Optimal Growth for Property and Casualty Insurance Companies

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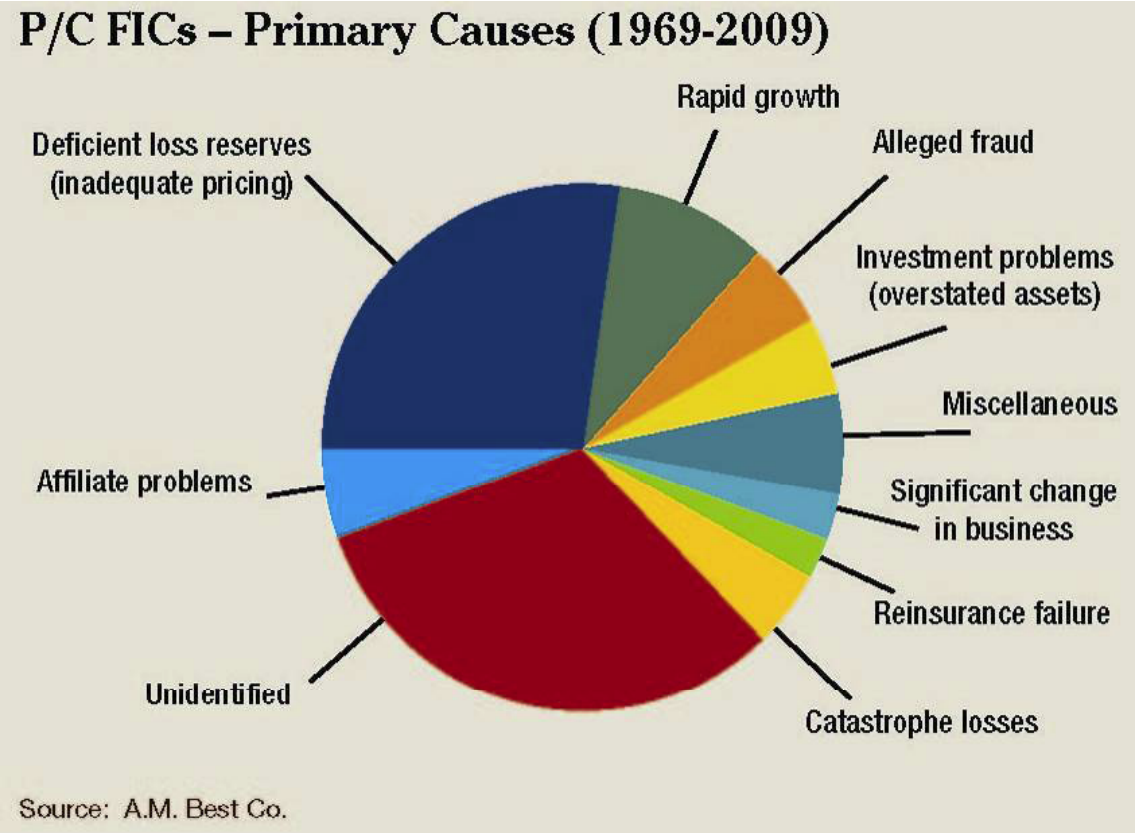
Agenda

- Introduction
- Equilibrium New Business Percentage
- Growth Impact Curve
- Growth Limit Curve
- Constrained Maximum Growth
- Optimal Growth
- Case Study

Introduction

Rapid Growth is one of top causes of financial impairment



Introduction

Conflicts between Growth and Profitability

- Faster growth may reduce profitability
  - Lower price
  - Loose underwriting
  - Attract more NB, NB has higher loss and expense ratios

Introduction

Conflicts between Growth and Profitability

- Aghion and Stein (2008): constraints on management time and other resources
- Harrington, Danzon, and Epstein (2008):insurance companies often sacrifice profit margins by cutting price excessively in the soft market to maintain sales volume
- Ma (2009): profitability will be eroded significantly when a high growth target is achieved by lowering underwriting standards

Introduction

Aging Phenomenon

- D'Arcy and Doherty (1989; 1990): loss ratio improves with policy age
- Cohen(2005): Evidence from personal auto
- Wu and Lin (2009)
  - 8 lines of business, 25 books, $29 billion premium
  - New business has loss ratios 7% (GL) to 18% (BOP) higher than renewal business
  - New business has retentions 3% (personal auto) to 19% (personal home) lower than renewal business

