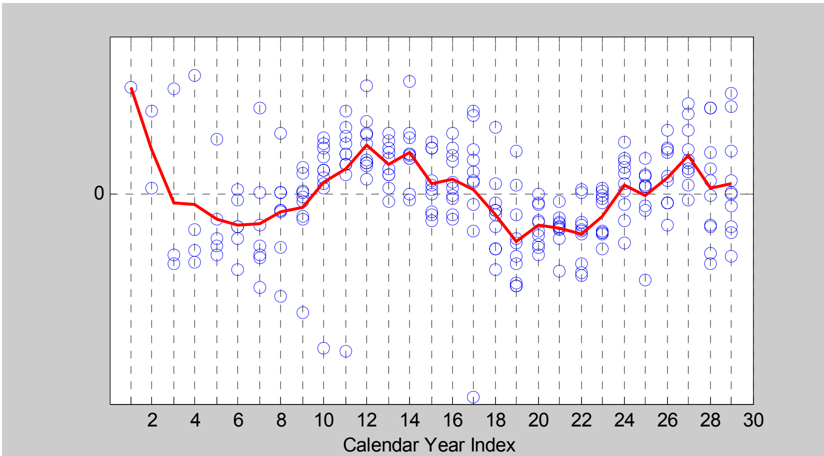
The Stochastic Trend Structure

$$y_{w,d} = \exp\left(\alpha_w + \sum_{i=1}^{d} \gamma_i + \sum_{j=1}^{w+d} \iota_j \right) + e_{w,d}$$

- α's, γ's and ι's are the 3 types.
- To make them not all different, we have 2 approaches:
  - *Breaks*: The same parameter for a number of years. Then, at a break point, a new parameter.
  - *Filtering*: The parameter value changes continuously, but the changes are smooth. The individual values are not independent.

The Fitted MFE2/ODP

- AIC: 6,244 BIC: 6,377 # Parameters: 38
- Deviance residuals:



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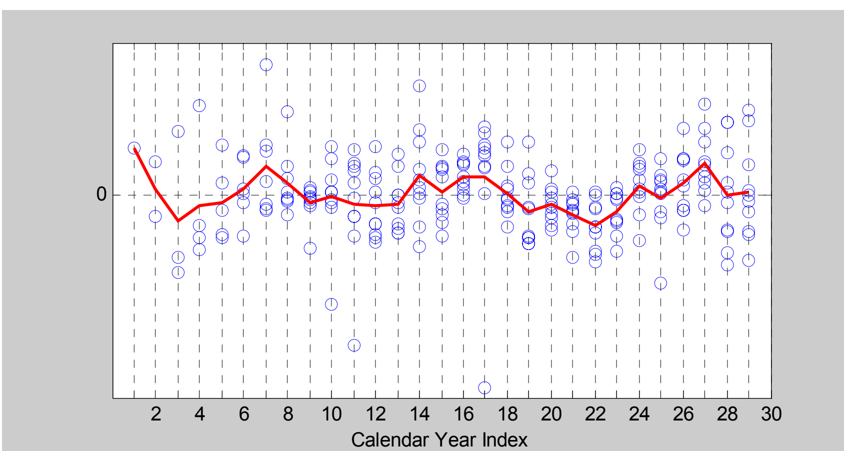
Add CY Trends with Breaks at 7, 12, 19

- AIC: 6,157 BIC: 6,301 # Parameters: 41
- Deviance residuals:



