Optimal Layers for Catastrophe Reinsurance

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Preliminaries - 1

- The reinsurance transaction generally
- Seeks to balance many factors:

-- Risk appetite of management

-- Market appetite for risk

-- Recent catastrophe experience (of reinsurer)

-- Recent catastrophe experience (of reinsured)

-- Underlying rate adequacy (primary rates)

-- Historical reinsurer/reinsured relationship

-- Costs and benefits

-- And many other factors

Preliminaries - 2

- Usually a reinsurance broker is involved
- The process easily is more art than science
- Some strange combinations of factors are brought together seemingly mysteriously on the way to a final catastrophe reinsurance arrangement
- Decision-making is generally based on the idea that a reinsurance transaction is evaluated on its own (irrespective of the underlying business)
- This paper attempts to improve on this condition

Preliminaries - 3

Two ideas are advanced in this paper:

1. We propose that the reinsurance decision incorporate the risk characteristics of the underlying book of business

2. We propose the optimization (at least the technical optimization) decision be based on maximizing downside risk-adjusted profit.

Preliminaries - 4

Input items

1 Loss ratio distribution (expected) of the reinsured book of business
2 Price quotes obtained for various combinations of reinsurance retentions and participations
3 Distribution of the number of catastrophes
4 Distribution of amount of gross loss arising from a catastrophe event

Preliminaries - 5

Process

1 Fit a distribution to reinsurance prices at various coverage/participation levels

2 Create the convolution distribution that combines the distributions of (a) loss ratio of the primary business, (b) the reinsurance layers and prices, (c) the number of catastrophe events, and (d) amount of gross/net loss arising from a catastrophe event (thus the distribution of risk adjusted underwriting profit)

Preliminaries - 6

Output

A probability distribution of various risk-adjusted profit rates (with associated statistics, including the associated semi-variance), at different risk-appetite assumptions

Agenda

- Introduction
- Optimal reinsurance: academics
- Optimal reinsurance: RAROC
- Optimal reinsurance: our method
- A case study
- Conclusions
- Q&A

1. Introduction

Reinsurance decision is a balance between cost and benefit

- Cost : reinsurance premium – loss recovered
- Benefit : risk reduction
  - Stable income stream over time
  - Protection again extreme events
  - Reduce likelihood of a rating downgrade

1. Introduction

How to measure risk reduction

- Variance and standard deviation
  - Not downside risk measures
  - Desirable swings are also treated as risk
- VaR (Value-at-Risk), TVaR, XTVaR
  - VaR: predetermined percentile point. PML (probable maximum loss per event) is a VaR measure at event level
  - TVaR: expected value when loss>VAR
  - XTVaR: TVaR-mean

1. Introduction

How to measure risk reduction

- Lower partial moment and downside variance

$$LPM(L \mid T, k) = \int_T^{\infty} (L - T)^k \, dF(L)$$

  - L is the amount of gross loss
  - T is the maximum acceptable losses, the benchmark for "downside"
  - k is the risk perception parameter to large losses, the higher the k, the stronger risk aversion to large losses
  - When k=1 and T is the 99th percentile of loss, LPM is equal to 0.01*VaR
  - When K=2 and T is the mean, LPM is semi-variance
  - When K=2 and T is the target, LPM is downside variance