Introduction

D'Arcy and Gorvett (2004) Optimal Growth Paper

- A milestone: first study
- Three-factor econometrics model
  - Market value=a+b*surplus+c*NWP+d*combined ratio
  - 15 companies: b=2.13, c=1.57, d=-23,878,168
  - 14 companies (Excluding AIG): b=1.85, c=0.28, d=-2,076,192
- Run DFA simulations
- Does optimal growth rate exist?
  - Using 14-company parameters: optimal growth = 0%
  - Using 15-company parameters: optimal growth = 10%

Introduction

Practical concerns of applying D'Arcy and Gorvett (2004)

- Data availability: Mutual, reciprocal, subsidiary, and privately-held companies do not have observed market values
- Parameter Risks: Volatile results by including AIG or not
- Complicated DFA simulations: not easy to understand and apply.

Introduction

Improvements from Fu (2012):

- Data availability: traditional actuarial database.
- Parameter Risks: no regression which is subject to volatility of equity market.
- Deterministic: easy to understand and apply.
- Study the conditions for the existence of optimal positive growth.

Disadvantages of Fu (2012): no stochastic insights

- Cannot be analyzed in the classical mean-variance framework of modern financial economics.
- No risk frontier.

Equilibrium New Business Percentage

Required NB percentage to achieve 15% overall growth when 10% of the current book of business consists of NB

Year (t)	RB Exposure (1)	NB Exposure (2)	RB % (3)=(1)/(5)	NB % (4)=(2)/(5)	Exposure (5)=(1)+(2)
1	0.900	0.100	90.0%	10.0%	1.000
2	0.890	0.260	77.4%	22.6%	1.150
3	1.009	0.314	76.3%	23.7%	1.323
4	1.159	0.362	76.2%	23.8%	1.521
5	1.333	0.416	76.2%	23.8%	1.749

Assume RB retention ratio 90% and NB retention 80%
0.89= 0.9*90%+0.1*80%; 0.26=1.15-0.89
1.009=0.89*90%+.26*80%; 0.314=1.15^2-1.009

Equilibrium New Business Percentage

Required NB percentage to achieve 15% overall growth when 25% of the current book of business consists of NB

Year (t)	RB Exposure	NB Exposure	RB %	NB %	Exposure
1	0.750	0.250	75.0%	25.0%	1.000
2	0.875	0.275	76.1%	23.9%	1.150
3	1.008	0.315	76.2%	23.8%	1.323
4	1.159	0.362	76.2%	23.8%	1.521
5	1.333	0.416	76.2%	23.8%	1.749

Equilibrium New Business Percentage

- Percentage of NB exposure converges at 23.8%.
- Notation: Q - exposure; G - growth rate; A - NB percentage
  - Exposure = prior (1+growth): $Q_t = Q_{t-1}(1+G_t)$
  - NB= total * NB percentage: $Q_{n,t} = Q_t A_t = Q_{t-1}(1+G_t)A_t$
  - RB = prior NB Renewal+ prior RB renewal: $Q_{r,t} = Q_{t-1}A_{t-1}R_{n,t} + Q_{t-1}(1-A_{t-1})R_{r,t}$
  - Total = NB+RB: $Q_{t-1}(1+G_t) = Q_{t-1}(1+G_t)A_t + Q_{t-1}A_{t-1}R_{n,t} + Q_{t-1}(1-A_{t-1})R_{r,t}$
- Solving for ENBP:

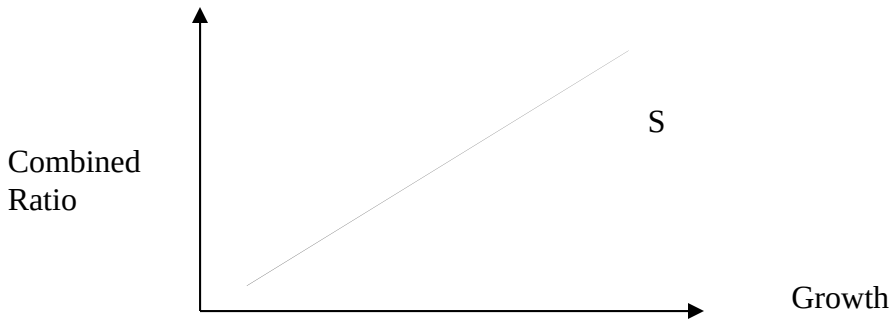
$$A_t = \frac{1+G_t-R_{r,t}}{1+G_t+R_{n,t}-R_{r,t}} = 1-\frac{R_{n,t}}{1+G_t+R_{n,t}-R_{r,t}}$$

