Recognizing LDF Variability: A Credibility Approach

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Cape Cod Method: Expected Pure Premium

$$\tilde{E}[PP] = \frac{\sum_i \left[\left(LTD_i \times DF_i / E_i \right) \times \left(E_i / DF_i \right) \right]}{\sum_i \left(E_i / DF_i \right)}$$

$$= \frac{\sum_i LTD_i}{\sum_i \left(E_i / DF_i \right)}$$

Where:

LTD = Loss to date (adjusted for trend).

DF = Ultimate loss development factor

E = Exposure (adjusted for trend).

Summed over each accident period *i*

Credibility

Total Variance = Variance of the Hypothetical Means + [Expected Value of the Process Variance]/N

Therefore, Variance of the Hypothetical Means = Total Variance - [Expected Value of the Process Variance]/N

It can be shown that[4]:

$$Z = \frac{VHM}{VHM + \frac{EPV}{N}} = \frac{N}{N + \frac{EPV}{VHM}}$$

Total Variance

(1) Accd Period	(2) Accd Period Subject Premium	(3) On-Level Factor	(4) On-Level Subject Premium	(5) On Level Losses	(6) Cumulative LDF's	(7) Developed & Trended Loss	(8) Trended Reported LR	(9) **Ultimate LR**
						(5)x(6)	(5)/(4)	(7)/(4)
2002	184,841,752	1.458	269,544,295	107,935,638	1.401	151,269,545	40.04%	56.12%
2003	258,499,389	1.043	269,544,295	84,122,238	1.754	147,587,305	31.21%	54.75%
2004	293,407,886	0.919	269,544,295	67,551,133	2.618	176,869,827	25.06%	65.62%
2005	303,248,288	0.889	269,544,295	44,663,392	4.176	186,535,040	16.57%	69.20%
2006	290,398,667	0.928	269,544,295	5,247,632	47.973	251,744,418	1.95%	93.40%
Total/Avg			1,347,721,477	309,520,034		914,006,136		**67.82%**

Weighted Mean:	**67.82%**
Total Variance:	**0.02421**
Cape Cod:	**59.63%**

Expected Process Variance

The first moment of the lognormal distribution is

$$u_1 = e^{u+\sigma^2/2}$$

The second moment about the mean is

$$u_2 = \left(e^{\sigma^2} - 1\right) e^{2u+\sigma^2}$$

Maximum likelihood estimators for the parameters u and σ of the age to age loss development factor at age i are:

$$\tilde{u}_i = \frac{\sum_k \ln x_k}{n}$$

$$\tilde{s}_i^2 = \frac{\sum_k \left(\ln x_k - \tilde{u}_i\right)^2}{n}$$

Expected Process Variance - Continued

Assuming serial independence for development factors, age to ultimate development parameters are additive so the estimators are

$$\tilde{U}_i = \sum_{j>=i} \tilde{u}_j$$

$$\tilde{S}_i^2 = \sum_{j>=i} \tilde{s}_i$$

Estimators for each accident period's mean ultimate development factor, $u(\theta_i)$, and process variance, $\sigma^2(\theta_i)$ are

$$Mean_i = \bar{X}_i = e^{\tilde{U}_i + \tilde{S}_i^2/2}$$

$$Variance_i = EPV_i = \left(e^{\tilde{S}_i^2} - 1 \right) e^{2\tilde{U}_i + \tilde{S}_i^2}$$

$$Z_i = \frac{VHM_i}{VHM_i + \frac{RLR_i^2 \; x \; EPV_i}{N}}$$

Variance of the Hypothetical Mean

Exhibit 2							
Accd Period		**6:18**	**18:30**	**30:42**	**42:54**	**54:66**	**66:78**
1996		**20.777**	**1.100**	**1.193**	**1.364**	**1.134**	**1.052**
1997		**7.700**	**1.248**	**1.238**	**1.143**	**1.091**	**1.057**
1998		**10.021**	**1.474**	**1.394**	**1.178**	**1.075**	**1.104**
1999		**6.551**	**1.282**	**1.377**	**1.262**	**1.140**	**1.111**
2000		**20.851**	**1.210**	**1.575**	**1.244**	**1.251**	**1.154**
2001		**16.804**	**1.622**	**1.479**	**1.263**	**1.217**	
2002		**11.692**	**1.481**	**1.481**	**1.238**		
2003		**14.991**	**1.608**	**1.438**			
2004		**8.367**	**1.676**				
2005		**11.807**					
"Last"		**6:18**	**18:30**	**30:42**	**42:54**	**54:66**	**66:78**
4	$\bar{x}_i^2$	0.0431	0.0021	0.0011	0.0001	0.0035	0.0010
	s_i^2	0.052	0.009	0.007	0.006	0.006	0.002
	a_i	2.440	0.467	0.400	0.225	0.156	0.101
	v_i	3.869	1.429	0.963	0.562	0.338	0.182
	Mean	47.973	4.176	2.618	1.754	1.401	1.199
Expected	**Process variance (EPV):**	129.498	0.161	0.049	0.018	0.012	0.003
	N	4	4	4	4	4	4
	EPV:	129.49770	0.16059	0.04854	0.01834	0.01155	0.00340
	Reported LR (RLR)	0.01947	0.16570	0.25061	0.31209	0.40044	-
	(RLR)²xEPV/N	0.01227	0.00110	0.00076	0.00045	0.00046	-
	Total Variance ITER 1	0.02421	0.02421	0.02421	0.02421	0.02421	0.02421
	VHM:	**0.01194**	0.02311	0.02345	0.02377	0.02375	0.02421
	Z:	0.4932	0.9545	0.9685	0.9816	0.9809	1.0000
	Scaled Z:	0.1126	0.2180	0.2212	0.2242	0.2240	0.2284

