Stochastic Reserving

CAE Spring 2006 Meeting

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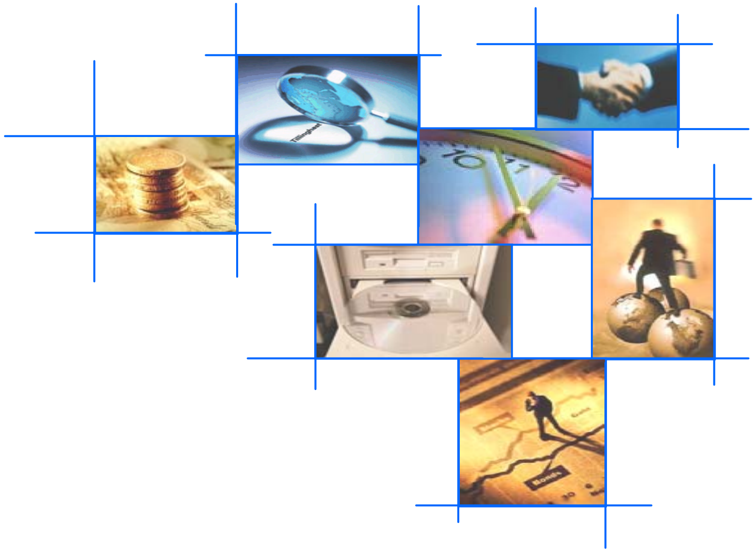
7 April 2006