Growth Impact Curve

Growth Impact Curve shows the underwriting fact:

- Combined ratio is an increasing function of growth
- Growth reduces underwriting profit margin

$$C_t = A_t C_{n,t} + (1 - A_t) C_{r,t} = \frac{(1 + G_t - R_{r,t}) C_{n,t} + R_{n,t} C_{r,t}}{1 + G_t + R_{n,t} - R_{r,t}}$$



Growth Limit Curve

Surplus capacity can constrain the growth of an insurance company

- Premium-to-surplus ratio
  - Regulator;
  - Rating agency
  - Internal ERM
- To avoid over-leverage, the profit growth after tax and dividend has to keep up the pace with sales growth

Growth Limit Curve

- Surplus constraints on the growth: evidence from academia
  - Davis 1979;
  - Hagstrom 1981;
  - Gron 1994;
  - Winter 1994;
  - Cummins and Danzon 1997;
  - Wang et al. (2011)

Growth Limit Curve

- I is investment, lambda is found-generating coefficient, S is surplus; Investment Asset is:

$$I_t = \lambda * WP_t + S_t$$

- t is tax and Y is investment yield; retained profit after tax and dividend is:

$$\pi_t = [EP_t * (1 - C_t) * (1 - t_u) + I_t Y_t * (1 - t_I)] * (1 - D_t)$$

- To maintain target premium-to-surplus ratio K:

$$\frac{WP_{t+1}}{S_{t+1}} = \frac{WP_t * (1 + G_{t+1})}{S_t + \pi_t} \leq K_t$$

Growth Limit Curve

To maintain a target premium-to-surplus ratio K

- Combined Ratio needs to be below a threshold

$$C_t \le 1 - \frac{WP_t * (1 + G_{t+1}) - K_t * S_t - K_t * I_t * Y_t * (1 - t_I) * (1 - D_t)}{EP_t * (1 - t_u) * (1 - D_t) * K_t}$$

- Or, the growth has to be below a threshold under certain profit level

$$G_{t+1} \le \frac{K_t * (S_t + \pi_t)}{WP_t} - 1$$

Growth Limit Curve

Growth limit curve shows the capital constraint

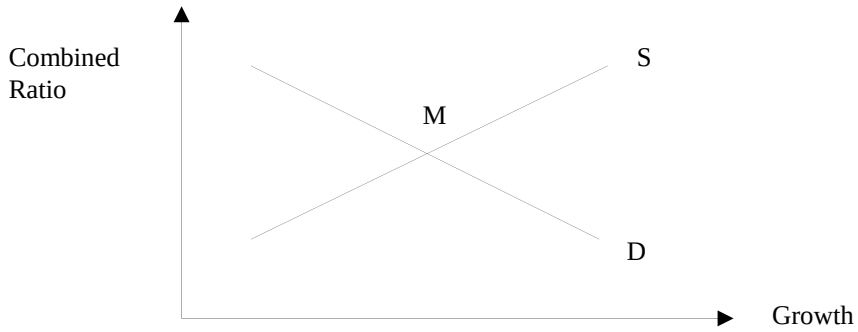
- Faster growth requires lower combined ratio to generate extra capital to support such growth
- Do not cross the line: if the combined ratio is over the curve, premium growth > surplus growth, the leverage ratio will increase and penetrate the “target”.

$$C_t \le 1 - \frac{WP_t * (1+G_{t+1}) - K_t * S_t - K_t * I_t * Y_t * (1-t_I) * (1-D_t)}{EP_t * (1-t_u) * (1-D_t) * K_t}$$

Constrained Maximum Growth

Balance two conflicting goals:

- Growth Impact curve – faster growth drives up combined ratio from the perspective of underwriting performance
- Growth Limit curve – faster growth requires lower combined ratio from the perspective of capital management
- Max growth rate under the capital constraint: the intersection M between two curves.



