On the Optimality of Multiline Excess of Loss Covers

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Agenda

- Optimal reinsurance old fashion
- Modern optimal reinsurance
- Numerical application : the multiline excess of loss treaty

Optimal reinsurance

- Aim of reinsurance : reducing the retained risk of the cedant.
- Unfortunately, ceding business usually implies ceding a portion of the profit.
- The decision-maker is faced with the choice between :
  - ceding huge amounts of premium $\Rightarrow$ reducing volatility but also reducing profits.
  - ceding small amounts of premium $\Rightarrow$ keeping in retention huge profits but also huge volatility .

Notations

- P : premium charged by the insurer (excluding administrative expenses and taxes).
- S : random aggregate loss for the insurer.
- P^{Re} : premium charged by the reinsurer (including administrative expenses).
- S^{Re} : random aggregate loss for the reinsurer.
- u : initial capital of the insurer.

Old fashion

- Define the retention :

$$Retention = S - S^{Re}.$$

- Look for a measure of risk :
  - Standard deviation : $\sigma(Retention)$.
  - Ruin probability : $\mathbb{P}[Retention > u + P - P^{Re}]$.
  - Lundberg bound for the probability of ruin in infinite time : e^{-Ru}.

Old fashion

- Define the profit :

$$Profit = P - S - (P^{Re} - S^{Re}).$$

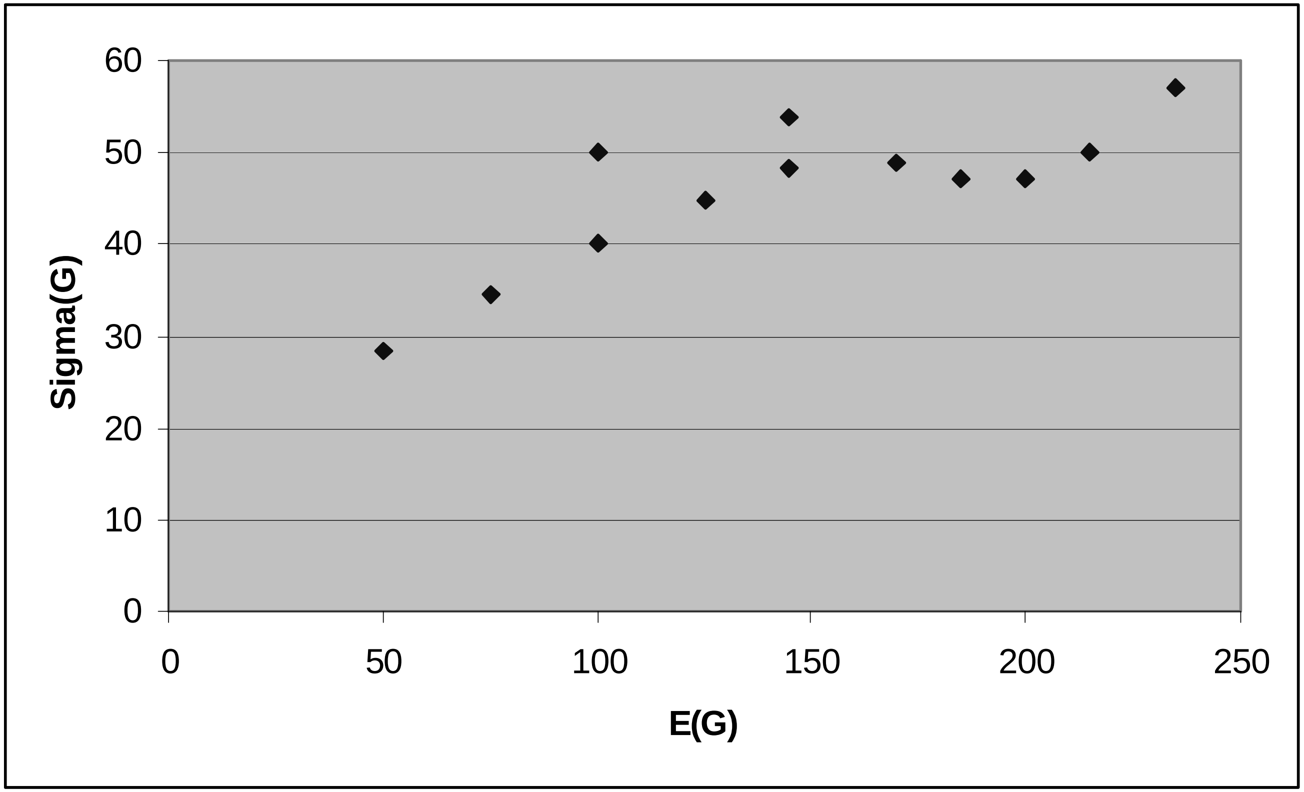
- Expected profit :

$$\mathbb{E}Profit = P - \mathbb{E}S - (P^{Re} - \mathbb{E}S^{Re}).$$

Theoretical results

- Let two reinsurance structures have the same expected loss and the same loading. Then excess of loss reinsurance is optimal between all individual reinsurances.
- Let two reinsurance structures have the same expected loss and the same loading. Then stop loss reinsurance is optimal.

