GUY CARPENTER

Capital Modeling and Loss Reserve Distributions

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MARSH & McLENNAN COMPANIES

AGENDA

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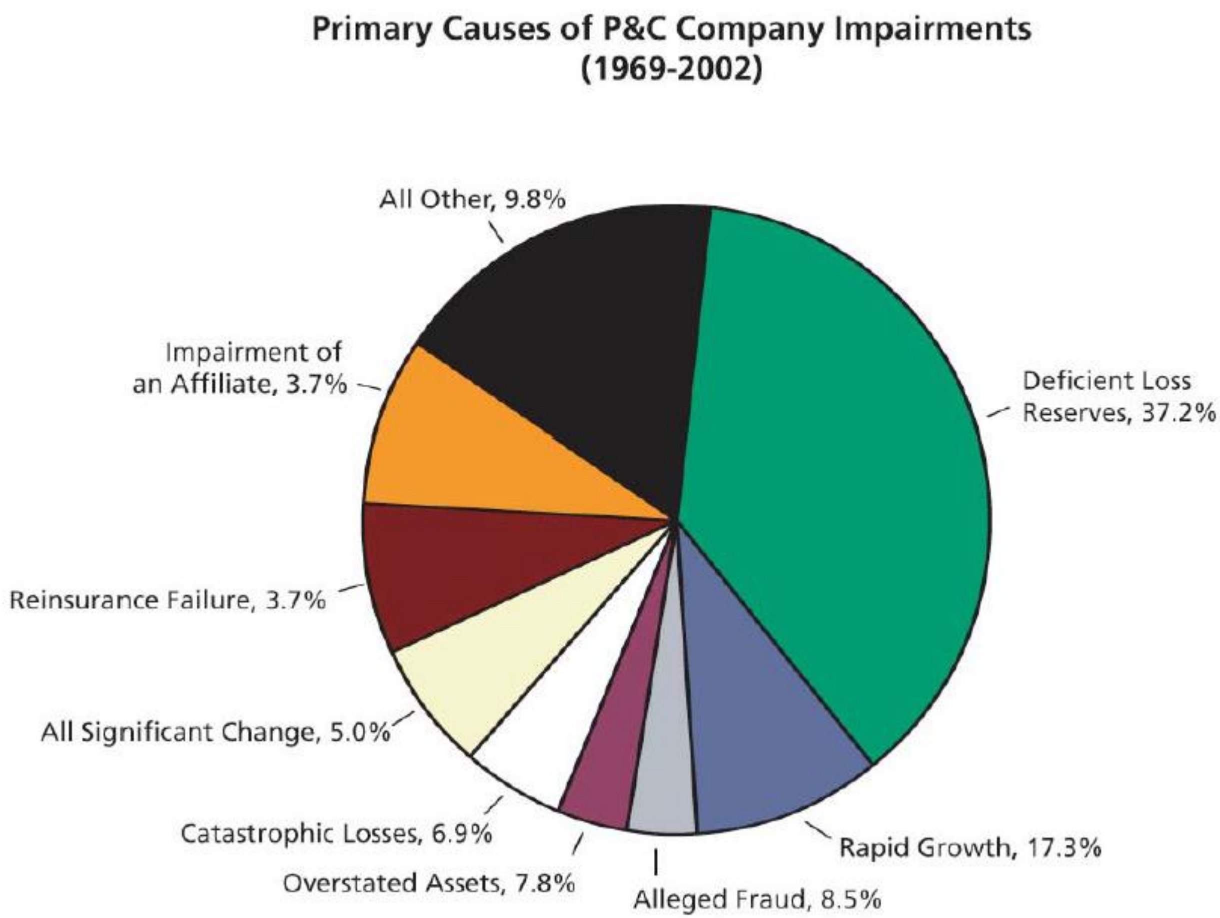
1 | **Capital Modeling Overview**
WHAT ARE THE RISKS FACING AN INSURANCE COMPANY

2 | **Model Aggregation**
INCLUDING CORRELATIONS AND DEPENDENCIES

3 | **Stochastic Reserving**
LIMITATIONS AND APPLICATIONS

Primary Causes of US Non-Life Insurance Insolvencies



The A.M. Best findings are consistent with those in "Failed Promises: Insurance Company Insolvencies," a 1990 U.S. Congressional. That report attributed insurer failures to under-reserving, underpricing, insufficiently supervised delegation of underwriting authority, rapid expansion, reckless management and abuse of reinsurance.

US Industry RBC Requirements

Source: NAIC

In Millions of USD

Category		No Explicit Cat Charge Amount (%)		100 Year EQ & 100 Wind Amount (%)		250 Year EQ & 100 Wind Amount (%)	
Insurance Affiliates	R0	49,825	(15%)	52,950	(13%)	52,950	(12%)
Fixed Income	R1	8,650	(3%)	8,514	(2%)	8,514	(2%)
Equity Investments	R2	95,576	(28%)	95,423	(23%)	95,423	(22%)
Credit	R3	13,675	(4%)	13,675	(3%)	13,675	(3%)
Reserve	R4	106,208	(31%)	106,208	(25%)	106,208	(25%)
Premium	R5	67,574	(20%)	62,079	(15%)	62,079	(14%)
Earthquake	R6	0	(0%)	28,687	(7%)	40,855	(10%)
Hurricane	R7	0	(0%)	49,006	(12%)	49,006	(11%)
Required Capital		208,706		219,454		221,976	
Actual Capital		826,627		826,627		826,627	
Required / Actual Capital		3.96		3.77		3.72	

Required Capital = $R0 + (R1^2 + R2^2 + R3^2 + R4^2 + R5^2 + R6^2 + R7^2)^{1/2}$

Capital Requirements in Japan

Japan FSA Solvency Margin Ratio Calculations (FY 2014)

In Millions of of USD (1 USD = 100 JPY)

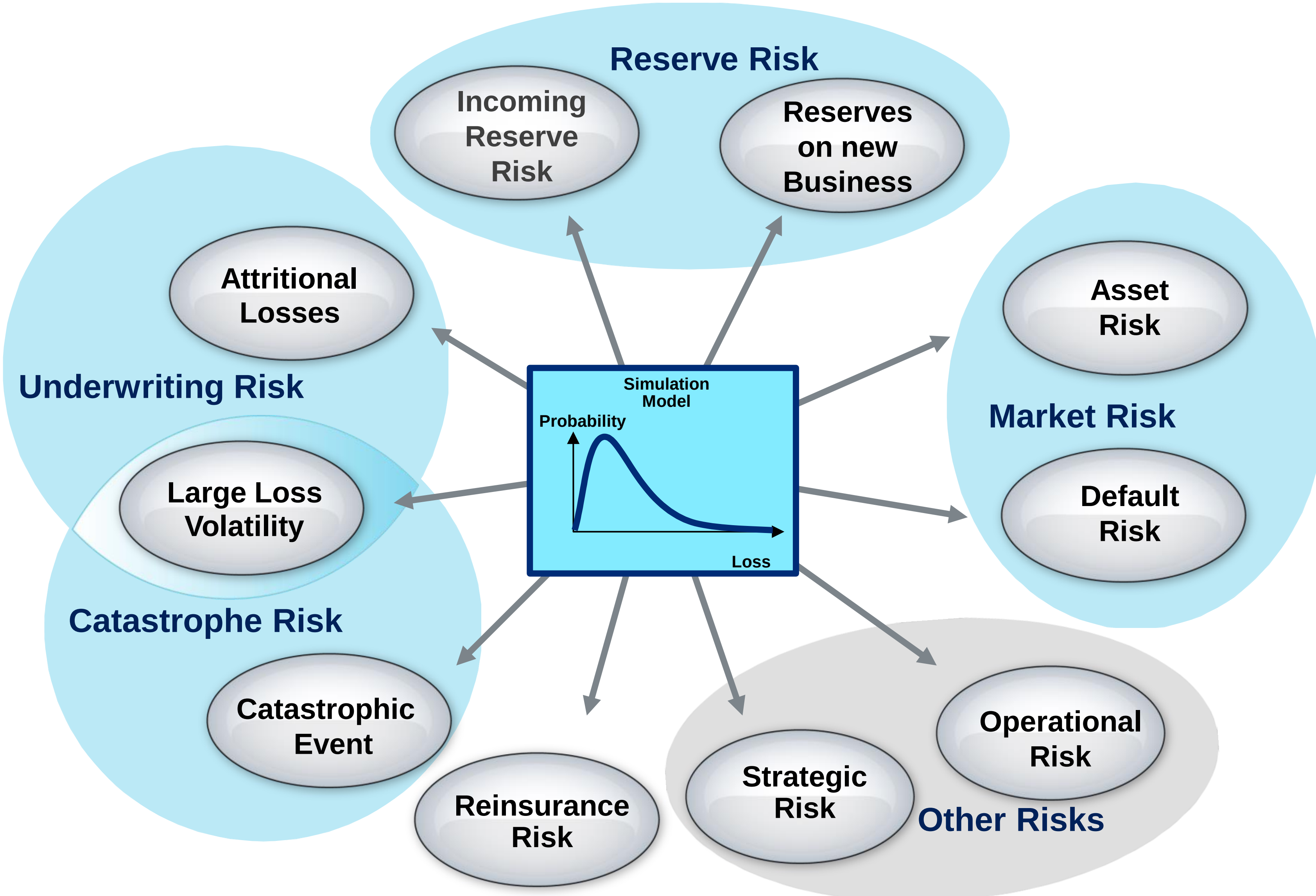
Category		Company A Amount (%)		Company B Amount (%)		Company C Amount (%)		Company D Amount (%)	
Insurance Risk	R1	1,627	(12%)	1,767	(20%)	1,153	(12%)	985	(25%)
3rd Sector (Medical)	R2	0	(0%)	0	(0%)	0	(0%)	0	(0%)
Interest Rate Risk	R3	256	(2%)	233	(3%)	184	(2%)	94	(2%)
Investment Risk	R4	8,603	(64%)	5,121	(57%)	6,140	(66%)	2,273	(57%)
Risk Management Control	R5	262	(2%)	176	(2%)	181	(2%)	78	(2%)
Cat Risk	R6	2,604	(20%)	1,678	(19%)	1,580	(17%)	553	(14%)

	Company A	Company B	Company C	Company D	Total
Required Capital	11,873	7,492	8,189	3,194	30,749
Actual Capital	44,626	26,833	26,679	12,856	110,995
Required / Actual Capital	3.76	3.58	3.26	4.02	3.61

Required Capital = $((R1 + R2)^2 + (R3 + R4)^2)^{1/2} + R5 + R6$

Individual or Organization	Number of shares held (thousands)
Japan Trustee Services Bank, Ltd. (Trust Account)	104,755
Moxley & Co	83,945
The Master Trust Bank of Japan, Ltd. (Trust Account)	70,922
State Street Bank and Trust Company 505223	52,503
Meiji Yasuda Life Insurance Company	51,199
State Street Bank and Trust Company	43,820
Tokio Marine & Nichido Fire Insurance Co., Ltd.	42,553
The Bank of Tokyo-Mitsubishi UFJ, Ltd.	36,686
Nippon Life Insurance Company	27,066
Mitsui Sumitomo Insurance Co., Ltd.	25,739

Capital Model



- Underwriting and reserve risk typically done by line of business.