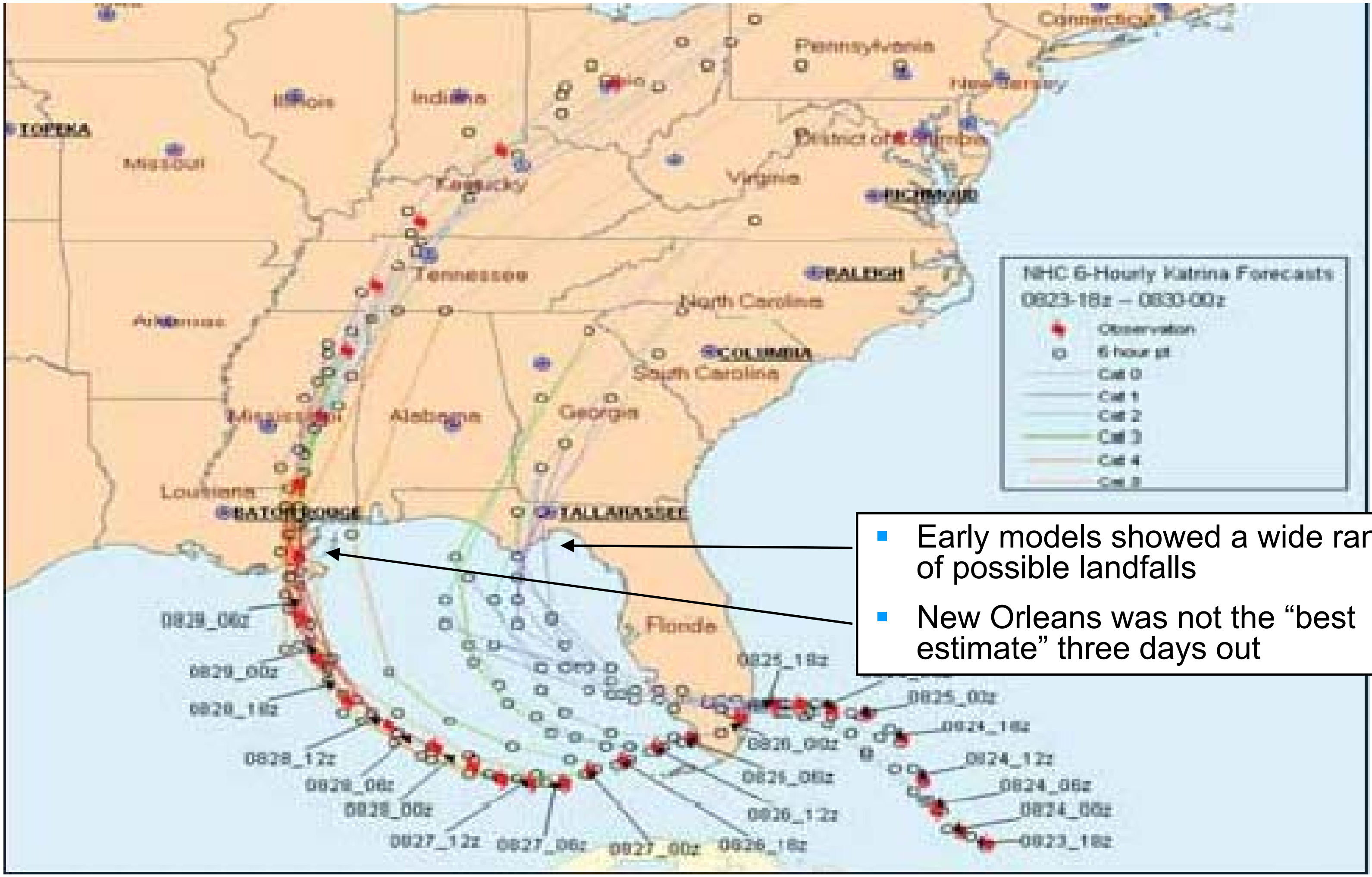
INTRODUCTION

OVERVIEW AND
BASIC TERMINOLOGY

STOCHASTIC RESERVING
METHODOLOGY

APPLICATIONS

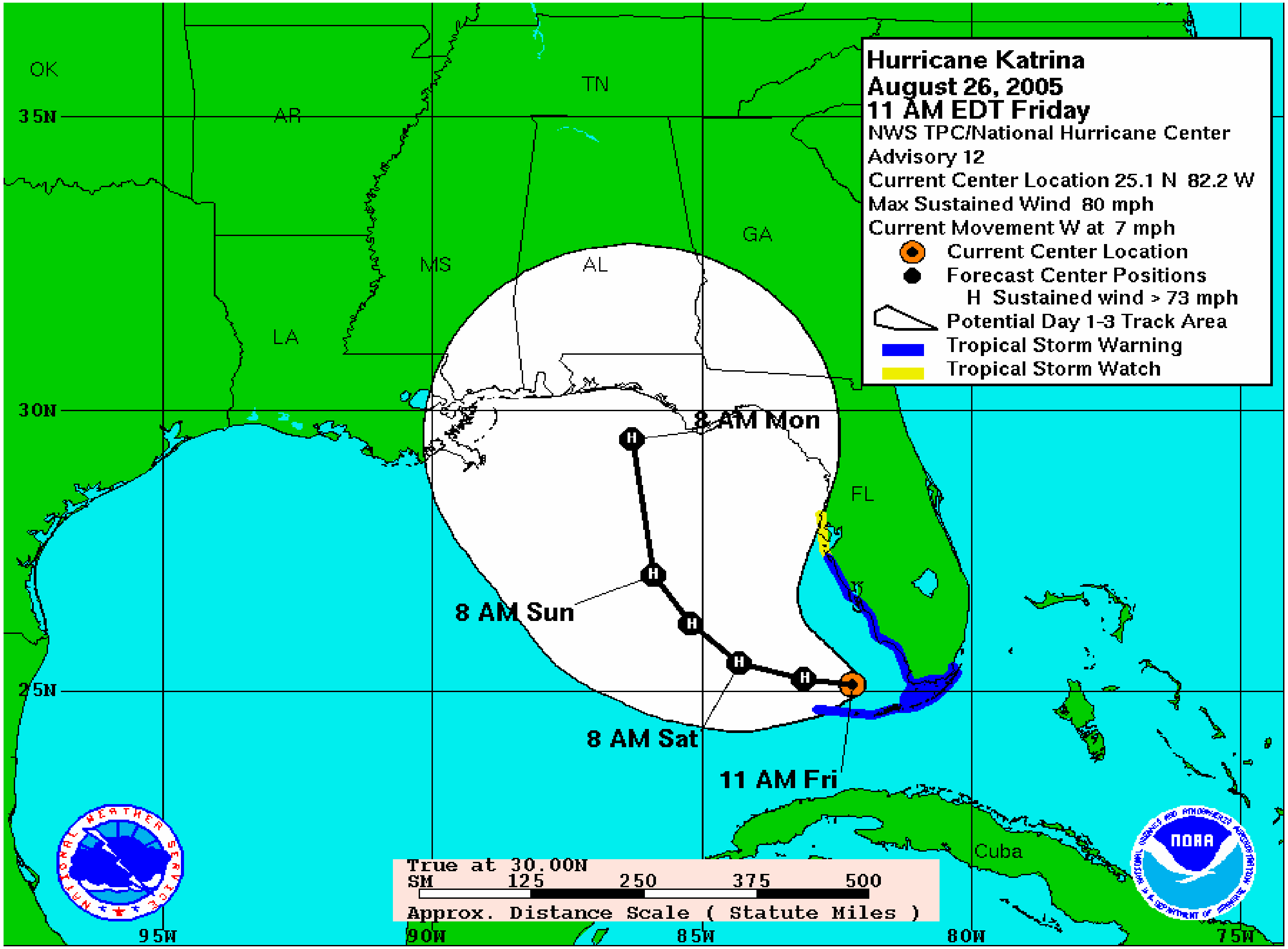
Estimating reserves is uncertain, just like predicting the path of a hurricane



- Early models showed a wide range of possible landfalls
- New Orleans was not the "best estimate" three days out

Source: RMS Report, "Hurricane Katrina: Profile of a Super Cat" October 2005.

