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# What about Underevaluating Operational Value at Risk in the Banking Sector?

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## Outline

- Introduction
- Motivation
- Literature review
- Objectives
- Contributions
- Data
- Methodology
- Results
- Conclusion and extensions

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## Introduction

- Huge operational losses have occurred recently in the financial sector (Barings, Allied Irish Bank, Enron, Société Générale…)
- Basel II accord : regulatory capital requirement for operational risk.
- Operational risk is defined as the loss resulting from inadequate or failed internal processes, people and systems or from external events
- 3 methods for the measurement of regulatory capital :
  - Basic Indicator Approach (BIA)
  - Standardized Approach (SA)
  - Advanced Measurement Approach (AMA)

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## Motivation

- An AMA has many advantages to banks:
  - a capital measure more precise and sensitive to risk exposure
  - a potential economy of capital to banks, compared to the standardized approach
- Research is still in its embryonic stage.
- Operational risk has several distinctive characteristics different from market risk and credit risk. The VaR calculation method is also different.

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## Literature Review

- The AMA draws heavily on the actuarial model: modeling losses in the insurance field (Cummins and Freifelder, 1978; Klugman et al. 1998…)
- Modeling operational losses :
  - Without taking the collection threshold into consideration (Dutta and Perry, 2006; de Fontnouvelle et al, 2004…)
  - Using the extreme value theory (Embrechts et al, 2007; Ebnother et al., 2001…)
  - Using a simple distributions lognormal-Poisson model (Frachot et al, 2005c; Chernobai et al (2005a)…
- Integration of external losses in the VaR measurement (Frachot et al., 2002; Alexander, 2003;…)

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## Objectives

- Develop a precise and rigorous measurement of a bank's operational VaR by choosing the best fitting to the data (severity and frequency)
- Combine internal and external loss data to take into consideration extreme losses which have not yet occurred.
- Compare our model to the standard model (lognormal-Poisson model) to show how this model frequently used underestimates the operational VaR.

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## Contributions

- Testing more complex but flexible distributions like the GB2 in order to fit well the loss data
- Conditional estimation of the severity distribution in the case of truncated data
- Correcting the parameters of the frequency distribution to take into consideration the number of losses not collected.
- Combining internal and external loss data to calculate the unexpected loss.
- Application of the model to real data of a Canadian bank

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## Data

- Canadian Bank's internal losses grouped by risk type (we exclude the risk type business disruptions and systems failures)
- The loss data cover a 3-year period (1 Nov 2001 to 31 Oct 2004)
- The data are collected over a collection threshold *s* (except for the risk types: external fraud and internal fraud)
- We exclude all the internal losses of more than $1 million US from the sample, that we add to external loss data (Fitch's OpVaR) containing extreme losses of over $1 million US, scaled to the Canadian Bank (using the model developed by Dahen and Dionne, 2007)

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## Descriptive Statistics on the loss amounts

**Table 1**: This table presents the descriptive statistics for loss amounts by event and type of risk over a 3-year period.

| | DPA* | CPBP* | EPWS* | EF* | IF* | EDPM* |
|---|---|---|---|---|---|---|
| **Mean** | $20,797 | $25,582 | $34,220 | $2,049 | $13,882 | $7,479 |
| **Median** | $7,728 | $3,169 | $12,520 | $677 | $2,975 | $2,000 |
| **Standard deviation** | $30,110 | $76,304 | $54,868 | $13,272 | $71,602 | $33,012 |
| **Kurtosis coefficient** | 9.25 | 47.66 | 12.54 | 2,722.29 | 78.57 | 341.00 |
| **Asymmetric coefficient** | 2.80 | 6.25 | 3.22 | 46.12 | 8.80 | 15.94 |
| **Maximum** | $157,138 | $819,717 | $334,034 | $1,003,045 | $645,700 | $863,876 |
| **Number of observations** | 53 | 509 | 97 | 22,178 | 81 | 1,547 |

*DPA: Damage to Physical Assets
*CPBP: Clients, Products and Business Practices
*EPWS: Employment, Practices, and Workplace Safety
*EF: External Fraud
*IF: Internal Fraud
*EDPM: Execution, Delivery and Process Management

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## Estimation of the severity distribution

- Estimation by risk type (6 risk types)
- Testing 4 distributions: exponential, lognormal, Weibull and GB2 (4 parameters)
- Depending on the structure of the data, we estimate the parameters of the distributions:
  - Unconditional estimation when the data are complete (case of internal and external fraud)
  - Conditional estimation of parameters when the data exceed the collection threshold (4 risk types)

$$f(y,\theta \mid y \geq s) = \frac{f(y,\theta)}{1 - F(s,\theta)} \quad \text{for} \quad y \geq s$$

s: the collection threshold
y: the observed loss



## Estimation of the severity distribution

- We construct goodness of fit tests (Kolmogorov-Smirnov, Anderson Darling, Cramèr-von-Mises) with parametric bootstrap.
  - The results show that the GB2 (the family distribution with 4 parameters) offers an excellent fitting to the data



## Estimation of the severity distribution

If the fitting of the distribution is rejected

- We divide the empirical distribution into parts and fit a parametric distribution to each part.
- Fit the Pareto distribution to the tails
  - Estimate the parameter of the Pareto for different samples (losses > different tail threshold)
  - The goodness of fit tests allows us to choose the threshold providing the best fit for the tail.