2. Optimal reinsurance: academics

- Borch, K., 1982, “Additive Insurance Premium: A Note”, *Journal of Finance* 37(5), 1295-1298
- Froot, K. A., *2007,* “Risk Management, Capital Budgeting, and Capital Structure Policy for Insurers and Reinsurers,” *Journal of Risk and Insurance 74, pp. 273—299.*
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- Gajek, L., and D. Zagrodny, 2000, “Optimal Reinsurance Under General Risk Measures”, *Insurance: Mathematics and Economics*, 34, 227-240.
- Lane, M. N., 2000, “Pricing Risk Transfer Functions”, *ASTIN Bulletin* 30(2), 259-293.
- Kaluszka M., 2001, “Optimal Reinsurance Under Mean-Variance Premium Principles”, *Insurance: Mathematics and Economics*, 28, 61-67
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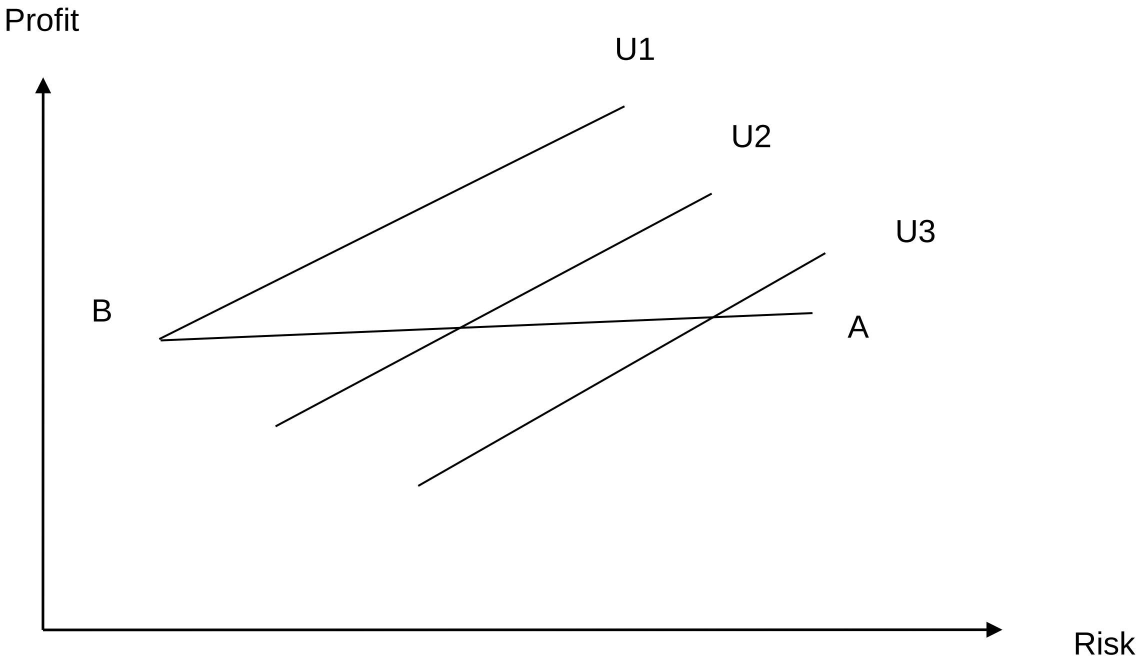
2. Optimal reinsurance: academics

- Cat reinsurance has zero correlation with market index, and therefore zero beta in CAPM.
- Because of zero beta, reinsurance premium should be a dollar-to-dollar trade of loss recovered.
- Reinsurance reduces risk at zero cost. Therefore optimizing profit/risk tradeoff implies minimizing risk
    - buy largest possible protection without budget constraints
    - buy highest possible retention with budget constraints