Just like with hurricanes, it is prudent to consider more than just the best estimate in planning actions



Source: National Hurricane Center. www.nhc.noaa.gov

Why use stochastic reserving methods?

- Improve understanding of estimates of claim liabilities
- Formal method for measuring confidence intervals around the best estimate
- Application is driven by
  - Regulatory and compliance (Accounting and Solvency)
  - Financial and capital management
  - Operational/strategic excellence

There is increasing industry demand for stochastic techniques

- Evolving financial reporting requirements
  - IFRS / Fair value accounting
  - Public companies (SEC)
- Evaluation of capital requirements (Solvency II)
  - Quantitative Information Surveys
  - Commonly used to support UK ICA
- Actuarial Profession wants better practice
  - Institute of Actuaries (GRIT) task force
  - Highlighted by CAS Task Force on the Credibility of Actuaries

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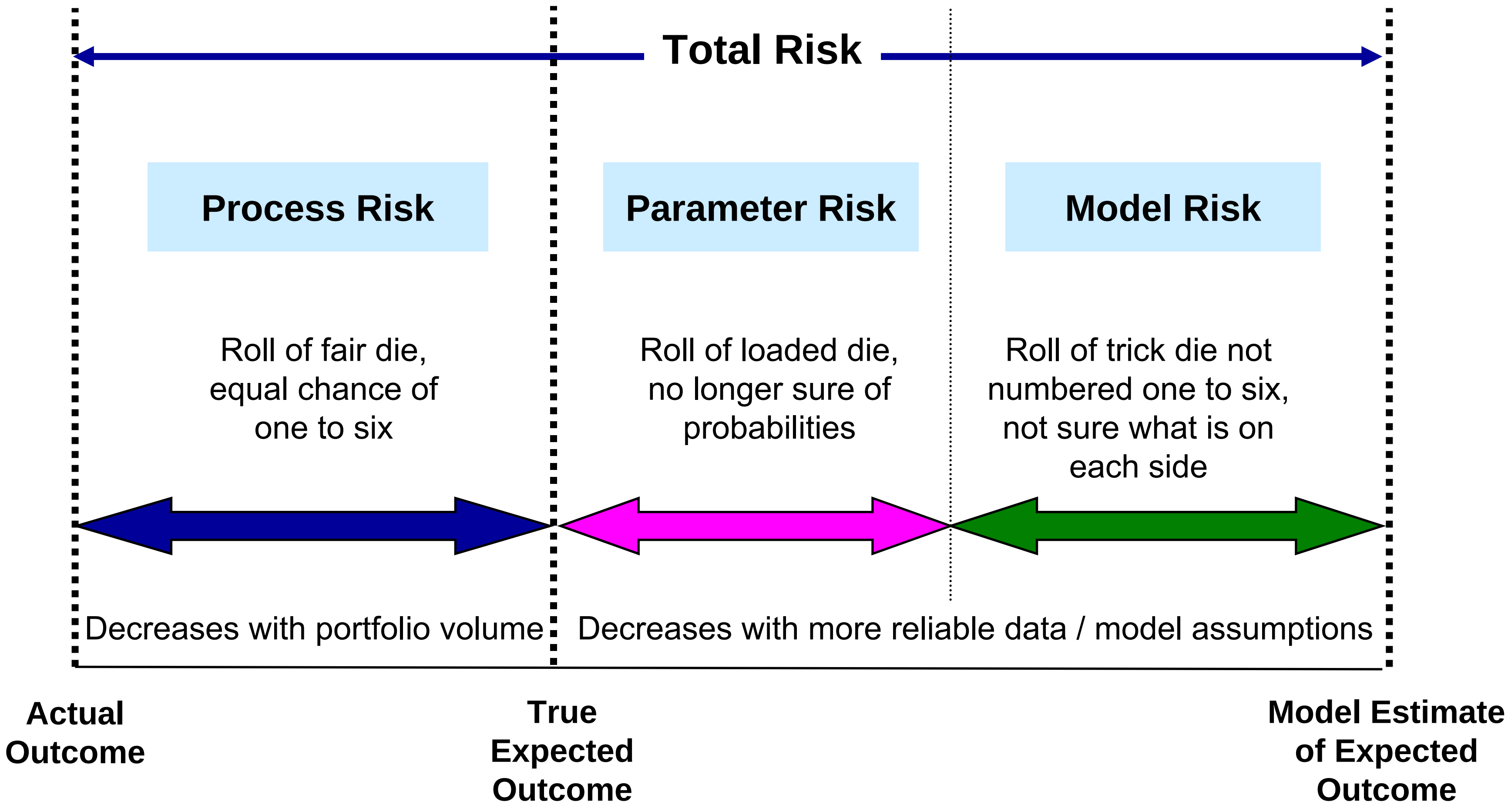
Stochastic techniques might best be thought of relative to our traditional, deterministic techniques