Optimal reinsurance



Modern optimal reinsurance

- First compute the economic allocated capital (Risk Adjusted Capital : RAC).
- Candidates :
  1. Value at Risk :

     $$VaR_\alpha(X) = \min\{x | F(x) \geq \alpha\}.$$

     is not a coherent measure of risk.
  2. TailVar :

     $$TCE_\alpha(X) = \mathbb{E}[X | X > VaR_\alpha(X)].$$

     is a coherent measure of risk.

Modern optimal reinsurance

- A portion of the RAC is provided by the policyholders.
- Better candidates :
  1. based on Value at Risk :

$$RAC = \min\{x|F(x) \geq \alpha\} - P.$$

  2. based on TailVar :

$$RAC = \mathbb{E}[X|X > VaR_\alpha(X)] - P.$$

Modern optimal reinsurance

- Then compute the profit :

$$Profit = P - S - (P^{Re} - S^{Re}).$$

- Then compute the return on risk adjusted capital :

$$\frac{Profit}{RAC}.$$

- Then compute the expected return on risk adjusted capital, also known as RORAC, for Return On Risk Adjusted Capital :

$$RORAC = \frac{\mathbb{E}Profit}{RAC}.$$

Multiline excess of loss treaty

- It is well known that spreading (diversifying) the risk between independent policies reduces the total risk in the sense that less deviations around the aggregate mean loss are expected.
- In other words, less capital has to be allocated due to the diversification effect.

Multiline excess of loss treaty

- The same effect can be obtained when an insurance company buys an excess of loss cover.
- Instead of buying different independent covers for different lines of business, it is acceptable to believe that the insurance company has an interest in diversifying by buying a multiline excess of loss cover.

Lines of business

- We will assume two lines of business : Fire and MTPL.
- Let us define
  - X_i^{Fire} as the i^{th} claim amount of type Fire,
  - X_i^{MTPL} as the i^{th} claim amount of type MTPL.
- It is assumed that the X_i^{Fire}'s are independent and identically distributed as well as the X_i^{MTPL}'s. X_i^{Fire}'s and X_i^{MTPL}'s are assumed to be mutually independent.

Lines of business

- We also define
  - N as the number of claims of type Fire,
  - M as the number of claims of type MTPL.
- We assume that N and M are independent and that N and the X_i^{Fire}'s on the one hand and M and the X_i^{MTPL}'s on the other hand are also independent.

Lines of business

- Let us define the liability of the excess of loss reinsurer for each claim :

$$
\begin{aligned}
R_i^{Fire} &= \min(L^{Fire}, \max(0, X_i^{Fire} - D^{Fire})), \\
R_i^{MTPL} &= \min(L^{MTPL}, \max(0, X_i^{MTPL} - D^{MTPL})).
\end{aligned}
$$

- where
  1. D^{Fire} is the deductible for fire claims
  2. L^{Fire} is the cover for fire claims
  3. D^{MTPL} is the deductible for MTPL claims
  4. L^{MTPL} is the cover for MTPL claims

Liabilities

- Let us define the liability of the ceding company for each claim :

$$
\begin{aligned}
C_i^{Fire} &= X_i^{Fire} - R_i^{Fire}, \\
C_i^{MTPL} &= X_i^{MTPL} - R_i^{MTPL}.
\end{aligned}
$$

- Let us define the aggregate liability of the reinsurer for each line :

$$
\begin{aligned}
S^{Fire} &= \sum_{i=1}^{N} R_i^{Fire}, \\
S^{MTPL} &= \sum_{i=1}^{M} R_i^{MTPL}.
\end{aligned}
$$

Aggregate liabilities

- Let us define the aggregate liability of the ceding company for each line :

$$
\begin{aligned}
T^{Fire} &= \sum_{i=1}^{N} C_i^{Fire}, \\
T^{MTPL} &= \sum_{i=1}^{M} C_i^{MTPL}.
\end{aligned}
$$

Reinsurer's liability

- Now let us assume that the ceding company buys a multiline excess of loss cover as follows

$$Cover = \max(0, S^{Fire} + S^{MTPL} - GAAD)$$

  where $GAAD$ is a global annual aggregate deductible applicable to both lines of business.

