Tail Risk, Systemic Risk and Copulas

2010 CAS Annual Meeting

Andy Staudt

09 November 2010

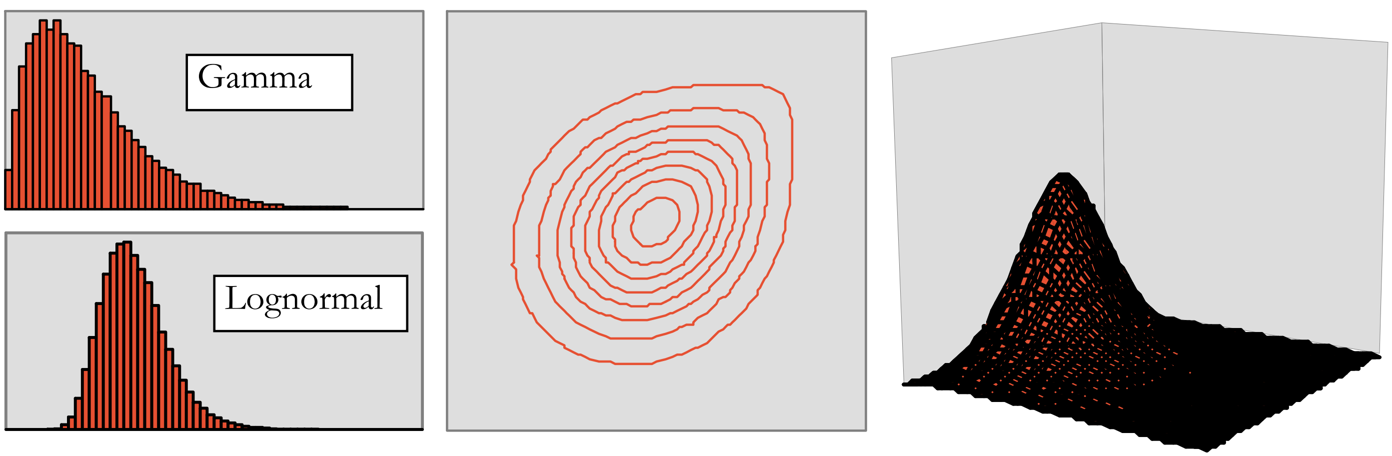
TOWERS WATSON

Outline

- Introduction
  - Motivation – flawed assumptions, not flawed models
  - Structure – non-technical with examples
- Definitions
- 4 aspects of copula specification within context of tail risk/systemic risk
  - *Correlation*
  - *Marginal distributions*
  - *Tail dependence*
  - *(A)symmetry*
- Parting thoughts

Some definitions

- *Tail risk*. Tail risk is the risk of an extreme event
- *Systemic risk*. Systemic risk is the risk of simultaneous extreme events
- *Copulas*. Copulas are a mathematic tool for modeling the joint distribution of random events. The key is that they allow us to separate the marginal distributions from the dependence structure and model each separately.



