EMB

A presentation to the Casualty Actuarial Society

How to Model a Crash
Relative Entropy Modelling (REVO)

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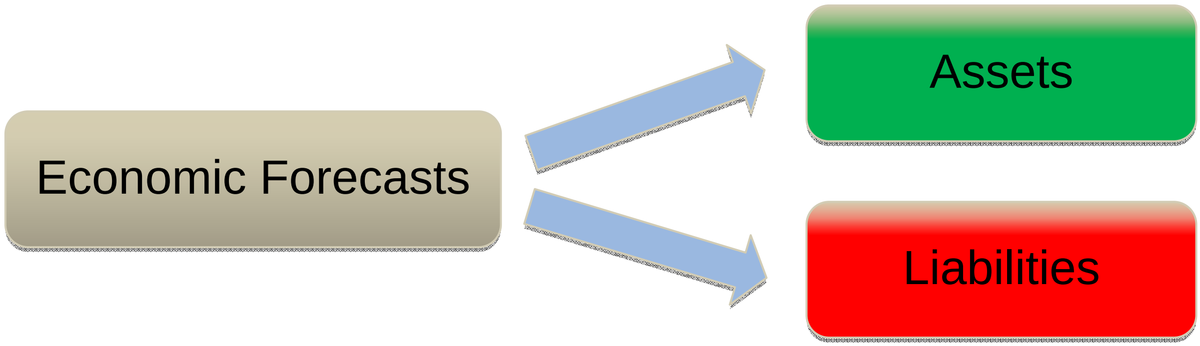
Introduction



- In order to perform realistic asset-liability modelling, it is often necessary to make use of future projections for economic variables such as inflation or return on equities.

- One desirable feature of such projections is that they are able to incorporate the possibility of future **shocks**, such as economic crashes.

- However, there may not be sufficient historical data on shocks on which to base projections.

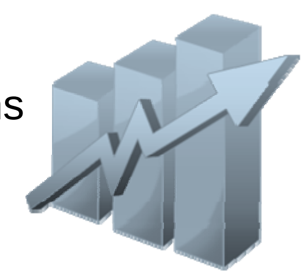
- This talk will propose one possible method of accounting for such shocks, by applying views to existing projections in order to modify those projections.

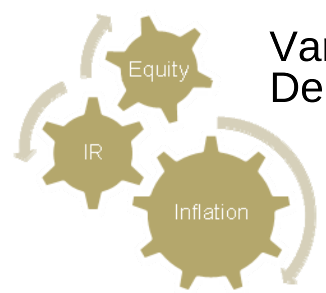
- This is known as the **Relative Entropy View Optimisation** approach, or **REVO**.

Economic Scenario Generators – Monte Carlo Approach

- The ESG we consider here is a **stochastic** model of the integrated global economy.
- Various **scenarios** are considered – typically 10,000. Each scenario is regarded as being equally likely.
- In each scenario, the model projects forward values for each of the economic measures we require, e.g. equity returns, interest rate yields, inflation, GDP, etc.
- In so doing, our model of the economy is developed to create a large range of scenarios that we believe spans the range of possibilities that may actually occur in practice.
- Hence, in using an ESG for risk analysis, we are implicitly requiring the model to behave as we expect the real world economy to evolve, namely both in terms of
  - the projected path (or level) of each econometric series
  - the dynamic interactions (or dependencies) between them

