CHAIN LADDER RESERVING METHODS FOR LIABILITIES WITH PER OCCURRENCE LIMITS

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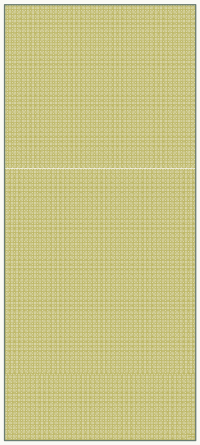


BACKGROUND

- Response to CAS Call for Non-Technical Papers 2013

- As a regulator I see many actuarial reports and many techniques. I wrote this paper to improve the quality of actuarial work. Don't remove large losses when LDFs come from net/limited triangles. Schedule P triangles are often limited.

INTRODUCTION

- Case Inc. Limited Losses: $3M
- Insured Limit: $500K
- Losses exceeding 100K: 120K, 450K, 500K
- Applicable Incurred LDF: 1.2
- Accident Year: 20XX

450K and 500K multiplied by the LDF exceed the limit. What should you do?

Multiply all by the LDF

$3M x 1.2 = $3.6M

OR

($3M - $.45-$.5)*1.2 + $.5 + $.5 =
$2.05*1.2 + $1M = $3.46M

Assume that 450K and 500K develop to 500K then stop.

SOME ASSUMPTIONS/METHODS

- There is some set of "best" LDFs that is applicable to losses that do not reach the self-insured retention or retained limit.
- Used 500K limit/retention throughout the paper
- LDFs are calculated using an all year weighted average.

DIFFERENT TRIANGLES FROM THE SAME DATA

Limited LDFS

A) Unlimited Triangle

	1	2	3	4	5
2009	415	853	1,258	1,654	2,051
2010	180	370	546	717	-
2011	580	1,192	1,758	-	
2012	180	370	-	-	
2013	415	-	-	-	

	1	2	3	4	5
f(d)	2.06	1.48	1.32	1.24	1.00
F(d)	4.94	2.41	1.63	1.24	1.00

B) Limited Triangle 500K per Occ

	1	2	3	4	5
2009	415	839	1,000	1,158	1,316
2010	180	370	546	717	-
2011	580	1,178	1,500	-	
2012	180	370	-	-	
2013	415	-	-	-	

	1	2	3	4	5
f(d)	2.03	1.28	1.21	1.14	1.00
F(d)	3.58	1.76	1.38	1.14	1.00

C) Small Only - Only Losses <= 450K

	1	2	3	4	5
2009	165	339	500	658	816
2010	180	370	546	717	-
2011	180	370	546	-	
2012	180	370	-	-	
2013	415	-	-	-	

	1	2	3	4	5
f(d)	2.06	1.48	1.32	1.24	1.00
F(d)	4.94	2.41	1.63	1.24	1.00

D) Large Only Limited to 500K per Occ

	1	2	3	4	5
2009	250	500	500	500	500
2010	-	-	-	-	-
2011	400	808	955	-	
2012	-	-	-	-	
2013	-	-	-	-	

	1	2	3	4	5
f(d)	2.01	1.11	1.00	1.00	1.00
F(d)	2.24	1.11	1.00	1.00	1.00

METHOD-A VS. METHOD-X

Method-A multiplies all losses by the development factors

Method-X tests each loss. If the loss is “large enough” then the ultimate loss is set equal to the limit.

The result from method X is always less than or equal to the result from method A.

CONCLUSIONS - CONTINUED

- If the LDFs were created using a limited triangle, then it is more accurate to multiply ALL the losses by the LDF.
- If the LDFs were created using a limited triangle, then removing large losses understates the reserve. *This is due to the fact that losses reaching the limits no longer develop over time and hence the LDFs estimated using all losses are smaller than the LDFs estimated using only the losses below the limits.*
- If you can separate large losses from small losses then consider creating two sets of LDFs.

CONCLUSION

- Case Inc. Limited Losses: $3M
- Insured Limit: $500K
- Losses exceeding 100K: 120K, 450K, 500K
- LIMITED LDF: 1.2
- Accident Year: 20XX

450K and 500K multiplied by the LDF exceed the limit. What should you do?

