Modelling claims reserving risks for solvency purposes

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Overview

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4. Controlling Financial Risks

0. Announcement

Lecture at ETH Zürich SS2006 on

"Market-consistent Actuarial Valuation"

Mario Wüthrich (ETH Zürich) and Hansjörg Furrer (SwissLife)

Place: ETH Zürich, HG D3.2

Time: Mondays, 16:15pm - 18pm

Start: Monday, April 3, 2006

1. Motivation

Solvency - What's that?

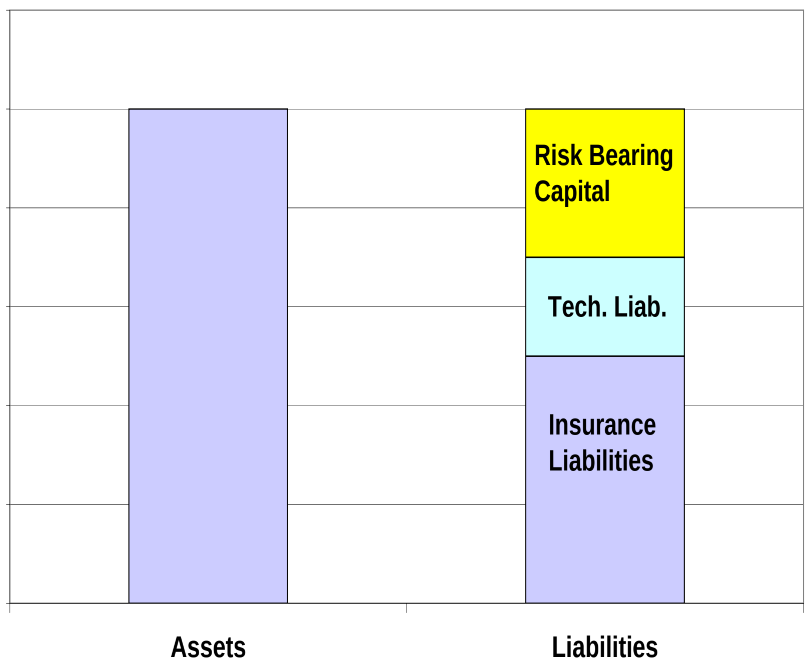
International Association of Insurance Supervisors IAIS says:

"the ability of an insurer to meet its obligations (liabilities) under all contracts at any time".

- Risk Bearing Capital RBC = Available Solvency Surplus = Assets - Liabilities

- Target Capital TC = Required Solvency Margin

$$\text{TC} \overset{!!!}{\leq} \text{RBC}$$



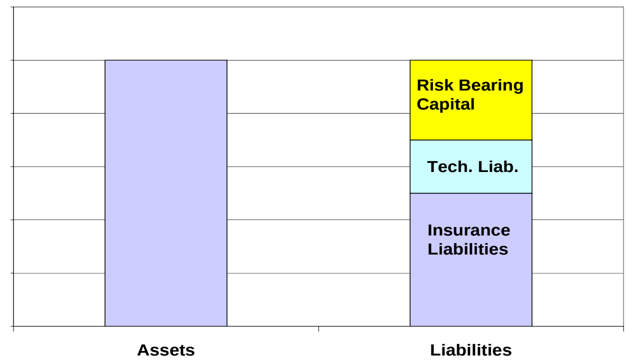
Sharma [6]: *"Financial strength is only the 2nd best strategy"*.

Calculation of RBC and TC

- **Past**
  - ⋆ Evaluation of RBC was not based on a market-consistent valuation.
  - ⋆ Target Capital was not risk-adjusted: E.g.

$$\text{Target Capital } = 16\% \text{ of premium.}$$

- **Future**
  - ⋆ Market-consistent valuation of Risk Bearing Capital
  - ⋆ Target Capital reflects risks in the portfolio $\Rightarrow$ risk-adjusted.



2. Construction of the Valuation Portfolio (VaPo)

Insurance liabilities are studied in **loss development triangles**:

Development triangle of incremental payments

Accident	Loss development periods									
years	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1996	357'848	766'940	610'542	482'940	527'326	574'398	146'342	139'950	227'229	67'948
1997	352'118	884'021	933'894	1'183'289	445'745	320'996	527'804	266'172	425'046	
1998	290'507	1'001'799	926'219	1'016'654	750'816	146'923	495'992	280'405		
1999	310'608	1'108'250	776'189	1'562'400	272'482	352'053	206'286			
2000	443'160	693'190	991'983	769'488	504'851	470'639				
2001	396'132	937'085	847'498	805'037	705'960					
2002	440'832	847'631	1'131'398	1'063'269						
2003	359'480	1'061'648	1'443'370			**Unknown Triangle**				
2004	376'686	986'608								
2005	344'014									

Incremental payments are denoted by $X_{i,j}$ with indices

$i = 1996, \ldots, 2005$ is the accident year,

$j = 0, \ldots, 9$ is the development year.