Building in Correlation

- For Catastrophe losses, the catastrophe models take care of this.
- Economic scenario files are used to model the asset risk and can include inflation indexes for wages, medical costs, construction costs, etc.
  - Higher or lower than expected inflation can be used to adjust future payments up or down
  - These indexes can be used to correlate assets and liabilities
- For non-catastrophe losses, correlation is typically modelled using **copulas**, **indexes**, and inflation.

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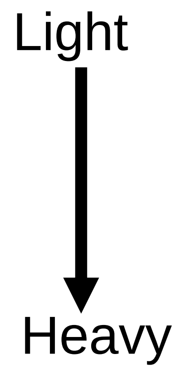
Introduction to Copulas

- Copulas are used to generate correlated random deviates
- Copulas can be used to correlate:
  - two or more losses caused from the same event
  - aggregates losses
  - claim count distributions
  - loss reserves

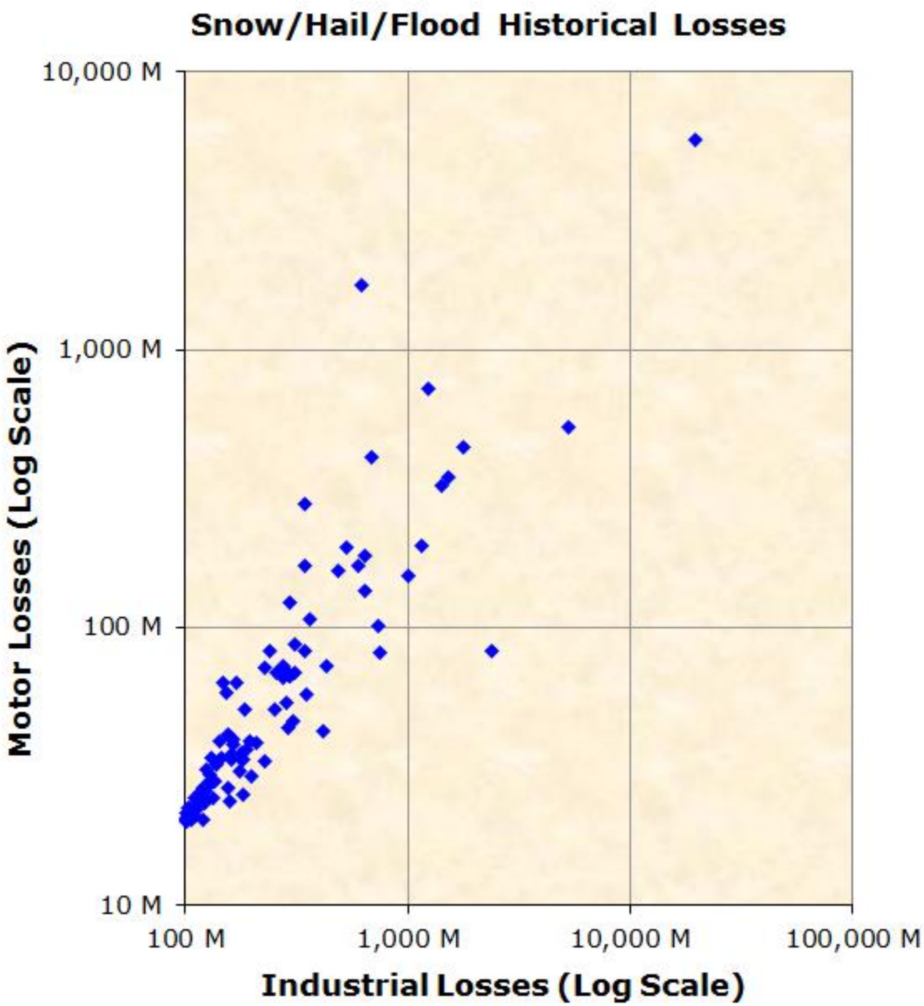
The Names

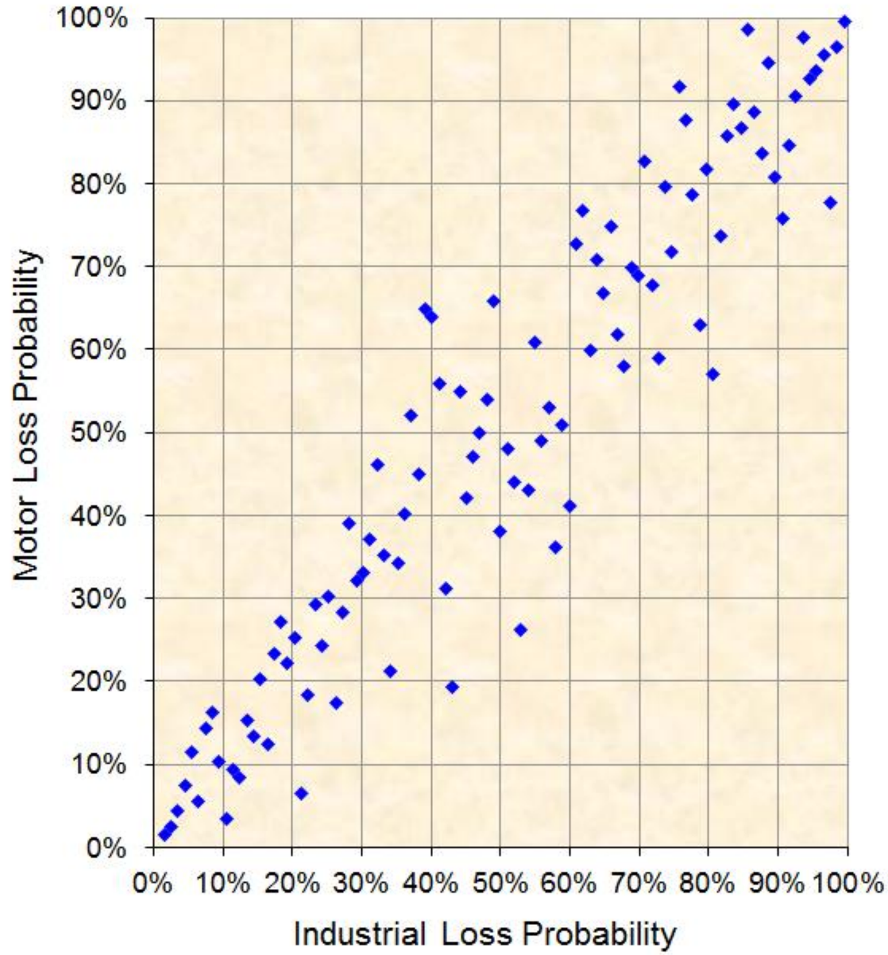
- Right Tailed
  - Frank
  - Normal
  - T
  - Gumbel
  - HRT (Heavy Right Tail)



- Left Tailed
  - Flipped Gumbel
  - Flipped HRT (Clayton)

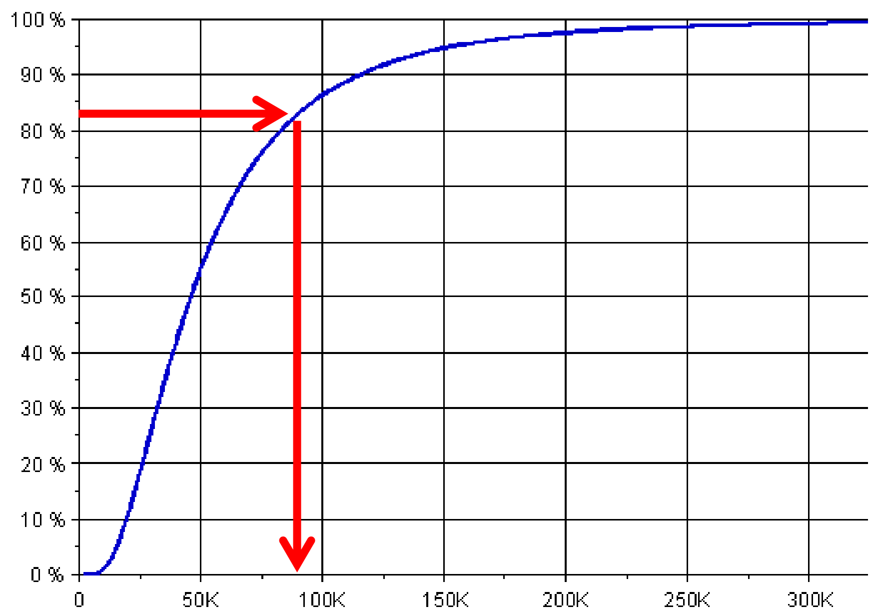
Sample Data


Snow/Hail/Flood Historical Losses
10,000 M
1,000 M
100 M
10 M
Motor Losses (Log Scale)
100 M
1,000 M
10,000 M
100,000 M
Industrial Losses (Log Scale)


100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
Motor Loss Probability
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%
Industrial Loss Probability