$3M x 1.2 = $3.6M

Multiply all by the LDF

OR

($3M - $.45-$.5)*1.2 + $.5 + $.5 =
$2.05*1.2 + $1M = $3.46M

Assume that 450K and 500K develop to 500K then stop.

METHOD USED IN PAPER

- Simulate many incurred losses and associated triangles where the losses have per occurrence limits.
- Apply the limited LDFs to the incurred losses both with and without large losses
- See which one is more accurate

SAMPLE SIMULATION - UNLIMITED

Year	d=1	d=2	d=3	d=4	d=5	d=6	d=7	d=8
2006	250	514	758	996	1,236	1,483	1,742	1,995
2006	150	308	455	598	741	890	1,045	1,197
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
Total	580	1,194	1,753	2,314	2,865	3,441	4,047	4,632

One Accident Year in an Unlimited Triangle

d	1	2	3	4	5	6	7	8	9	10
$f^T(d)$	2.055	1.475	1.315	1.240	1.200	1.175	1.145	1.125	1.110	1.000
$F^T(d)$	9.964	4.849	3.287	2.500	2.016	1.680	1.430	1.249	1.110	1.000

UNLIMITED - TO- LIMITED

Unlimited Occurrences

Year	d=1	d=2	d=3	d=4	d=5	d=6	d=7	d=8
2006	250	514	758	996	1,236	1,483	1,742	1,995
2006	150	308	455	598	741	890	1,045	1,197
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
				Several Rows				
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
Total	580	1,194	1,753	2,314	2,865	3,441	4,047	4,632

Limited Occurrences

Year	d=1	d=2	d=3	d=4	d=5	d=6	d=7	d=8
2006	250	500	500	500	500	500	500	500
2006	150	308	455	500	500	500	500	500
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
				Several Rows				
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
Total	580	1,180	1,495	1,720	1,888	2,068	2,260	2,440

EXAMPLE LIMITED TRIANGLE & LIMITED LDFS

	1	2	3	4	5	6	7	8	9	10
2004	180	370	546	[illegible]	890	1,068	1,252	1,436	1,616	[illegible]
2005	180	370	546	[illegible]	890	1,068	1,252	1,436	1,616	-
2006	[illegible]	1,178	1,500	[illegible]	1,890	2,068	2,252	2,436	-	
2007	180	370	546	[illegible]	890	1,068	1,252	-		
2008	[illegible]	839	1,000	1,158	1,316	1,479	-			
2009	180	370	546	[illegible]	890	-	-			
2010	225	462	682	897	-	-	-			
2011	[illegible]	808	955	-						
2012	180	370	-	-						
2013	285	-	-	-						

	1	2	3	4	5	6	7	8	9	10
f(d)	2.04	1.33	1.24	1.18	1.15	1.12	1.11	1.13	1.11	1.00
F(d)	7.19	3.53	2.66	2.15	1.83	1.59	1.42	1.25	1.11	1.00

AN ITERATION OF METHOD A

Accident Year	Age - d	Incurred $(000)	F(d)	Method A Estimate $(000)	True Ultimate $(000)	Method A IBNR $(000)	True IBNR $(000)	Error $(000)	Error as a % of True IBNR
		(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
		modeled	modeled	=(a)*(b)		=(c) -(a)	=(d)-(a)	=(f)-(e)	=(g)-(f)
2004	10	19,794	1.000	19,794	19,794	-	-	-	-
2005	9	19,616	1.000	19,794	19,794	178	178	-	0%
2006	8	19,436	1.015	19,842	19,794	606	358	248.7	
2007	7	19,254	1.039	19,746	19,794	491	539	(47.7)	-9%
2008	6	19,179	1.059	20,350	20,144	871	666	205.6	
2009	5	890	1.823	19,624	19,794	735	904	(169.2)	-19%
2010	4	897	2.115	19,928	20,242	19,031	19,345	(314.1)	-2%
2011	3	955	2.666	20,541	19,995	19,586	19,040	546.0	
2012	2	570	3.553	19,305	19,794	935	19,424	(488.7)	-34%
2013	1	285	7.119	20,050	19,845	19,765	19,560	204.4	
Total		11,978		20,173	19,988	8,198	8,013	185.1	2%
2004-2011		11,320		16,818	16,349	5,498	5,029	469.3	9%

