CAS Ratemaking and Product Management Seminar - March 2013

CP-2. Catastrophe Pricing : Making Sense of the Alternatives

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- There may be a quiz at the end – take notes!

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CAT Pricing Overview

- CAT Loss Simulation Software
  - Generate thousands of simulated years of results
- Now What?
  - Easy to compute expected CAT Loss
  - What about risk load?
- Risk Load based on RORAC
- Required capital?
  - Standalone vs portfolio
  - Incremental vs Allocation
  - Tail vs Adverse vs All loss scenarios
- Understanding the Alternatives
  - TVaR, Incremental VaR,
  - Co-Var, Co-TVaR
  - Order Independence and Coherence?
  - De-worsification?

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CAT Result Basics

Event Loss Table
Event Exceeding Probability Calculation
Simulated years
AEP and OEP TVaR Calculations

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Event Loss Table

Event Rank	Peril	Region	Annual Prob	Event Return period	Risk A Loss	Risk B Loss	Risk C Loss	…	Total Portfolio Loss
1	EQ	CA	0.021%	4,762	300	1,200	0	…	125,000
2	EQ	CA	0.040%	2,500	0	1,000	0	…	100,000
3	HU	FLA	0.080%	1,250	0	0	3,000	…	90,000
4	EQ	CA	0.070%	1,429	900	400	0	…	80,000
5	HU	LA	0.045%	2,222	0	0	2,100	…	75,000
6	EQ	CA	0.055%	1,818	700	0	700	…	70,000
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
998	HU	NC	0.015%	6,667	0	2	0	…	2
999	HU	FL	0.400%	250	0	2	1	…	2
1,000	HU	SC	0.200%	500	0	1	0	…	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	
4,998	EQ	NM	0.100%	1,000	0	0	0	…	0
4,999	HU	FLA	0.400%	250	0	0	0	…	0
5,000	EQ	AK	0.500%	200	0	0	0	…	0

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Portfolio Event Exceeding Probability Table

k Event Rank	Peril	Region	p(k) Annual Prob	Event Return period	EP(k) Exceeding Probability	Portfolio EP Return Period	Portfolio Event Loss
1	EQ	CA	0.021%	4,762	0.021%	4,762	125,000
2	EQ	CA	0.040%	2,500	0.061%	1,640	100,000
3	HU	FLA	0.080%	1,250	0.141%	710	90,000
4	EQ	CA	0.070%	1,429	0.211%	474	80,000
5	HU	LA	0.045%	2,222	0.256%	391	75,000
6	EQ	CA	0.055%	1,818	0.311%	322	70,000
⋮	⋮	⋮	⋮				
998	HU	NC	0.015%	6,667	24.000%	4	2
999	HU	FL	0.400%	250	24.304%	4	2
1,000	HU	SC	0.200%	500	24.455%	4	1
⋮	⋮	⋮	⋮				
4,998	EQ	NM	0.100%	1,000	83.000%	1	-
4,999	HU	FLA	0.400%	250	83.068%	1	-
5,000	EQ	AK	0.500%	200	83.153%	1	-

Exceeding Probability and Return Period

- $EP(k+1) = EP(k) + p(k+1)\big(1 - EP(k)\big)$
- EP(k) = Probability that over one year there will be a loss bigger than or equal to the k^{th} largest loss in the event loss table
- Return period = 1/EP(k)
- The event associated with the 100 year return period has annual probability, p(k), less than 1/100

Simulation Trials

Trial Year	Event 1	Event 2	Event 3	…		Largest Event over the Year	Total Annual Loss
1	40,000	-	-	-		40,000	40,000
2	1	3,500	9	-	-	3,500	3,510
3	-	-	-			0	0
4	10	27,550	-			27,550	27,560
5	700	400	50			700	1,150
6	1,250	4	25			1,250	1,279
7	-	-	-			0	0
8	75	45	70,000			70,000	70,120
9	-	-	-			0	0
10	15	3,500	45			3,500	3,560
⋮	.	.	.			⋮	⋮
9,998	2	-	-			2	2
9,999	550	7,750	-			7,750	8,300
10,000	650	-	-			650	650