(a) Marginals (b) Gumbel copula (c) Joint distribution

Topic

Correlation

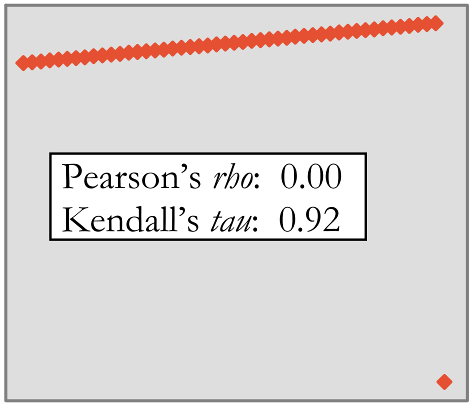
Marginal distributions

Tail dependence

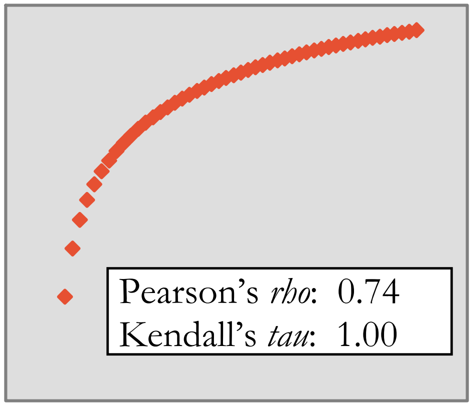
(A)symmetry

The trouble with correlation

- *Short answer*. Correlation only tells one part of the story
  - *Correlation.* Correlation generally specifically refers to the Pearson correlation coefficient which is a measure of *linear* association between random variables
  - *Dependence.* Dependence is a more general concept which refers to any type of association between random variables. Alternate measures include rank correlations such as Kendall’s tau and Spearman’s rho as well as tail dependence (discussed in more detail later)
- *Another short answer*. Correlation is easily distorted



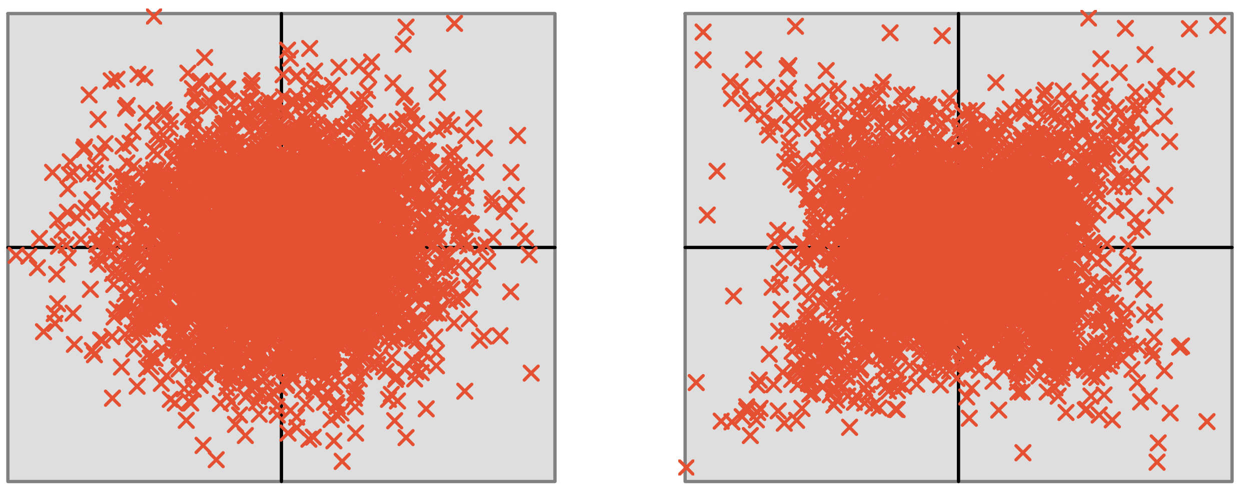
(a) Outliers



(b) Non-linear relationships

The trouble with correlation (continued)

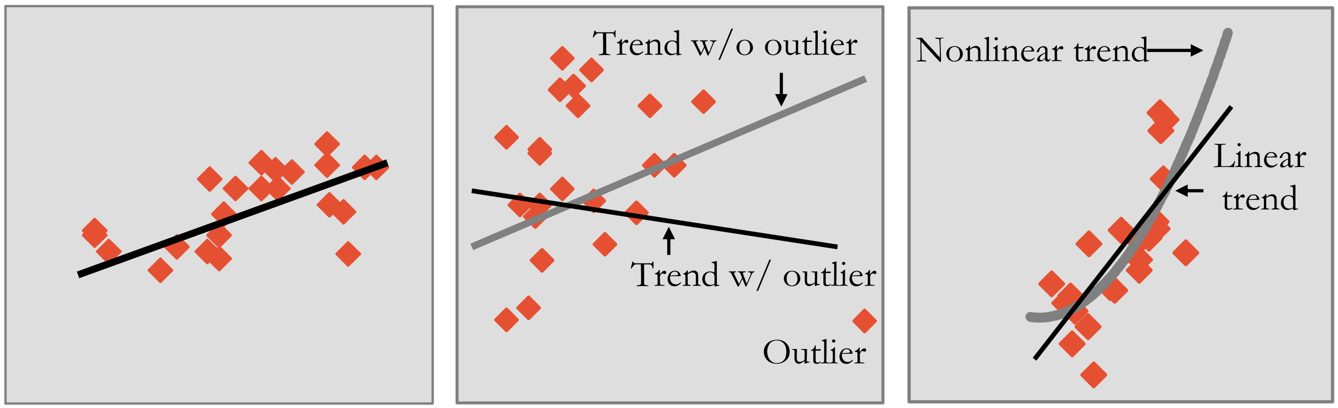
- *A long-winded answer*. Correlation does not (necessarily) uniquely define the dependence structure (i.e., knowing the correlation between two risks doesn't tell us how they are related)