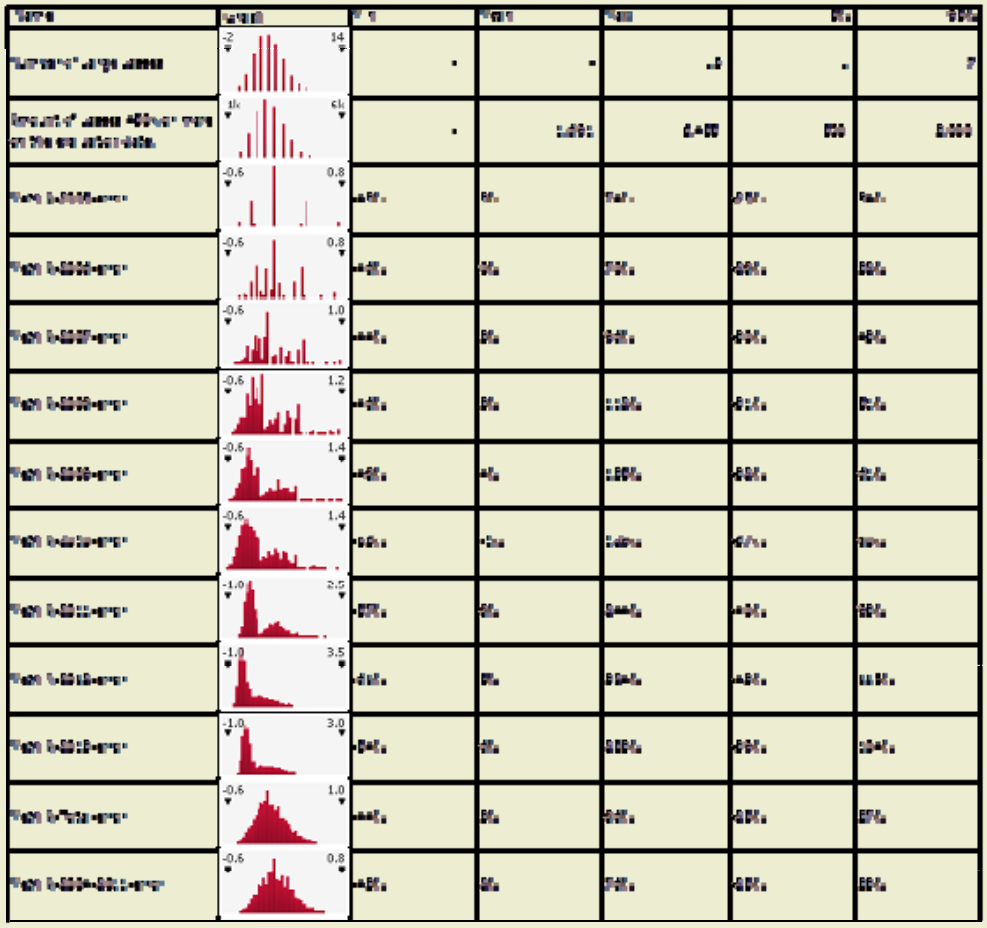
AN ITERATION OF METHOD X

Accident Year	Age - d	F(d)	Incurred $(000)	Large Losses	Inc X Known Large Losses	Method X Estimate $(000)	True Ultimate $(000)	Method X IBNR $(000)	True IBNR $(000)	Error $(000)	Error as a % of True IBNR
		(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
		modeled	modeled	modeled	=(b)-(c)	=(a)*(d) - (c)		=(e)-(b)	=(f)-(b)	=(g)-(h)	=(i)/(h)
2001	10	1.00	1,721		1,721	1,721	1,721				0%
2002	9	1.11	1,515	-	1,515	1,724	1,724	173	173	-	0%
2003	8	1.25	2,484	1,000	1,484	2,734	2,734	357	357	(0)	0%
2004	7	1.43	1,254	-	1,254	1,745	1,724	491	539	(48)	-9%
2005	6	1.67	1,472	500	972	2,033	2,144	561	672	(111)	-17%
2006	5	2.00	930	-	930	1,824	1,724	736	794	(58)	-7%
2007	4	2.50	587	-	587	1,223	2,342	1,031	1,755	(724)	-41%
2008	3	3.33	855	500	455	1,716	1,365	761	510	251	49%
2009	2	5.00	870	-	870	1,305	1,724	435	854	(419)	-49%
2010	1	10.00	355	-	355	2,058	1,545	1,703	1,190	513	43%
Total			11,973	2,000		18,799	17,988	6,824	6,013	(1,189)	-19%
2004-2011			11,320	2,000		15,444	15,349	4,124	5,029	(905)	-18%

