

MANAGING EXTREMES

Willis Re

# AN ACTUARIAL MODEL OF EXCESS OF POLICY LIMITS LOSSES

Seminar on Reinsurance

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# Agenda

- Goal
- Background & motivation
- Proposed model
- Analysis and discussion
- Numerical example
- Conclusion

# Goal

- Calculate XPL loss cost in XOL layer
- Practical
- Integrated into
  - Traditional exposure rating
  - Standard XOL reinsurance pricing software

# BACKGROUND & MOTIVATION

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# Background & motivation

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- You
  - Are a reinsurance pricing actuary
  - Price treaties covering Excess of Policy Limits (XPL) losses
  - Do not have credible data
  - Use exposure rating
- Does your exposure rating tool calculate a loss cost for XPL?

# Background & motivation

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- How to price for XPL exposure: are there any actuarial papers?
- Braithwaite and Ware
  - Build a standalone, specialized curve for XPL/ECO
  - Get lots of claims data, fit frequencies and severities
  - How to price a clash cover; not what we're interested in
    - Not practical
    - Not connected to working layer loss curves

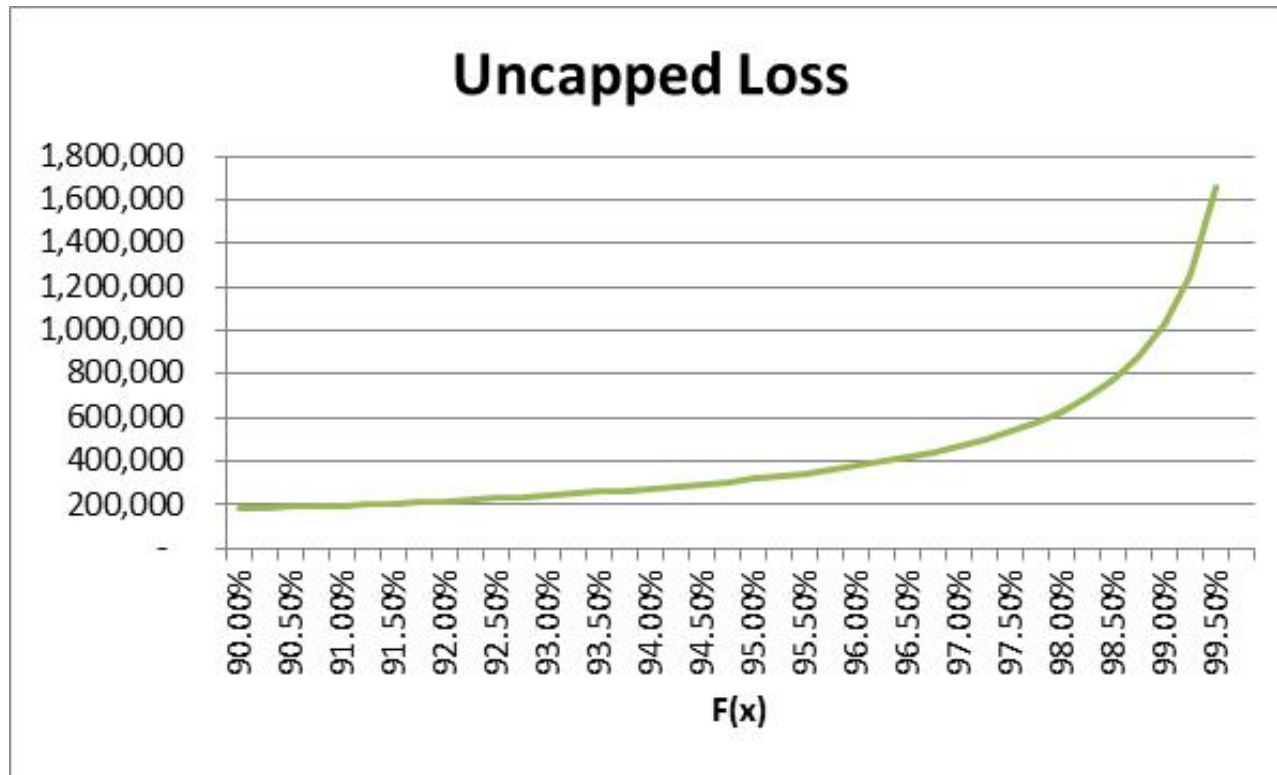
# PROPOSED MODEL

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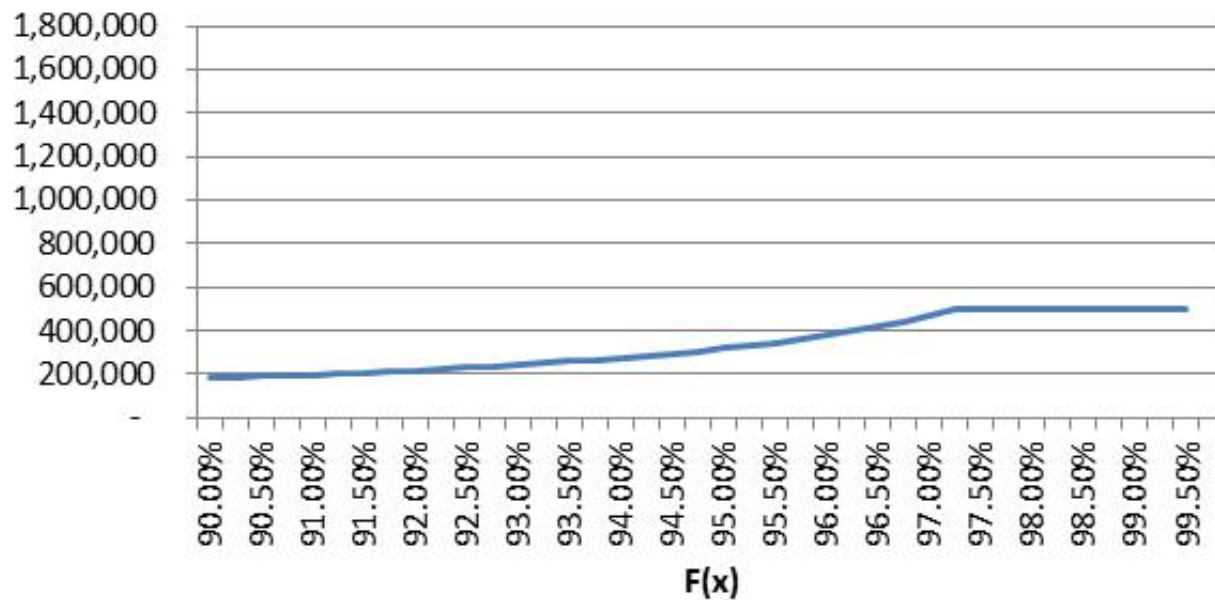
# Traditional model