2. Optimal reinsurance: academics

Academic Assumption



2. Optimal reinsurance: academics

Those studies do not help practitioners

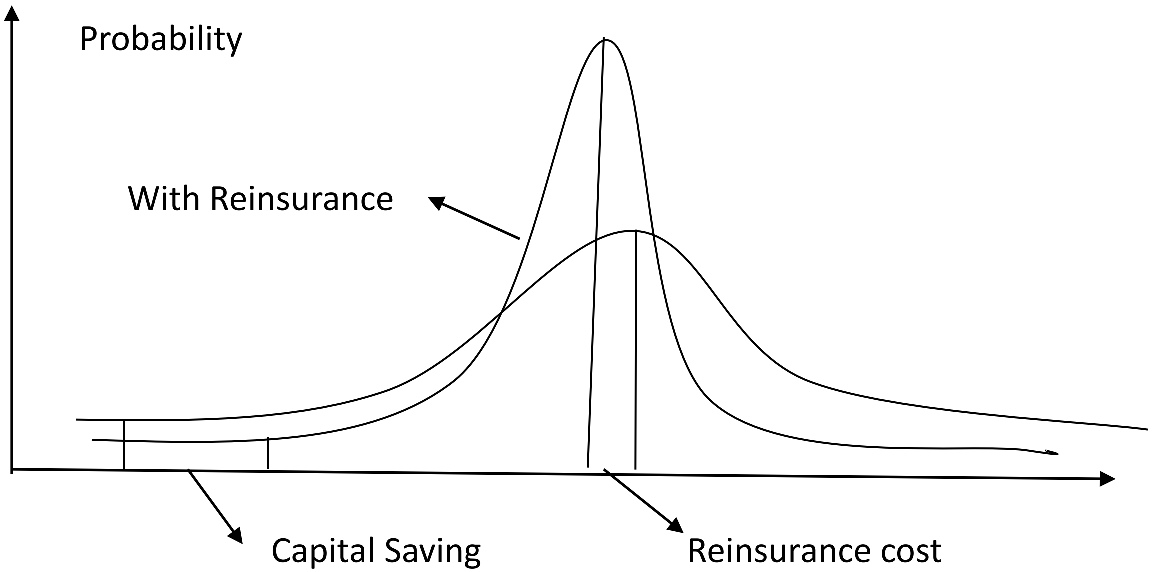
- Reinsurance is costly.
  - Reinsurers need to hold a large amount of capital and require a market return on such a capital.
  - Reinsurance premium/Loss recovered can be over 10 in reality
- No reinsurers can fully diversify away cat risk
- Only consider the risk side of equation and ignore cost side.

3. Optimal reinsurance: RAROC

RAROC (Risk-adjusted return on capital) approach is popular in practice

- Economic capital (EC) covers extreme loss scenarios
- Reinsurance cost = reinsurance premium – expected recovery
- Capital Saving = EC w/o reinsurance – EC w reinsurance
- RAROC=Expected Profit / Economic Capital
- Cost of Risk Capital (CORC) = Reinsurance cost / Capital Saving
- CORC and RAROC balance profit (numerator) and risk (denominator)

3. Optimal reinsurance: RAROC



- No universal definition of economic capital
- Use VaR or TVaR to measure risk
  - Only consider extreme scenarios.
  - Linear risk perception.

4. Optimal Reinsurance: DRAP Approach

Downside Risk-adjusted Profit (DRAP)

$$DRAP = Mean(r) - \theta * LPM(r \mid T, k)$$

$$LPM(r \mid T, k) = \int_{-\infty}^{T} (T - r)^k dF(r)$$

- r is underwriting profit rate
- θ is the risk aversion coefficient
- T is the bench mark for downside
- k measures the increasing risk perception toward large losses

4. Optimal Reinsurance: DRAP Approach

Loss Recovery

$$G(x_i, R, L) = \begin{cases} 0 & if \quad x_i <= R \\ (x_i - R) * \phi & if \quad R < x_i <= R + L \\ L * \phi & if \quad x_i > R + L \end{cases}$$

- R is retention
- L is the limit
- Φ is the coverage percentage
- x_i is cat loss from the ith event