(a) Normal copula (b) *t* copula

Case study.
Texas loss ratios by line (1986 – 2008)

- Data



(a) GL by CAL (b) CMP-Property by GL (c) CAL by CMP-Liability

- Capital allocation

#	Copula	Calibration	CTE(95^{th})	Capital Allocation: CAL	CMP Liability	CMP Property	GL	Cramer-von-Mises Goodness of Fit Statistic*
1	Normal	Pearson's *rho*	1.30	28%	35%	12%	25%	0.11
2	*t* (df=8.5)	Pearson's *rho*	1.35	28%	35%	12%	25%	0.11
3	*t* (df=11.0)	Kendall's *tau*	1.50	28%	40%	10%	22%	0.05

*Smaller values indicate a better fit.

Topic

Correlation

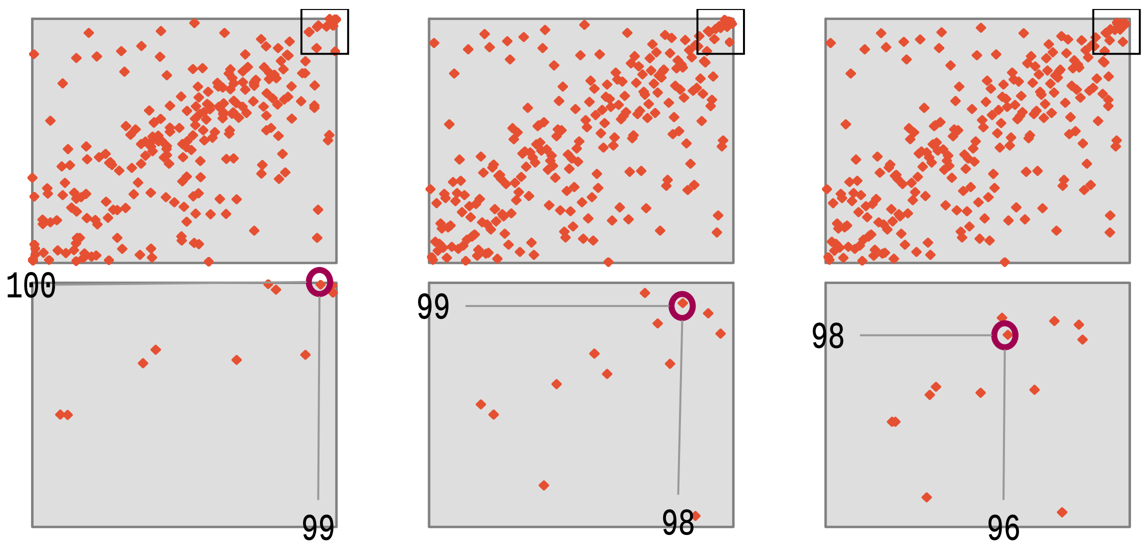
Marginal distributions

Tail dependence

(A)symmetry

The Goldilocks approach to tail risk

- Some types of marginal distributions
  - *Empirical.* Too unimaginative, history repeats itself, nothing new ever happens
  - *Parametric*. Too rigid, will work well in some places and fail in other places
  - *Mixed*. Just right, model the central and extreme data separately
- Pseudo-observations