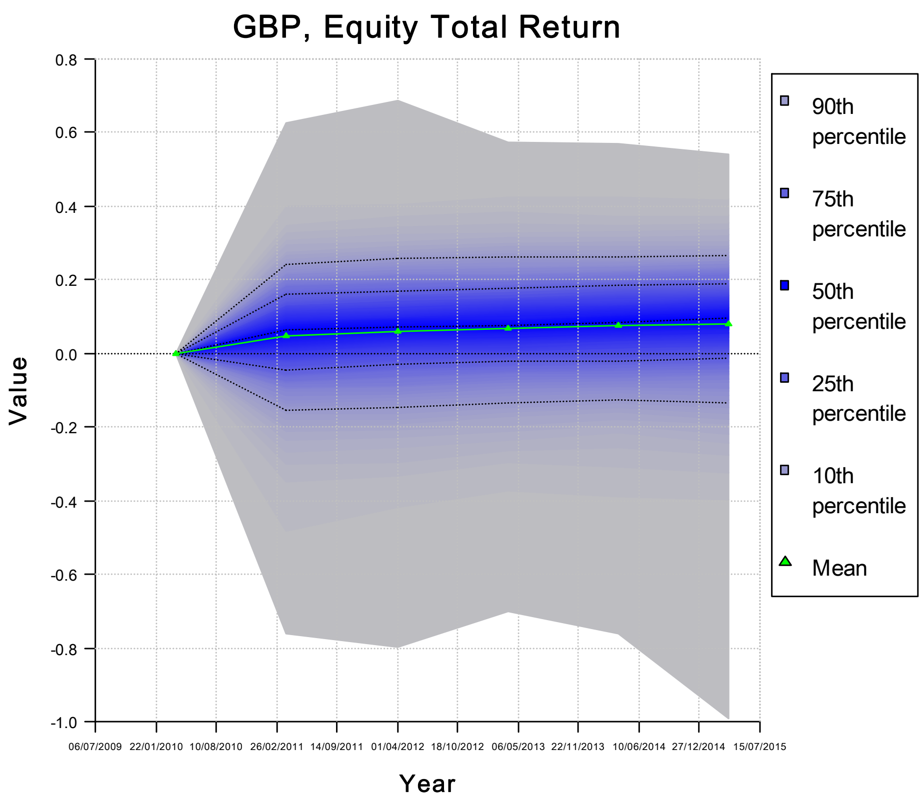
Variable Projections

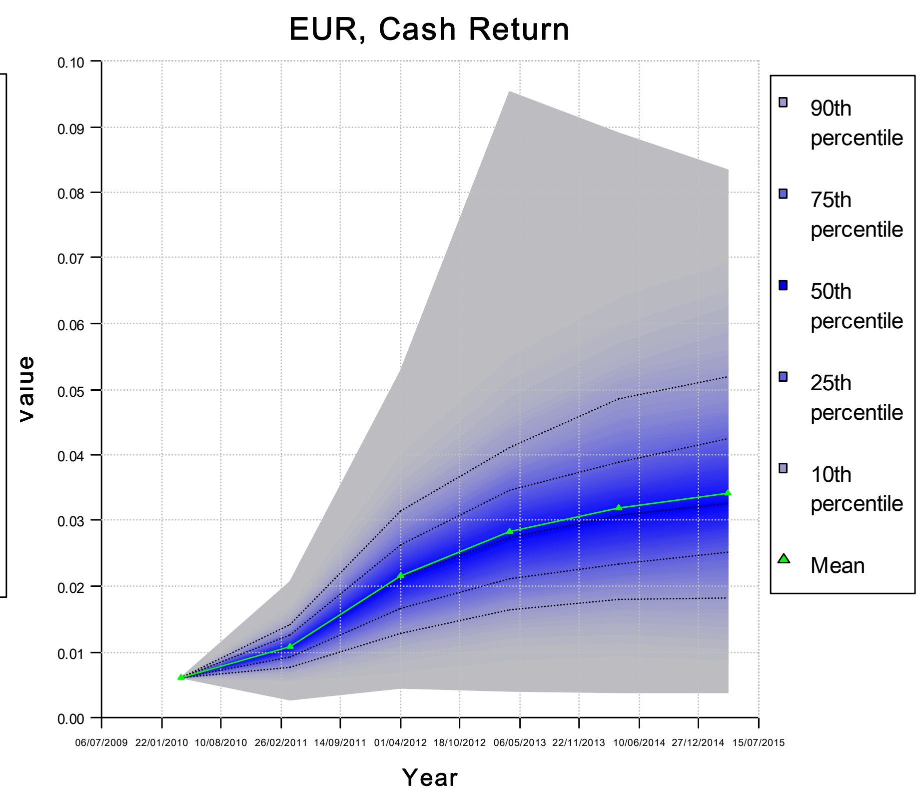




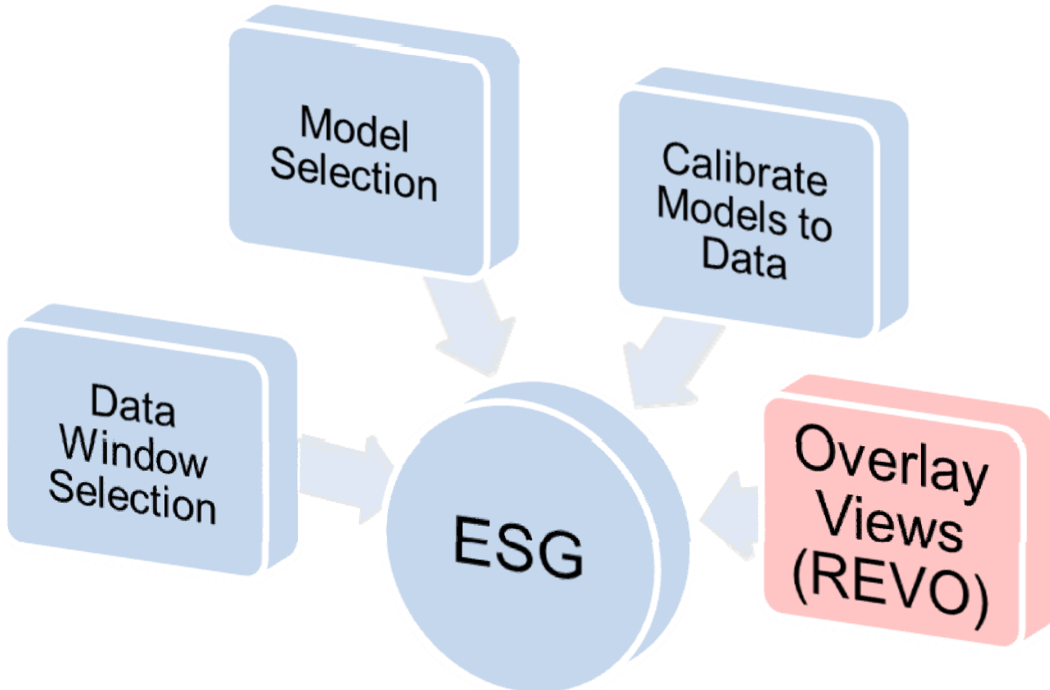
Variable Dependencies

Economic Scenario Generators – Typical Results





The ESG Parameterisation Process



Question: Which historical window of data do we select ? The window selection implicitly overlays *some* element of judgement – but it is very hard to quantify exactly what view is being proposed

For each data series we wish to model, there are various phases in the modelling process.

1) Data Window Selection

User selects historic window on data series believed to contain the dynamic behaviour that best represents *expected* future behaviour – but this also introduces a "drift" assumption.

2) Model Selection

User selects a stochastic model for data items that, based on past data, seems to best recreate the expected evolution of the series going forward.

3) Calibration

Statistical & Qualitative tests used to "fit" model to historical window of data.

4) REVO Overlay

The method by which judgement is imposed on the simulations. To be discussed…

History can be poor guide to the future …

- How can we incorporate the possibility of future shocks in our projections?
- One way is to **overlay** information from other sources.

"View Overlay" - Objectives

- Our objective is to take a fully calibrated ESG and overlay **new information** as required.

Source of "New" Information

- Examples of information sources are:
  - fundamental economic forecast providers
  - a **stress test** provided by an internal risk management team (e.g. possibly as required by Solvency II)
  - stress tests may include the possibility of an **economic crash**
- We may attempt to constrain the ESG output so as to focus on those simulations that meet all our specified criteria

"View Overlay" – The Options

Direct Parameter Adjustment

- Determine which parameter(s) influence the desired output.
- Modify each econometric model parameter directly