10,000 SIMULATIONS

		Error as a Percentage of IBNR						Error as % of Ult
		10th Percentil	25th Percentil	Mean Error	75th Percentil	90th Percentil	Std Dev	Mean
All Years	A	-19%	-10%	3.4%	15%	28%	19%	1.35%
2004-2011	A	-20%	-11%	1.9%	14%	25%	17%	0.54%
All Years	X	-34%	-27%	-20%	-12%	-5%	11%	-7.90%
2004-2011	X	-32%	-25%	-18%	-9%	-3%	11%	-5.54%

@RISK – GRAPHS OUTPUT



SENSITIVITY ANALYSIS – 5,000 SIMULATIONS

Table 1- All Years Combined – Sensitivity of Mean Error to LDF and Percentage of Large Losses

		Highest LDF		High LDF		Moderate LDF	
Ratio of Large Losses to Total Losses - Ultimate Limited Basis	**Method**	**Mean Error as % of IBNR**	**Mean Error as % of Ultimate**	**Mean Error as % of IBNR**	**Mean Error as % of Ultimate**	**Mean Error as % of IBNR**	**Mean Error as % of Ultimate**
15%	A	7%	3%	4%	1%	2%	0%
	X	-33%	-12%	-23%	-6%	-16%	-3%
10%	A	4%	1%	2%	1%	1%	0%
	X	-25%	-10%	-16%	-5%	-12%	-2%
5%	A	1%	1%	1%	0%	0%	0%
	X	-14%	-6%	-9%	-3%	-6%	-1%

QUESTIONS

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OTHER SLIDES - VARIOUS LDFS

Highest

d	1	2	3	4	5	6	7	8	9	10
f(d)	2.055	1.475	1.315	1.240	1.200	1.175	1.145	1.125	1.110	1.000
F(d)	9.964	4.849	3.287	2.500	2.016	1.680	1.430	1.249	1.110	1.000

High

d	1	2	3	4	5	6	7	8	9	10
f(d)	1.541	1.263	1.179	1.138	1.116	1.102	1.085	1.073	1.065	1.000
F(d)	3.973	2.579	2.042	1.733	1.523	1.365	1.239	1.143	1.065	1.000

Moderate

d	1	2	3	4	5	6	7	8	9	10
f(d)	1.296	1.150	1.104	1.081	1.068	1.060	1.050	1.043	1.038	1.000
F(d)	2.289	1.766	1.536	1.391	1.287	1.205	1.137	1.083	1.038	1.000

OTHER SLIDES - NOTATION

- ***The Analysis and Estimation of Loss & ALAE Variability…*** by the CAS Working Party on Quantifying Variability in Reserve Estimates.
- *w*: The accident year
- *d*: The age of the losses. If the accident year is 2010, then *d*=1 at 12/31/2010 and *d*=2 at 12/31/2011
- f(*d*) : Incremental LDF
- F(*d*) : Cumulative LDF.
- $f^T(d)$: true value of f(*d*) for unlimited losses.
- $F^T(d)$: true value of F(*d*) for unlimited losses.

OTHER SLIDES - MORE NOTATION AND METHODS

- $f^T(d)$: best value of $f(d)$ for unlimited losses.
- $F^T(d)$: best value of $F(d)$ for unlimited losses.
- Losses are in 000's
- The retained limit is always 500K
- LDFs are calculated using an all year weighted average