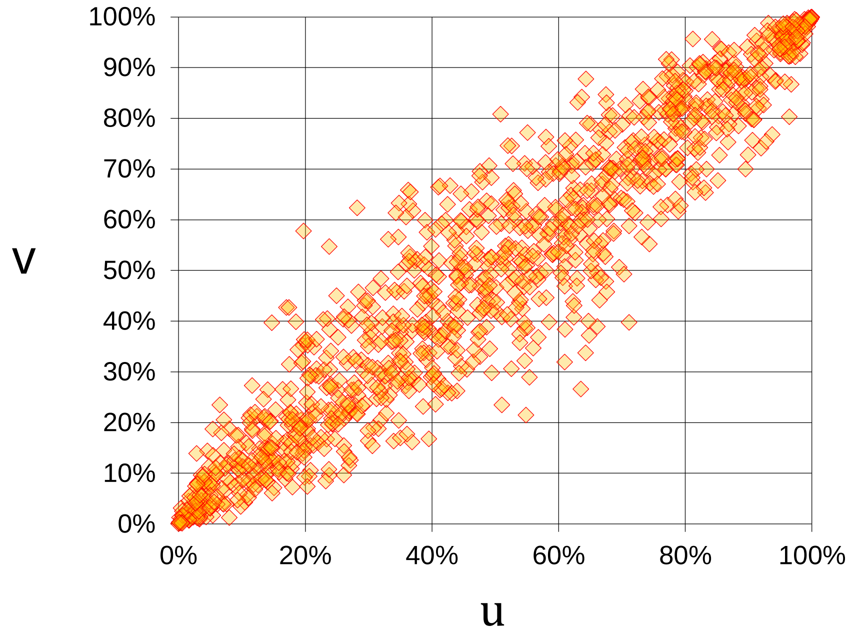
Running a Simulation Model

- To simulated losses, you generate a random number, u, and then find the corresponding loss value using $F^{-1}(u)$, where $F(x)$ is the cumulative distribution function of x.
- For example, if u=0.83, the corresponding loss would be roughly 90,000.

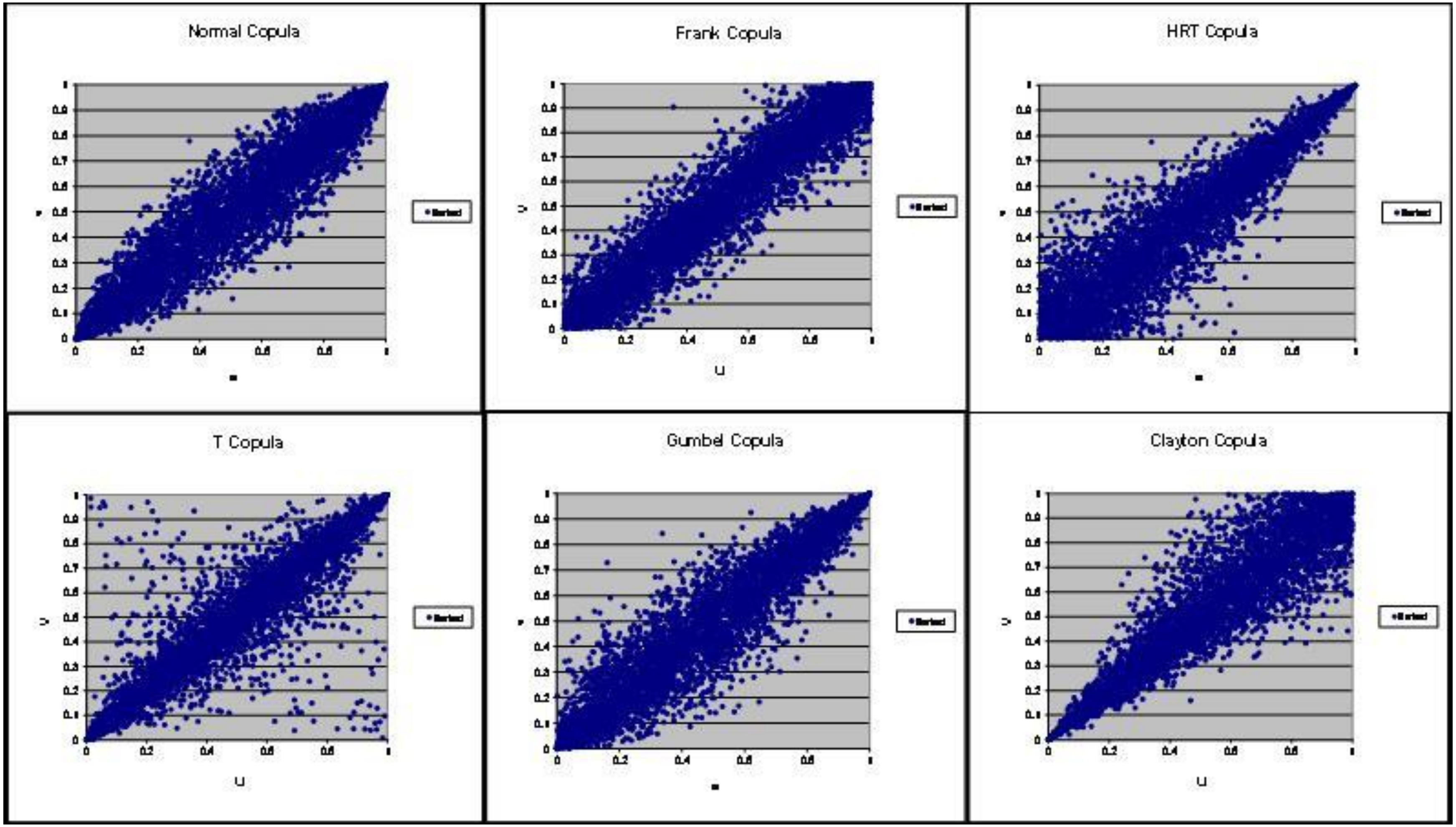


Running a Simulation Model

- When you have two losses that are correlated, you generate two random numbers, u and v, that are correlated. For example, u=.83 and v=.88.
- You then calculated $F^{-1}(u)$ and $G^{-1}(v)$ where $F(x)$ and $G(x)$ are the two cumulative loss distributions.
- The correlated pair of random numbers are generated using copulas.

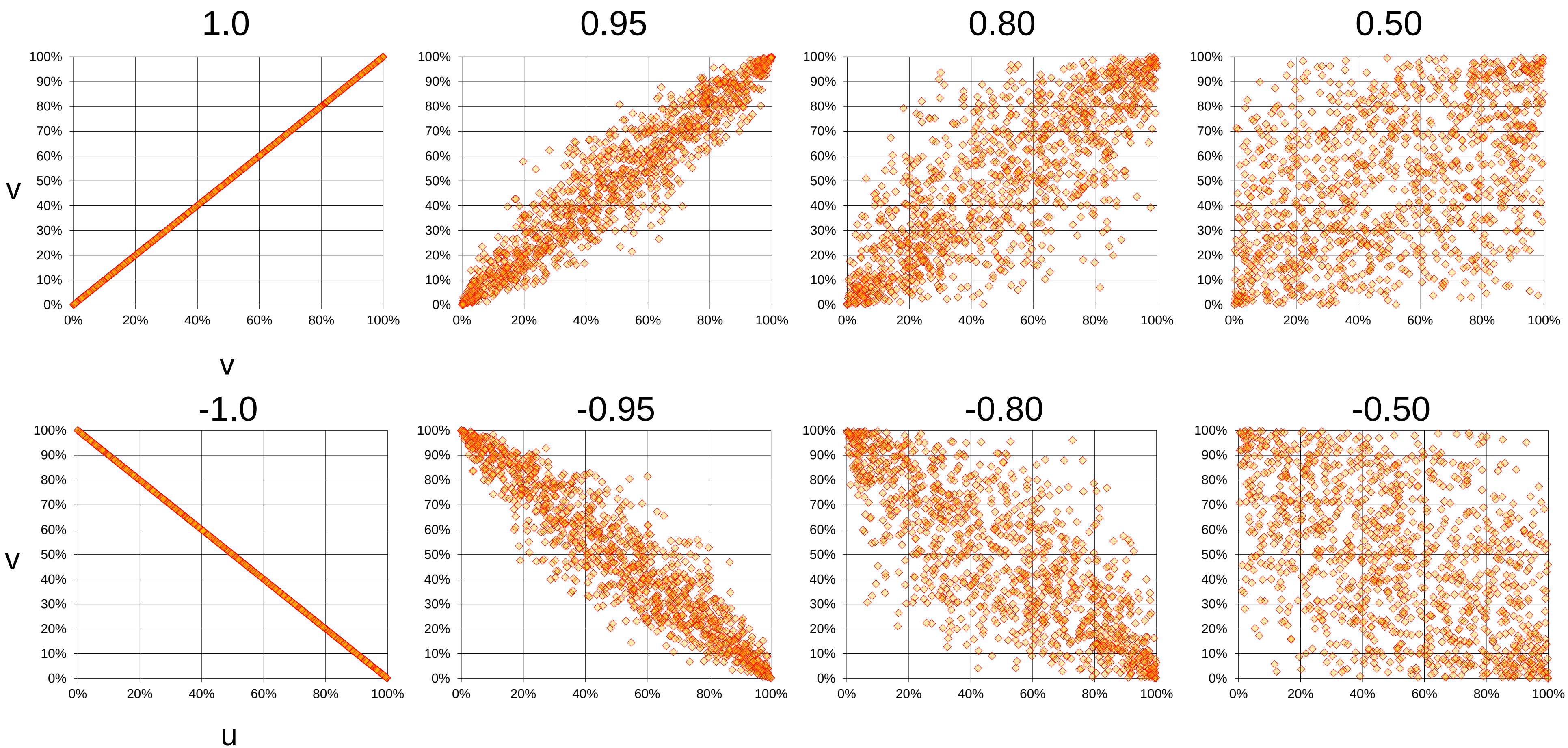


Six Copulas with the Same Correlation



The above copulas all have the same level of positive correlation as measured by Kendall's tau and shows the effect of choosing different copulas