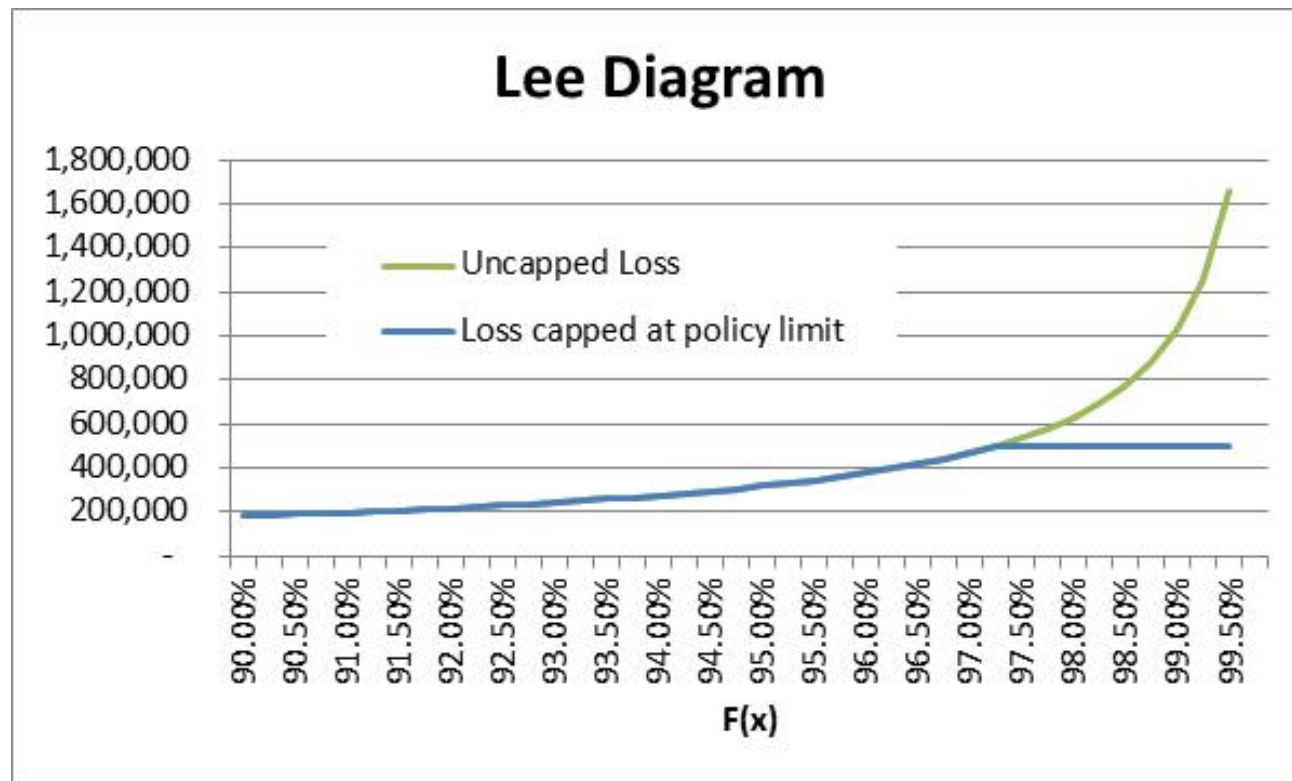


# Traditional model

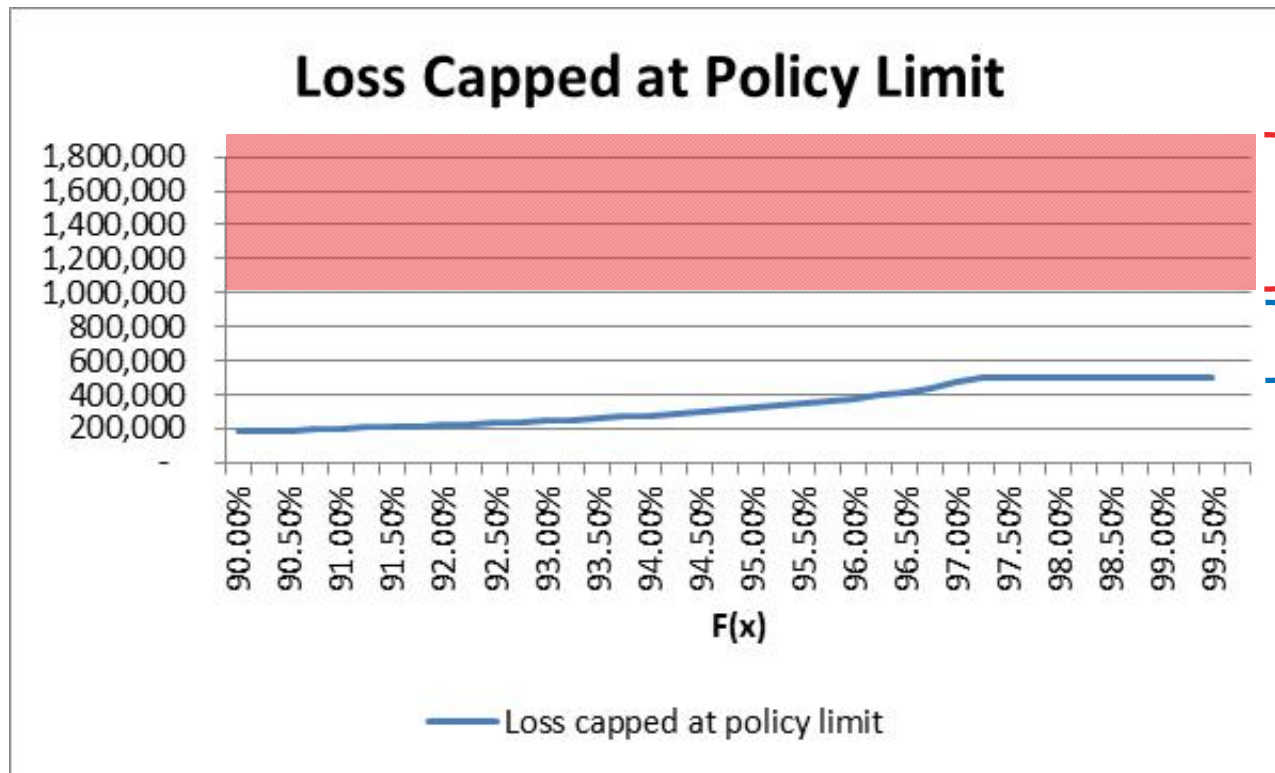
### Loss Capped at Policy Limit



# Traditional model



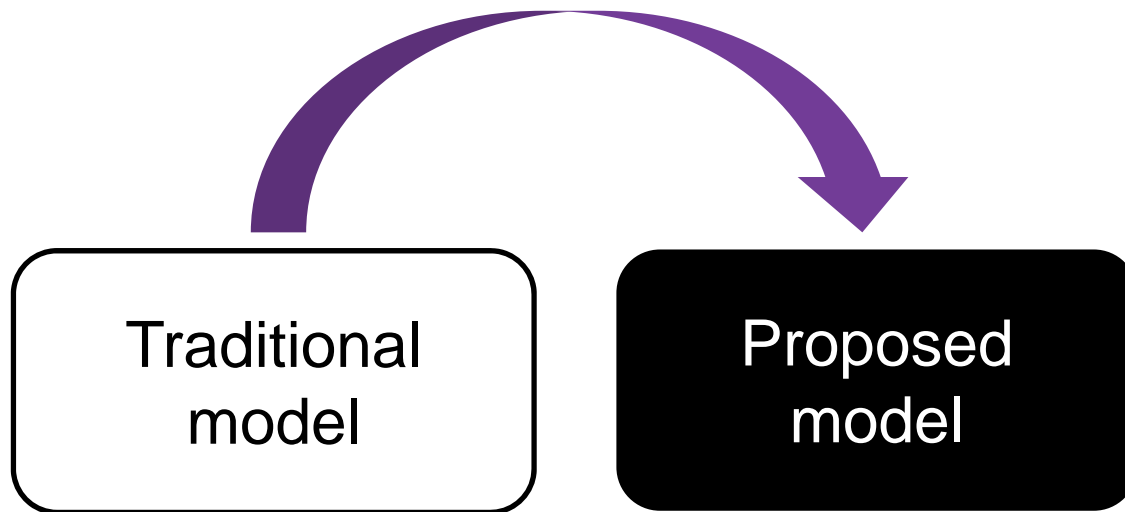
# Traditional model



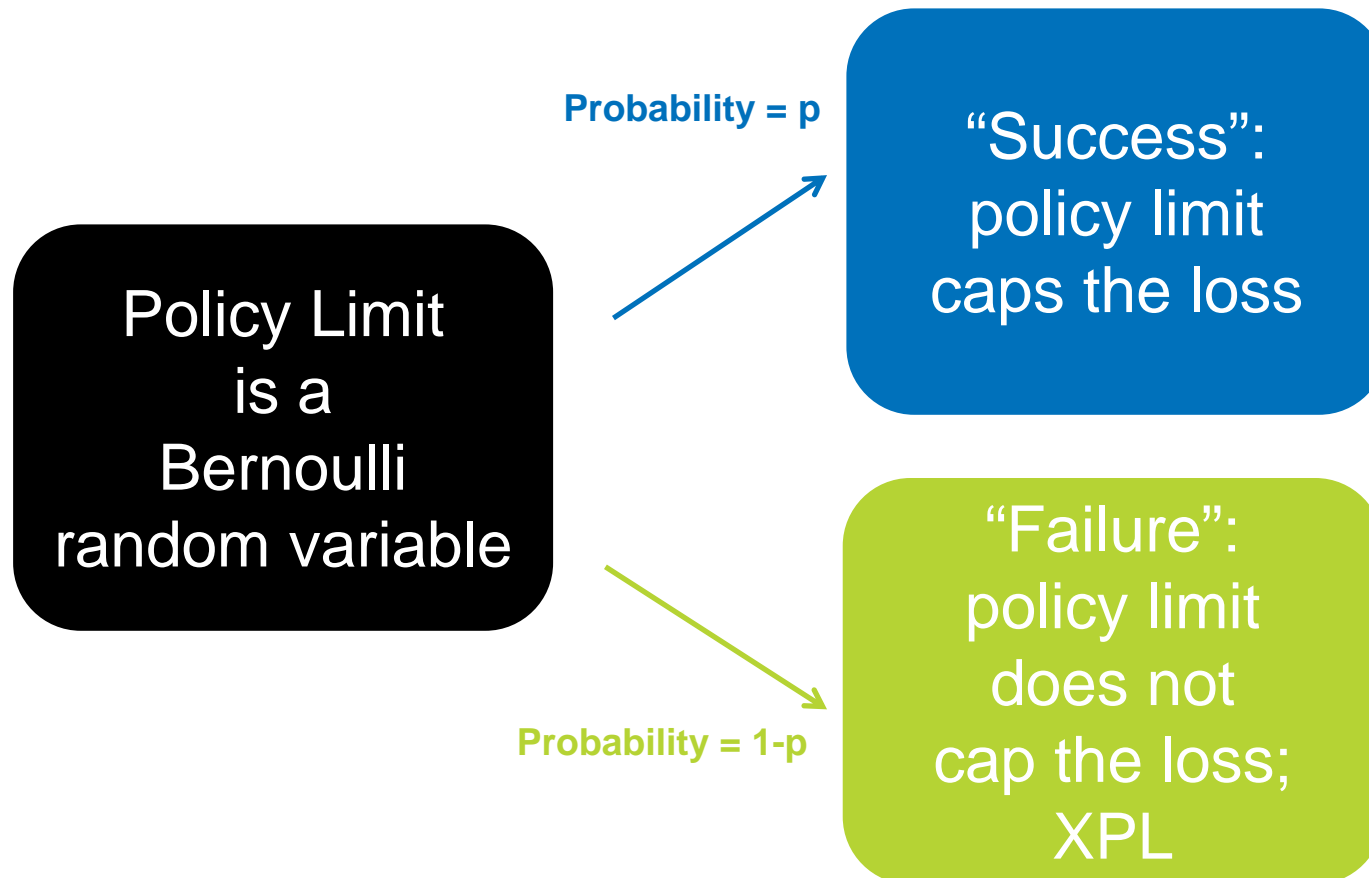