- Deterministic methods provide a point estimate of claim liabilities
  - Best estimate usually chosen judgmentally
  - No assessment of the difference to be expected between the estimate and the real future payments.
- Loss development is a stochastic process; the historical data is a specific realization
- Stochastic methods are more informative
  - Produce a full distribution of possible outcomes
    - — Confidence levels of held reserves
  - Consider the volatility of the unpaid claims
    - — Individual lines
    - — Correlation across the various lines

Several distinct types of risks are inherent in the measurement of claim liabilities; the role of each depends on the context



Types of stochastic reserving methods: (1) Cumulative methods

- Chain-ladder-type approach
- Methods that measure the variability of the link ratios
- Generally require user to select the form of the distribution (e.g., normal or lognormal)
- These models produce mean estimates consistent with the deterministic chain-ladder algorithm
- Example:
  - Mack method

Types of stochastic reserving methods: (2) Simulation methods

- It is often easier to use simulation techniques rather than deriving a mathematical formula
- The underlying assumption is that the simulated data has the same statistical properties as the observed data
- Examples:
  - The Feldblum method assumes that historical link ratios are realizations from an assumed distribution
  - The Bootstrapping method assumes that the residuals of the actual triangle and the chain ladder implied triangle are distributed around zero
  - Frequency/Severity methods that employ separate assumptions regarding the frequency and severity distributions of claims

Types of stochastic reserving methods: (3) Bayesian methods

- The other methods derive parameters directly from historical data
- Bayesian methods instead, like Bornhuetter-Ferguson, employ the data as a second step to refine the user’s initial expectations of the parameters
  - User makes assumptions regarding the “prior/initial” distribution of the parameters (for example the link ratios and ultimate losses)
- Examples:
  - Verrall 2000
  - Practical Method

Types of stochastic reserving methods: (4) Incremental Approaches

- Methods based on incremental claim payments
- These methods fit curves across accident/development/calendar year dimensions
- Examples:
  - GLM-type approaches
  - Christofides method
  - Mack's additive model

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MACK METHOD

GLM-BASED METHODS

PRACTICAL METHOD

VERRALL'S BAYESIAN METHOD

BOOTSTRAPPING

AGGREGATION

COMMENTS

Mack Method: Basics & Assumptions

- The Mack Method estimates the prediction error of chain-ladder estimates.
- It is probably the most common stochastic reserving method.
- The basic assumptions underlying the stochastic Mack method are the same as the ones for the deterministic chain-ladder.
- Chain-Ladder Basic Assumptions:

(CL1)	$E(F_{ik} \mid C_{i1}, \ldots, C_{ik}) = f_k,$	$1 \le i \le n,\ 1 \le k \le n-1,$
(CL2)	$\mathrm{Var}(F_{ik} \mid C_{i1}, \ldots, C_{ik}) = \frac{\sigma_k^2}{w_{ik} C_{ik}^{\alpha}},$	$1 \le i \le n,\ 1 \le k \le n-1,$
(CL3)	The accident years $(C_{i1}, \ldots, C_{in})$, $1 \le i \le n$, are independent.	

(Source: Mack, Th. (1999), The Standard Error of Chain Ladder Reserve Estimates: Recursive Calculation and Inclusion of a Tail Factor)

Mack method: Steps

- Analytical approach using a recursive formula:
  - $$(\text{s.e.}(\hat{C}_{i,k+1}))^2 = \hat{C}_{ik}^2\left((\text{s.e.}(F_{ik}))^2 + (\text{s.e.}(\hat{f}_k))^2\right) + (\text{s.e.}(\hat{C}_{ik}))^2\hat{f}_k^2$$
- It is a distribution-free approach as only the mean and the standard error are estimated.
- The mean estimate produced by the Mack Method is equal to the standard chain-ladder point estimate (volume-weighted-all-years).
- The estimates for the standard error of the claim liabilities are based on the variability of the triangle.
- To derive percentiles or ranges, an additional assumption has to be made on the distribution of future payments (e.g., normal or lognormal)

Mack method: Split of process and parameter risk

- The total risk can be split into process error and estimation error. The mean squared error of the estimator $\hat{C}_{iI}$ for the ultimate cumulative claims C_{iI} for accident year *i* is defined as

$$mse(\hat{C}_{iI}) = E((\hat{C}_{iI} - C_{iI})^2 \mid D)$$
$$= \underbrace{Var(C_{iI} \mid D)}_{\text{Process Risk}} + \underbrace{(E(C_{iI} \mid D) - \hat{C}_{iI})^2}_{\text{Parameter Risk}}$$

where *D* is the set of all data observed so far.