We define cumulative payments as follows

$$C_{i,j} = \sum_{k=0}^{j} X_{i,k}. \qquad (1)$$

Expected Payments and Reserves

$X_{i,j}$ are known in the upper triangle $\mathcal{D} = \{(i,j) : \ i + j \leq 2005\}$, and have to be estimate in the lower triangle $\mathcal{D}^c = \{(i,j) : \ i + j > 2005\}$.

There are various methods to estimate the expected incremental (cumulative, resp.) payments, i.e. estimate for $i + j > 2005$

$$E\left[X_{i,j} \middle| \mathcal{D}\right] \quad \text{and} \quad E\left[C_{i,j} \middle| \mathcal{D}\right], \text{ resp.} \tag{2}$$

Methods: Chain-ladder methods, Bornhuetter-Ferguson methods, GLM methods, credibility methods, bootstrap methods, etc.

For illustrative purposes we choose the Chain-ladder method on cumulative payments.

Example chain-ladder method on $C_{i,j}$

Loss development factors on cumulative payments

Accident years	Loss development periods 0	1	2	3	4	5	6	7	8	9
1996		3.1432	1.5428	1.2783	1.2377	1.2092	1.0441	1.0404	1.0630	1.0177
1997		3.5106	1.7555	1.5453	1.1329	1.0845	1.1281	1.0573	1.0865	
1998		4.4485	1.7167	1.4583	1.2321	1.0369	1.1200	1.0606		
1999		4.5680	1.5471	1.7118	1.0725	1.0874	1.0471			
2000		2.5642	1.8730	1.3615	1.1742	1.1383				
2001		3.3656	1.6357	1.3692	1.2364					
2002		2.9228	1.8781	1.4394						
2003		3.9533	2.0157			**Unknown Triangle**				
2004		3.6192								
2005										
CL Factors		**3.4906**	**1.7473**	**1.4574**	**1.1739**	**1.1038**	**1.0863**	**1.0539**	**1.0766**	**1.0177**

Estimated incremental payments

Accident years	Loss development periods 0	1	2	3	4	5	6	7	8	9
1996										
1997										94'634
1998									375'833	93'678
1999								247'190	370'179	92'268
2000							334'148	226'674	339'456	84'611
2001						383'287	351'548	238'477	357'132	89'016
2002					605'548	424'501	389'349	264'121	395'534	98'588
2003				1'310'258	725'788	508'792	466'660	316'566	474'073	118'164
2004			1'018'834	1'089'616	603'569	423'113	388'076	263'257	394'241	98'266
2005		856'804	897'410	959'756	531'636	372'687	341'826	231'882	347'255	86'555

Expected payments per accounting year

Assume that the latest given accounting year is $I = 2005$ ($\Rightarrow \mathcal{D}$).

Payments in the subsequent accounting years (diagonals) are $k \geq 1$

$$Y_k = \sum_{i+j=I+k} X_{i,j}. \qquad (3)$$

I.e. the insurance liabilities generate a random cashflow $\{Y_k\}_{k \geq 1}$.

Task: Value this cashflow!

1st approach VaPo construction

Construct a multi-dimensional liability portfolio:
Choose a basis/units $Z^{(k)}$ (financial instruments).

Multidimensional Valuation Portfolio (VaPo):

period	instrument	cashflow		number of units
$t = 1$	$Z^{(1)}$	Y_1	$\longrightarrow$	$E\left[Y_1 \mid \mathcal{D}\right]$
$t = 2$	$Z^{(2)}$	Y_2	$\longrightarrow$	$E\left[Y_2 \mid \mathcal{D}\right]$
$\vdots$	$\vdots$	$\vdots$		$\vdots$
$t = k$	$Z^{(k)}$	Y_k	$\longrightarrow$	$E\left[Y_k \mid \mathcal{D}\right]$
$\vdots$	$\vdots$	$\vdots$		$\vdots$

$\Longrightarrow$ look for independent decoupling basis/number of units.

The basis could be zero-coupon bonds, inflation protected zero-coupon bonds, etc.