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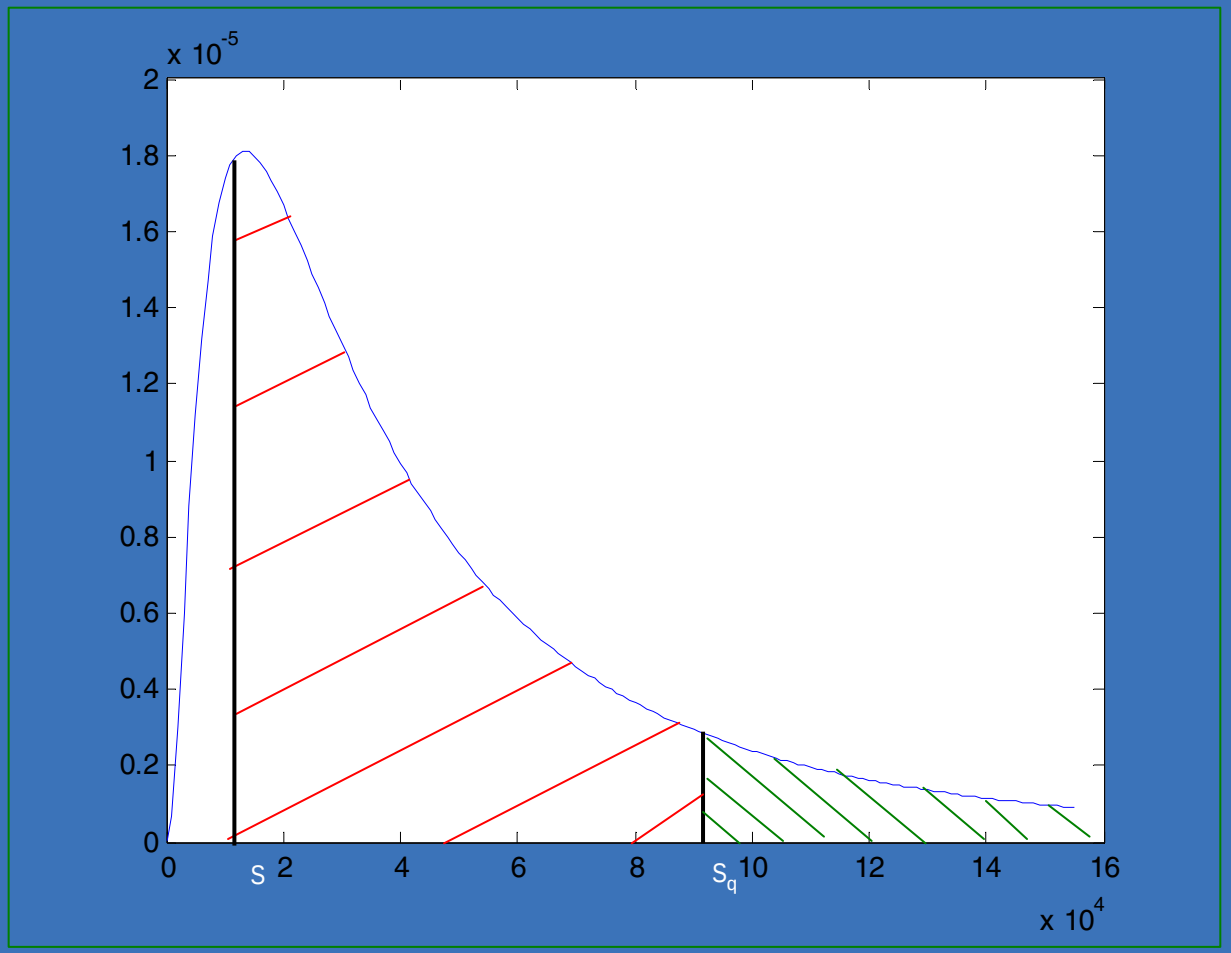
## Estimation of the severity distribution



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## Estimation of the frequency distribution

- Modeling the number of losses per day or per week depending on the risk type in order to correct a potential collection bias in the data.
- Estimating the parameters of the Poisson and the negative binomial distributions
- Testing the degree of fitting of the distributions estimated for the different periods chosen with a $\chi^2$ test with parametric bootstrap

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## Estimation of the frequency distribution

- Correction of the estimated parameters to consider for the number of losses below the collection threshold.

Poisson

$$\lambda_{real} = \frac{\lambda_{sample}}{1 - \hat{F}(s)}$$

$\lambda_{sample}$ is the estimated parameter

$\hat{F}(s)$ is the estimated cumulative severity distribution evaluated at the collection threshold

Negative binomial

$$p_{real} = \frac{p_{sample}\left(1 - \hat{F}(s)\right)}{1 - p_{sample}\hat{F}(s)}$$

$p_{sample}$ is the estimated parameter

$\hat{F}(s)$ is the estimated cumulative severity distribution evaluated at the collection threshold

- The results show that only the negative binomial fits well to the number of operational losses

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## Calculation of the Operational VaR by risk type

- Aggregation of the estimated distribution of severity and frequency to obtain the annual losses
- Monte Carlo simulation to derive the non parametric distribution of the annual losses
- Determination of the annual Value at Risk at a 99,9% confidence level.