4. Optimal Reinsurance: DRAP Approach

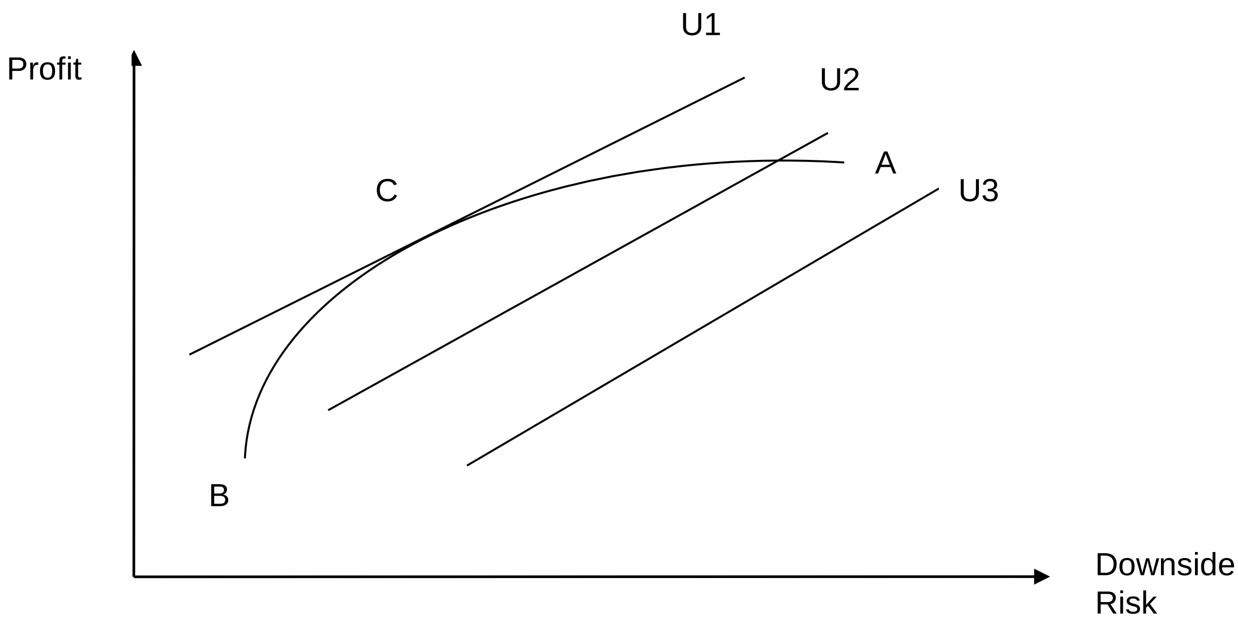
Underwriting profit

$$r = 1 - \frac{EXP + Y + RP(R,L)}{EP} - \frac{\sum_{i=1}^{N} x_i - G(x_i, R, L) + RI(x_i, R, L)}{EP}$$

- EP: gross earned premium
- EXP: expense
- Y non cat losses
- RP(R, L): reinsurance premium
- RI (xi, R, L): reinstatement premium
- N: number of cat events

4. Optimal Reinsurance: DRAP Approach

$$\underset{R,L}{Max} \quad Mean(r) - \theta * LPM(r \mid T, k)$$



AB is efficient frontier

U1, U2, U3 are utility curves

C is the optimal reinsurance that maximizes DRAP

4. Optimal Reinsurance: DRAP Approach

Advantages over conventional mean-variance studies in academic studies:

- An ERM approach.
  - Considers both catastrophe and non-catastrophe losses simultaneously
  - Overall profitability impacts layer selection. High profitability enhances an insurer's ability to retain more cat risk.
- Use a downside risk measure (LPM) other than two-sided risk measure (variance)

4. Optimal Reinsurance: DRAP Approach

Theta estimations

$$DRAP = Mean(r) - \theta * LPM(r \mid T, k)$$

- Theta may not be constant by size of loss
- Theta is time variant
- Theta varies by individual institution
- How much management is willing to pay to mitigate risk?
- How much do investors require to take the risk?
  - index risk premium = index return – risk free rate
  - Insurance risk premium = insurance return-risk free rate
  - cat risk premium= cat bond yield- expected loss-risk free rate

4. Optimal Reinsurance: DRAP Approach

K and T estimations

$$LPM(r \mid T, k) = \int_{-\infty}^{T} (T - r)^k \, dF(r)$$

- k may not be constant by the size of loss
  - For smaller loss, loss perception is close to 1, k=1; for severe loss, k>1
  - Academic tradition: k=2
- T is the bench mark for “downside”
  - Zero: underwriting loss is risk
  - Zero ROE: underwriting loss larger than investment income is risk
  - Large negative: severe loss is treated as risk

5. Case Study

A hypothetical company

- Gross earned premium from all lines:10 billion
- Expense ratio: 33%
- Lognormal non-cat loss from actual data

mean=5.91 billion; std=402 million

- Lognormal cat loss estimated from AIR data
  - mean # of event=39.7; std=4.45
  - mean loss from an event=10.02 million; std=50.77 million
  - total annual cat loss mean=398 million; std=323 million