Bayesian Approach

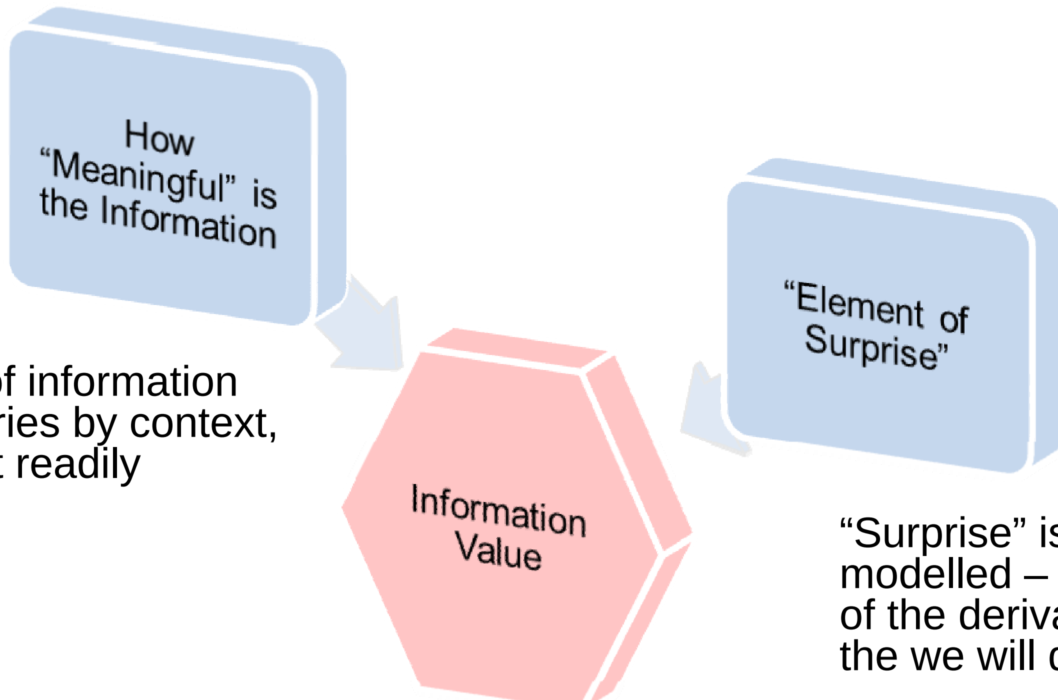
- Determine which parameter(s) influence the desired output.
- Introduce parameter uncertainty.
- "Bias" the relevant parameter draws in such a way to skew the outcomes as required

Maximum Entropy (REVO)

- In contrast, this approach *does not* attempt to modify the underlying ESG parameters.
- Instead, the ME approach *rescales* the ESG output simulations to increase the likelihood of some scenarios over others

Quantifying Information (1) – Shannon, 1948

- We want to consider the level of **information** present in a set of ESG predictions.
- In order to define information, we follow Claude Shannon, who founded Information Theory.