## Comparison of our model with the standard model

- Standard model :
  - Lognormal-Poisson model
  - No consideration to the missing losses
  - Frequently used in practice and subject of several researches
  - We compare our model to the standard model for the risk types IF, EPWS and EF

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## Comparison of our model with the standard model

**Table 2:** We compare the mean of annual losses as well as the annual VaR, at the 90; 95; 99 and 99,9% level of confidence, as calculated based on the standard model and our model.

| | Variation of the standard model compared to our model (%) | | | | |
|---|---|---|---|---|---|
| **Types of risk** | **Variation of losses /year** | **Variation of VaRs at 90%** | **Variation of VaRs at 95%** | **Variation of VaRs at 99%** | **Variation of VaRs at 99.9%** |
| **IF*** | -0.06 | -2.06 | -2.18 | -1.68 | -1.34 |
| **EPWS*** | -4.67 | -9.40 | -11.89 | -17.07 | -27.32 |
| **EF*** | 0.95 | -5.62 | -13.50 | -40.28 | -81.59 |

*IF: Internal Fraud
*EPWS: Employment, Practices, and Workplace Safety
*EF: External Fraud



## Results of the comparison

A poor specification of severity and frequency distributions (standard model) result in serious biases in the calculation of VaR (operational capital)

- For IF: a small difference between the VaR (2%) due to a poorly specified frequency distribution.
- For EPWS: an underestimation of a variation of about 27% measures the impact of the poorly specified frequency distribution and the no consideration of the collection threshold.
- For EF: an important underestimation especially for the VaR at 99.9% shows the impact of poor choice of the severity and the frequency distributions

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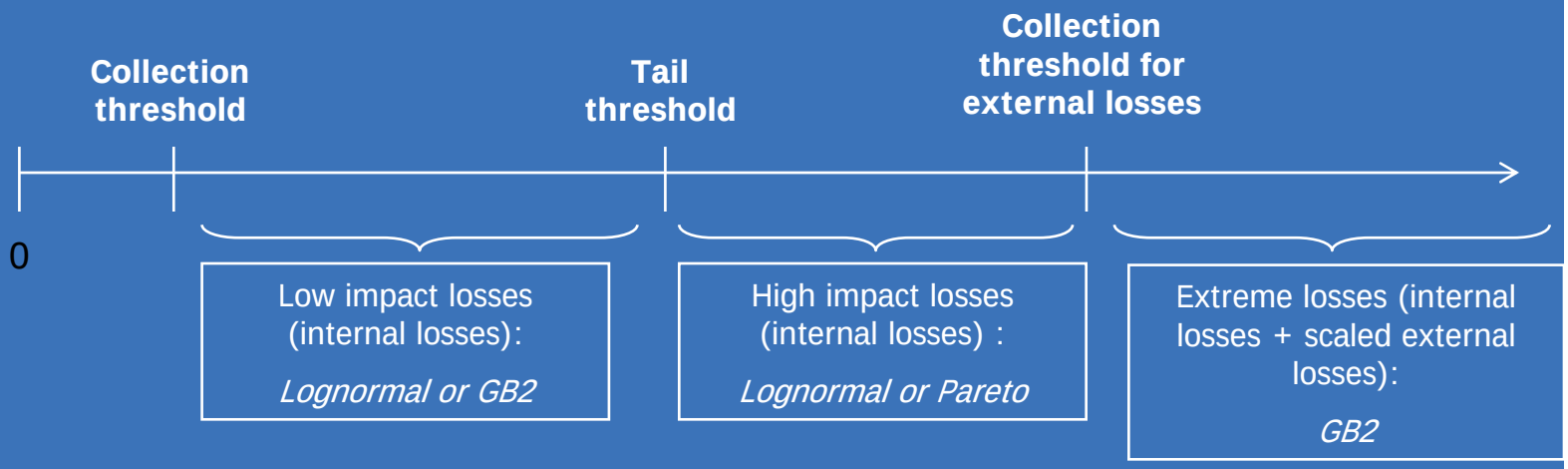
## Combination of the internal and external data

- Using scaled external losses (Fitch data base) to the Canadian bank with the scaling method developed in Dahen and Dionne (2007)
- We generate extreme losses (over $ 1M) according to the estimated frequency (Dahen and Dionne, 2007) and the estimated severity (Dahen, 2007)
- We add these simulated losses to the annual loss generated from the internal data
- We calculate the mean and the 99.9 percentile of the simulated distribution.