Annual Loss Rank Ordered Simulation Trials

Trial Year Rank	Ranking based on total annual loss	Largest Event	Total Annual Loss
1		125,000	175,000
2		125,000	170,000
3		90,000	155,000
4		100,000	137,500
5		100,000	135,000
6		100,000	130,000
7		90,000	125,000
8		90,000	115,000
9		100,000	105,000
10		90,000	102,500
:		:	:
99	100/10000 = 1.0%	21,250	37,500
100	100 year return period	21,000	36,675
101	AEP VaR = 36,675	35,000	35,950
:		:	:
9,998		-	0
9,999		-	0
10,000		-	0

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Largest Event Rank Ordered Simulation Trials

Trial Year Rank	Ranking based on largest event loss	Largest Event	Total Annual Loss
1		125,000	175,000
2		125,000	170,000
3		100,000	137,500
4		100,000	135,000
5		100,000	130,000
6		100,000	100,000
7		95,000	97,500
8		92,500	102,000
9		90,000	155,000
10		90,000	125,000
:		:	:
99	100/10000 = 1.0%	35,125	35,250
100	100 year return period	35,000	35,950
101	OEP VaR = 35,000	35,000	35,125
:		:	:
9,998		-	0
9,999		-	0
10,000		-	0

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Premium, Risk Measures, and Required Capital

Basic Equations
Basic Properties
Coherence
Three Paradigms
Portfolio Dependent Methods

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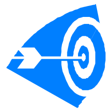
Basic Equations

- P= E[X]+ RL(X)

  P = Indicated premium prior to expense loading

  X = CAT Loss

  RL(X) = Risk Load
- RL(X) = r_{target}*C(X)
- C(X) = Required Capital
- RORAC Approach used by most everyone in actual CAT Treaty pricing
- CAPM not used
  - since CATs independent of stock market, CAPM risk load should be zero ?



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Premium – Basic Properties

1. ***Monotonic: If $X_1 \leq X_2$, then $P(X_1) \leq P(X_2)$***
2. ***Pure: If $X \equiv \alpha$ then P(X) =E[X]***
3. ***Bounded: If X ≤ k, then P(X) ≤ k***
4. ***Continuous (Stable): P(X) is continuous***
   - small changes in X do not cause large changes in P(X)

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Premium – Coherence Properties

1. ***Scalable: $P(\lambda X) = \lambda \cdot P(X)$***
2. ***Translation Invariant: $P(X+\alpha) = P(X) + \alpha$***

   when $0 \leq \alpha$.
3. ***Subadditive: $P(X_1 + X_2) \leq P(X_1) + P(X_2)$***

   A failure of subadditivity means there is consolidation penalty instead of a benefit

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Risk Measure

- A **risk measure**, ρ, maps a real-valued random variable, X, to a non-negative number, ρ (X)
- Risk Measure Basic Properties

1. ***Monotonic:***
   If $X_1 \leq X_2$, then $E[X_1]+\rho(X_1) \leq E[X_2]+\rho(X_2)$
2. ***Pure: If $X \equiv \alpha$ then $\rho(X) = 0$***
3. ***Bounded: If $X \leq k$, then $\rho(X) \leq k$***
4. ***Continuous (Stable): $\rho(X)$ is continuous***
   1. small changes in X do not cause large changes in $\rho(X)$

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Risk Measure – Coherence Properties

1. ***Scalable***: $\rho(\lambda X) = \lambda \cdot \rho(X)$
2. ***Translation Invariant***: $\rho(X+\alpha) = \rho(X)$ *when* $0 \leq \alpha$.
3. ***Subadditive***: $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$

A failure of subadditivity means there is consolidation penalty instead of a benefit

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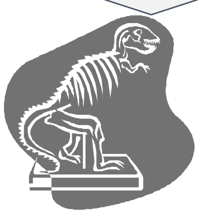
What is the right way to compute Required CAT Capital?



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Required Capital Paradigms

Standalone: C(X) = ρ (X)
ρ(X) is a risk measure.