VaPo 1st approach (1/2)

Estimated incremental payments

Accident years	Loss development periods 0	1	2	3	4	5	6	7	8	9	Accounting Year Payments
1996											
1997										94'634	5'226'536
1998									375'833	93'678	4'179'394
1999								247'190	370'179	92'268	3'131'668
2000							334'148	226'674	339'456	84'611	2'127'272
2001						383'287	351'548	238'477	357'132	89'016	1'561'879
2002					605'548	424'501	389'349	264'121	395'534	98'588	1'177'744
2003				1'310'258	725'788	508'792	466'660	316'566	474'073	118'164	744'287
2004			1'018'834	1'089'616	603'569	423'113	388'076	263'257	394'241	98'266	445'521
2005		856'804	897'410	959'756	531'636	372'687	341'826	231'882	347'255	86'555	86'555

Present value of Reserves, monetary value of VaPo

										monetary value of VaPo
Constant interest rate	1.50%									**17'873'967**
Risk-free rate 2005	0.88%	1.14%	1.36%	1.57%	1.75%	1.91%	2.05%	2.18%	2.29%	**17'847'512**
Nominal										**18'680'856**

Observe:

Our instruments $Z^{(k)}$ are evaluated with 3 different accounting principles (constant interest rate, risk-free rate 2005, nominal).

The resulting differences in the monetary reserves are substantial.

VaPo 1st approach (2/2)

- Application of accounting principle gives a monetary value to VaPo.
- VaPo contains only best estimate reserves (estimated conditional expectations, pure risk premium).
- Add a protection against technical risks for:
  - ⋆ process variance, pure stochastic error, volatility of claims
  - ⋆ estimation error, parameter error, model error, e.g. in the chain-ladder model we need to estimate parameters.
- The protection margin in the example is calculated with the help of the mean square error of prediction (MSEP) for accounting year payments (see [1]) and an appropriate risk measure.

2nd approach VaPo protected against technical risks

To avoid a.s. ruin we charge a loading for the protection against technical risks (Y_k are random variables).

period	instrument	cashflow		number of units
$t = 1$	$Z^{(1)}$	Y_1	$\longrightarrow$	$E[Y_1 \vert \mathcal{D}] + i \cdot \rho_1$
$t = 2$	$Z^{(2)}$	Y_2	$\longrightarrow$	$E[Y_2 \vert \mathcal{D}] + i \cdot \rho_2$
$\vdots$	$\vdots$	$\vdots$		$\vdots$
$t = k$	$Z^{(k)}$	Y_k	$\longrightarrow$	$E[Y_k \vert \mathcal{D}] + i \cdot \rho_k$
$\vdots$	$\vdots$	$\vdots$		$\vdots$

• ρ_k is a risk measure corresponding to $Y_k|\mathcal{D}$ denoting the "uncertainty" in our best estimates $E[Y_k|\mathcal{D}]$, $k \geq 1$.
• i denotes the cost-of-capital spread we have to pay for the risk margin on the financial market.

VaPo protected against technical risk

	Accounting Year	VaPo 1st approach	Protection against technical risks	VaPo protected 2nd approach
i=8%	2006	5'226'536	123'866	5'350'402
	2007	4'179'394	123'620	4'303'015
rho=	2008	3'131'668	117'133	3'248'801
99% VaR	2009	2'127'272	91'842	2'219'113
	2010	1'561'879	73'114	1'634'993
	2011	1'177'744	53'326	1'231'069
	2012	744'287	32'492	776'779
	2013	445'521	28'665	474'186
	2014	86'555	13'829	100'383

	monetary value	**cost for protection**	**monetary value**
Constant interest rate	**17'873'967**	**624'031**	**18'497'998**
Risk-free rate 2005	**17'847'512**	**620'657**	**18'468'169**
Nominal	**18'680'856**	**657'886**	**19'338'741**

Observe:

We have replaced $E\left[Y_k \middle| \mathcal{D}\right]$ by $E\left[Y_k \middle| \mathcal{D}\right] + i \cdot \rho_k$, i.e.

charge the cost-of-capital for the security margin.

We do not say anything about the **availability** of the capital!

Valuation Portfolio conclusions

- On the liability side of our balance sheet we have constructed a valuation portfolio VaPo with deterministic cashflows.
- The technical risks are now absorbed by the investor/shareholder.
- This VaPo is a multidimensional vector which consists of financial instruments (comparable to asset side of balance sheet).
- So far, we have not considered financial and ALM risks.
- Important: ρ_k should also charge for model and parameter risk! Pay attention to dependencies!
- Only an accounting principle gives a monetary value to VaPo.