# Combination of the internal and external data





## Conclusion

- We develop a model that :
  - Offers a good fitting to all data (distribution with 4 parameters, division of the distributions, goodness of fit tests with parametric bootstrap)
  - Takes into consideration the collection threshold (severity and frequency distributions)
  - Integrates scaled external losses in the VaR calculation

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## Extensions

- Calculate the aggregate VaR by modeling the dependence between the risk types
- Calculate the operational capital when the internal and external losses are insufficient (ex. business disruptions and systems failures) by scenario analysis

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# White Paper Briefing: ERM And Other Considerations Related To A Principles-Based Approach


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## ERM And Other Considerations Related To A Principles-Based Approach

**PREFACE**

**This presentation is intended to track key discussion points related to the white paper submitted for the ERM Symposium to educate and inform its audience regarding important considerations related to the transition from a rules-base to a principles-based approach, in line with changes in regulations and internal business requirements. The paper focuses on the role of actuaries and use of a centralized enterprise risk management platform in facilitating this transition, as well as other organizational considerations necessary to ensure that objectives are met.**

**The paper is not intended to comprehensively cover broad scope of principles-based regulation, nor is it intended to offer legal, financial, or technical advice regarding the subject. Readers are encouraged to contact the authors for further information regarding any aspect of the subject matter covered.**

**The views expressed are held by the authors and are not necessarily representative of FTI Consulting, Inc., DFA Capital Management, Inc., or AmeRisk Consulting LLC.**

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**ERM And Other Considerations Related To A Principles-Based Approach**

## Agenda

- INTRODUCTION
- FINANCIAL INNOVATION AND GLOBAL CONVERGENCE
- GLOBAL GUIDELINES - IAIS RESERVES AND SOLVENCY
- STRATEGIC ISSUES – RESERVES
- ENTERPRISE RISK MANAGEMENT
- EDUCATION AND TRAINING
- TRACKING REGULATORY PROGRESS
- IMPACT ASSESSMENT
- IMPLEMENTATION ISSUES
- GOVERNANCE AND OVERSIGHT
- SUMMARY



# Introduction

- Harmonization of regulations and standards in a global market
  - IAIS advocating a common regulatory platform for all insurers
  - Principles-based vs. rules based
  - ERM – core component of principles-base approach
  - Impacts to Actuaries & other functional units (accounting, audit, investment management) with increased governance from the Board of Directors
  - This paper addresses applications of ERM to this operational change process – financial and accounting



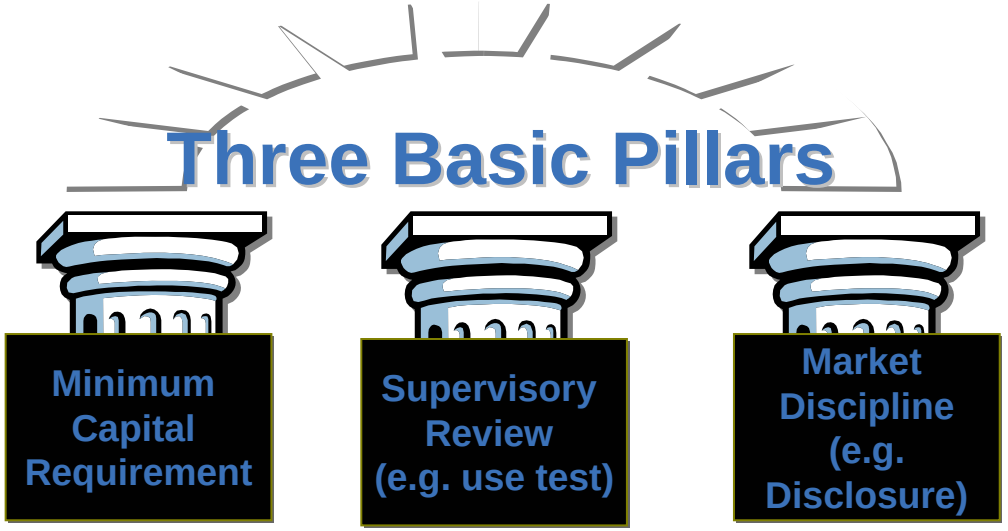
# Financial Innovation and Global Convergence

## Modernizing for the future

Graham Leach Bliley Act

Sarbanes Oxley (SOX)

Basel II Accord / Solvency II / IAIS / NAIC



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## Global Guidelines – IAIS Reserves & Solvency

- ERM is a core principle under IAIS regulatory guidelines
- Board of Directors accountable



# Strategic Issues for Actuaries - Reserves

- Principles-based reserve methodology
  - Property/Casualty reserve procedures
  - Life reserve procedures
- Need for operational transparency
  - Requires an extension of cross-department efforts – accounting, audit, treasury, investment, Executive Committee, BOD – relating to ERM of reserve levels and regulatory capital requirements