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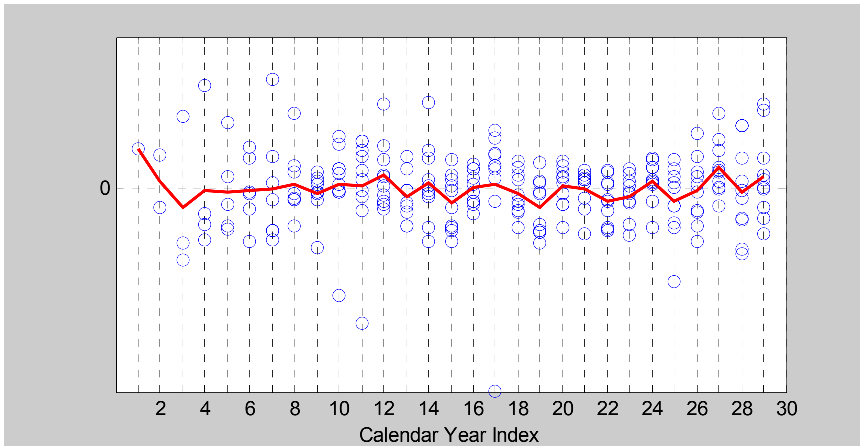
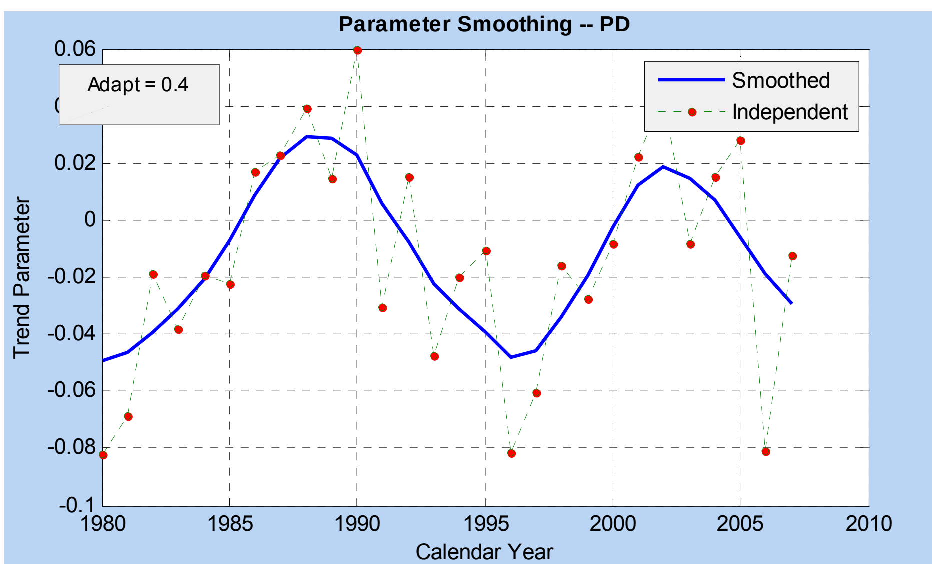
CY Filter with 40% adaptation coefficient

- AIC: 6,127 BIC: 6,284 # Parameters: 44.7
- Deviance residuals:



Independent and Smoothed Parameters



Overview of General Weighted Average (GWA) Filter

- Solution is provided for one parameter "type" – a time-ordered series of parameter estimates.
  - Multiple parameter types are solved recursively.
- The true (unknown) parameters are assumed to be the result of a stationary random walk.
  - The single parameter of the random walk, deemed the "adaptive variance" is a user input.
- The parameters are first estimated as separate parameters.
- The smoothed parameter at each point is a GWA of the individual parameters, where:
  - A GWA is an average weighted by a covariance matrix;
  - The covariance matrix reflects the sum of the covariances due to (1) parameter estimation error and (2) the random walk process.

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Model for Mortality

$m_{w,d}$ is mortality for
- year of birth ***w***,
- year of death ***d***

$$m_{w,d} = \exp\left(c_d u_w + a_d + b_d h_{w+d}\right) + e_{w,d}$$

- Note there are 5 parameter types
- This data can handle many more parameters
  - Some parameter reduction still may be desirable

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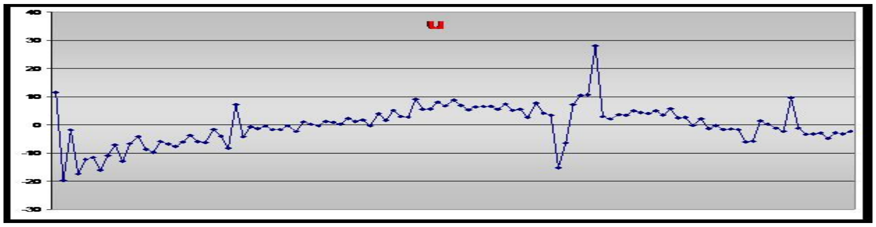
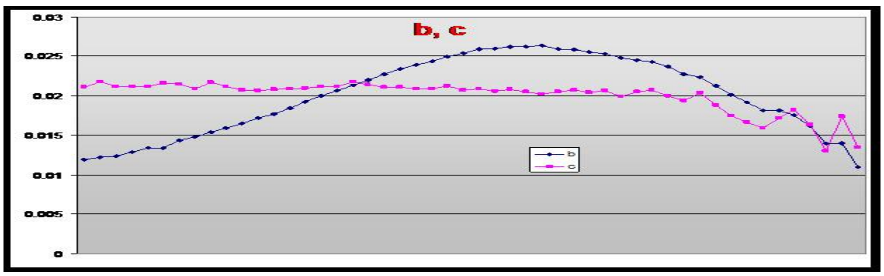
Some smoothing for *u* and *c* parameters?