Mack Method: Summary

- **Advantages**
  - Intuitive, chain-ladder-like parameters
  - Most commonly used
  - Widely accepted
  - Distribution-free approach (basic model only)
  - Easy & fast implementation
  - Can be extended to a consistent model for multiple lines
    (Model by Braun)

- **Disadvantages**
  - Does not allow for input of actuarial judgement
  - Percentiles only available if additional assumption on distribution is made
  - Works best on paid claims

GLM-based methods: Basics and Assumptions

- The most popular GLM model assumes that each cell of the incremental claims triangle is distributed over-dispersed Poisson:
  - C_{ij} ~ Poisson(m_{ij}) ; E[C_{ij}] = x_i*y_j ; Var(C_{ij}) = φ*E[C_{ij}],
  - Overdispersion means that the variance is proportional to the mean (instead of identical)
- The basic GLM-model assumes an accident year (*x*) and a calendar year (*y*) dimension and a logarithmic link-function
- Luckily, this boils down again to a standard chain-ladder approach!
- As a second step, bootstrapping techniques can be used to evaluate the distribution of future payments
- Other GLM-based approaches:
  - Negative binomial distribution
  - Normal distribution
  - Generalized additive methods (GAM)

GLM-based methods: Summary

- **Advantages**
  - Commonly accepted
  - Consistency
    (The Overdispersed Poisson model exactly replicates the Chain-Ladder-estimate)

- **Disadvantages**
  - Rather scientific approach
  - Not very intuitive
  - No possibility to include prior knowledge
  - Overdispersed Poisson model cannot handle negative payments
    (→ assume Normal distribution instead, but cannot replicate chain-ladder results any more then)

Practical Method: Basics and Assumptions

- Belongs to the Bayesian family of stochastic reserving methods
- Created in 2000 by a working party of the UK Institute of Actuaries chaired by Graham Lyons of Tillinghast
- Underpinnings are the standard deterministic reserving methodologies
- A stochastic version of the B-F methodology and the standard chain ladder methodology
- Requires heavy use of actuarial judgment

Practical Method: Steps

- STEP 1: Start with a standard chain ladder and BF-reserve analysis, allowing for hand-smoothed development factors, based on observed averages and other available information
- STEP 2: Selection of standard errors for each development period, incl. tail standard error
- STEP 3: Selection of standard error for initial expected loss ratios (IELR)
- STEP 4: Running the simulation

Practical Method: Standard Error Selections

- The standard errors are proportional to R(j), where 1+R(j) is the selected age-to-age development factor
- For older accident years, there is still variation even though R(j) are close to 0. For these years it is assumed that the standard deviation is constant
- Distribution for each column of development factors – lognormal assumption for early development ages, normal assumption for later development ages
- Stochastic IELR's based on normal distribution with deterministic values as means and calculated standard deviation (use latest 5 to 7 years)

Practical Method: Summary

- **Advantages**
  - Intuitive, chain-ladder-like parameters
  - Consistent to deterministic approach including actuarial judgement
  - B-F approach produces stable results even when chain ladder results are highly skewed by particular outliers
  - Guidance on tail standard error selection
  - Flexibility
- **Disadvantages**
  - Exposure information needed (like BF-approach)
  - Leveraged to tail standard error selection
  - Does not differentiate between process and parameter risk
  - Flexibility
  - Results can be leveraged by subjective judgement

Verrall's Bayesian method: Basics and Assumptions

- The method starts with an overdispersed Poisson GLM-approach, which is then extended to a Bayesian model by introducing prior distributions on the row and column parameters x and y.
- The actuary makes additional assumptions regarding the "prior/initial" distribution of the parameters
- Based on the historical data and Bayes theorem a "posterior/final" distribution of the parameters is produced
- Typically, it is too complex to calculate the mean or variance analytically. So called Markov Chain Monte Carlo (MCMC) simulations produce the distribution of the parameters and the future payments

Verrall‘s Bayesian method: Summary

- **Advantages**
  - Very promising approach,
  - Can be interpreted as a stochastic BF
  - Excellent possibility to include prior knowledge and actuarial judgement
- **Disadvantages**
  - Not very much used in practice (yet?!)
  - Sophisticated model, Non-intuitive parameters
  - Not suitable for Excel, professional programming environment needed

Bootstrapping: Basics and Assumptions

- Bootstrapping is a general approach of estimating the distribution of parameters used in a variety of contexts
- Creating many sets of “pseudo data” by sampling with replacement from the observed data results in a simulated distribution of the parameters
- Bootstrapping does not care about the underlying distribution – instead bootstrapping assumes that the historical observations contain sufficient variability in their own right to help us predict the future
- But the resampling algorithm calculates parameter risk only!
  - To take into account process risk as well, an additional simulation is carried out for the future incremental payments, based on the assumed distribution in the underlying model (e.g. GLM overdispersed Poisson).