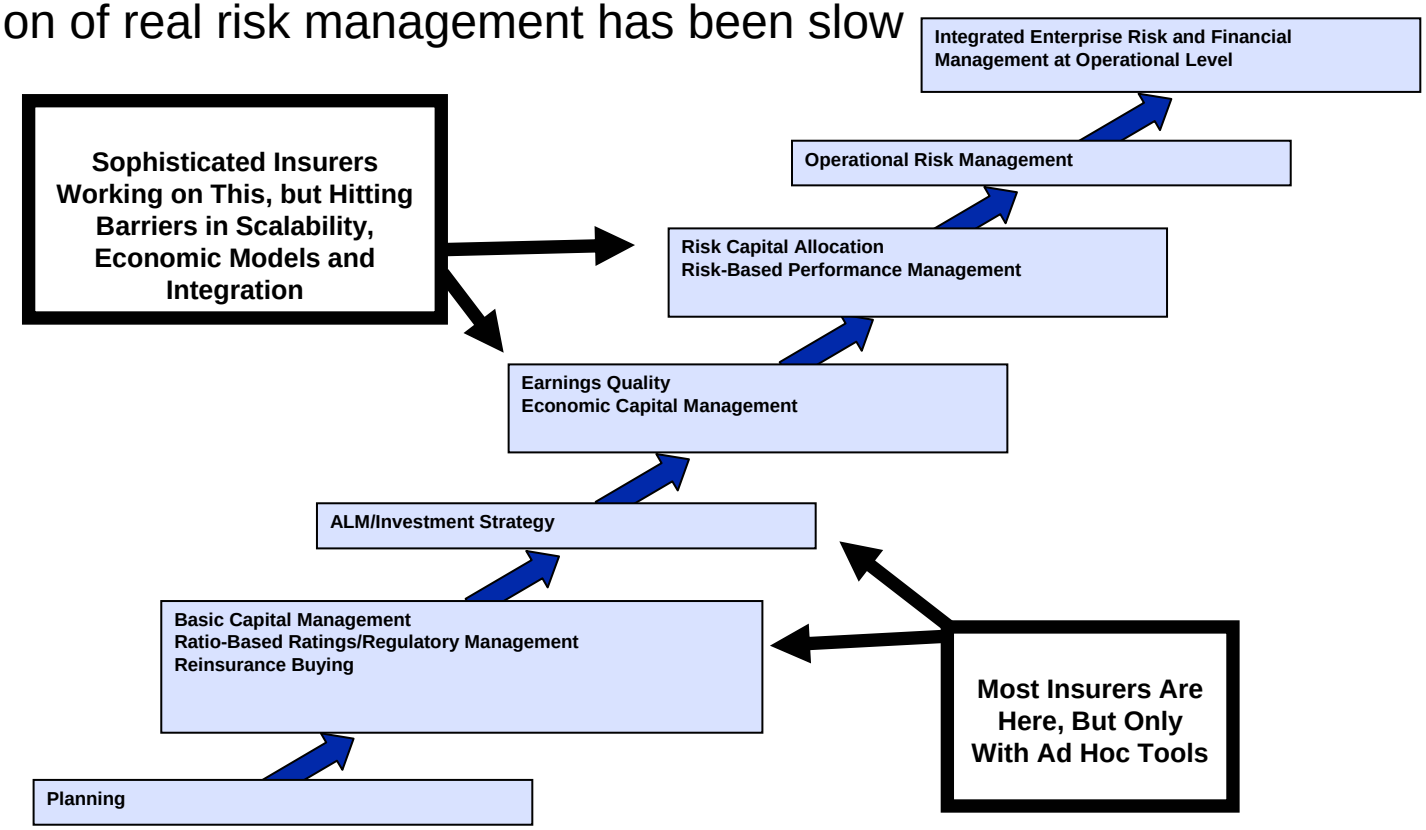
# Enterprise Risk Management

- Proven benefits of ERM
  - Alignment of the strategic aspects of risk with day-to-day operational activities
  - Facilitation of more transparency for investors and regulators
  - Enhancement of revenue and earnings growth
  - Control of downside risk potential



# ERM Drivers

Adoption of real risk management has been slow






However…



# Change is coming

- Ratings agencies and regulatory bodies are catalysts for change
- As companies adopt a principles-based approach, they'll need to focus more on integrating ERM into corporate DNA



# Risk Integration Imperatives

- Harmonizing business decisions and risk modeling
- Understanding methodologies and organizational implications
- Pricing
- Product Development
- Performance Measurement



# Holistic Scenario Analysis

- Ability to model the entire company
  - Better understand and manage risk
  - Improve asset-liability management
  - More efficiently allocate capital
  - Optimize reinsurance purchasing
  - Enhance performance management
  - Maintain or improve ratings
  - Increase shareholder value
- Aggregated and business line views



# Migration to Central Risk Management Platform

- Leveraging legacy infrastructure and point solutions
- Key requirements for group-wide risk models:
  - Consistent scenarios
  - Dependencies modeled by common underlying drivers
  - Proper accounting aggregation to entity and group level
  - Group level management decisions
  - Local decisions reflecting global situation
  - Robust simulation modeling engine and output database technology
  - Leverage existing models, parameters and data



# Scalability Challenges of Combining Non-Life and Life Models

- Non-life models ➔large # of paths
- Life models ➔large # of time periods
- Model technology should provide:
  - Scalability to support massive models without requiring a compromise in modeling accuracy
  - Integrated management decision feedback mechanism



# Education and Training

- ERM is transformational process
  - Alignment of all resources with enterprise objectives
- Stakeholders requiring education and training
  - Accounting Department
  - Internal Auditing Department
  - Asset/Liability Management
  - Executive Committee and Board of Directors
  - Actuaries
  - External Reporting
  - Investment Community Relations