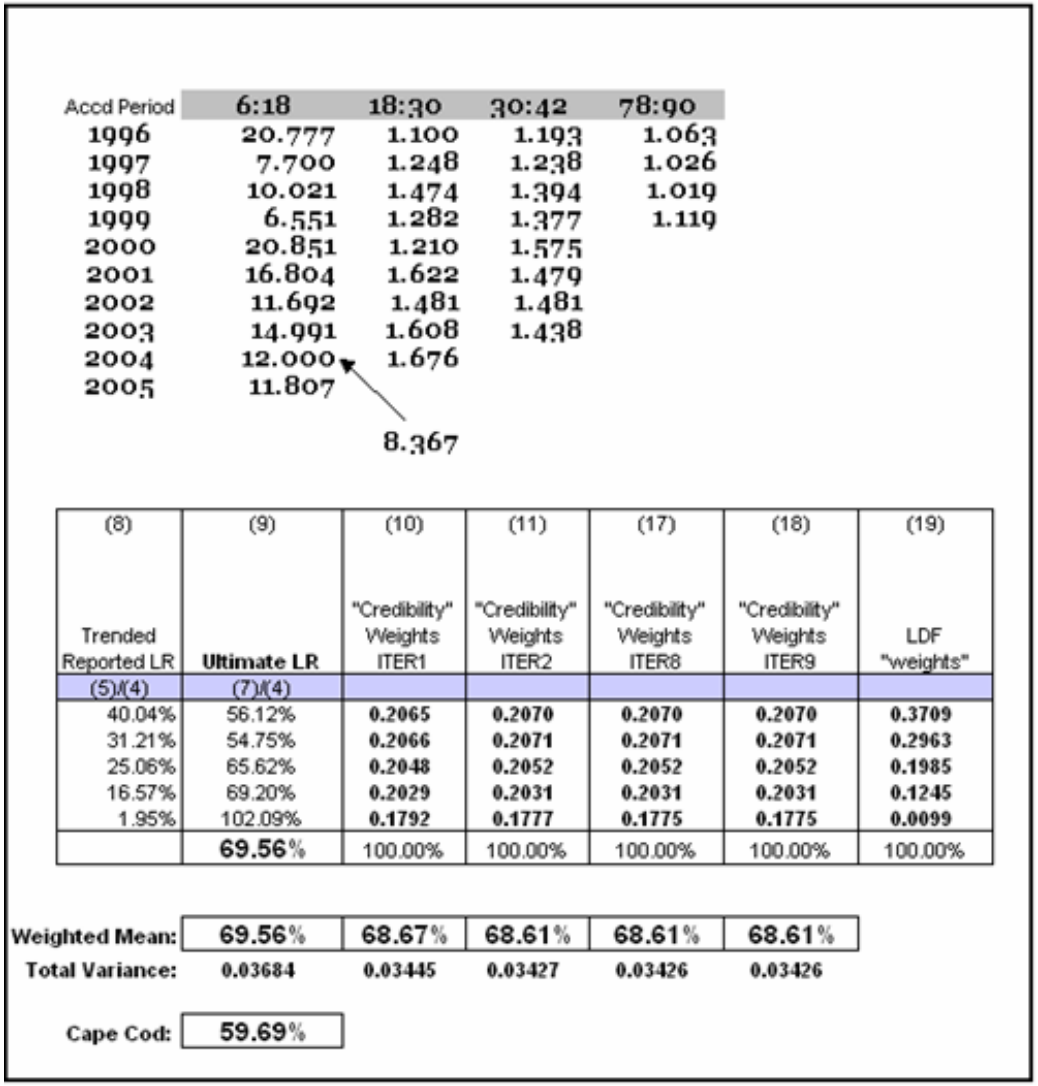
Iteration

(8)	(9)	(10)	(11)	(17)	(18)	(19)
Trended Reported LR	**Ultimate LR**	"Credibility" Weights ITER1	"Credibility" Weights ITER2	"Credibility" Weights ITER8	"Credibility" Weights ITER9	LDF "weights"
(5)/(4)	(7)/(4)					
40.04%	56.12%	**0.2240**	**0.2335**	**0.2557**	**0.2558**	**0.3705**
31.21%	54.75%	**0.2242**	**0.2337**	**0.2562**	**0.2562**	**0.2960**
25.06%	65.62%	**0.2212**	**0.2295**	**0.2483**	**0.2483**	**0.1983**
16.57%	69.20%	**0.2180**	**0.2251**	**0.2398**	**0.2398**	**0.1243**
1.95%	93.40%	**0.1126**	**0.0783**	**0.0000**	**0.0000**	**0.0108**
	67.82%	100.00%	100.00%	100.00%	100.00%	100.00%