5. Case Study

- K=2
- T=0%
- Theta is tested at 16.71, 22.28, and 27.85, which represents that primary insurer would like to pay 30%, 40%, and 50% of gross profit to hedge downside risk, respectively.
- UW profit without Insurance is 3.92%
- Variance 0.263%
- Downside variance is 0.07% (T=0%)
- Probability of underwriting loss is 18.41%
- Probability of severe loss (<-15%) is 0.48%

5. Case Study

Reinsurance quotes (million)

Retention	Upper Bound of Layer	Reinsurance Limit	Reinsurance Price	Rate-on-line
305	420	115	20.8	18.09%
420	610	190	21.7	11.42%
610	915	305	19.8	6.50%
610	1,030	420	25.2	5.99%
1,030	1,800	770	28.7	3.72%
1,800	3,050	1,250	39.1	3.13%

5. Case Study

Recoveries and penetrations by layers

Retention (million)	Upper Limit (million)	Mean	Standard Deviation	Recovery/reinsurance Premium	Penetration Probability
305	420	8,859,074	29,491,239	42.59%	10.18%
420	610	8,045,968	35,917,439	37.08%	6.04%
610	915	6,496,494	41,009,356	32.81%	3.15%
610	1,030	7,923,052	51,899,244	31.44%	3.15%
1,030	1,800	4,858,545	55,432,115	16.93%	1.11%
1,800	3,050	2,573,573	48,827,021	6.58%	0.40%

5. Case Study

Reinsurance Price Curve Fitting

- (x1, x2) represents reinsurance layer
- f(x) represent rate-on-line

$$p(x_1, x_2) = \int_{x1}^{x2} f(x)dx$$

- Add quadratic term. Logarithm, and inverse term to reflect nonlinear relations

$$f(x) = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 \log(x) + \beta_4 x^{-1}$$

$$p(x_1, x_2) = \beta_0 (x_2 - x_1) + \frac{1}{2}\beta_1 (x_2^2 - x_1^2) + \frac{1}{3}\beta_2 (x_2^3 - x_1^3)$$
$$+ \beta_3 (x_2 \log(x_2) - x_1 \log(x_1)) + \beta_4 (\log(x_2) - \log(x_1))$$

5. Case Study

Reinsurance Price Fitting

Retention	Upper Bound of Layer	Reinsurance Limit	Reinsurance Price	Rate-on-line	Fitted rate	Fitted Rate-on-line
305	420	115	20.8	18.09%	20.84	18.12%
420	610	190	21.7	11.42%	21.69	11.41%
610	915	305	19.8	6.50%	19.87	6.51%
610	1,030	420	25.2	5.99%	25.18	6.00%
1,030	1,800	770	28.7	3.72%	28.73	3.73%
1,800	3,050	1,250	39.1	3.13%	39.10	3.13%
305	610	305	42.5	13.93%	42.52	13.94%
305	915	610	62.3	10.22%	62.39	10.23%
305	1,030	725	67.7	9.33%	67.70	9.34%
305	1,800	1,495	96.5	6.45%	96.43	6.45%
305	3,050	2,745	135.6	4.94%	135.53	4.94%
420	915	495	41.5	8.39%	41.55	8.39%
420	1,030	610	46.9	7.68%	46.87	7.68%
420	1,800	1,380	75.6	5.47%	75.60	5.48%
420	3,050	2,630	114.7	4.36%	114.69	4.36%
610	1,800	1,190	53.9	4.53%	53.91	4.53%
610	3,050	2,440	93	3.81%	93.01	3.81%
915	1,030	115	5.3	4.64%	5.32	4.62%
915	1,800	885	34	3.85%	34.04	3.85%
915	3,050	2,135	73.1	3.42%	73.14	3.43%
1,030	3,050	2,020	67.8	3.36%	67.83	3.36%