Portfolio Incremental: C(X) = C(X|R)
= ρ(R+X) - ρ(R)

Portfolio Allocation C(X) =C(X|R)
= A(X,R) *ρ(R+X)

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Portfolio Dependent Capital Properties

- Standalone Capital Cap
  - Portfolio dependent capital ≤ Standalone capital
- Automatic Calibration
  - $\sum C(X|R) = C(R)$
- Order Dependent
  - Required capital for an account may depend on the order in which it was written or renewed.
  - Portfolio optimization difficulties: getting rid of the account that used the most order dependent capital may not reduce portfolio capital very much.

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Risk Measure, Required Capital and Risk Load

- Risk measures properties can be translated into properties of required capital algorithms.
  - Example: C(X) is scalable if $C(\lambda X) = \lambda \cdot C(X)$
- Risk measure properties can also be translated into properties of risk loads and can be used to define properties of indicated premiums
- Be clear as needed about whether risk measures, required capital algorithms, or risk load calculations are being discussed.
  - Example: C(X) = TVaR(X) is required capital,
    RL(X) = 10%· TVaR(X) is risk load

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Required Capital and Risk Measure Alternatives

Alternatives
Discrete definitions

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Risk Measures: Variance and Stnd Dev

- **Variance**: $Var(X) = E[(X-\mu)^2]$
- **Semivariance**: $Var^{+}(X) = E[(X-\mu)^2 \mid X \geq \mu] * Prob(X \geq \mu)$
- **Standard Deviation**: $\sigma = Var^{1/2}(X)$
- **Semi Standard Deviation**: $\sigma^{+} = Var^{+\,1/2}(X)$

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Risk Measures: VaR, TVaR

- **Value at Risk**: $VaR(\theta) = \sup\{x \mid F(x) \leq \theta\}$
- **Excess Value at Risk**: $XVaR(\theta) = Var(\theta) - \mu$
- **Tail Value at Risk**: TVaR(θ) = conditional mean of x values in the tail, 1 - θ, of probability
- **Excess Tail Value at Risk**: $XTVaR(\theta) = TVaR(\theta) - \mu$

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Risk Measures: Distortion

Distortion Risk Measure: $E^*[X] = E[X^*]$ where $F^*(x) = g(F(X))$
where g is a distortion function

Excess Distortion Risk Measure: $E^*[X] - E[X]$

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Variance and Stnd Dev Example

Statistic	Value	Statistic	Value
Trials	10	Variance	166.4
Average	10.0	Standard Dev	12.9
		Semivariance	122.0
		SemiStnd Dev	11.0

Ordered Loss Data			
Rank	Loss	Variance Contribution	Semivariance Contribution
1	40.0	900	900
2	26.0	256	256
3	18.0	64	64
4	6.0	16	0
5	4.0	36	0
6	2.0	64	0
7	2.0	64	0
8	2.0	64	0
9	0.0	100	0
10	0.0	100	0

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VaR and TVaR Example

Statistic	Value	Statistic	Value
Trials	10	Rank for VaR	3.0
Average	10.0	VaR	18.0
Percentage	70.00%	TVaR	28.0
		XTVaR	18.0

Ordered Loss Data			
Rank	Loss	VaR Percentage	Conditional Tail Avg
1	40.0	90%	40.0
2	26.0	80%	33.0
3	18.0	70%	28.0
4	6.0	60%	22.5
5	4.0	50%	18.8
6	2.0	40%	16.0
7	2.0	30%	14.0
8	2.0	20%	12.5
9	0.0	10%	11.1
10	0.0	0%	10.0

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Wang Shift Example

Statistic	Value	Statistic	Value
Trials	10	Wang Shift Parameter	0.500
Average	10.0	Transformed Mean	16.3
Percentage	n/a	XS Transformed Mean	6.3