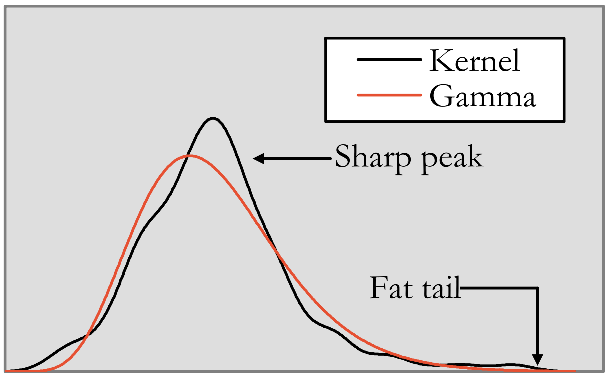
(a) Gamma (b) Empirical (c) Empirical + GPD

But the marginal distributions do affect systemic risk

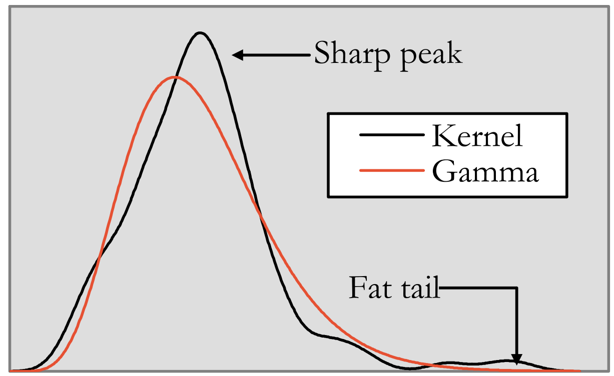
- Advantage of copulas. The major advantage of copulas is that they allow us to separate the marginal distribution from the dependence structure and model these independently…but that doesn't mean these components are independent
- Selecting the right marginal
  - *Tail risk*. Obviously, selecting the right marginal is crucial to adequately model the tail risk
  - *Systemic risk.* However, selecting the right marginal can also be crucial to appropriately model the systemic risk
- Inference functions for margins (IFM). Approach to parameterizing a copula which relies on fitting to the psuedo-observations; if the psuedo-observations understate the tail risk, the copula will understate the systemic risk

Case study.
Federal crop insurance corn & soybean losses (1989 – 2008)

- Data



(a) Corn



(b) Soybeans

- Benefit to diversification

Marginals	Copula	Copula Parameter	CTE(95^{th})	Benefit to Diversification	Cramer-von-Mises Goodness of Fit Statistic*
Gamma	Gumbel	1.88	58.7	5.7%	0.036
Empirical	Gumbel	1.89	82.4	5.6%	0.035
Mixed Empirical-GPD	Gumbel	1.93	106.6	4.8%	0.031