Bootstrapping: Steps

- STEP 1: Start with a cumulative triangle. Keep current diagonal intact. Apply average link ratios to “back-cast” a series of fitted historical payments
- STEP 2: Convert both actual and fitted triangles to incrementals
- STEP 3: Look at difference between fitted and actual payments to develop a set of residuals: r_p= (P-m) / SQRT(m)
- STEP 4: Residuals adjusted for degrees of freedom
- STEP 5: Create a “false history” by making random draws, with replacement, from the triangle of residuals. Combine the random draws with the recast historical data to come up with the “false history”.
- STEP 6: Calculate link ratios from the data in the cumulated false history triangle and use the link ratios to square the false history data triangle
- STEP 7: Simulate from original model and results from step 6
- Iteration of resampling and simulation (steps 5 to 7); keep reserve estimates
- Prediction error is then the standard deviation of results

Bootstrapping: Summary

- **Advantages**
  - Easy to implement
  - Commonly used
  - Method can be applied to incomplete data triangles (i.e. trapezoids)

- **Disadvantages**
  - No possibility to include prior knowledge or actuarial judgement
  - Method does not work well with negative loss development (due to underlying ODP model)
  - Data outliers can have a leveraged effect on the results

Aggregation: Correlation between Lines of Business

- Strength of the correlation is irrelevant if we only care about the mean reserve indication for two lines A and B:
  - mean(A + B) = mean(A) + mean(B)
- Strength of correlation matters when we look towards the ends of the aggregate distribution of (A+B)!
- Generally, the aggregate distribution is less risky than the distribution of the individual lines:
  - 75^{th}percentile(A + B) < 75^{th}percentile(A) + 75^{th}percentile(B)
  - Equality only occurs in the case of perfect correlation across lines (this is very unlikely!)
- The volatility of the aggregate distribution increases:
  - By the volatility of the individual lines
  - By the correlation between the lines

Aggregation: Correlation Modelling

- Summing up all triangles and projecting just one big segment
  - ➔ Inconsistent approach, not recommended
- The Mack method can be extended to a multi-class model
- Bootstrapping can also be done simultaneously for various lines
  - ➔ Ok, but only applicable when the same approach is used for all classes (and all accident years)
- Construction of an aggregated distribution from the marginal distributions and the correlation between classes of business by means of a copula approach
  - ➔ Flexible, reliable approach, but
    - — "Correlation" must be well-defined
    - — Estimating correlation from the existing data is not always straight-forward
- Copulas vary the degree of association over the aggregate distribution (e.g.: for property losses, the correlation is higher in the tail of the distribution)

Comments (1 of 2)

- Calendar year effects can be critical and may cause an overstatement of the reserve uncertainty.
- If possible, one-time effects should be taken out of the data and the triangles be transferred to a stationary basis before carrying out stochastic calculations.
- Large claims can distort the calculations and sometimes need be simulated separately.
- Tail volatility is an important factor. It can be assessed externally (benchmarks) or estimated from the data

Comments (2 of 2)

- No one-size-fits-all approach
- No single method can handle all situations
(This is true for deterministic methods as well!)
- Essential to check model assumptions and reasonableness of results
- Benchmarks on variance and correlation are an important part of the process

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Purposes of evaluating loss reserve uncertainty

Compliance/Financial Reporting

- Regulators
  - NAIC
  - SEC
  - CEIOPS
- Rating Agencies
  - AM Best
  - S&P
- Actuarial Profession
  - CAS
  - GRIT

Financial/Capital Management

- Anticipate potential for "bad news"
- Capital management
  - Capital needs
  - Capital Allocation
  - Optimize Reinsurance

Operational/Strategic Excellence

- Monitor results
  - Early warning system
  - What deviations from plan are significant?
- Growth strategies
- Evaluate investments

Originally applications were in this category (Compliance/Financial Reporting)