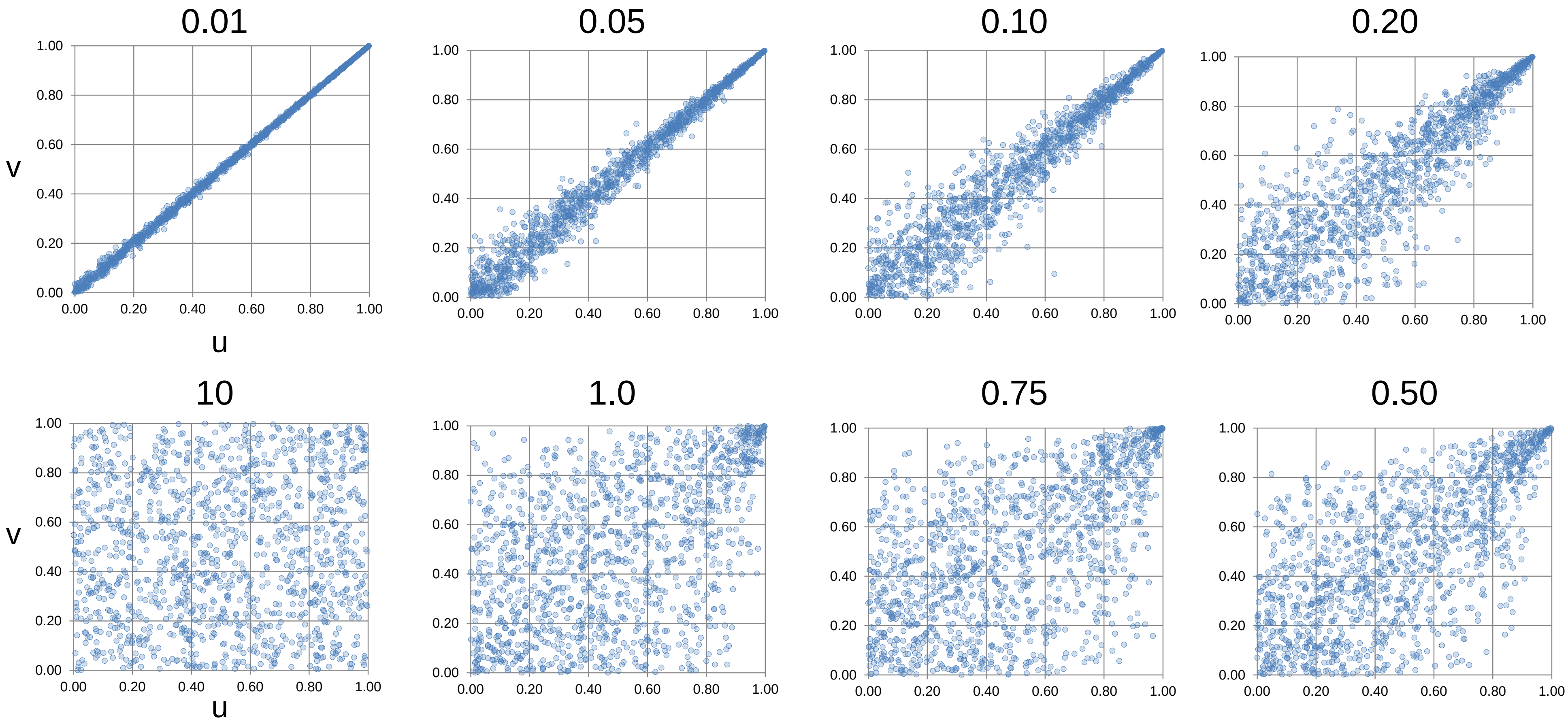
Normal Copula with different Parameters, “a”



u = random number, p = random number, v uses the formula below

v = NORMDIST(NORMINV(u,0,1)*a + NORMINV(p,0,1)*(1-a^2)^0.5,0,1,1)

HRT Copula with tail Parameters, “a”



u = random number, p = random number, v uses the formula below

$$v = 1-\{1-(1-u)^{-1/a}+[(1-p)(1-u)^{1+1/a}]^{-1/(a+1)}\}^{-a}$$

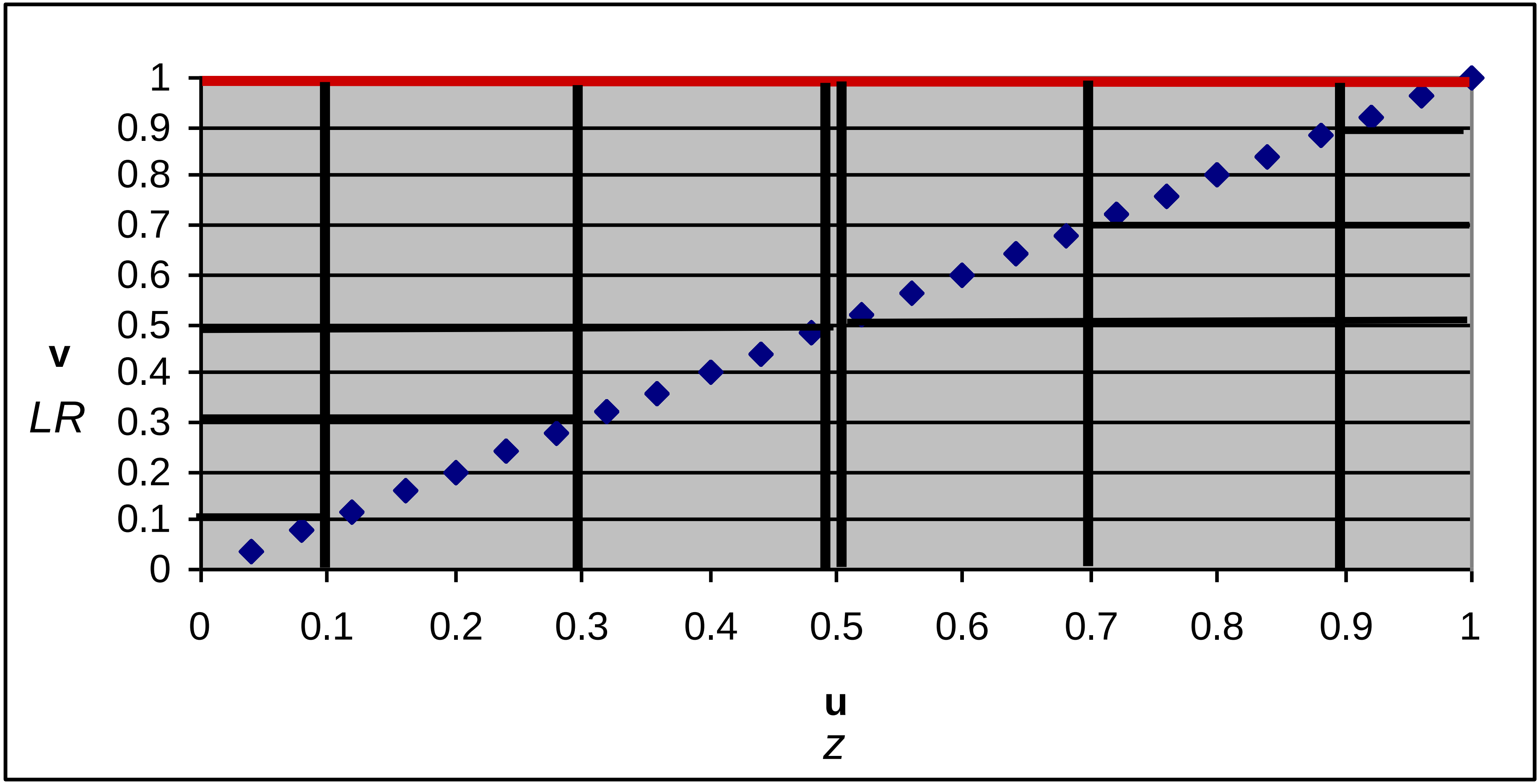
Visually Comparing Fits to Data

- Visually – hard to compare “Side-by-Side”
- Visual Solution: Left Right Tail Concentration Functions that graph the coordinates of $(\mathrm{z}, \mathrm{LR}(\mathrm{z}))$ where

$$\mathrm{LR}(\mathrm{z}) = (\mathrm{z}<=.5)*\mathrm{Pr}(\mathrm{v}<\mathrm{z}|\mathrm{u}<\mathrm{v}) + (\mathrm{z}>.5)*\mathrm{Pr}(\mathrm{v}>\mathrm{z}|\mathrm{u}>\mathrm{z})$$

$$\mathrm{LR}(\mathrm{z}) = (\mathrm{z}<=.5)*\mathrm{Pr}(\mathrm{v}<\mathrm{z},\mathrm{u}<\mathrm{z})/\mathrm{z} + (\mathrm{z}>.5)*\mathrm{Pr}(\mathrm{v}>\mathrm{z},\mathrm{u}>\mathrm{z})/(1\text{-}\mathrm{z})$$