# Tracking Regulatory Progress

- No definitive decision
  - Specific methodologies being considered are subject to change
  - Develop & continue education/training programs and strategies for implementation, but remain flexible
- Readiness reviews and discussions



# Impact Assessment

- ERM methodology
- Principles-based reserving approach
- Internal controls
- Leveraging Sarbanes-Oxley infrastructure
- Reserve Committee
- New risks: principles-based approach and IFRS

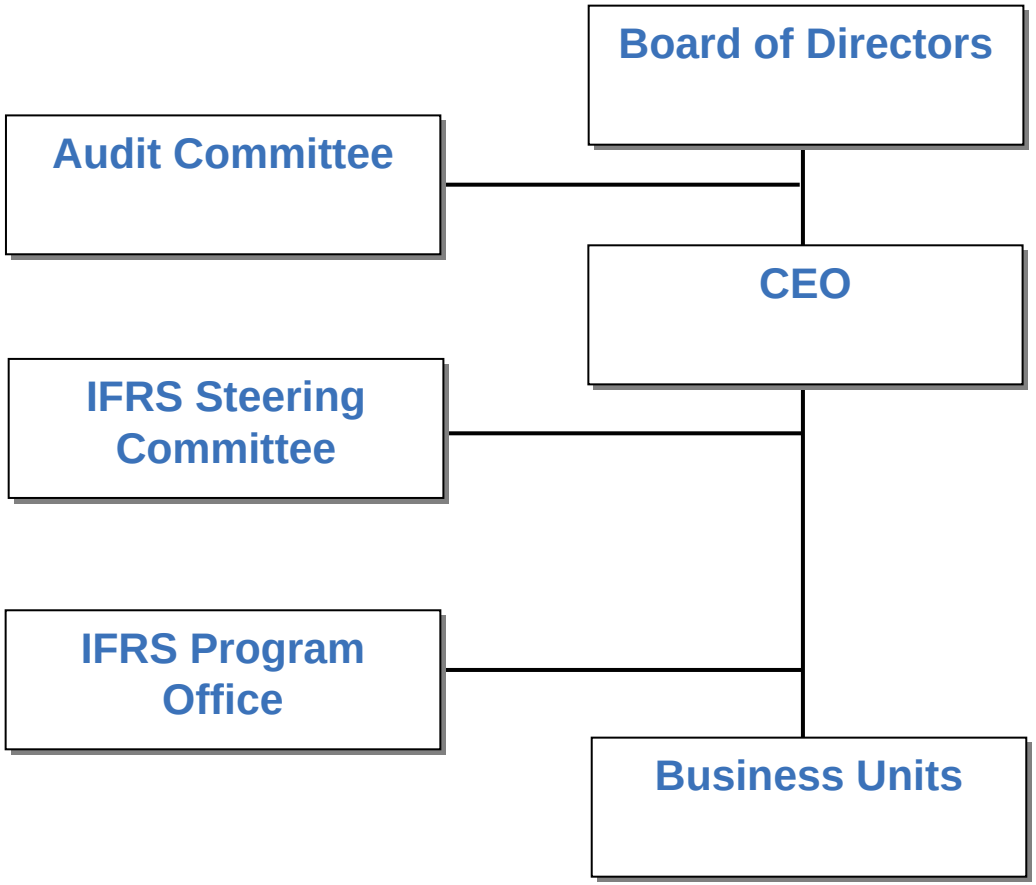


# Implementation Issues

- Legal component
- Ethical component
- Political component
- Business component
- Strategies and Plans



# Governance and Oversight



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## Summary

- Growing support for principles-based approach within U.S.
- Leadership requirements
- ERM component
- Readiness important to achieving benefits

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## ERM And Other Considerations Related To A Principles-Based Approach

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# ERM for Strategic Management – Status Report

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## History of ERM

- Started with banking regulation
  - Banks were not quantifying risk, going broke
  - Basic risk quantification modeling developed
  - Can't say this has been an unqualified success, as risks have become more complex
- Expanded to insurance regulation
- Attempts for application to strategic management
  - Manufacturing looking at supply chains, etc.
  - Financial institutions model financial risk
    - Basically shifting from policyholder viewpoint to shareholder viewpoint



## For Today:

- Two key areas of ERM and strategic management:
  - Capital need
  - Risk adjusted profitability
- Status:
  - Where we were
  - Where we are
  - What hasn't been done
  - What might be done



## What ERM Does and What It Does Not Yet Do

- Does
  - Quantify risk level of capital
  - Quantify riskiness of business units
- Does not yet
  - Show optimal level of capital
    - Would like to be able to say what capital maximizes franchise value of the firm
  - Show which business units are most profitable on a risk basis
    - Comparing profit to allocated capital does not necessarily tell which units are adding most value to the firm



# Capital Needs of an Insurer

## Where We've Been

- Capital = 1/3rd of premium
- As reserves became more important, maybe
  - Capital = ¼ of liabilities, including reserves and UEPR
- But not all liabilities have equal risk, so RBC tried:
  - Capital = sum of factors times income statement and balance sheet items
- Problems
  - These are only very rough measures of risk and capital need
  - Resulting capital not sensitive to many risk variables
  - For instance, increasing adequacy of premium or reserves typically increases the RBC stated capital need

