


**CP-1: Beyond the Cost Model: Understanding Price Elasticity and Its Applications**

**CAS 2013 Ratemaking and Product Management**

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**Agenda**

- › What is elasticity?
- › What affects elasticity?
- › How to model elasticity?

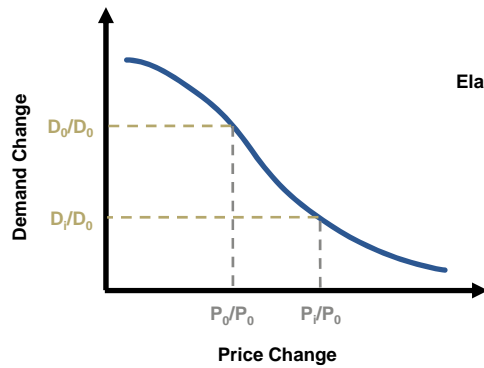
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## What is Elasticity?

Elasticity defined

- Rate of response in quantity demanded given a specified change in price



$$\text{Elasticity} = \frac{\% \text{ Change in Demand}}{\% \text{ Change in Price}}$$

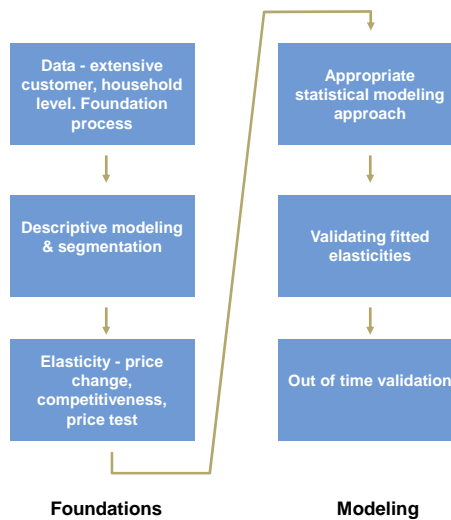
$$= \frac{D_1/D_0 - D_0/D_0}{P_1/P_0 - P_0/P_0}$$

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3

## Elasticity modeling - a rigorous approach



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## How to model elasticity?

Modeling tools

### • Classification and Regression Trees

- Interpolate missing data
- Identify initial main effects
- Identify key segments for models
- Identify complex interactions

### Generalized Linear Models

- Parameterizes model structure
- Complex interaction strategies
- Issues
  - Possible "negative" elasticity

### Generalized Non Linear Models

- Interacts price factors with all non price factors
- Non linear element forces positive elasticity
- Issues
  - Tendency to overfit
  - Ignores real "negative" elasticity

➤ Modeling requires flexibility in choosing the right strategy for the right data set

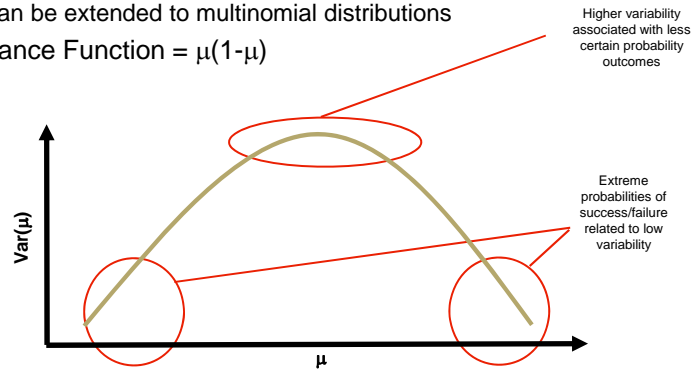
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## Distribution Function

- Binomial
  - Basic functional form in decision modeling
  - Belongs to the exponential family of distributions
  - Can be extended to multinomial distributions
- Variance Function =  $\mu(1-\mu)$



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## Link Function

- Recall the following basic model form

$$\hat{Y} = \mu = h(X\beta)$$

Link function (g=h<sup>-1</sup>)  
Links random and systematic component  
↓

- Standard link functions used in loss cost models

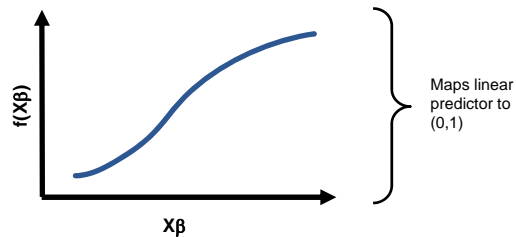
- Multiplicative:  $\exp(X\beta)$
- Identity:  $X\beta$
- Reciprocal:  $1/(X\beta)$

## Link Function

- Logit link function used in binomial models:

$$\frac{1}{1 + \frac{1}{\exp(X\beta)}}$$

- Properties of the logit link function:



- S-shape curve “traps” the predictive value to the probability range

## What do we mean by “price elasticity”?

- Most people define elasticity as ....
  - **Percentage change in demand / percentage change in price**
  - “Classical elasticity”
  - Definition found in economics textbooks
- But sometimes ...
  - **Absolute change in linear predictor / percentage change in price**
  - “Linear predictor elasticity”
  - Doesn't vary with demand

## Price Elasticity Definitions

- Classical

$$Demand_1 = \frac{1}{1 + \frac{1}{\exp(\beta_0 + \alpha_1 \times \frac{P_1}{P_0})}}$$

$$Demand_2 = \frac{1}{1 + \frac{1}{\exp(\beta_0 + \alpha_1 \times \frac{P_2}{P_0})}}$$

$$Classical = \frac{\frac{Demand_2 - Demand_1}{Demand_1}}{\frac{P_2 - P_1}{P_1}}$$

## Price Elasticity Definitions

- Linear Predictor

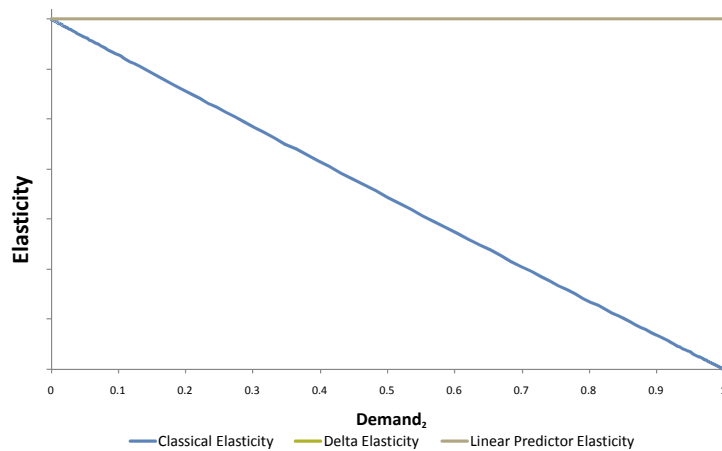
$$Demand_1 = \frac{1}{1 + \frac{1}{\exp(\beta_0 + \alpha_1 \times \frac{P_1}{P_0})}}$$

$$Demand_2 = \frac{1}{1 + \frac{1}{\exp(\beta_0 + \alpha_1 \times \frac{P_2}{P_0})}}$$

$$Linear = \frac{\beta_0 + \alpha_1 \times \frac{P_2}{P_0} - \beta_0 + \alpha_1 \times \frac{P_1}{P_0}}{\frac{P_2 - P_1}{P_1}} = \alpha_1$$

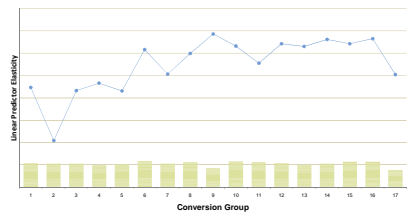
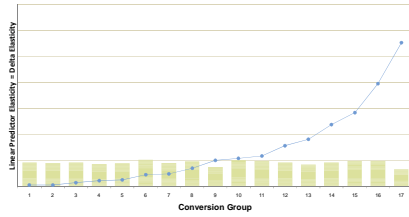
## What does logit imply about elasticity?

- If there are **no interactions with price change factors**



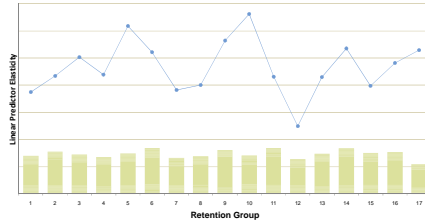
## The case for logistic GLMs

Normal



Probit

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Logistic

13

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## How to model elasticity?

Generalized Non-Linear Models

- Allow models to be fitted where the linear predictor is not a linear combination of factors

Generalized Linear Models

$$y = \frac{1}{1 + \exp(-X \beta_{segments} + \Delta P \beta_{\Delta P})} + error$$

ΔP can either be represented by a categorical factor or by a curve

Generalized Non-Linear Models

$$y = \frac{1}{1 + \exp(-X \beta + \Delta P e^{Zz})} + error$$

Forces elasticity to be positive

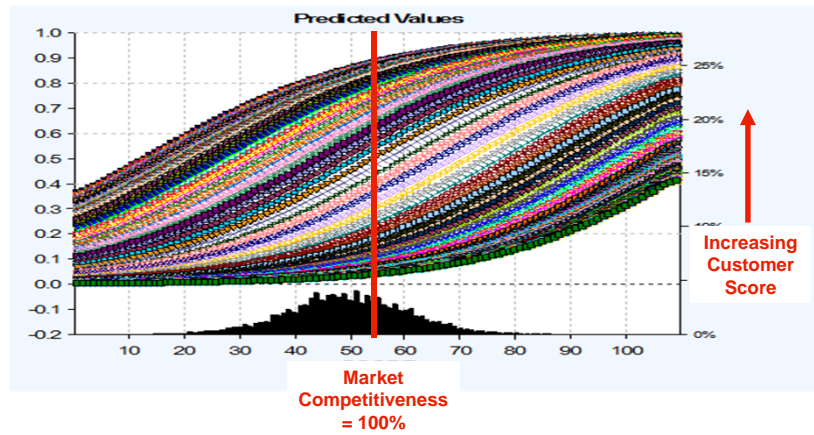
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### Competitive Demand: US Auto New Business