Optimal Growth

Insurance Company Valuation

- ϕ is expected price-to-book ratio;
- η is the expected price-to-sales ratio
- *w* is weight given to surplus-indicated company value

$$V_{t+n} = W * \phi * S_{t+n} + (1-W) * \eta * WP_{t+n}$$

To maximize the company value

$$\underset{G}{Max} \quad W * \phi * S_{t+n} + (1-W) * \eta * WP_t * (1+G)^n$$

Case Study

Assumptions

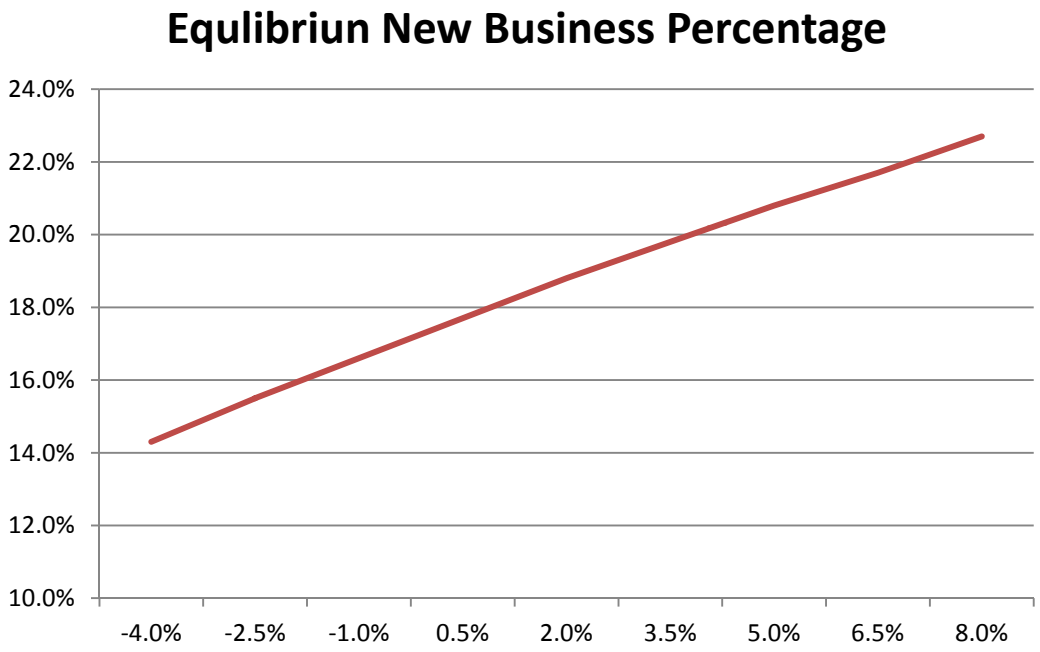
At market price level,

- NB loss ratio is 75%, RB loss ratio is 62%;
- NB retention is 78%, RB retention is 84%;
- NB expense ratio is 37%, RB expense ratio is 32%;

G is the exposure growth rate, R is the retention ratio, dp is the rate difference from market

- $G_t = 2\% - 1.5 * dp_t$, the lower the price, the faster the growth
- $R_{r,t} = 84\% - 0.2 * dp_t$ $R_{n,t} = 78\% - 0.3 * dp_t$, the lower the price, the higher the retention