**Smaller values indicate a better fit.*

Topic

Correlation

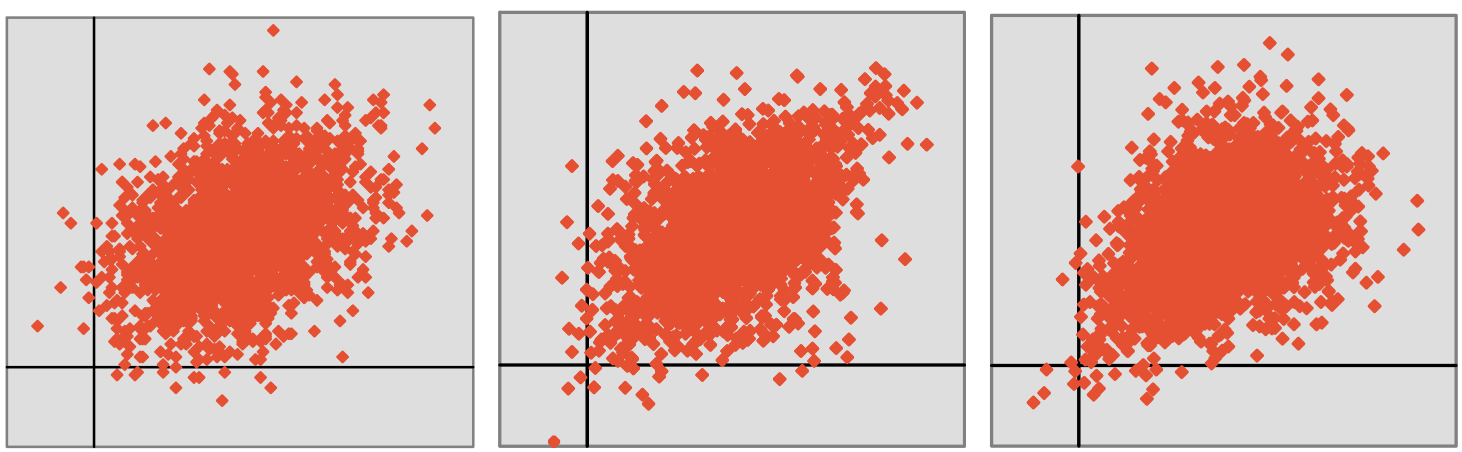
Marginal distributions

Tail dependence

(A)symmetry

There's dependence…and then there's tail dependence

- Central vs. extreme dependence
  - *Pearson's correlation, Kendall's tau, Spearman's rho*. These are all measures of association which focus on central dependence
  - *Tail dependence*. Tail dependence is another measure of association however it specifically looks for extreme or tail dependence



(a) Normal Copula (b) *t* Copula (c) Clayton Copula

Not all copulas allow for tail dependence

- Examples
  - *Normal*. Has NO tail dependence
  - *t*. Has some lower tail dependence and some upper tail dependence
  - *Clayton*. Has loads of lower tail dependence and no upper tail dependence

	Kendall's	Tail Dependence	
Copula	*tau*	Lower	Upper
Normal	0.25	0.00	0.00
t (df=4.45)	0.25	0.17	0.17
Clayton	0.25	0.35	0.00

Case study.
Counterparty default risk

- Hypothetical. 1M in recoverables from each of 2 reinsurers each with a 3% chance of default and a 25% dependence
- What is the probability of joint default

Probability of:	Normal Copula	Extreme Value Copulas: Galambos	Gumbel	Husler Reiss
No Defaults	94.4%	95.0%	95.0%	95.0%
One Default	5.2%	4.0%	4.0%	4.0%
Both Default	0.4%	1.0%	1.0%	1.0%

- What is the modeled loss in default

Threshold	Normal Copula	Extreme Value Copulas: Galambos	Gumbel	Husler Reiss
50^{th}	120K	120K	120K	120K
75^{th}	240K	240K	240K	240K
90^{th}	600K	600K	600K	600K
95^{th}	1.10M	1.20M	1.20M	1.20M
97.5^{th}	1.16M	1.39M	1.40M	1.40M
99.9^{th}	1.41M	1.97M	1.98M	1.97M