Weighted Mean:	**67.82%**	**64.97%**	**63.84%**	**61.27%**	**61.27%**
Total Variance:	**0.02421**	**0.01823**	**0.01594**	**0.01072**	**0.01072**

Cape Cod:	**59.63%**

Examples (1)



Accd Period	6:18	18:30	30:42	78:90
1996	20.777	1.100	1.193	1.063
1997	7.700	1.248	1.238	1.026
1998	10.021	1.474	1.394	1.019
1999	6.551	1.282	1.377	1.119
2000	20.851	1.210	1.575	
2001	16.804	1.622	1.479	
2002	11.692	1.481	1.481	
2003	14.991	1.608	1.438	
2004	12.000	1.676		
2005	11.807			

8.367

(8) Trended Reported LR (5)/(4)	(9) **Ultimate LR** (7)/(4)	(10) "Credibility" Weights ITER1	(11) "Credibility" Weights ITER2	(17) "Credibility" Weights ITER8	(18) "Credibility" Weights ITER9	(19) LDF "weights"
40.04%	56.12%	**0.2065**	**0.2070**	**0.2070**	**0.2070**	**0.3709**
31.21%	54.75%	**0.2066**	**0.2071**	**0.2071**	**0.2071**	**0.2963**
25.06%	65.62%	**0.2048**	**0.2052**	**0.2052**	**0.2052**	**0.1985**
16.57%	69.20%	**0.2029**	**0.2031**	**0.2031**	**0.2031**	**0.1245**
1.95%	102.09%	**0.1792**	**0.1777**	**0.1775**	**0.1775**	**0.0099**
	69.56%	100.00%	100.00%	100.00%	100.00%	100.00%

Weighted Mean:	**69.56%**	**68.67%**	**68.61%**	**68.61%**	**68.61%**
Total Variance:	**0.03684**	**0.03445**	**0.03427**	**0.03426**	**0.03426**

Cape Cod: **59.69%**

Examples (2)

Accd Period	6:18	18:30	30:42	78:90
1996	20.777	1.100	1.193	1.063
1997	7.700	1.248	1.238	1.026
1998	10.021	1.474	1.394	1.019
1999	6.551	1.282	1.377	1.119
2000	20.851	1.210	1.575	
2001	16.804	1.622	1.479	
2002	11.692	1.100	1.481	
2003	14.991	1.608	1.438	
2004	8.367	1.676		
2005	11.807			

1.481

(8) Trended Reported LR (5)/(4)	(9) **Ultimate LR** (7)/(4)	(10) "Credibility" Weights ITER1	(11) "Credibility" Weights ITER2	(17) "Credibility" Weights ITER8	(18) "Credibility" Weights ITER9	(19) LDF "weights"
40.04%	56.12%	**0.2658**	**0.2905**	**0.2863**	**0.2863**	**0.3667**
31.21%	54.75%	**0.2661**	**0.2911**	**0.2870**	**0.2870**	**0.2930**
25.06%	65.62%	**0.2609**	**0.2778**	**0.2750**	**0.2750**	**0.1963**
16.57%	64.29%	**0.2073**	**0.1406**	**0.1517**	**0.1517**	**0.1325**
1.95%	86.86%	**0.0000**	**0.0000**	**0.0000**	**0.0000**	**0.0115**
	65.53%	100.00%	100.00%	100.00%	100.00%	100.00%

Weighted Mean:	**65.53%**	**59.93%**	**59.51%**	**59.58%**	**59.58%**
Total Variance:	**0.01653**	**0.00732**	**0.00814**	**0.00799**	**0.00799**

Cape Cod: **59.02%**

Reserving

Accd Period	Reported LR	Developed Reported LR	**Ultimate LR**	Credibility	Credibility weighted Loss Ratio
	6/2	14x16	15/4		
2002	49.59%	69.51%	56.12%	**0.9568**	**56.34%**
2003	28.20%	49.47%	54.75%	**0.9583**	**55.03%**
2004	20.63%	54.03%	65.62%	**0.9289**	**65.31%**
2005	13.71%	57.28%	69.20%	**0.8972**	**68.39%**
2006	1.74%	83.66%	93.40%	**0.0000**	**61.27%**
Total/Avg	61.73%	89.98%			

CCC LR: **61.27%**