Case Study



$$A_t = \frac{1 + G_t - R_{r,t}}{1 + G_t + R_{n,t} - R_{r,t}}$$

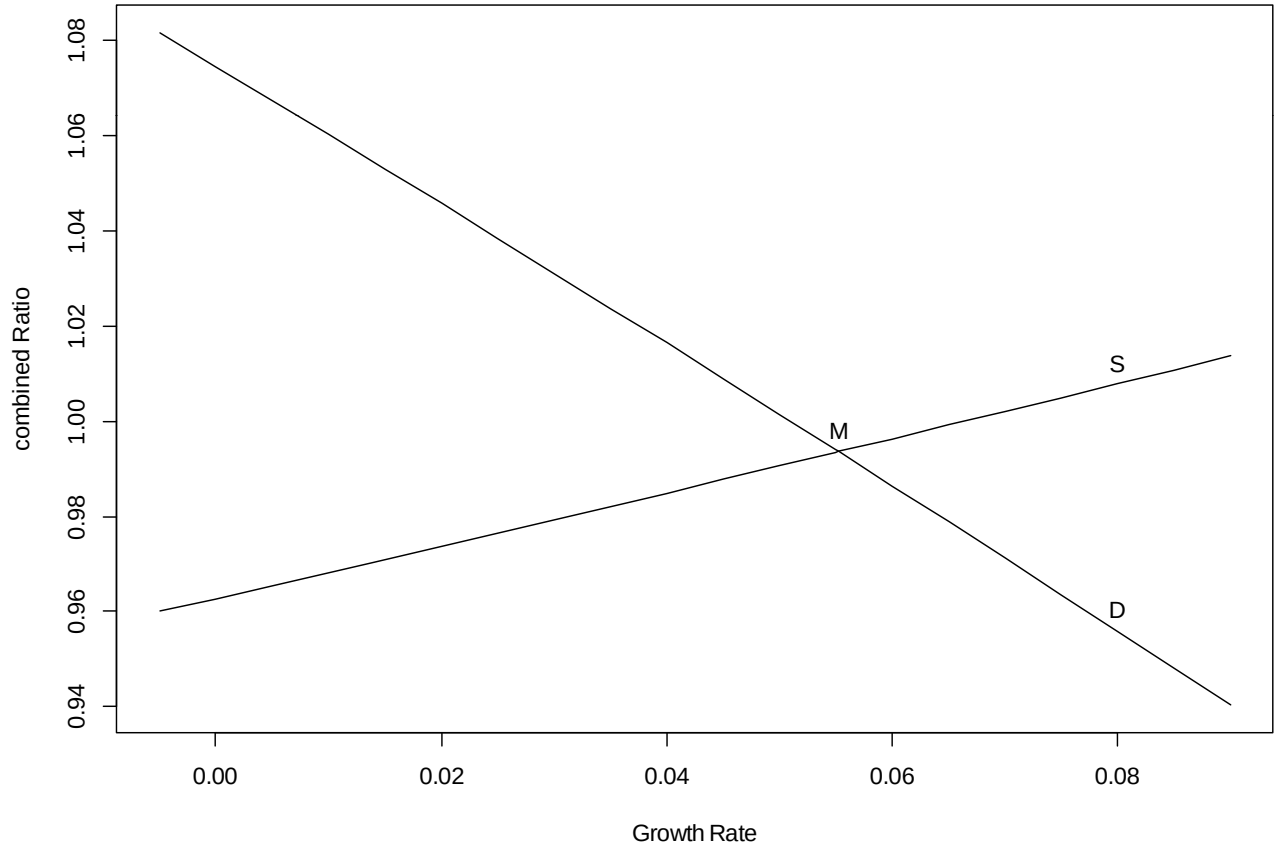
Case Study

Equilibrium NB percentages, loss and combined ratios by growth

Exposure Growth	dp	NB Retention	RB Retention	ENBP	NB LR	RB LR	NB CR	RB CR	whole Book CR
8.0%	-4%	79.2%	84.8%	22.7%	78.1%	64.6%	115.1%	96.6%	100.8%
6.5%	-3%	78.9%	84.6%	21.7%	77.3%	63.9%	114.3%	95.9%	99.9%
5.0%	-2%	78.6%	84.4%	20.8%	76.5%	63.3%	113.5%	95.3%	99.1%
3.5%	-1%	78.3%	84.2%	19.8%	75.8%	62.6%	112.8%	94.6%	98.2%
2.0%	0%	78.0%	84.0%	18.8%	75.0%	62.0%	112.0%	94.0%	97.4%
0.5%	1%	77.7%	83.8%	17.7%	74.3%	61.4%	111.3%	93.4%	96.5%
-1.0%	2%	77.4%	83.6%	16.6%	73.5%	60.8%	110.5%	92.8%	95.7%
-2.5%	3%	77.1%	83.4%	15.5%	72.8%	60.2%	109.8%	92.2%	94.9%
-4.0%	4%	76.8%	83.2%	14.3%	72.1%	59.6%	109.1%	91.6%	94.1%