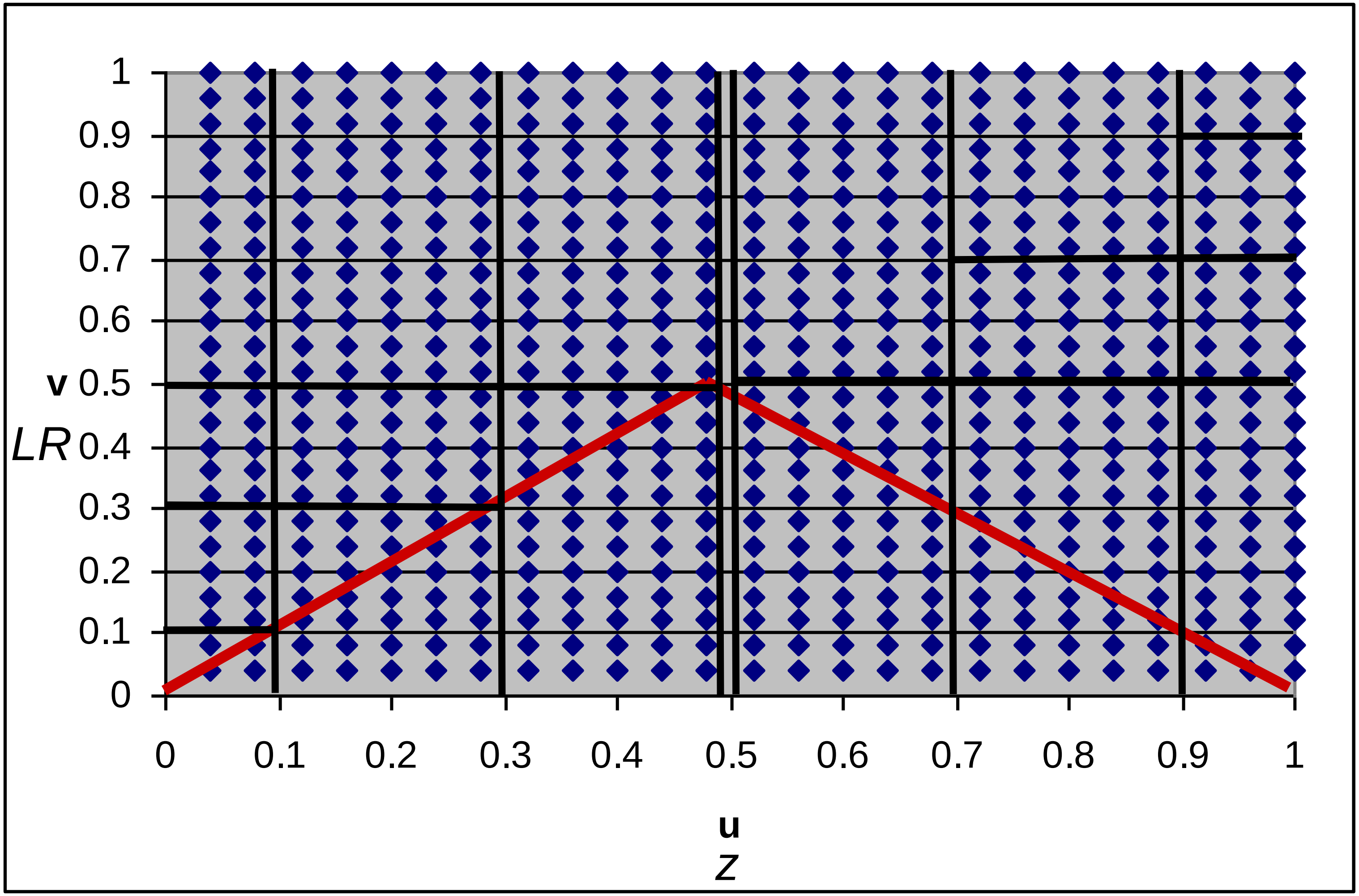
Understanding Left Right (LR) Graphs
Correlated data



For 100% correlated data LR function = 100%

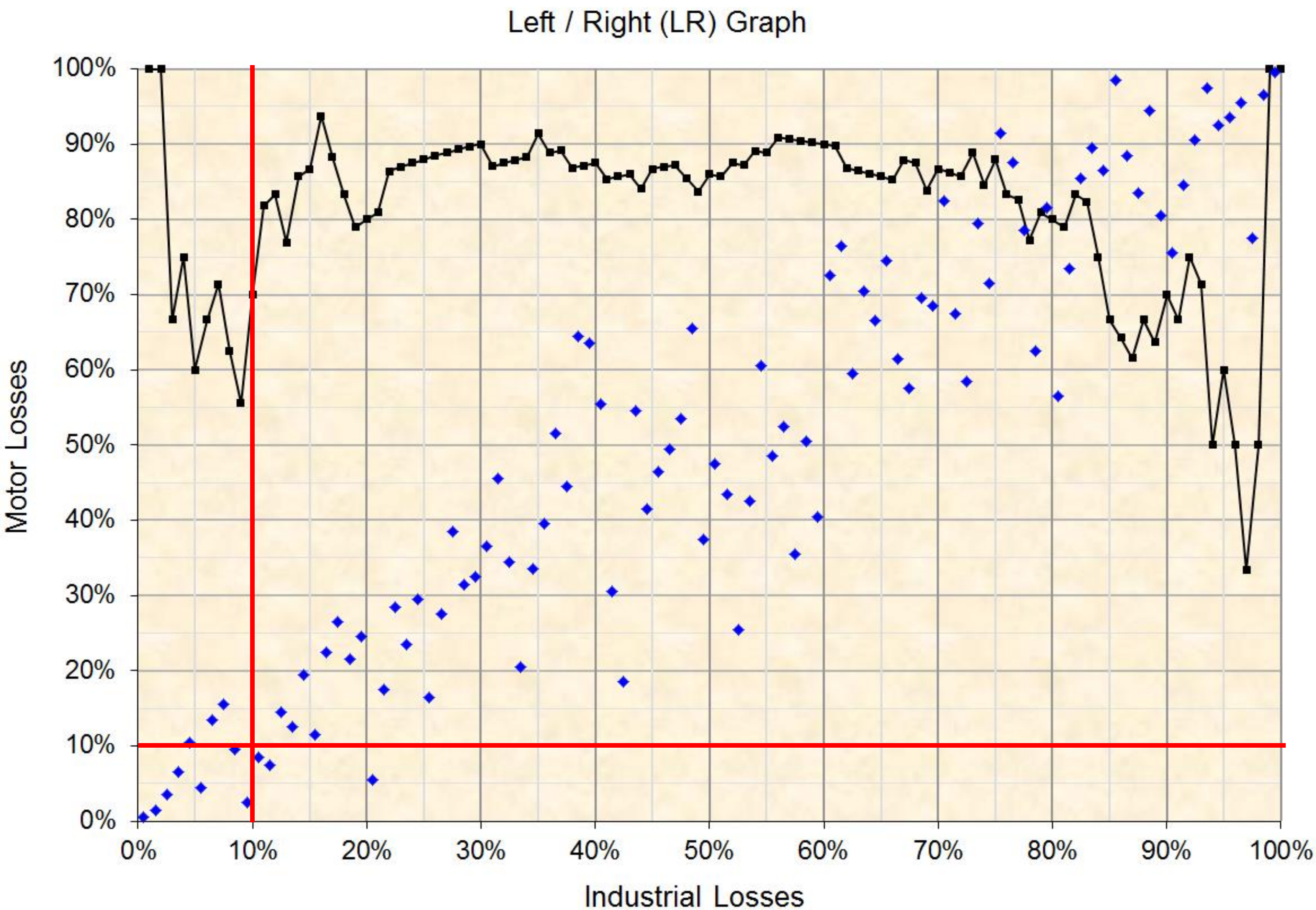
Understanding LR Graphs

Uncorrelated data

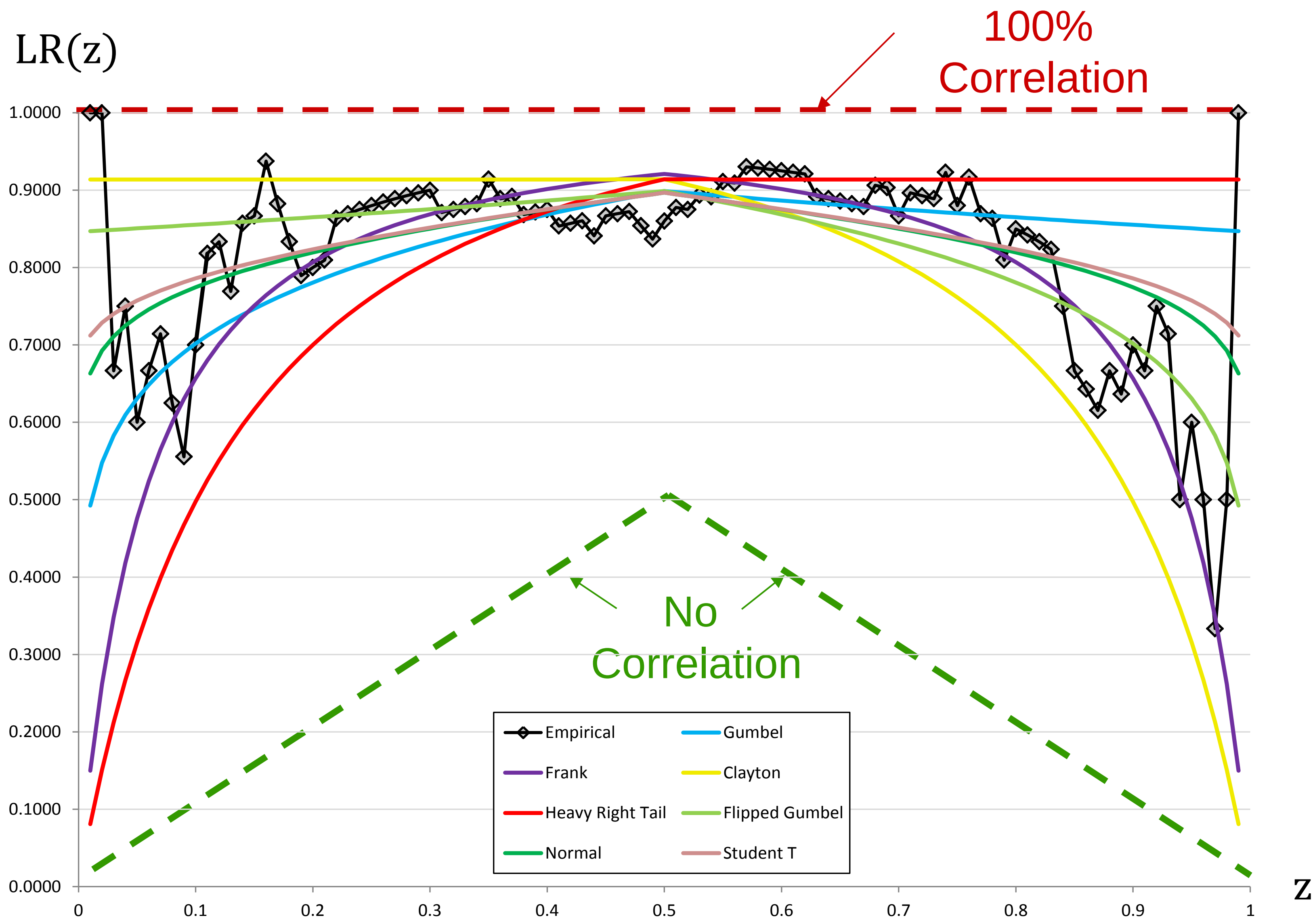


For uncorrelated data L(z) = z and R(z) = 1 - z

Fitting a Copula



Comparing Copula Fits



Indexes

- An index be used to correlate many different lines of business
- An index can be applied to claim count distributions (contagion)
- An index can be used to correlate severity distributions
- An index can be used to aggregate losses
  - Such as a lognormal, with a mean of 1 and cv of 5% that applies to multiple lines of business
  - Such as applying one index to two or more severity distributions
- An index can be used to correlate premiums