Reinsurance layer

Capping by policy limit suppresses losses from exposing the reinsurance layer

Now, shift your paradigm



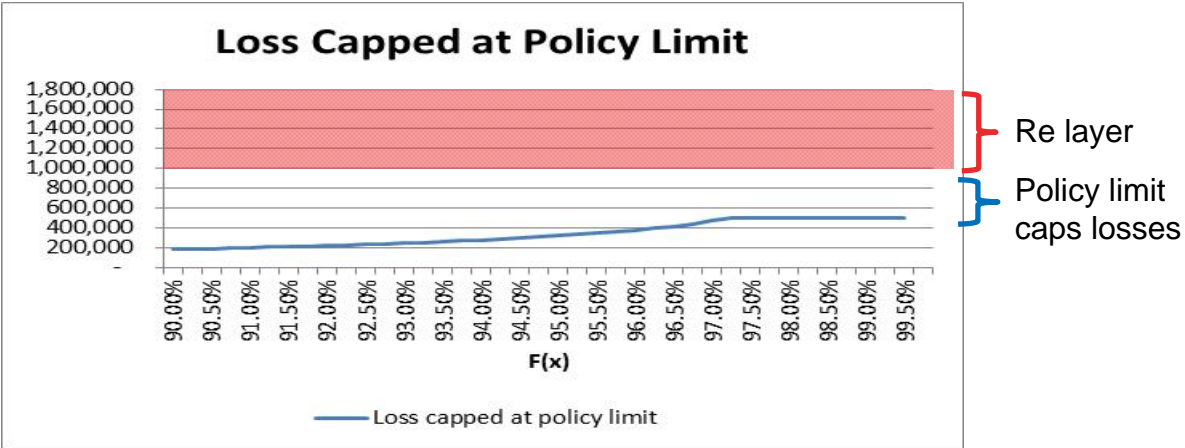
# Proposed model



# Proposed model