Case Study

Empirical Growth Impact and Limit Curves
and Constrained Maximum Growth



Case Study

Five-year profits, surplus, and leverage ratios
at constrained maximum growth 5.52%

	Beginning Surplus	WP	EP	Investment	Inv Profit	UW Profit	Total Profit	Tax rate	After-Tax Profit	Payout %	Year	End Surplus	Prem/ Surplus
0	0.667	1.000	0.974	1.867	0.075	0.006	0.081	35%	0.053	30%	0.037	0.703	1.500
1	0.703	1.055	1.028	1.970	0.079	0.007	0.085	35%	0.056	30%	0.039	0.742	1.500
2	0.742	1.114	1.084	2.079	0.083	0.007	0.090	35%	0.059	30%	0.041	0.783	1.500
3	0.783	1.175	1.144	2.193	0.088	0.007	0.095	35%	0.062	30%	0.043	0.827	1.500
4	0.827	1.240	1.207	2.315	0.093	0.008	0.100	35%	0.065	30%	0.046	0.872	1.500
5	0.872	1.308	1.274	2.442	0.098	0.008	0.106	35%	0.069	30%	0.048	0.920	1.500

Assume 4% investment yield, 35% tax rate, 30% dividend payout ratio, 1.2 fund generating coefficient

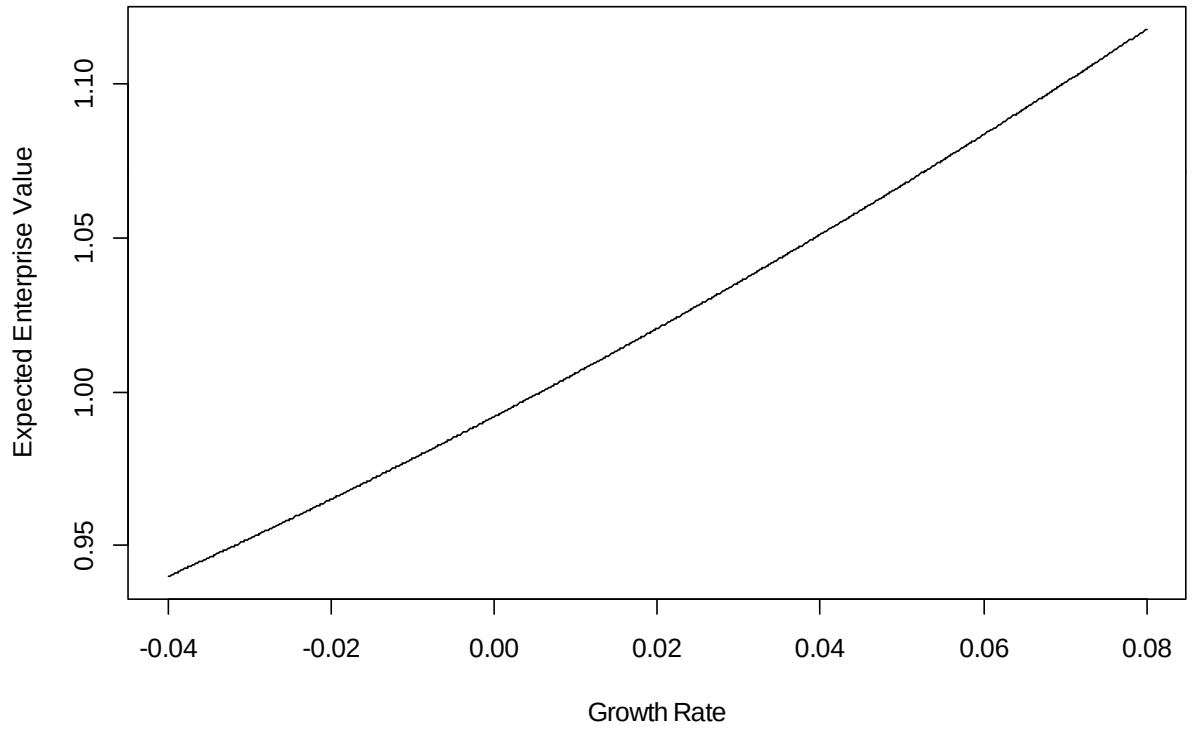
Case Study

Five-year profits, surplus, and leverage ratios
at 8% growth

	Beginning Surplus	WP	EP	Investment	Inv Profit	UW Profit	Total Profit	Tax rate	After-Tax Profit	Payout %	Year	End Surplus	Prem/ Surplus