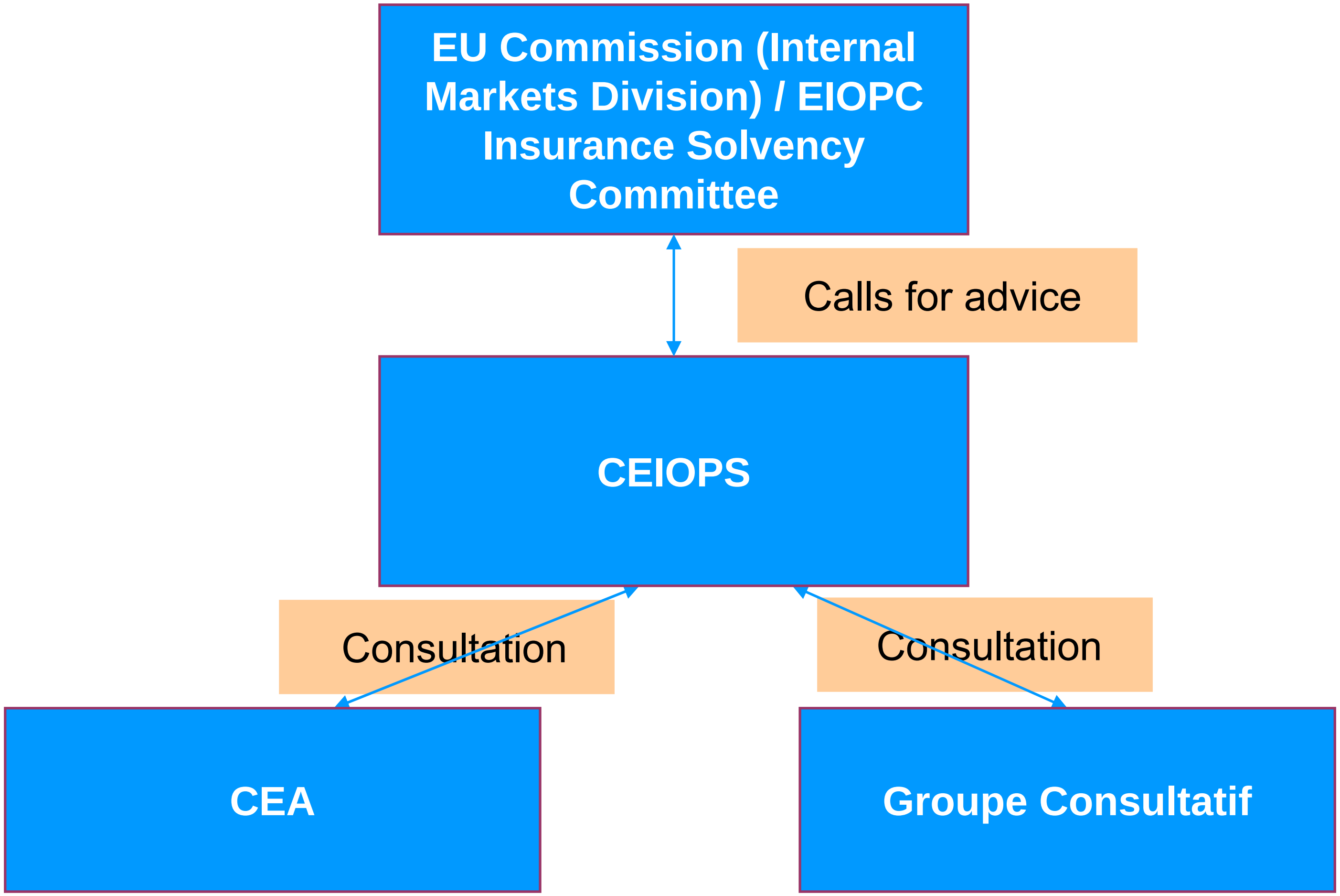
Increasingly used for value-added applications in these categories (Financial/Capital Management; Operational/Strategic Excellence)

International Accounting Standards

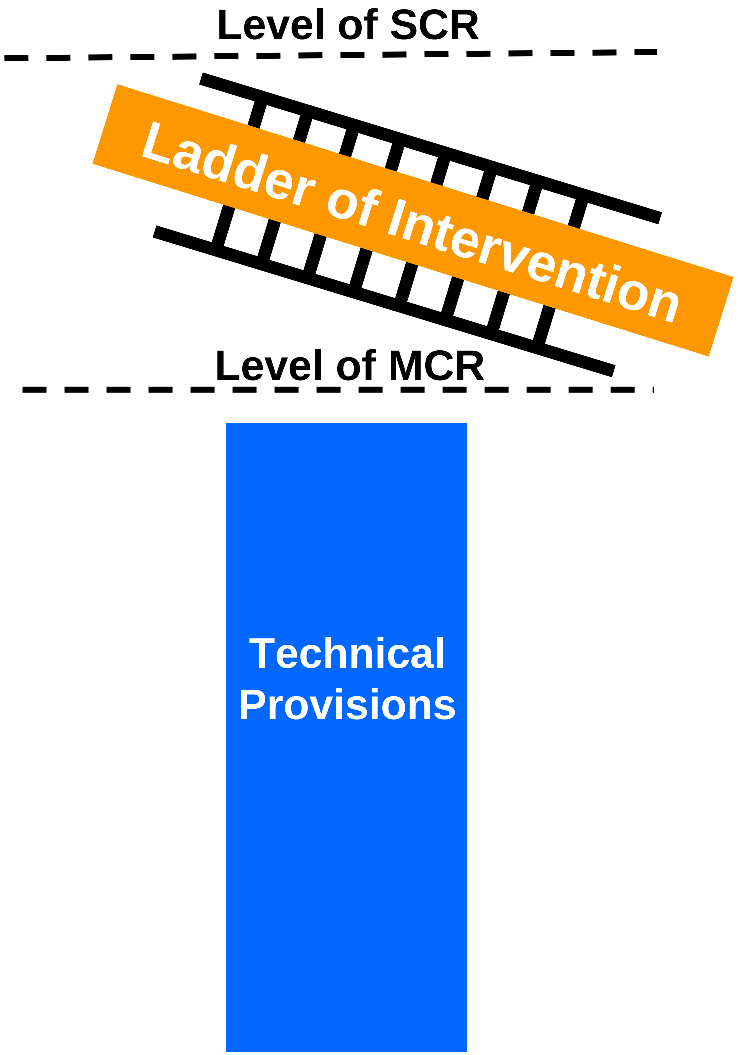
- Current proposals for new accounting standards in IFRS Phase II on market-consistent valuation of claims reserves required:
  - Discounted claims reserves…
  - risk margins based on uncertainty
- The form of the risk margins is still under consideration
- A number of countries (Australia, Malaysia, Canada) require (discounted) reserves to include explicit prudence levels (e.g. 75th percentile).