- We are interested in analysing the retention's risk of the ceding company :

$$Retention = T^{Fire} + T^{MTPL} + \min(S^{Fire} + S^{MTPL}, GAAD).$$

Dependencies generated by the model

- The random variables R_i^{Fire}, C_i^{Fire} depend on X_i^{Fire} whereas R_i^{MTPL}, C_i^{MTPL} depend on X_i^{MTPL}.
- This means that even though N, M, X^{Fire}, and X^{MTPL} are mutually independent, S^{Fire}, S^{MTPL}, T^{Fire}, T^{MTPL} are not, which makes the calculation of the distribution of $Retention$ difficult.
- We need to obtain the joint distribution of

$$(S^{Fire}, S^{MTPL}, T^{Fire}, T^{MTPL}).$$

Numerical application

- The distribution of the fire claim amounts, X^{Fire}, is limited Pareto with parameters $A = 400$, $B = 2000$ and $\alpha = 1.50$. The distribution of the MTPL claim amounts, X^{MTPL} is limited Pareto with parameters $A = 700$, $B = 2000$ and $\alpha = 2.50$.
- The distribution of the fire claim numbers, N is Poisson with parameter $\lambda = 2.5$. The distribution of the MTPL claim numbers, M is Poisson with parameter $\lambda = 5$.

Numerical application

- For each treaty we will compute the following elements :

1. $\mathbb{E}Retention$
2. $\sigma(Retention)$
3. $TCE_{0.95}(Retention)$
4. $TCE_{0.99}(Retention)$
5. $RORAC_{0.95}(Ret) = \frac{P(Ret)-\mathbb{E}ret}{TCE_{0.95}(Ret)-P(Ret)}$
6. $RORAC_{0.99}(Ret) = \frac{P(Ret)-\mathbb{E}ret}{TCE_{0.99}(Ret)-P(Ret)}$

where $P(Ret) = 1.1\mathbb{E}Ret$.

Treaty 1

$$
\begin{aligned}
D^{Fire} &= 500 \\
L^{Fire} &= 1500 \\
D^{MTPL} &= 800 \\
L^{MTPL} &= 1200 \\
GAAD &= 0 \\
\mathbb{E}(Retention) &= 3949.62 \\
\sigma(Retention) &= 1655.30 \\
TCE_{0.95}(Retention) &= 7659.65 \\
TCE_{0.99}(Retention) &= 8964.04 \\
RORAC_{0.95} &= 11.91\% \\
RORAC_{0.99} &= 8.55\%
\end{aligned}
$$

Treaty 2

$$
\begin{aligned}
D^{Fire} &= 800 \\
L^{Fire} &= 1200 \\
D^{MTPL} &= 1000 \\
L^{MTPL} &= 1000 \\
GAAD &= 0 \\
\mathbb{E}(Retention) &= 4642.69 \\
\sigma(Retention) &= 1949.41 \\
TCE_{0.95}(Retention) &= 9029.28 \\
TCE_{0.99}(Retention) &= 10568.84 \\
RORAC_{0.95} &= 11.84\% \\
RORAC_{0.99} &= 8.50\%
\end{aligned}
$$

Treaty 3

$$
\begin{aligned}
D^{Fire} &= 500 \\
L^{Fire} &= 1500 \\
D^{MTPL} &= 800 \\
L^{MTPL} &= 1200 \\
GAAD &= 1000 \\
\mathbb{E}(Retention) &= 4756.58 \\
\sigma(Retention) &= 1822.77 \\
TCE_{0.95}(Retention) &= 8660.21 \\
TCE_{0.99}(Retention) &= 10038.44 \\
RORAC_{0.95} &= 13.88\% \\
RORAC_{0.99} &= 9.90\%
\end{aligned}
$$

Discussion

- The loading applied by the insurer and reinsurer may be very different. In that case the optimality should be examined with respect to the expected gain and not with respect to the expected retention.
- We have now analysed the large claims that are reinsured through an excess of loss treaty. Obviously we should take account of the small claims in order to compute the risk measures.

Discussion

- We have argued that it is better for the ceding company to buy excess of loss treaties with small priorities and with a global annual aggregate deductible. Administrative reasons may go against these solutions. Indeed, small priorities imply that a large number of claims are expected to hit the layers, which creates a lot of administration for both the insurer and the reinsurer. This is in particular true for long-tail business like MTPL where a stability clause is generally in use.
- In practice, the reinsurer would limit its annual liability through a global annual aggregate limit.

Discussion

- When the priorities of the treaties tend to 0 and the limits tend to ∞, the cover becomes a multiline stop-loss treaty.
- The hypothesis of independence between the lines of business may be relaxed for the case of umbrella covers where correlations exist between the covered lines of business. In such a case, copulas may help in order to price the cover. However, in order to analyse the retained risk of the cedant, only simulations would help and one should be aware of the fact that a huge number of simulations will be necessary to correctly catch the dependencies in the tails.