Ordered Loss Data						
Rank	Loss	Empirical CDF	Normal Inv	Shifted	Trnsfrmd CDF	Trnsfrmd Density
1	40.0	100.0%			100.0%	21.7%
2	26.0	90.0%	1.28	0.78	78.3%	14.9%
3	18.0	80.0%	0.84	0.34	63.4%	12.4%
4	6.0	70.0%	0.52	0.02	51.0%	10.7%
5	4.0	60.0%	0.25	-0.25	40.3%	9.4%
6	2.0	50.0%	0.00	-0.50	30.9%	8.3%
7	2.0	40.0%	-0.25	-0.75	22.6%	7.3%
8	2.0	30.0%	-0.52	-1.02	15.3%	6.3%
9	0.0	20.0%	-0.84	-1.34	9.0%	5.2%
10	0.0	10.0%	-1.28	-1.78	3.7%	3.7%

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Ranking Definition of VaR and TVaR

- Let $X_1 \geq X_2 \ldots \geq X_n$ *be an ordering of n trials of X*
- *Suppose* $k = (1 - \theta)n$, *then*

$$VaR(\theta) = X_k$$

$$TVaR(\theta) = \frac{1}{k}\sum_{j=1}^{k} X_j$$

- Note TVaR is <u>not</u> necessarily equal to the Conditional Tail Expectation (CTE) when the data is discrete.

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TVaR and CTE are Not the Same!

- CTE = Conditional Tail Expectation for **<u>points</u>** larger than the corresponding VaR
- *CTE(θ) = E[X|X>VaR(θ)] {or E[X|X≥VaR(θ)]}*
  - When there are mass points, the CTE may not necessarily capture the exact (1- θ) tail of probability
- TVaR is defined as the average of x values over the (1 – θ) tail of probability

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Example: TVaR and CTE are not the same

Statistic	Value
Trials	10
Pct	50%
Rank	5

Results	A	Ref	A+Ref
Mean	2.80	26.00	28.80
VaR	2.00	33.00	34.00
TVaR	5.00	34.80	35.40
CTE (>)	5.75	36.00	35.75
CTE (≥)	4.50	34.80	35.40

Loss Data by Trial				Separately Ordered Loss Data			
Trial	A	Ref	A+Ref	Rank	A	Ref	A+Ref
1	8	12	20	1	8	37	37
2	0	37	37	2	7	36	36
3	0	36	36	3	4	35	35
4	0	35	35	4	4	33	35
5	1	33	34	5	2	33	34
6	2	17	19	6	2	27	31
7	7	16	23	7	1	17	23
8	2	33	35	8	0	16	20
9	4	27	31	9	0	14	19
10	4	14	18	10	0	12	18

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Incoherent, Impure, Non-monotonic, Uncalibrated, and Unstable

- What is: "The five most common phrases used by your friends to describe you"?
- Some required capital formulas fail coherence
  - Variance and Incremental VaR are not scalable
  - VaR is not subadditive
- Some are impure including VaR and TVaR
- CTE non-monotonic with ">" or "≥" definition
- Most incremental formulas need calibration
- Co-VaR is not stable

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Incremental VaR not scalable: A

Statistic	Value		Mean	VaR
Trials	10	Risk A Standalone	10.00	11.00
Percentage	50.00%	Reference Portfolio	100.00	96.00
Rank	5	Sum	110.00	107.00
		Combined Portfolio	110.00	105.00
		Incremental VaR for A		9.00

Loss Data by Trial				Separately Ordered Loss Data			
Trial	A	Ref	A+Ref	Rank	A	Ref	A+Ref
1	11	52	63	1	28	148	149
2	1	148	149	2	20	140	144
3	0	140	140	3	16	128	140
4	0	128	128	4	13	124	128
5	4	96	100	5	11	96	105
6	28	68	96	6	7	92	100
7	16	64	80	7	4	88	96
8	20	124	144	8	1	68	95
9	7	88	95	9	0	64	80
10	13	92	105	10	0	52	63

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Incremental VaR not scalable: 2*A

Statistic	Value		Mean	VaR
Trials	10	Risk 2A Standalone	20.00	22.00
Percentage	50.00%	Reference Portfolio	100.00	96.00
Rank	5	Sum	120.00	118.00
		Combined Portfolio	120.00	124.00
		Incremental VaR for 2A		28.00