3. Financial Risks and Solvency

We denote by S the existing portfolio on the asset side.

Financial risks derive from the fact that S and VaPo differ.

Definition: A company is solvent if (accounting condition)

$$\mathcal{A}_0(S) \geq \mathcal{A}_0(\mathsf{VaPo}) \tag{4}$$

and (insurance contract condition)

$$\mathcal{A}_k(S) \geq \mathcal{A}_k(\mathsf{VaPo}) \qquad \text{for all } k > 0, \tag{5}$$

where $\mathcal{A}_k$ is the appropriate (linear) accounting principle at time k.

Asset-Liability Matching

a) Prudent solution:

- Choose $S = \text{VaPo} + F$.
  F is free reserve/excess capital, s.t. $\mathcal{A}_k(F) \geq 0$ for all $k \geq 0$.

b) Realistic situation:

- S does not contain VaPo.

- ALM mismatch is often wanted: Additional risks imply additional chances to earn money.

- ALM mismatch asks for additional protection against financial risks.

4. Controlling Financial Risks

Decompose the asset portfolio in 3 parts:

$$S = \widetilde{S} + M + F, \tag{6}$$

- $\mathcal{A}_0(\widetilde{S}) = \mathcal{A}_0(\text{VaPo})$ (accounting condition),
- M is the margin for financial risks,
- F is the free reserve with $\mathcal{A}_k(F) \geq 0$ for all $k \geq 0$.

Interpretation of M?

Margrabe option

Since an insurance company closes its books once a year, we want to have a switching option at the end of each accounting year:

$\Rightarrow$ we need to buy a Margrabe option [5] for each accounting year:

Step $k \to k+1$: Assume $\mathcal{A}_k(\widetilde{S}) = \mathcal{A}_k(\text{VaPo})$, then M is the option which allows for switching from $\widetilde{S}$ to VaPo at $k+1$, whenever

$$\mathcal{A}_{k+1}(\widetilde{S}) < \mathcal{A}_{k+1}(\text{VaPo}). \tag{7}$$

This option can be priced using classical financial mathematics.

Example: Pricing Margrabe option for BS model

In the 2-dim. Black-Scholes model:

The price of the Margrabe option is (see e.g. Gerber-Shiu [4])

$$\mathcal{A}_k(\text{VaPo}) \cdot \left(\Phi\left(\sigma/2\right) - \Phi\left(-\sigma/2\right) \right), \tag{8}$$

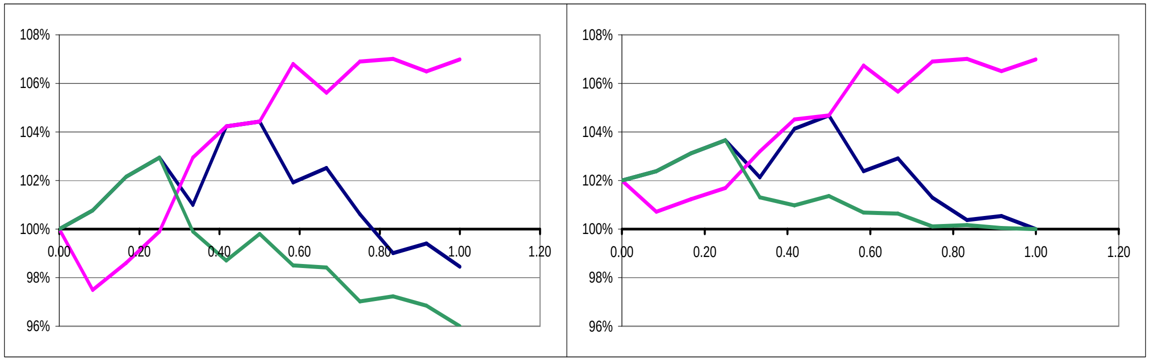
where σ is the appropriate volatility (depending on the ALM missmatch).

Henceforth:

σ	price relative to $\mathcal{A}_k(\text{VaPo})$
0.05	1.99%
0.10	3.99%
0.20	7.97%
0.30	11.92%

Conclusions

Margrabe option:



The VaPo protected against financial risks will provide us each year with the price of a Margrabe option, which can be used to:

- Buy the option (not realistic),
- Hedge the option,
- Cover costs of target capital for financial risks (approach used in practice, not satisfactory from a theoretical point of view).

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