Frequency Correlation

- If you mix a Poisson distribution with a mean, λ, by a Gamma distribution with a mean=1 and variance, *c*, the resulting distribution is equivalent to a Negative Binomial with a mean of λ and a variance to mean ratio of $1 + c\lambda$

  – The Gamma distribution can be shared across multiple Poisson distributions. In this case, the new frequencies coefficient of variations are:

  – The CV's are $CV_{N_i} = \sqrt{1/\lambda_{N_i} + c} = \sqrt{(CV_{N_i})^2 + c}$

  – And the correlation between the two frequency distributions is:

$$\rho_{N_1,N_2} = \sqrt{\frac{c\lambda_{N_1}}{1 + c\lambda_{N_1}}}\sqrt{\frac{c\lambda_{N_2}}{1 + c\lambda_{N_2}}} \qquad \rho_{N_1,N_2} = \frac{c}{\sqrt{{}^{1}/_{\lambda_{N_1}} + c}\sqrt{{}^{1}/_{\lambda_{N_2}} + c}}$$

  – *c* is sometime referred to as a contagion.

Correlating two claim count distributions using shared contagion

- Contagion parameter c = 0.20
- Loss Cause 1 has a Poisson distribution with frequency of 2
- Loss Cause 2 has a Poisson distribution with frequency of 10

	Loss Cause 1	Loss Cause 2	Combined
Mean	$2.0027	$9.9976	$12.0003
Standard Deviation	$1.6697	$5.4645	$6.3771
CV	83.375231 %	54.658342 %	53.141295 %
Minimum	$0.0000	$0.0000	$0.0000
Maximum	$14.0000	$49.0000	$58.0000
Samples	100000	100000	100000
Non-Zero Probability	81.564000 %	99.612000 %	99.794000 %
Variance	2.7879729244	29.8610642879	40.6673899271

- New variance to mean ratios should be
  - Loss Cause 1: 1 + 0.2 x 2 = 1.4
  - Loss Cause 2: 1 + 0.2 x 10 = 3.0

$$\rho_{N_1,N_2} = \sqrt{\frac{c\lambda_{N_1}}{1 + c\lambda_{N_1}}}\sqrt{\frac{c\lambda_{N_2}}{1 + c\lambda_{N_2}}}$$

$$\rho_{1,2} = \sqrt{\frac{.2 \times 2}{1 + .2 \times 2}}\sqrt{\frac{.2 \times 10}{1 + .2 \times 10}} = 43.64\%$$

$$\tilde{\rho}_{1,2} = \frac{40.67 - 2.79 - 29.86}{2\sqrt{2.79 \times 29.86}} = 43.94\%$$

How Correlated are Two Frequency Distributions that share the same Contagion?

- If the frequency of one of the distributions is zero, then the correlation is zero
- As the frequency of one of the distributions gets close to zero, the correlation gets smaller.
- As the frequency increases the correlation increases.

- Resulting Correlation

c = 0.10	0.1	1	10	100	1000
0.1	1.0%	3.0%	7.0%	9.5%	9.9%
1	3.0%	9.1%	21.3%	28.7%	30.0%
10	7.0%	21.3%	50.0%	67.4%	70.4%
100	9.5%	28.7%	67.4%	90.9%	94.9%
1000	9.9%	30.0%	70.4%	94.9%	99.0%

c = 0.20	0.1	1	10	100	1000
0.1	2.0%	5.7%	11.4%	13.7%	14.0%
1	5.7%	16.7%	33.3%	39.8%	40.7%
10	11.4%	33.3%	66.7%	79.7%	81.4%
100	13.7%	39.8%	79.7%	95.2%	97.3%
1000	14.0%	40.7%	81.4%	97.3%	99.5%

Example of a Severity Mixing

- An index can be applied to the severity distribution or the aggregate distribution (called a mixing distribution)
- For the mixing index, M, assume it follows a lognormal distribution with mean = 1 and variance of *m*
- The resulting CV of the mixed distribution is

$$CV_{M \cdot S} = \sqrt{\left(CV_S{}^2 + m + CV_S{}^2 m\right)}$$

Correlation Resulting from a Shared Mixing Distribution

Severity / Aggregate Correlation

The correlation is:

$$\rho_{S_1,S_2} = \frac{m}{\sqrt{CV^2{}_{S_1}(1+m)+m}\sqrt{CV^2{}_{S_2}(1+m)+m}}$$

If you divide the top and bottom by $(1+m)$ you get the following

$$\rho_{S_1,S_2} = \frac{\left(\frac{m}{(1+m)}\right)}{\sqrt{CV^2{}_{S_1}+\frac{m}{(1+m)}}\sqrt{CV^2{}_{S_2}+\frac{m}{(1+m)}}}$$

This looks like the contagion correlation with $\mathrm{c} = \left(\frac{m}{(1+m)}\right)$

Example Correlation Resulting from a Shared Mixing Distribution
Impact of increasing the Aggregate CV

- For a fixed mixing parameter, m (mean of 1, variance of m)
  - Correlation decreases as the CV of the severity distributions increase.

$$\rho_{S_1,S_2} = \frac{\left(\frac{m}{(1+m)}\right)}{\sqrt{CV^2{}_{S_1} + \frac{m}{(1+m)}}\sqrt{CV^2{}_{S_2} + \frac{m}{(1+m)}}}$$

m: 0.1

CV2 \ CV1	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
0%	100.00%	94.92%	83.33%	70.89%	60.19%	51.64%	44.90%	39.56%	35.27%	31.77%	28.87%
10%	94.92%	90.09%	79.10%	67.28%	57.13%	49.01%	42.62%	37.55%	33.47%	30.15%	27.40%
20%	83.33%	79.10%	69.44%	59.07%	50.16%	43.03%	37.42%	32.97%	29.39%	26.47%	24.06%
30%	70.89%	67.28%	59.07%	50.25%	42.67%	36.61%	31.83%	28.04%	25.00%	22.52%	20.46%
40%	60.19%	57.13%	50.16%	42.67%	36.23%	31.08%	27.03%	23.81%	21.23%	19.12%	17.38%
50%	51.64%	49.01%	43.03%	36.61%	31.08%	26.67%	23.19%	20.43%	18.21%	16.40%	14.91%
60%	44.90%	42.62%	37.42%	31.83%	27.03%	23.19%	20.16%	17.76%	15.84%	14.26%	12.96%
70%	39.56%	37.55%	32.97%	28.04%	23.81%	20.43%	17.76%	15.65%	13.95%	12.57%	11.42%
80%	35.27%	33.47%	29.39%	25.00%	21.23%	18.21%	15.84%	13.95%	12.44%	11.20%	10.18%
90%	31.77%	30.15%	26.47%	22.52%	19.12%	16.40%	14.26%	12.57%	11.20%	10.09%	9.17%
100%	28.87%	27.40%	24.06%	20.46%	17.38%	14.91%	12.96%	11.42%	10.18%	9.17%	8.33%

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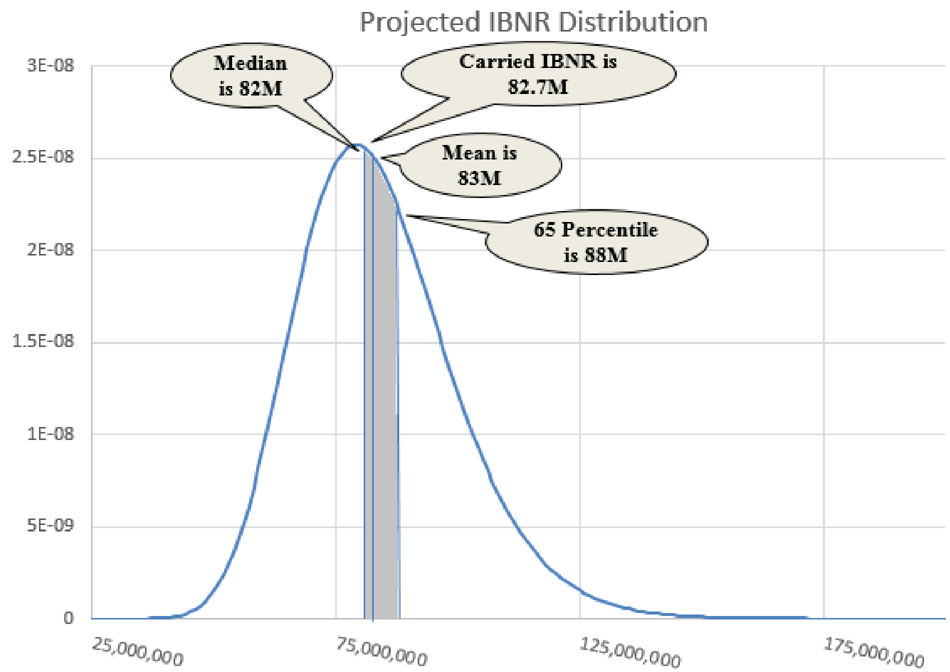
Stochastic Reserving

- Goal of Stochastic Reserving
  - Reserve should not be just a point estimation
  - Stochastic reserving provides a predictive distribution
  - Useful in capital modeling, reserve adequacy analysis, and loss reserve margins



Popular Methods

- Various stochastic reserving methods and authors
  - Mack
  - Bootstrapping (England and Verrall)
  - Generalized Linear Modeling (GLM)
  - Merz - Wüthrich
  - Rehman - Klugman
  - Roger Hayne
  - Daniel Murphy
  - Gary Venter

Mack

- Mack method is one of the most commonly used stochastic reserving methods.
  - Based on chain-ladder Method
  - Easy to implement
  - No distribution generated
  - Assumes accident years (AY) are independent