0	0.667	1.000	0.963	1.867	0.075	-0.008	0.067	35%	0.044	30%	0.031	0.697	1.500
1	0.697	1.080	1.040	1.993	0.080	-0.008	0.072	35%	0.047	30%	0.033	0.730	1.549
2	0.730	1.166	1.123	2.129	0.085	-0.009	0.076	35%	0.050	30%	0.035	0.765	1.598
3	0.765	1.260	1.213	2.276	0.091	-0.010	0.082	35%	0.053	30%	0.037	0.802	1.648
4	0.802	1.360	1.310	2.434	0.097	-0.010	0.087	35%	0.057	30%	0.040	0.841	1.697
5	0.841	1.469	1.415	2.604	0.104	-0.011	0.093	35%	0.061	30%	0.042	0.884	1.747

Case Study

Expected Company Values after 5 years by Growth Rate
When W=50%



Assume price-to-book ratio =1.2, price-to-sales ratio 0.8:

$$\underset{G}{Max} \quad 50\% * 1.2 * S_{t+n} + 50\% * 0.8 * WP_t * (1+G)^5$$

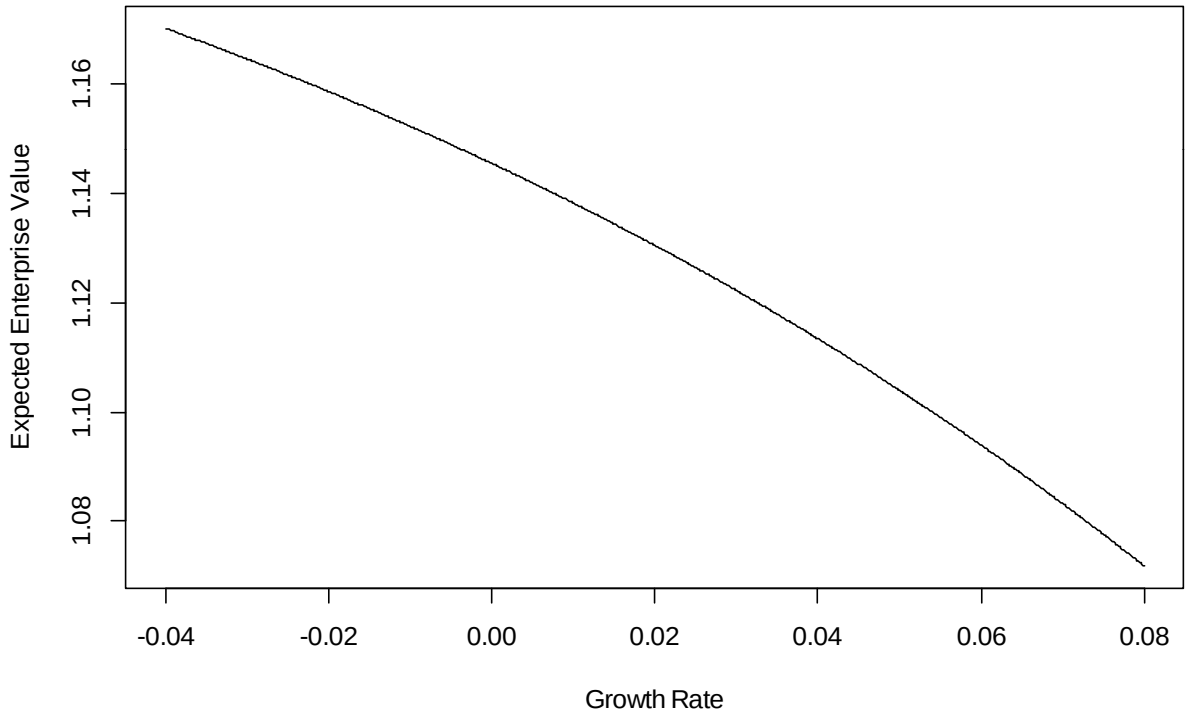
Case Study

When the weight on surplus is 50%

- Growth dominates the surplus (it is easier to grow sales volume than to grow underwriting profit).
- The company will grow as fast as possible if no capital constraint
- The optimal growth is the constrained maximum growth under the leverage constraint on capital.