## ERM Initial Attempt at Capital Analysis

- Capital = enough to make one-year probability of ruin small enough
- Often called "economic capital"
- Advantages
  - Consistent probabilistically across firms
  - Low enough in information content that public disclosure will not help competitors
    - Good criterion for regulatory measure



## Disadvantages of Economic Capital

- Arbitrary choice of probability
- Only one way to measure risk
- Inaccuracy of models in extremes
- Not the capital a company needs in practice
  - Want to optimize value
  - Can be affected by perceptual issues, competitors, quality of risk management, reputation of management …



## What ERM Does Well: Quantifies Risk Level of Actual Capital

- Initial attempt: probability of default
  - Problem: models not very accurate at extreme probabilities
- What can be done:
  - Capital at multiples of various risk measures
    - Capital = (perhaps)
      - 4 standard deviations of earnings
      - 3 times 1-in-100 TVaR
      - 4 times 1-in-100 cat occurrence
      - 1.5 times 2 1-in-100 aggregate loss years in a row
  - Useful for comparisons of how different strategies that change risk can be imputed to change capital needs
- Does not tell you how much capital is optimal



## Live Examples

- Capital = Multiple of probability of loss at quantifiable levels
- Swiss Re 12/31/2006: Times 99% VaR, 99.5% VaR and 99% TVaR
  - Capital is: 3.6x 2.9x 2.4x
  - Year over year comparisons in annual report
- Munich Re: Base capital = 2 aggregate 1-in-100 years in a row; actual capital is a multiple of base – maybe 1.5x
- Endurance: Maximum capital loss of 25% at 1-in-100 event, or… economic capital is 4 times the 1-in-100 loss
- ERM moves us from ¼ of liabilities to 4 times 1-in-100 loss level
  - A real improvement in probabilistic quantification



## Optimal Capital Issues

- Basic
  - You need enough to keep them buying, i.e., enough to support growth
  - Can be affected by perceptual issues, competitors, reputation of management …
- Reputation important
  - Role of risk management, stability, etc.
- Competitive analysis
  - What is capital level of competition?
  - What sectors of business to compete for?
  - How much does capital affect winning or losing quoted-for business?
    - Might have to talk to underwriters



## What Might Be Done in Optimal Capital: Flavored Models

- Find capital to optimize franchise value: market – book
- Model value of firm as the expected present value of future cash flows to shareholders (at least since de Finetti in 1957)
  - Look for capital level that maximizes value – capital in this sense
    - That is really finding optimum dividend policy
      - Dividend paid earlier has higher present value
      - But paying it could increase probability of ruin, ending future dividends
- Part of general theory of stochastic control
  - You have a stochastic process (total capital) which you can control to some extent (paying dividends) and seek optimum strategy

## Actuarial Progress in Stochastic Control of Insurer Capital

- Gerber and Shiu in NAAJ 2006 assume loss process is compound Poisson and severity is a mixture of exponentials, and find the optimum strategy
  - Turns out to be a barrier strategy
    - Pay out all capital over a certain level in dividends and none below it
- Another possible control is ceding reinsurance
  - Can seek optimal strategy for reinsurance and capital
  - Bather 1969 addressed this for proportional reinsurance
  - Asmusson et al. 2000 try excess reinsurance
  - Pricing rule for excess cover becomes an issue to address
  - Costly reinsurance can be valuable to stave off ruin



## Enter Finance

- De Finetti was in late 1950's where finance world (MM) was saying risk transfer is not worthwhile
- One assumption of that is that external finance is always available and not costly
- Actuarial literature did not consider refinancing, so basically had infinite cost of external capital
  - Peura 2003 Univ Helsinki thesis maybe first exception
- Froot and others introduced costly but possible external finance in the financial literature – like new shares 40% below latest price of existing for Societe Generale
- Major adding costly external finance to actuarial approach
  - Calls it Flavored as an acronym for some of the contributors



# Risk-Adjusted Profit of Business Units

## Traditional Risk Measurement

- Premium volume
- Exposures in force
- Limits in force
- Number of policies
- PML
- These don't compare well across businesses
- Not really probabilistically based



## ERM Risk Measures

- Value at Risk (VaR)
  - $Pr(Y>VaR_\alpha) = 1 - \alpha$
- Tail Value at Risk (TVaR)
  - $E[Y|Y>VaR_\alpha] = TVaR_\alpha$
- Both give probabilistically consistent ways of comparing units
- Both are tail measures
  - Ignore risk that is probably important, in that you would charge for it
  - Lower $\alpha$'s probably better, like 85% or even 60%



## Issues with VaR and TVaR

- Sum of VaR's can be greater than VaR of sum
  - Pool of risks each with loss probability of ½% each has $VaR_{0.99} = 0$, so sum = 0, but $VaR_{0.99}$ of pool > 0
  - Actually not so critical in practice
- TVaR is linear in large losses, which is opposed to usual risk attitudes