Bootstrapping

- Bootstrapping method is a very versatile model for estimating reserve distribution
  - No distributional assumption
  - Level of skewness in the data is automatically reflected
  - More complex to build
  - A deep understanding of underlying model and data is required

GLM

- GLM method is a flexible generalization of ordinary linear regression
  - Allows various distribution assumptions from exponential family
  - Able to view trends in three different directions
  - Requires manual adjustments after initial fitting
  - Has more flexibility in reserve mean selection

Merz - Wüthrich

- Merz – Wüthrich method produces one year reserve risk
  - Definition: The variance of difference between expected ultimate losses at time t and t + 1
  - Based on chain – ladder model assumptions
  - Useful for Solvency II

Rehman - Klugman

- Rehman – Klugman method produces reserve risk based on ultimate loss triangle instead of paid/incurred loss triangle
  - Assume age-to-age ratios of estimated ultimates follow lognormal distribution
  - Consider correlation in development year (DY) direction
  - Not able to normalize each AY by exposure size

Practical Expectations from Stochastic Reserving

- Expectations of Stochastic Reserving Results from a Practical Reserving Actuary
  - **Stochastic mean should be close to deterministic mean**
    - **Otherwise stochastic distribution is not reliable**
  - CV should be stable from year to year when there is no significant change in the business nature
  - CV should decrease as loss data mature
    - Backtesting with calendar year data removed

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    - **Backtesting with calendar year data removed**

Backtesting

Original Loss Triangle

Loss Triangle

AY/DY	1	2	3	4	5	6	7	8	9	10	11	12	13	14
2001	*19,303,998*	*41,715,012*	*29,374,058*	*11,835,301*	*13,061,422*	*13,048,744*	*13,036,421*	*13,024,451*	*13,012,834*	*13,001,568*	*12,990,653*	*12,980,088*	*12,969,873*	*12,960,005*
2002	*59,626,420*	*128,774,467*	*90,676,813*	*100,044,603*	*110,411,411*	*110,305,867*	*110,203,317*	*110,103,753*	*110,007,166*	*109,913,549*	*109,822,895*	*109,735,196*	*109,650,445*	
2003	*3,165,898*	*6,834,223*	*13,180,672*	*14,542,236*	*16,048,990*	*16,033,878*	*16,019,201*	*16,004,957*	*15,991,146*	*15,977,766*	*15,964,816*	*15,952,296*		
2004	*1,019,259*	*6,021,750*	*11,613,405*	*12,812,964*	*14,140,428*	*14,127,254*	*14,114,463*	*14,102,054*	*14,090,026*	*14,078,378*	*14,067,108*			
2005	*4,569,334*	*26,995,449*	*52,062,955*	*57,440,425*	*63,391,256*	*63,332,775*	*63,276,008*	*63,220,952*	*63,167,602*	*63,115,954*				
2006	*19,167,417*	*113,240,163*	*218,393,728*	*240,950,468*	*265,912,203*	*265,669,299*	*265,433,585*	*265,205,042*	*264,983,651*					
2007	*2,704,564*	*15,977,804*	*30,816,164*	*33,999,200*	*37,521,629*	*37,487,640*	*37,454,666*	*37,422,702*						
2008	*2,096,923*	*12,392,068*	*23,901,097*	*26,370,276*	*29,102,776*	*29,076,566*	*29,051,142*							
2009	*8,224,572*	*48,606,168*	*93,749,306*	*103,436,501*	*114,156,977*	*114,054,393*								
2010	*1,692,907*	*10,005,388*	*19,298,004*	*21,292,853*	*23,500,559*									
2011	*6,897,527*	*40,720,852*	*78,616,961*	*86,739,070*										
2012	*8,426,937*	*49,750,011*	*96,048,939*											

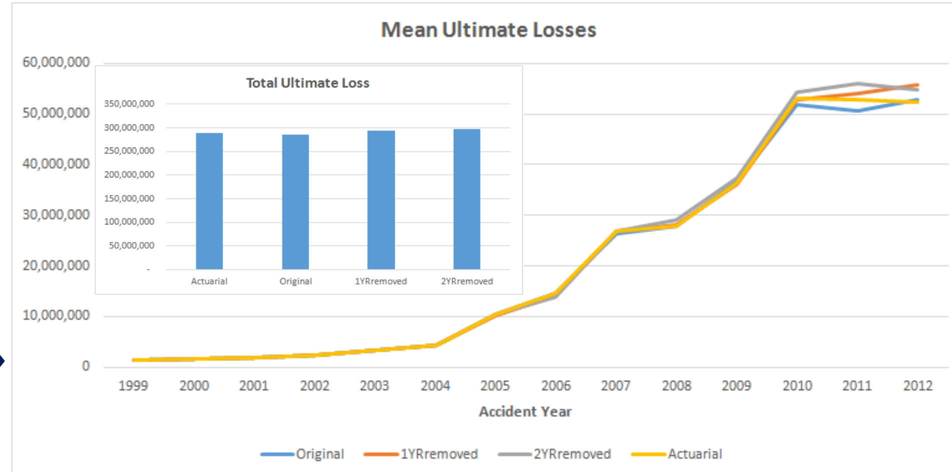
One Calendar Year Removed Loss Triangle

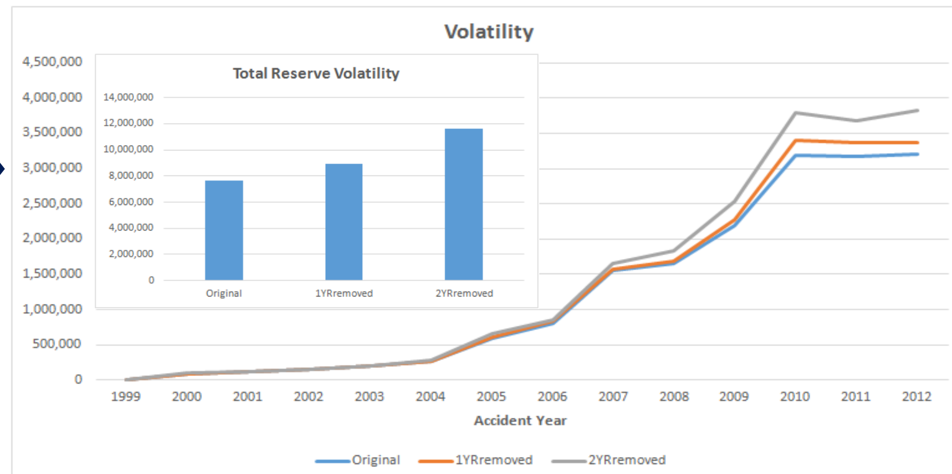
Loss Triangle

AY/DY	1	2	3	4	5	6	7	8	9	10	11	12	13	14
2001	*19,303,998*	*41,715,012*	*29,374,058*	*11,835,301*	*13,061,422*	*13,048,744*	*13,036,421*	*13,024,451*	*13,012,834*	*13,001,568*	*12,990,653*	*12,980,088*	*12,969,873*	
2002	*59,626,420*	*128,774,467*	*90,676,813*	*100,044,603*	*110,411,411*	*110,305,867*	*110,203,317*	*110,103,753*	*110,007,166*	*109,913,549*	*109,822,895*	*109,735,196*		
2003	*3,165,898*	*6,834,223*	*13,180,672*	*14,542,236*	*16,048,990*	*16,033,878*	*16,019,201*	*16,004,957*	*15,991,146*	*15,977,766*	*15,964,816*			
2004	*1,019,259*	*6,021,750*	*11,613,405*	*12,812,964*	*14,140,428*	*14,127,254*	*14,114,463*	*14,102,054*	*14,090,026*	*14,078,378*				
2005	*4,569,334*	*26,995,449*	*52,062,955*	*57,440,425*	*63,391,256*	*63,332,775*	*63,276,008*	*63,220,952*	*63,167,602*					
2006	*19,167,417*	*113,240,163*	*218,393,728*	*240,950,468*	*265,912,203*	*265,669,299*	*265,433,585*	*265,205,042*						
2007	*2,704,564*	*15,977,804*	*30,816,164*	*33,999,200*	*37,521,629*	*37,487,640*	*37,454,666*							
2008	*2,096,923*	*12,392,068*	*23,901,097*	*26,370,276*	*29,102,776*	*29,076,566*								
2009	*8,224,572*	*48,606,168*	*93,749,306*	*103,436,501*	*114,156,977*									
2010	*1,692,907*	*10,005,388*	*19,298,004*	*21,292,853*										
2011	*6,897,527*	*40,720,852*	*78,616,961*											
2012	*8,426,937*	*49,750,011*												

Backtesting Results

	Mean Ultimate Losses				Standard Deviation		
AY	Actuarial	Original	1YRremoved	2YRremoved	Original	1YRremoved	2YRremoved
1999	1,385,631	1,385,631	1,385,631	1,385,631	0	0	0
2000	1,632,333	1,632,332	1,632,332	1,632,332	93,156	94,252	96,777
2001	1,888,505	1,888,505	1,888,505	1,888,505	112,177	113,766	117,202
2002	2,417,283	2,417,283	2,417,283	2,417,283	144,678	146,827	151,507
2003	3,343,009	3,343,009	3,343,009	3,343,009	194,887	198,098	205,022
2004	4,305,538	4,301,237	4,301,237	4,380,954	263,387	268,167	280,845
2005	10,450,293	10,419,015	10,347,944	10,477,295	594,931	607,389	652,125
2006	14,703,562	14,598,229	14,265,147	13,926,305	800,542	832,008	859,923
2007	26,965,376	26,446,624	26,914,071	26,883,523	1,554,709	1,568,563	1,649,190
2008	27,885,793	27,894,614	27,987,319	28,965,962	1,650,713	1,693,468	1,832,646
2009	36,409,273	36,695,804	36,290,269	37,376,519	2,187,385	2,280,994	2,539,227
2010	52,978,716	51,793,479	52,933,577	54,313,720	3,181,078	3,398,056	3,790,854
2011	52,839,590	50,713,537	53,969,056	55,888,999	3,169,660	3,364,015	3,682,382
2012	52,454,173	52,831,730	55,810,282	54,873,864	3,202,132	3,362,646	3,822,556
Total	289,659,072	286,361,028	293,485,663	297,753,902	7,618,808	8,942,812	11,616,966