- Non price parameters compiled into a customer score interacted with competitive ratio

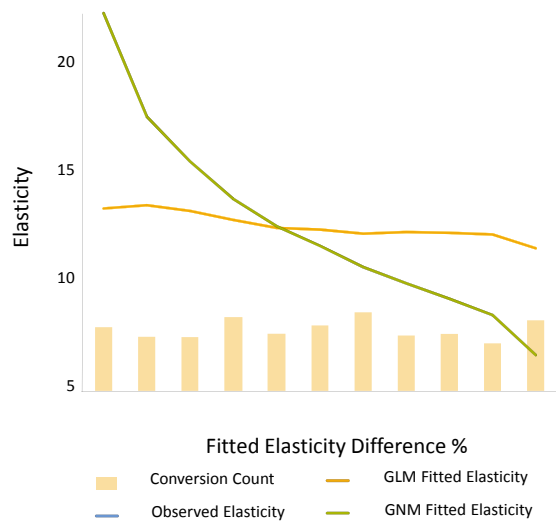


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### New business – linear vs. non-linear



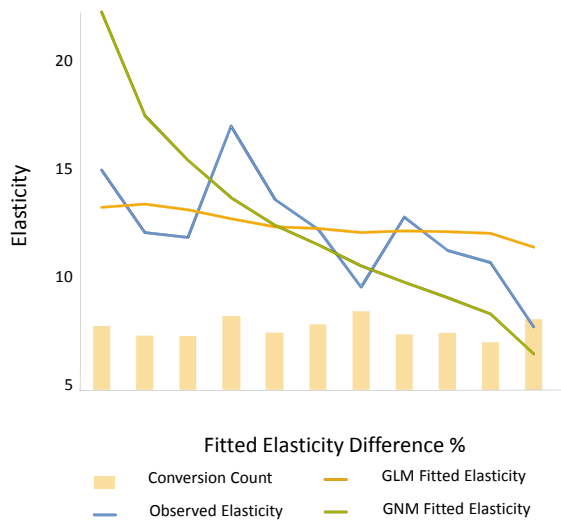
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### New business – linear vs. non-linear

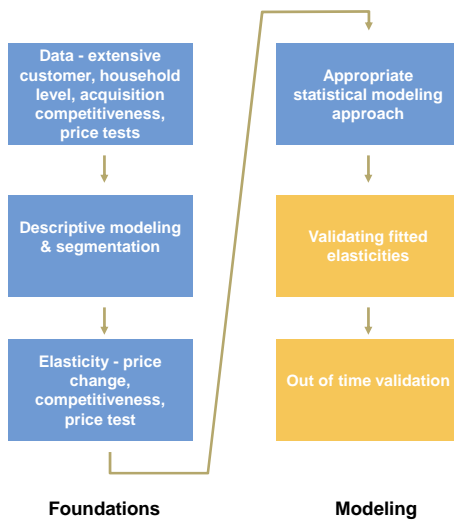


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### Elasticity modeling - a rigorous approach

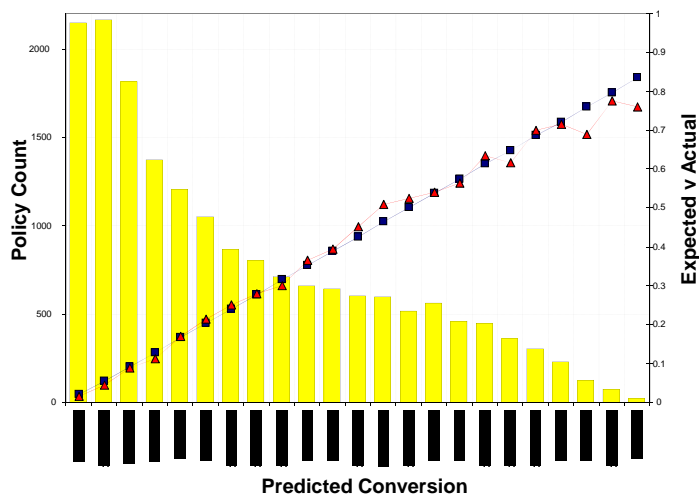


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### New business - out of time validation



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