Topic

Correlation

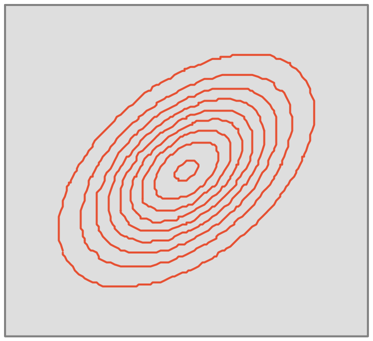
Marginal distributions

Tail dependence

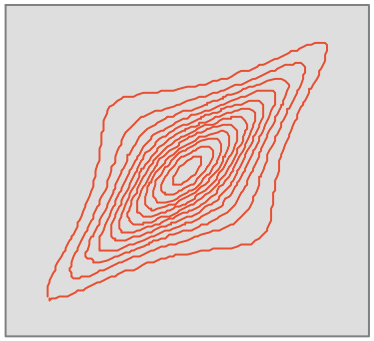
(A)symmetry

Denzel Washington's face

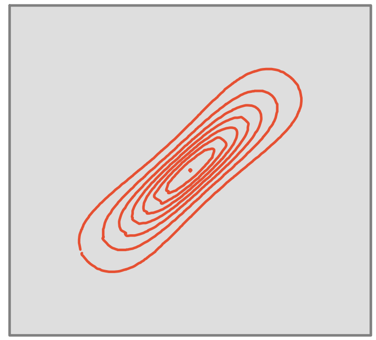
- Some copulas are symmetric…



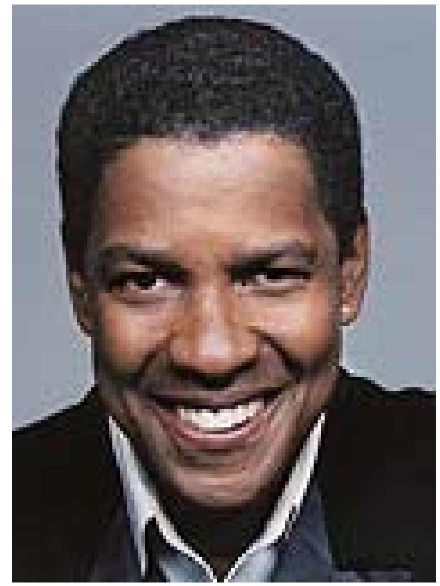
(a) Normal copula



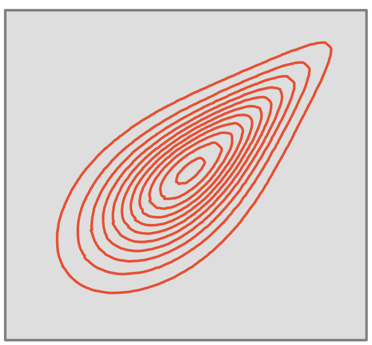
(b) t copula



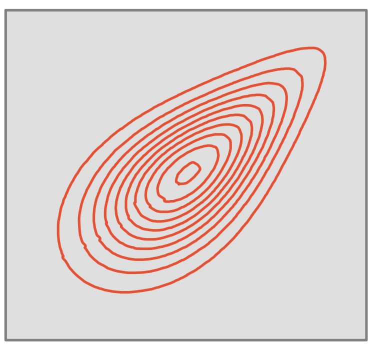
(c) Frank copula



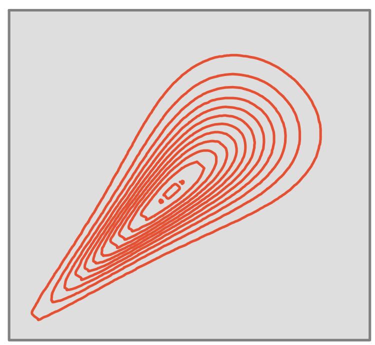
- Others are not…



(a) Galambos copula



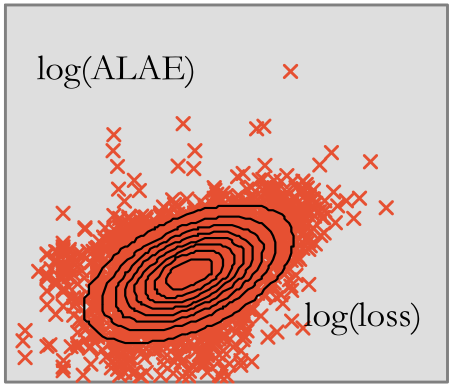
(b) Husler-Reiss copula



(c) Clayton copula

Case study.
Loss & ALAE components of Florida medical malpractice (2000 – 2009)

- Data



- Comparison of moments

Copula	Symmetry	Skewness	Excess Kurtosis
Actual	**Asymmetric**	**0.50**	**1.50**
Normal	Symmetric	0.00	0.00
Frank	Symmetric	0.00	0.10
t	Symmetric	0.00	0.25
Galambos	Asymmetric	0.10	0.15
Gumbel	Asymmetric	0.10	0.25
Skew *t*	Asymmetric	0.40	1.80

Parting thoughts

- *Correlation*. Correlation is easily distorted and not the only measure of association. Consider alternate measures of association.
- *Marginals*. Consider using an extreme value distribution to model events above a certain threshold. This will give you a better estimate of tail risk and systemic risk.
- *Tail dependence*. The normal copula does not allow for tail dependence but most other copulas do in some form or another
- (*A)symmetry*. Very little is symmetric; like you would with univariate distributions consider skewed copulas

Contact information

- Andy Staudt
  - +44 (0) 20 7170 3479
  - andy.staudt@towerswatson.com