5. Case Study

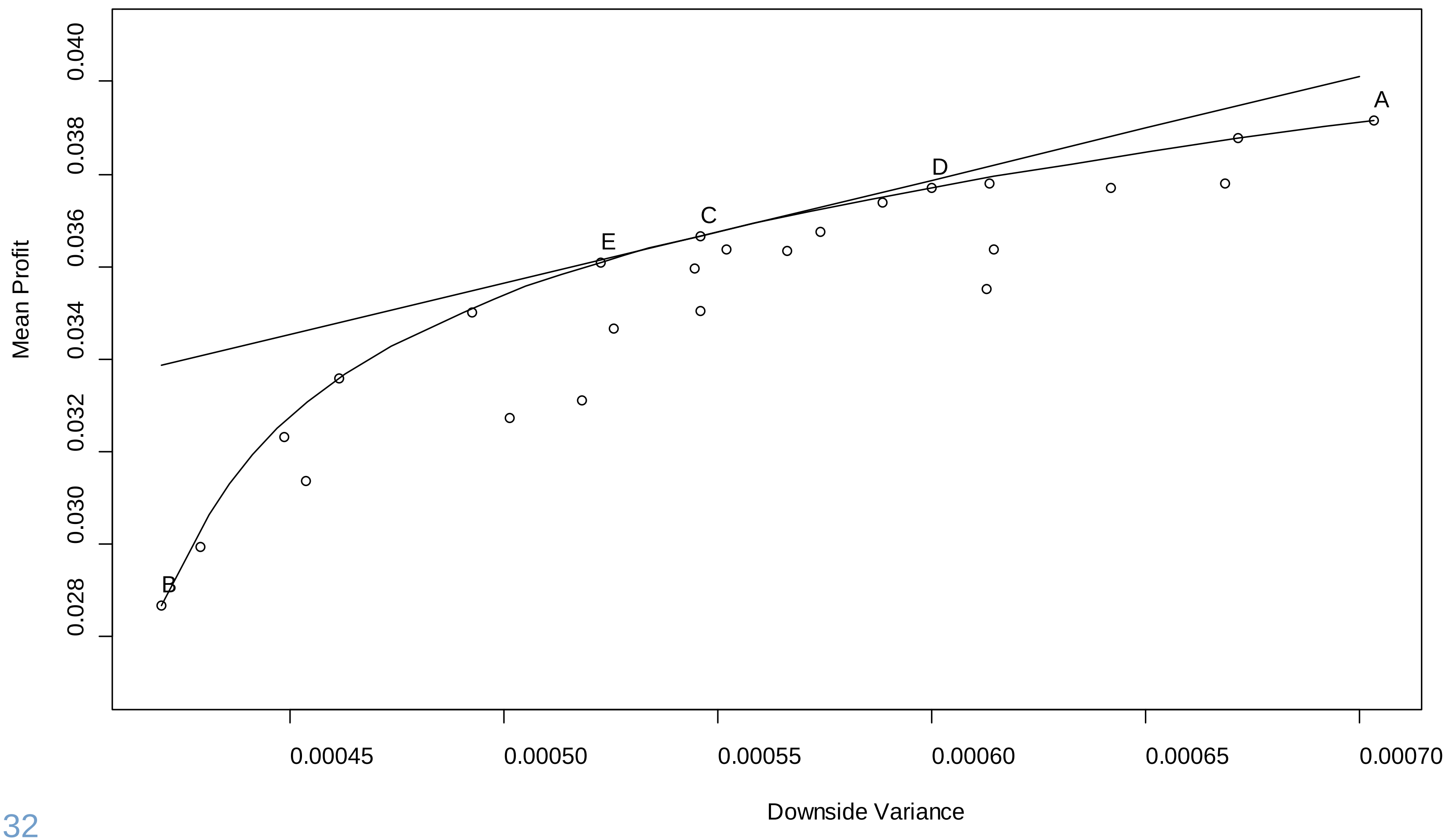
Performance of Reinsurance Layers theta=22.28