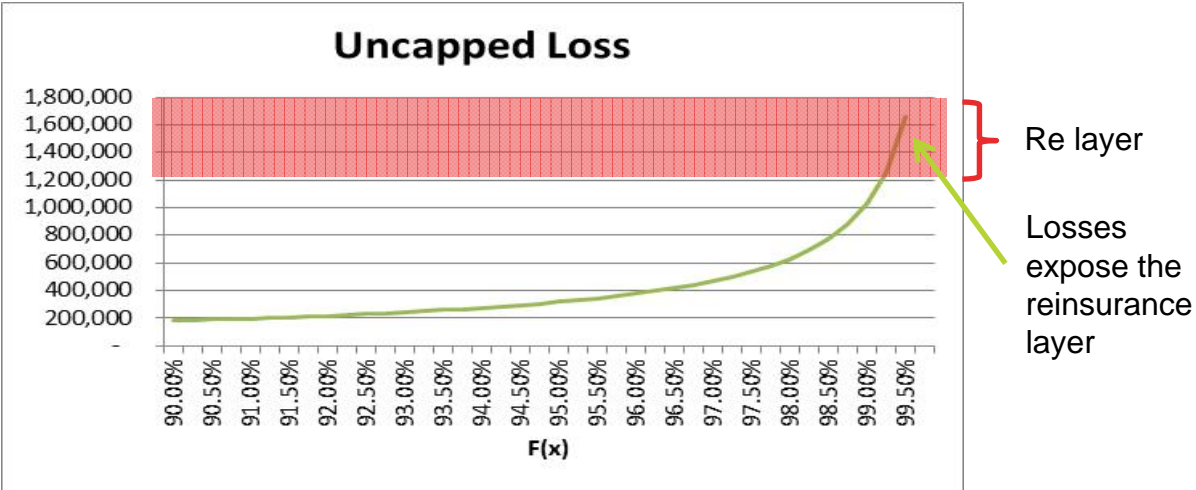
Probability =  $p$

“Success”:  
policy limit  
caps the loss



Probability =  $1-p$

“Failure”:  
policy limit  
does not  
cap the loss;  
XPL



# ANALYSIS AND DISCUSSION

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# Implementation options

- Simulation
- Analytic formula



# Simulation

- Suitable for
  - Cat models
  - Economic capital models
- Not suitable for
  - Reinsurance exposure rating
    - Use analytic formula