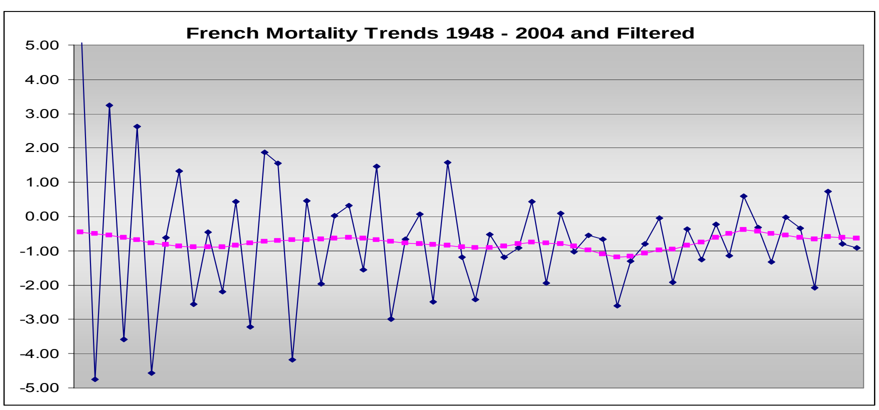
- ***u*** parameters for birth years 1848 to 1954



- ***b***, ***c*** parameters for age at death 50 to 99



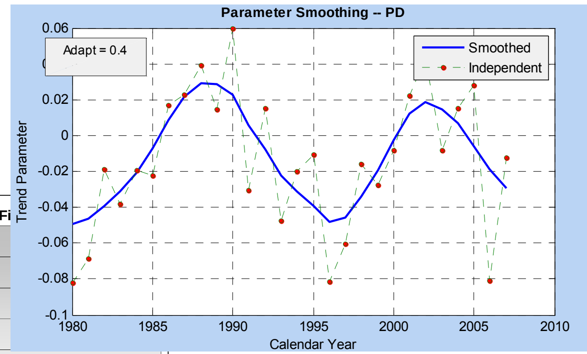
CY Mortality Trends are 1st differences of h parameters. Smoothed with GWA filter

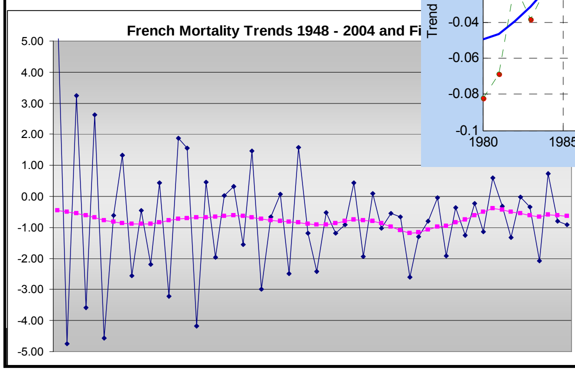


- What is the proper value for the smoothing coefficient?
- Is it clear that the trend is drifting at all?

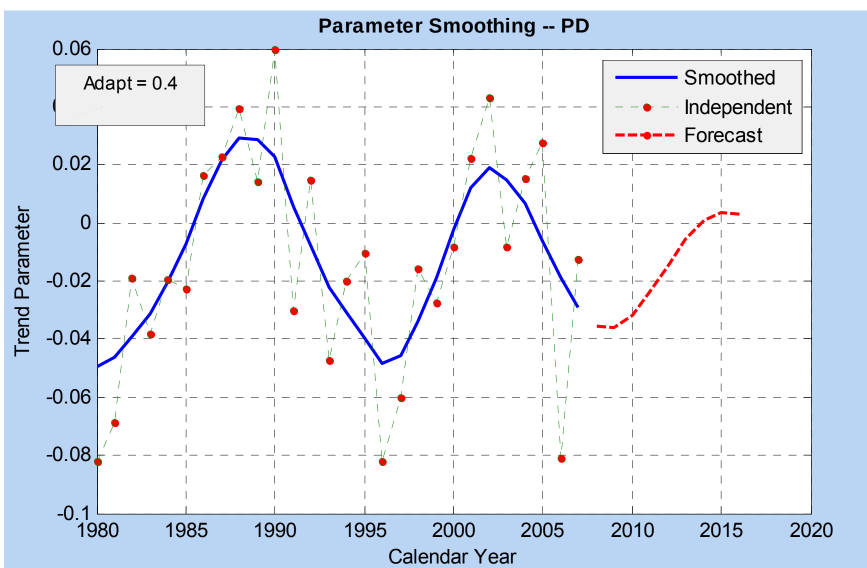
Beware of induced autocorrelation in successive trend parameter estimates!

- When parameters are trends, autocorrelation ≈ -50% is an artifact of parameter estimation error.
- GWA filter corrects for it.





Smoothed Parameters with AR – 2 Forecast



Conclusion:
What are the implications for forecast error?

- Trends that change over time are often evident in historical data.
- Models with static parameters may significantly underestimate time-related forecast error.
- The GWA filter can assist in detecting the true process through the noise of estimation error, while correcting for induced autoregressive effects.
- Estimates of forecast error will be realistic only when they include provisions for systemic time-related risk.
  - Some judgment will be required in selecting the form and parameters of the systemic risk model.