Stochastic reserving techniques can provide a basis to calculate risk margins

Solvency II – Organisational Structure



Main Reference Points for Solvency II Framework



- Technical Provisions – The amounts set aside to fulfil obligations towards policyholders and other beneficiaries - may include **some element of prudence**

- Solvency Capital Requirement (SCR) - The level of capital that enables an institution to absorb significant unforeseen losses and that gives reasonable assurance to policyholders and beneficiaries

- Minimum Capital Requirement (MCR) - A safety net that reflects a level of capital below which ultimate supervisory action would be triggered

Solvency II: QIS1 – Request from CEIOPS

- QIS 1 ran from October 2005 to December 2005 and asked for details of technical provisions incorporating risk margins for both life and general insurance
  - Inclusion of 75th percentile and 90th percentile risk margins in the insurance assumptions with the impact of the "deposit floor" shown – no group diversification allowed for in risk margins
  - In addition, participating firms were invited to bring to CEIOPS' attention the results of risk margins based on the 60th percentile and/or margins based on a cost-of-capital approach
  - Risk margins did not apply to financial elements of basis which were market-consistent with no allowance for "own credit risk".
- Instructions confirmed use of swap yields for risk free rates – no exemption for illiquid liabilities.

Solvency II: QIS2 – Request from CEIOPS

- QIS2 expected to run from May 2006 to July 2006
- Goals of QIS2 are threefold. CEIOPS hopes to:
  - Receive feedback on the practicality of the calculations
  - Learn what the possible impact on the balance sheets and the amount of capital that might be needed, will be
  - Look for qualitative and quantitative information about the suitability of the possible approaches to the calculation of the SCR
- QIS2 will ask participating companies to calculate Solvency Capital Requirements ("SCR")
  - The market value of the liabilities will be an important starting point in the calculations
    - — Inclusion of 75th percentile risk margins
    - — Optionally, the cost of capital approach, may be used to derive the risk margins. The insurance industry seems to favour this approach.

Conclusions

- Stochastic reserving provides insight into reserve uncertainty
- Stochastic models are more complex than traditional models but this makes actuarial analysis and judgment even more important
- Regulatory focus on risk management and disclosure is increasing demand for stochastic analysis
  - Solvency II encourages internal capital modelling
  - SEC disclosures on uncertainty
- Stochastic reserving is here to stay and usage will increase over time

Stochastic Reserving Methods - A Practical Comparison

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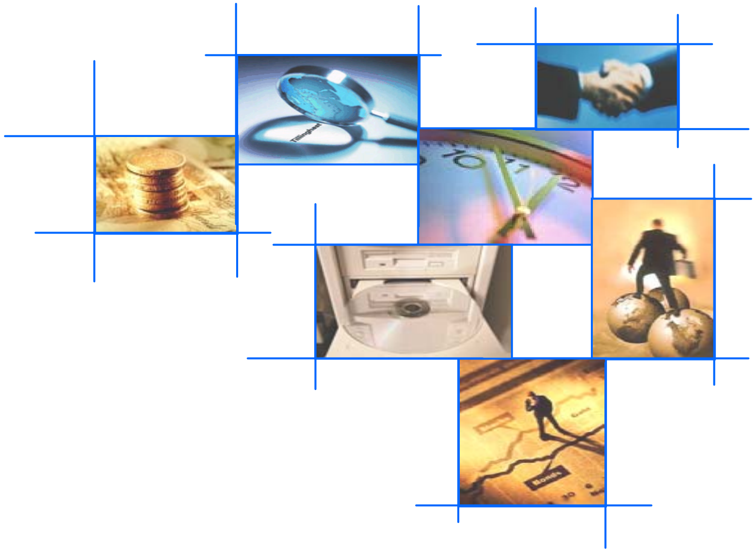

APPENDIX: Literature

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