# Reinsurance exposure rating

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- Traditional model
  - Use “exposure factor”
  - % exposure in layer =  $\text{layer LEV} / \text{policy LEV}$
  - Layer LEV reflects policy limit capping

# Traditional model

- Use Limited Expected Value (LEV)

$$LEV(X, k) = \int_0^k xf(x)dx + k[1 - F(x)]$$

# Reinsurance exposure rating

- Proposed method:
  - % exposure in layer = layer LEV / policy LEV
  - Layer LEV reflects
    - Probability  $p$ :
      - Success
      - Layer LEV if the policy limit caps the loss
    - Probability  $1-p$ :
      - Failure
      - Layer LEV if the policy limit does not cap loss

# Traditional model

- Use Policy Limited Expected Value (PLEV)

$$PLEV(X, k, Z) = p \left\{ \int_0^k xf(x)dx + k[1 - F(x)] \right\} + (1 - p) \left\{ \int_0^{\infty} xf(x)dx \right\}$$

$$PLEV(X, k, Z) = pLEV(X, k) + (1 - p)E[X]$$

# NUMERICAL EXAMPLE

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# Numerical example

- Severity curve
  - Pareto (2 parameters)
  - Theta = 50,000
  - Alpha = 1.5

Limits Profile		
Limit	% of Premium	ELR
50,000	1.0%	65.0%
100,000	1.0%	65.0%
500,000	2.0%	65.0%
1,000,000	80.0%	65.0%
2,000,000	10.0%	65.0%
3,000,000	1.0%	65.0%
4,000,000	1.0%	65.0%
5,000,000	3.0%	65.0%
10,000,000	1.0%	65.0%

## XPL Modeling Parameters

Amount	Probability of "success" = p	1 - p
50,000	99.0%	1.0%
100,000	99.0%	1.0%
200,000	99.0%	1.0%
300,000	99.0%	1.0%
400,000	99.0%	1.0%
500,000	99.0%	1.0%
600,000	99.0%	1.0%
700,000	99.0%	1.0%
800,000	99.0%	1.0%
900,000	99.0%	1.0%
1,000,000	99.0%	1.0%
2,000,000	99.0%	1.0%
3,000,000	99.0%	1.0%
4,000,000	99.0%	1.0%
5,000,000	99.0%	1.0%
6,000,000	99.0%	1.0%
7,000,000	99.0%	1.0%
8,000,000	99.0%	1.0%
9,000,000	99.0%	1.0%
10,000,000	99.0%	1.0%
25,000,000	100.0%	0.0%

# Output

Exposure Rating Output with XPL Loading						
1	2	3	4	5	6	
			Layer Losses as % of total ground up losses Traditional Exposure Rating	Layer Losses as % of total ground up losses Proposed Method Including XPL	Implied Loading for XPL Proposed / Traditional - 1	
Layer	Limit	Attachment				
1	500,000	-	88.420%	88.440%	0.02%	
2	500,000	500,000	10.067%	10.074%	0.07%	
3	1,000,000	1,000,000	1.150%	1.219%	5.99%	
4	3,000,000	2,000,000	0.333%	0.403%	21.06%	
5	5,000,000	5,000,000	0.031%	0.068%	119.37%	
6	15,000,000	10,000,000	0.000%	0.033%	#N/A	
Total	25,000,000	-	100.000%	100.237%	0.24%	

Proposed method generates layer loss cost including XPL



# CONCLUSION

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# Conclusion

- Treat policy limit as a Bernoulli random variable
  - Suitable for simulation modeling
    - Cat models
    - Economic capital models
  - Suitable for analytic formulas
    - Exposure rating for XOL reinsurance
    - Practical

# Questions & comments?

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- Workbook with implemented formulas is available upon request

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