Retention (million)	Upper Limit (million)	Prob r<0	Prob r<-15%	Mean	Variance	Downside Variance	Risk-adjusted Profit
No Reinsurance		18.41%	0.48%	3.916%	0.263%	0.070%	2.350%
305	420	19.02%	0.42%	3.781%	0.253%	0.067%	2.291%
420	610	19.17%	0.35%	3.771%	0.249%	0.064%	2.341%
610	915	19.31%	0.30%	3.779%	0.247%	0.061%	2.412%
610	1030	19.53%	0.27%	3.739%	0.243%	0.059%	2.428%
1030	1800	19.95%	0.26%	3.676%	0.243%	0.057%	2.397%
1800	3050	20.44%	0.41%	3.551%	0.247%	0.061%	2.186%
305	610	19.63%	0.33%	3.637%	0.241%	0.061%	2.268%
305	915	20.50%	0.25%	3.503%	0.228%	0.055%	2.287%
305	1,030	20.76%	0.22%	3.465%	0.224%	0.053%	2.293%
305	1,800	22.31%	0.13%	3.231%	0.210%	0.045%	2.231%
305	3,050	24.77%	0.04%	2.869%	0.200%	0.042%	1.934%
420	915	19.85%	0.25%	3.634%	0.235%	0.057%	2.373%
420	1,030	20.06%	0.22%	3.595%	0.232%	0.054%	2.382%
420	1,800	21.79%	0.14%	3.358%	0.216%	0.046%	2.330%
420	3,050	24.25%	0.05%	2.995%	0.206%	0.043%	2.038%
610	1,800	21.05%	0.16%	3.500%	0.226%	0.049%	2.402%
610	3,050	23.35%	0.11%	3.135%	0.215%	0.045%	2.124%
915	1,030	18.63%	0.40%	3.877%	0.258%	0.067%	2.380%
915	1,800	20.14%	0.21%	3.637%	0.239%	0.055%	2.407%
915	3,050	22.44%	0.17%	3.272%	0.226%	0.050%	2.155%
1030	3050	22.15%	0.20%	3.311%	0.230%	0.052%	2.156%
680	1390	20.00%	0.21%	3.667%	0.237%	0.055%	2.451%

5. Case Study

Efficient Frontier

Figure 3: Reinsurance Efficient Frontier



5. Case Study

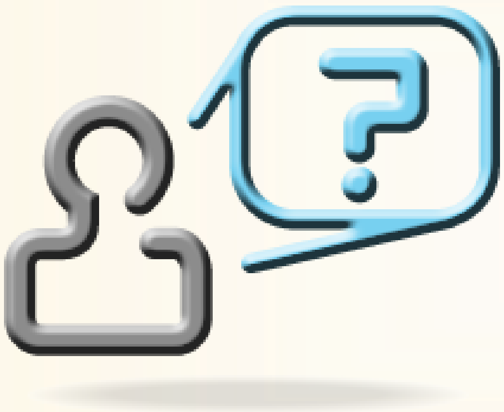
- Optimal Reinsurance Layers theta =16.71, 22.28, 27.85

Theta	Retention (million)	Upper Limit (million)	Mean	Downside Variance	Risk-Adjusted Profit theta=16.71	Risk-Adjusted Profit theta=22.28	Risk-Adjusted Profit theta=27.85
16.71	795	1220	3.771%	0.060%	<u>2.768%</u>	2.434%	2.100%
22.28	680	1390	3.667%	0.055%	2.755%	<u>2.451%</u>	2.147%
27.85	615	1460	3.610%	0.052%	2.736%	2.445%	<u>2.154%</u>

- If the overall profit rate increases 2% and theta remains at 22.28, the optimal layers becomes (740, 1420)

6. Conclusions

- The overall profitability (both cat and non-cat losses) impacts optimal insurance decision
- Risk appetites are difficult to measure by a single parameter.
- DRAP captures risk appetites comprehensively though theta (risk aversion coefficient), T (downside bench mark), and moment k (increasingly perception of risk arising from large loss)
- DRAP provides an alternative approach to calculate optimal layers.



Q & A