## Alternative Risk Measures

- Semi-standard deviation (adverse deviations only)
- $E[Ye^{cY/EY}]$ gives weight to tail but measures all risk
- Mean under transformed probabilities
  - Gives more weight to tail but all risk included
  - Financial value measures like CAPM and Black-Scholes can be put in this form
    - Useful for risk measure to give value of risk taken
- Can do TVaR with transformed probabilities
  - No longer linear in large losses



## Example of Transformed Mean

- Esscher transform for compound Poisson
- Severity density g(y) transformed to: $g^*(y) = g(y)Ee^{cy/EY} / Ee^{cY/EY}$
- Frequency: $\lambda^* = \lambda Ee^{cY/EY}$
- Constant c chosen to give overall load
- In one test, this fit well to reinsurance prices
- Is minimum entropy martingale transform
- Risk measure: transformed mean – mean



## Allocating Risk

- Allocating capital usually done by allocating risk
- Purpose is usually to get risk-adjusted returns by business unit
- Typical approach: proportional allocation
  - Measure risk for each unit and for firm, and allocate firm risk in proportion
- Problems with proportional allocation:
  - Does not show the contribution of each unit to the risk of the firm
  - Probably will not show marginal impact of change in unit on change in firm risk
  - Growing units with higher return on risk will not necessarily increase return on risk for the whole firm



## Contribution of Units to Firm

- Use co-measures
  - Prototype: sum of covariances of units with firm is the variance of the firm
- Co-$TVaR_\alpha$ for unit $X_j$ : $E[X_j|Y>VaR_\alpha]$
  - Shows contribution of $X_j$ to TVaR of firm
- Can be defined for any risk measure that can be expressed as a conditional expected value



## Riskiness of Business Units

- Real issue is what is the contribution of the unit to the riskiness of the firm
- For homogeneous risk measures (i.e., r[aY] = a r[Y]) this is done by Euler's Theorem
  - Total risk measure for company is a multivariate function of the losses of the business units
  - Euler: derivatives of risk measure with respect to units' volumes sum up to entire risk measure
  - Derivative is marginal impact of unit's volume of business on the entire company risk measure
  - So can allocate by marginal impact and still sum to total
  - Venter, Major, Kreps ASTIN 2006 do the derivatives for many risk measures



## Details

- For risk measure $\rho$, company variable Y = sum of units $X_j$
- $r(X_j) = \lim_{\varepsilon \to 0}[\rho(Y) - \rho(Y - \varepsilon X_j)]/\varepsilon$ .
- If $\rho(Y) = StD(Y)$,
  - $r(X_j) = Cov(X_j,Y)/StD(Y)$
- Co-TVaR is also marginal as is co-VaR

## Properties of Marginal Decomposition

- Better than separate individual unit measures
- For capital maintains pricing at the margin
- Growing units with higher profit/risk will increase profit/risk for the firm
- But only if growth is proportional, like reducing quota share or increasing shares of business written
- Approximation otherwise – unless risk unit is transformed mean, then exact



## Even so, Not Ideal Way to Risk-Adjust Profits

- Arbitrary and artificial
- Not likely to reflect risk pricing principles
  - Risk adjusting profit by allocating capital comes down to quantifying the value of the risk – really increase in firm value
  - Could work if risk pricing were the risk measure used in allocation, but allocation then not really needed
  - A good risk pricing theory is needed to do it right
- A 1st approximation is RTVaR – risk adjust TVaR
  - Conditional mean plus a percent of conditional standard deviation above the TVaR probability level
  - Standard deviation pricing of tail risk
  - Derivative for marginal calculation known
  - Can use at probability of not meeting plan



## Possible Next Step

- Model market value of firm
  - Growth prospects, profitability prospects
  - Risks to market value
- Allocate franchise value – market value less capital – to business units
- Stronger basis for strategic planning
- Need better theory of market value:
  - Higher co-moments, impact of jumps, customer attitudes to firm risk



## What Might Be Done for Risk Pricing

- Two financial paradigms: CAPM and arbitrage theory
- Venter ASTIN 1991 showed that CAPM can be incorporated into arbitrage theory
  - Transform the probabilities using conditional mean of market given the individual asset being priced to define new probabilities
- If returns are not normally distributed, it has been known since early 1970's that maximizing investor utility requires pricing higher co-moments than just covariance
  - Some evidence of this in market prices
  - Needed in insurance pricing
- Also jump risk may be an additional priced factor



## Jump Risk

- In single period model not important
- But in continuous model, its existence can make the market incomplete without hedges for some risks
- You would think this would be of concern to investors
- So seems logical to price jump risk over and above moments
- This is done in many arbitrage-free approaches
- Extending CAPM for jumps may be possible using co-jumps
  - That is jump for the company when there is a market jump
- Still some work needed to have a real pricing model for insurance risk



## In Summary

- Economic capital better defined as multiple of loss at lower risk level, but still does not give the true capital need
- Tail risk measures have weaknesses
- If you're going to allocate, be marginal
- Even then not likely to give value of risk
- Strategic risk analysis through ERM has come a long way, but is not nearly done
  - Modeling and optimizing franchise value of firm
  - Measuring value contribution of business units, perhaps through a pricing theory