Loss Data by Trial				Separately Ordered Loss Data			
Trial	2A	Ref	2A+Ref	Rank	2A	Ref	2A+Ref
1	22	52	74	1	56	148	164
2	2	148	150	2	40	140	150
3	0	140	140	3	32	128	140
4	0	128	128	4	26	124	128
5	8	96	104	5	22	96	124
6	56	68	124	6	14	92	118
7	32	64	96	7	8	88	104
8	40	124	164	8	2	68	102
9	14	88	102	9	0	64	96
10	26	92	118	10	0	52	74

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VaR Subadditivity-Epic Fail

Statistic	Value		Mean	VaR
Trials	10	Risk A	10	6
Percentage	50.00%	Reference Portfolio	100	124
Rank	5	Sum	110	130
		Combined Portfolio	110	148
		Consolidation Benefit	0	-18
		Incremental VaR for A		24

Loss Data by Trial				Separately Ordered Loss Data			
Trial	A	Ref	A+Ref	Rank	A	Ref	A+Ref
1	6	40	46	1	26	148	170
2	0	148	148	2	24	144	154
3	26	144	170	3	18	140	150
4	14	140	154	4	14	132	148
5	18	132	150	5	6	124	148
6	4	68	72	6	6	92	94
7	0	64	64	7	4	68	72
8	24	124	148	8	2	64	64
9	2	92	94	9	0	48	54
10	6	48	54	10	0	40	46

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Real Allocation Advantages

- Automatically calibrated (in equilibrium)
- Not order dependent if allocation method is not order dependent
- Easier to compare accounts

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Real Allocation Approaches

1. *Stand-alone Risk Measure as Allocation Base*
2. *Marginal Risk Measure as Allocation Base*
   - *Adjusted for Order Dependence (Mango)*
3. *Game theory –(LeMaire) Allocation of Portfolio Consolidation Benefit*
4. *Co-Measures – (Kreps)*
5. *Percentile Allocation (Bodoff)*

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Tail Focused Co-Measures

- Intuitive Appeal on First Look
  - Automatically calibrated
  - Focused on the tail events that consume capital
  - Penalizes accounts to the extent they contribute to severe portfolio hits
- On Closer Inspection
  - Some co-measures are unstable: co-VaR
  - Coherence not inherited: co-TVaR not subadditive

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Co-VaR Instability

- The 100 year return period Co-Var for A is $20
  - Slight portfolio change or new simulation could make it $0

Rank	VaR Percentage	Portfolio Loss	Risk A Loss
1			
98	99.02%	$422	$6
99	99.01%	$408	$0
100	99.00%	$405	$20
101	98.99%	$395	$0
102	98.98%	$390	$4
10,000			

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Co-TVaR A

Statistic	Value	Results	Mean	VaR	TVaR	Co-TVaR
Trials	10	Risk A	10.00	8.00	17.60	8.00
Pct	50%	Reference Portfolio	100.00	120.00	140.00	140.00
Rank	5	Sum	110.00	128.00	157.60	148.00
		Combined Portfolio	110.00	140.00	148.00	148.00
		Incremental	10.00	20.00	8.00	8.00

Loss Data by Trial				Separately Ordered Loss Data				Co-Stats	
Trial	A	Ref	A+Ref	Rank	A	Ref	A+Ref	Co- A	Co-Ref
1	8	32	40	1	32	156	156	0	156
2	0	152	152	2	28	152	152	0	152
3	28	120	148	3	12	140	148	28	120
4	0	140	140	4	8	132	144	12	132
5	12	132	144	5	8	120	140	0	140
6	8	60	68	6	8	100	132	32	100
7	0	156	156	7	4	64	72	8	64
8	8	64	72	8	0	60	68	8	60
9	32	100	132	9	0	44	48	4	44
10	4	44	48	10	0	32	40	8	32