Practical Limitation

- There is no one method that works in all of the situations. No perfect method!
  - Mack (Late Claim Development)
  - Bootstrapping (Over-skewed Loss Distribution)
  - GLM (Tail Factor & Recent AYs' Trends)
  - Merz – Wüthrich (One Year Risk vs. Ultimate Risk)
  - Rehman – Klugman (Covariance Calculation)

Practical Limitation - Mack

- Practical Challenges—large latent claim dev
  - Personal Auto Liability
  - 2 large claims happened in the second last dev period

AY/DY	1	2	3	4	5	6	7	8	9	10	11	12
1900	3,199	4,473	4,768	5,319	5,503	5,518	6,144	6,148	6,161	6,158	7,524	7,524
1901	3,309	4,077	4,637	4,901	4,991	5,016	5,013	6,260	6,259	6,259	6,266	
1902	3,504	4,316	5,168	5,828	6,377	6,505	6,502	6,499	6,497	6,497		
1903	3,670	4,531	4,759	4,821	4,916	5,139	5,137	5,137	5,144			
1904	3,789	4,448	4,874	5,119	5,360	5,355	5,411	5,412				
1905	3,731	5,550	6,591	6,953	7,012	7,004	7,038					
1906	3,336	3,869	4,877	4,980	5,775	5,807						
1907	2,591	3,975	5,173	5,657	5,750							
1908	2,964	3,756	4,115	4,381								
1909	2,773	3,966	4,892									
1910	3,065	4,169										
1911	3,193											

Practical Limitation - Mack

- Practical Challenges—large latent claim dev
  - Low probability of reemergence
  - Mack method recognizes those 2 large claims in loss development factor calculation, which produces huge mean and variance estimation of this line's reserve.
  - Estimated CV of reserve is close to 1

Practical Limitation - Mack

- Practical Challenges—large latent claim dev
  - Solution: GLM is one solution
  - GLM allows actuaries to avoid adding those 2 claims in trends calculation, but still consider them in the total error calculation
  - GLM produces reasonable mean and variance

Practical Limitation - Bootstrapping

- Practical Challenges – Over-Skewed Loss Distribution
  - Bootstrapping chain-ladder produces CV close to 10 for the D&O loss triangle below

AY/DY	1	2	3	4	5	6	7	8	9	10	11	12
1900	97	4,974	13,242	9,369	9,654	18,015	17,980	16,850	16,840	16,841	16,706	16,706
1901	980	5,635	13,108	10,425	9,914	7,992	7,992	7,991	7,970	7,971	8,091	
1902	628	22,282	29,083	39,559	45,828	48,182	60,920	60,858	60,858	62,421		
1903	1,852	17,540	51,949	57,417	60,211	97,983	97,332	97,719	80,366			
1904	10,144	28,404	30,611	38,209	63,015	59,884	59,946	56,120				
1905	6,539	47,918	74,266	77,827	125,302	123,730	137,666					
1906	23,995	70,670	150,215	152,032	151,312	152,992						
1907	8,243	51,259	70,450	89,498	106,331							
1908	16,851	52,722	86,783	100,701								
1909	10,948	43,225	58,153									
1910	11,892	46,385										
1911	5,396											

Practical Limitation - GLM

- Practical Challenges – Tail Factor & Recent AYs' Trends
  - Loss triangle is not a standardized data set for regression

AY/DY	1	2	3	4	5	6	7	8	9	10	11	12
1900	1,750	2,446	2,608	2,909	3,010	3,018	3,361	3,363	3,370	3,368	3,368	3,368
1901	1,810	2,230	2,537	2,681	2,730	2,744	2,742	3,424	3,424	3,424	3,428	
1902	1,917	2,361	2,827	3,188	3,488	3,558	3,557	3,555	3,554	3,554		
1903	2,007	2,478	2,603	2,637	2,689	2,811	2,810	2,810	2,814			
1904	2,073	2,433	2,666	2,800	2,932	2,929	2,960	2,961				
1905	2,041	3,036	3,605	3,803	3,836	3,831	3,850					
1906	1,825	2,117	2,668	2,724	3,159	3,176						
1907	1,417	2,174	2,830	3,094	3,145							
1908	1,621	2,054	2,251	2,397								
1909	1,517	2,169	2,676									
1910	1,677	2,280										
1911	1,746											

Practical Limitation - GLM

- Practical Challenges – Tail Factor & Recent AYs' Trends
  - Due to limited data and regression mechanism, late DYs' trends (tail factor) and recent AYs' trends are often not treated as significantly different from previous years
  - With GLM model, actuaries are not easy to insert a different opinion other than what the data says

Practical Limitation - Merz - Wüthrich

- Practical Challenges – One Year Reserve Risk vs. Ultimate Reserve Risk
  - The one year reserve risk from Merz – Wüthrich method is often very close to the ultimate reserve risk from Mack method
  - In many cases, one year paid out loss is 30% to 70% of total reserve, but one year reserve risk is more than 90% of ultimate reserve risk

Practical Limitation - Merz - Wüthrich

- Practical Challenges – One Year Reserve Risk vs. Ultimate Reserve Risk
  - In the following example, Merz - Wüthrich one year CV is about 97% of Mack ultimate CV
  - GLM and bootstrapping are other possible solutions for one year risk

AY/DY	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1900				16,203	16,528	16,678	16,653	16,676	16,689	16,689	16,703	16,705	16,704	16,708
1901			130,921	138,722	142,266	138,540	138,555	138,519	138,523	138,551	138,566	138,563	138,577	
1902		9,573	16,670	18,300	19,739	20,304	20,719	20,668	20,675	20,675	20,724	20,741		
1903	511	9,288	13,701	16,677	18,498	18,926	18,934	18,937	19,608	19,761	19,761			
1904	11,465	55,340	64,381	68,802	69,535	69,965	71,345	71,433	71,514	71,515				
1905	7,863	170,323	265,191	294,006	365,445	374,159	374,672	376,826	377,842					
1906	6,878	17,911	40,803	45,503	48,134	48,638	50,045	50,119						
1907	2,680	11,699	32,012	38,474	39,012	39,726	39,767							
1908	9,115	73,797	118,041	140,587	143,091	144,788								
1909	2,353	12,286	23,437	28,123	30,502									
1910	26,596	52,333	62,343	69,859										
1911	28,704	63,937	84,689											
1912	3,919	19,656												
1913	10,031													

Practical Limitation - Rehman - Klugman

- Practical Challenges – Covariance Calculation
  - One step of Rehman – Klugman method is to calculate covariance matrix by DY
  - However, loss triangle is not a standard data set to calculate covariance matrix

AY/DY	1	2	3	4	5
1900	53,812	53,807	53,807	53,881	53,981
1901	49,031	49,031	49,031	49,031	
1902	71,691	71,187	71,187		
1903	71,858	70,853			
1904	57,980				

Practical Limitation - Rehman - Klugman

- Practical Challenges – Covariance Calculation
  - The inconsistency in covariance calculation may result in negative variance
  - The loss triangle below produces negative variance for AY 1908 cumulative LDF

AY/DY	1	2	3	4	5	6	7	8	9	10	11	12
1900		55,153	55,305	53,948	53,185	52,472	52,817	53,812	53,807	53,807	53,881	53,981
1901	57,018	56,217	48,828	49,970	49,939	48,913	49,021	49,031	49,031	49,031	49,031	
1902	66,255	61,413	63,238	74,397	69,387	69,453	69,632	71,691	71,187	71,187		
1903	65,956	68,998	69,519	71,261	71,722	76,211	77,237	71,858	70,853			
1904	62,982	63,193	61,404	60,315	59,416	58,467	57,251	57,980				
1905	69,217	71,556	70,593	69,314	69,570	68,985	68,753					
1906	61,734	60,609	59,760	59,444	59,162	59,223						
1907	38,768	39,588	38,564	35,611	33,739							
1908	46,858	48,348	51,103	51,212								
1909	30,177	31,790	30,434									
1910	33,699	33,679										
1911	48,675											

Applications of Stochastic Reserving

- Reserve Adequacy Assessment
  - Required in some countries' statutory report
- Reserve Risk for Capital Modeling
  - Reserve risk accounts for a significant portion of overall insurance risk
- Loss Reserve Margins
  - 75% level required in some countries like Australia and Malaysia
- Estimate of One-Year change in loss reserves
- Risk Aggregation
  - Unsolved problem: correlation of reserve risk

Correlation of Reserve Risk

- Causes of Correlation of Reserve Risk
  - Inflation Risk
  - Claim Management Change
  - Legislative Risk
  - Clash Risk
  - Reserving Cycle
  - More…

Quantification of Reserve Risk Correlation

- In most of the capital models, reserve risk correlation is determined by expert opinion
  - None (e.g. ρ=0%)
  - Low (e.g. ρ=25%)
  - Medium (e.g. ρ=50%)
  - High (e.g. ρ=75%)

Quantification of Reserve Risk Correlation

- How to quantify reserve risk correlation from loss data?
  - Historical Booked Reserve Change
  - Paid/Incurred Loss Triangle

Quantification of Reserve Risk Correlation

- Historical Booked Reserve Change
  - Booked Reserve Change = (Booked Reserve - Paid Loss in next 12 months – Remaining Reserve after 12 months)/ Booked Reserve
  - Easy to calculate
  - Require 10+ years experience
  - Cannot reflect business nature/claim management change promptly

Quantification of Reserve Risk Correlation

- Paid/Incurred Loss Triangle
  - Reserving Model Residuals Correlation
    - Loss Triangle A + $\varepsilon 1$vs. Loss Triangle A + $\varepsilon 2$;
    - Assume that there is a reserving model X can model A with zero residuals
  - GLM Model Trends Correlation
    - How to combine AY/DY/CY trends correlations?
    - Same loss triangles & different model settings may result in significantly different correlations
  - Implied Reserve Risk Correlation
    - Model loss triangle A, B and A + B
    - May not be suitable for different LOBs

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