Case Study

Expected Company Values after 5 years by Growth Rate
When W=90%



$$\underset{G}{Max} \quad 90\% * 1.2 * S_{t+n} + 10\% * 0.8 * WP_t * (1+G)^5$$

Case Study

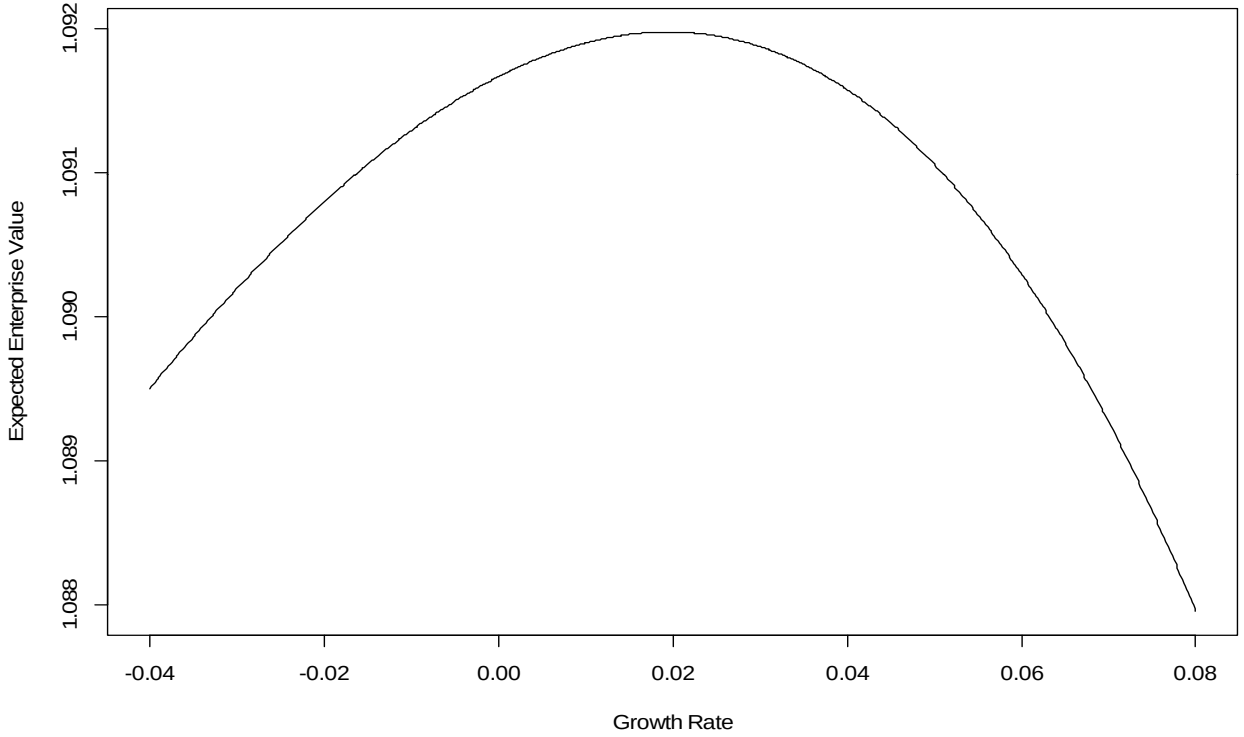
THE CINCINNATI INSURANCE COMPANIES

When the weight on surplus is 90%

- Surplus overweighs the growth.
- The company will not attempt to grow without growth constraints.
  - "If a company is not attempting to grow, its book will gradually ages, so the loss ratio declines. This generates a higher net income in the near future and increase policyholders' surplus"
- The optimal positive growth does not exist.
- This is equivalent to 14-company case in D'Arcy and Gorvett (2004)

Case Study

Expected Company Values after 5 years by Growth Rate
When W=76%



$$\underset{G}{Max} \quad 76\% * 1.2 * S_{t+n} + 24\% * 0.8 * WP_t * (1+G)^5$$

Case Study

When 74% <W<78%:

- There is a balance between surplus and growth.
- The expected company value is a bell curve of growth.
- The optimal positive growth exists.
- This is equivalent to 15-company case in D'Arcy and Gorvett (2004)