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Co-TVaR B

Statistic	Value	Results	Mean	VaR	TVaR	Co-TVaR
Trials	10	Risk B	10.00	8.00	16.80	11.20
Pct	50%	Reference Portfolio	100.00	120.00	140.00	140.00
Rank	5	Sum	110.00	128.00	156.80	151.20
		Combined Portfolio	110.00	136.00	151.20	151.20
		Incremental	10.00	16.00	11.20	11.20

Loss Data by Trial				Separately Ordered Loss Data				Co-Stats	
Trial	B	Ref	B+Ref	Rank	B	Ref	B+Ref	Co- B	Co-Ref
1	0	32	32	1	28	156	176	20	156
2	4	152	156	2	20	152	156	4	152
3	16	120	136	3	16	140	148	8	140
4	8	140	148	4	12	132	140	8	132
5	8	132	140	5	8	120	136	16	120
6	12	60	72	6	8	100	128	28	100
7	20	156	176	7	4	64	72	12	60
8	4	64	68	8	4	60	68	4	64
9	28	100	128	9	0	44	44	0	44
10	0	44	44	10	0	32	32	0	32

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Co-TVaR A+B

Statistic	Value	Results	Mean	VaR	TVaR	Co-TVaR
Trials	10	Risk A+B	20.00	20.00	32.80	29.60
Pct	50%	Reference Portfolio	100.00	120.00	140.00	132.00
Rank	5	Sum	120.00	140.00	172.80	161.60
		Combined Portfolio	120.00	152.00	161.60	161.60
		Incremental	20.00	32.00	21.60	29.60

Loss Data by Trial				Separately Ordered Loss Data				Co-Stats	
Trial	A+B	Ref	B+Ref	Rank	A+B	Ref	A+B+Ref	Co- A+B	Co-Ref
1	8	32	40	1	60	156	176	20	156
2	4	152	156	2	44	152	164	44	120
3	44	120	164	3	20	140	160	60	100
4	8	140	148	4	20	132	156	4	152
5	20	132	152	5	20	120	152	20	132
6	20	60	80	6	12	100	148	8	140
7	20	156	176	7	8	64	80	20	60
8	12	64	76	8	8	60	76	12	64
9	60	100	160	9	4	44	48	4	44
10	4	44	48	10	4	32	40	8	32

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Co-TVaR Subadditivity Fail

Results	Mean	VaR	TVaR	Co-TVaR
A	10.00	8.00	17.60	8.00
B	10.00	8.00	16.80	11.20
Sum A+B	20.00	16.00	34.40	19.20
Combined A+B	20.00	20.00	32.80	29.60
Ref	100.00	120.00	140.00	132.00
Sum A+B+Ref	120.00	140.00	172.80	161.60
Combined A+B+Ref	120.00	152.00	161.60	161.60

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Summary and Conclusions

Key distinctions
Practical fixes
Portfolio-dependent tail-focused methods Conclusions

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Key Distinctions

- Distribution region focus
  - Tail
  - Adverse events
  - Full distribution
- Portfolio dependence
  - Calibration
  - Order dependence
  - Incremental or allocation algorithm
- Theoretical strength
  - Basic – stable and monotonic
  - Coherent – scalable and subadditive

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Practical Fixes

Issue/problem	Practical solution
Order Dependence	Use Reference portfolio
Scale (Share) dependence of portfolio methods	Price initially at highest authorized share.
Co-Var instability	Average over events in neighborhood

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Portfolio Incremental Tail-Focused Methods

- Intuitively appealing
  - Strong belief existing portfolio should matter
  - Tail events drive overall capital requirement
- Bargain pricing of non-peak zone coverage
  - Non-peak zone events independent of portfolio
  - Pure algorithms give them $0 capital
  - Promoting de-worsification?
- Tail uncertainty
  - No way to empirically validate
  - Very sensitive to model changes
  - Cut-off problem - exclude giant meteor strikes?

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Conclusions

- Indicated pricing is based on target return on required capital.
- Debate is over required capital
- A profusion of methods and approaches
- Tail focus and portfolio dependence are key areas where methods differ
- Some of key methods used in practice do not satisfy all the desired conceptual properties
- Try any method yourself on simple examples- understand how it works and how it fails.

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