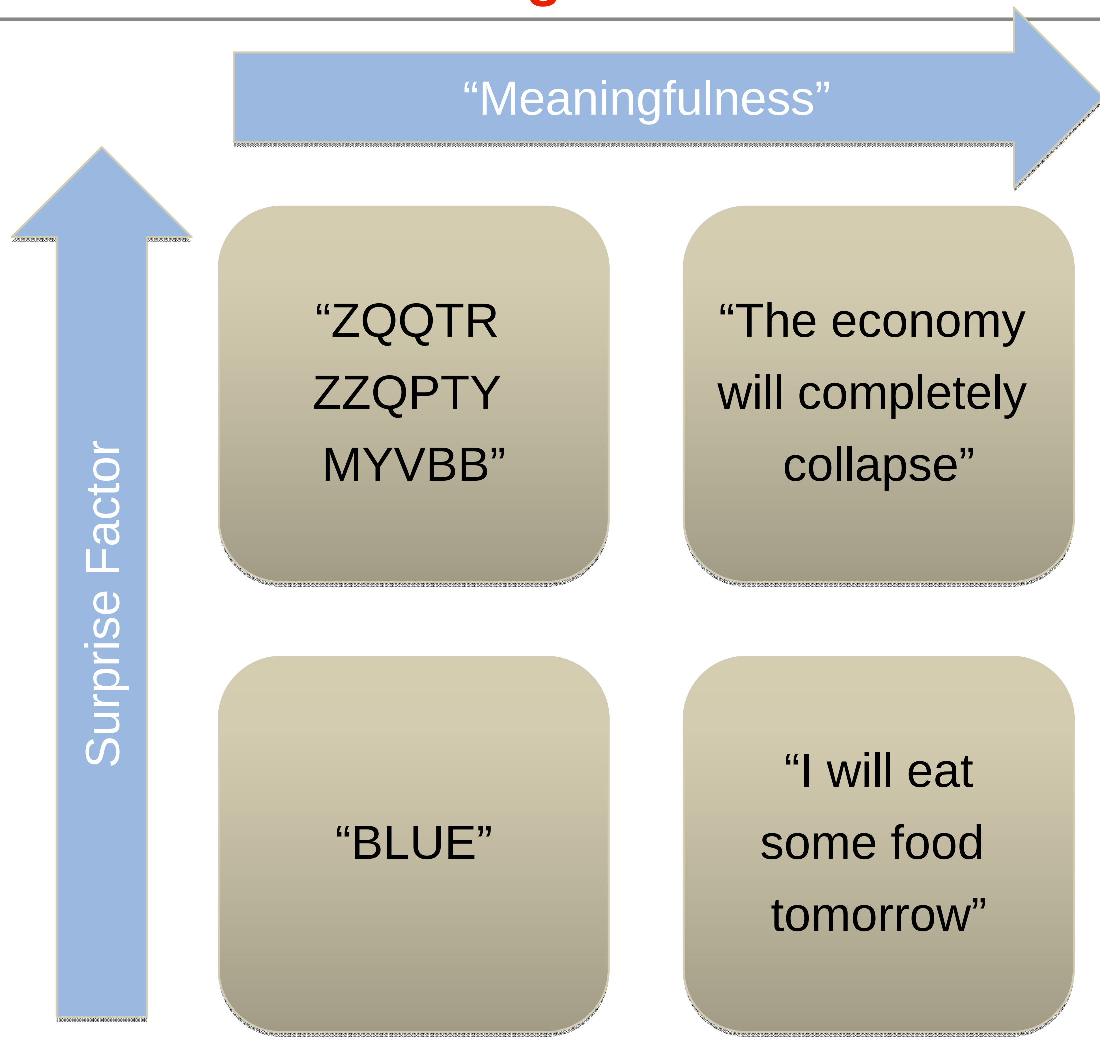


The "Meaning" of information is subjective, varies by context, and is hence not readily quantified.

"Surprise" is more easily modelled – and is the basis of the derivation of **entropy** the we will discuss today.

Broadly, the more **unlikely** an event the more surprising it is.

Surprise Factor vs "Meaningfulness"



- Example (idea courtesy of Applebaum [(1)])

Quantifying Information (2)

- We require our measure of information to exhibit some “intuitive” properties.
- Define **I(X)** as a measure of information where **X** is the event providing the information.
- To set up the problem, let’s imagine a deck of 52 unbiased cards, and possible events that may occur when we draw a card:

Description	Event	Probability	Information Content
Draw a Heart	x_1	1/4	$I(x_1)$
Draw a Seven	x_2	1/13	$I(x_2)$
Draw a Seven of Hearts	$x_1 \cap x_2$	1/52	$I(x_1 \cap x_2)$

3 Intuitive Properties of Information

- $I(x_1 \cap x_2) \geq I(x_2) \geq I(x_1)$
- $I(x_1 \cap x_2) = I(x_1) + I(x_2)$
- $I(X) \geq 0$

- The lower the probability of the event occurring, the greater the information imparted when the event actually occurs.
- Information from independent events is additive
- Information can never be “lost”

Information, (X) to Entropy, (X)

- One such potential candidate for a measure that satisfies these properties is the equation below – and it turns out that this is indeed the “only” such candidate:

$$I(X) = -K\log_{a} P(X)$$

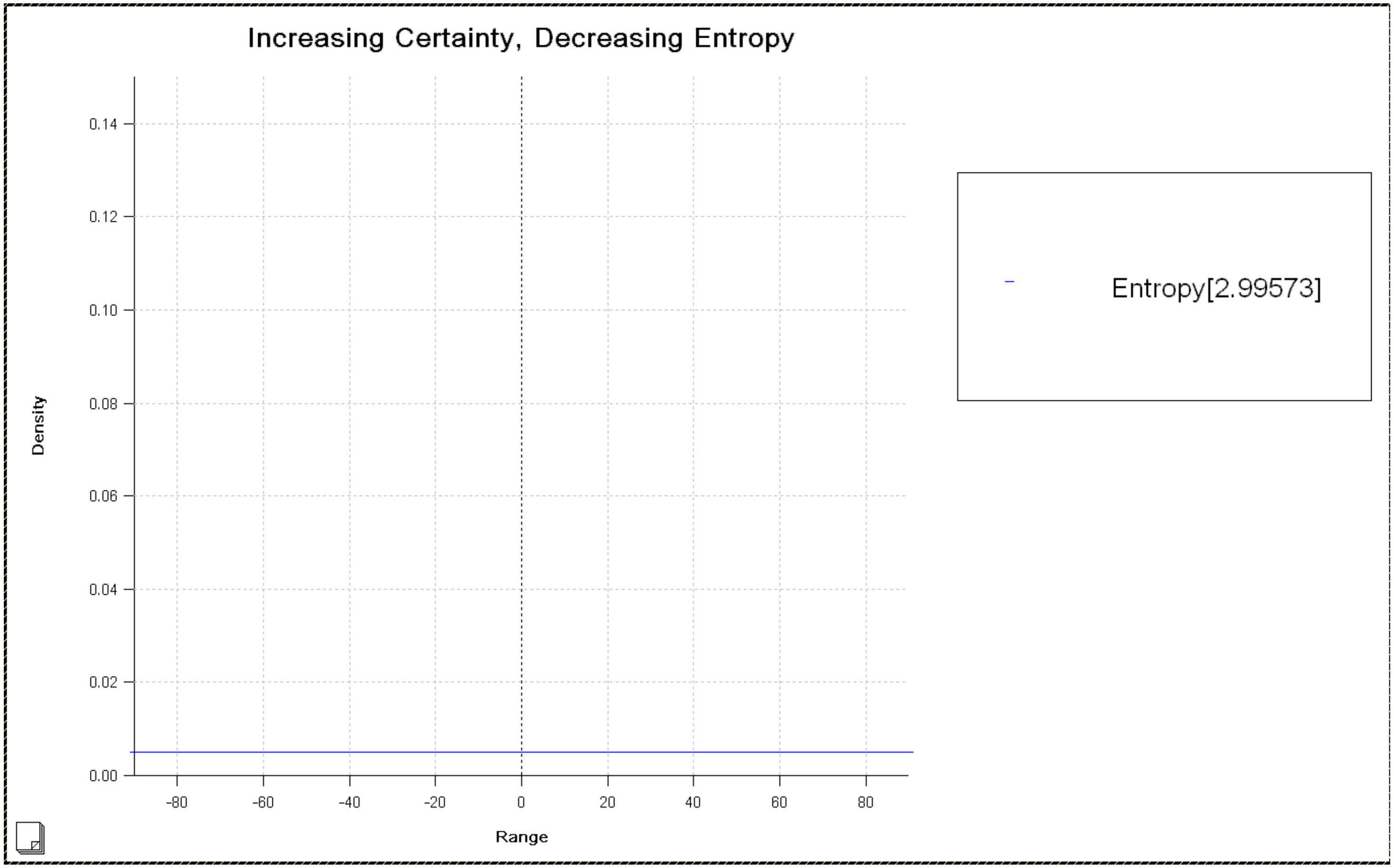
where K and a are arbitrary positive constants, P(X) is the Probability of event X, and I(X) represents a numerical quantity of “Information”.

- If we extend the above, we can calculate the “information” content of the random variable **X** itself, i.e. **I(X)**. **I(X)** is itself a random variable in this case.
- The mean (or expectation) of **I(X)** is known as the **entropy**, and is denoted by **H(X)**

$$\mathrm{H}(X) = E[I(X)] = -\sum_{i=1}^{n} p_i \log_{a} p_i$$

We typically assume K = 1, and use the natural logarithm.

Entropy & Information



- As the distribution becomes more concentrated around a point, the information content increases and the entropy decreases.

The Maximum Entropy Principle (ME)

Principle of Maximum Entropy

- In the absence of any other information, we are "most uncertain" and entropy is maximized.

Why is the Principle of Maximum Entropy helpful ?

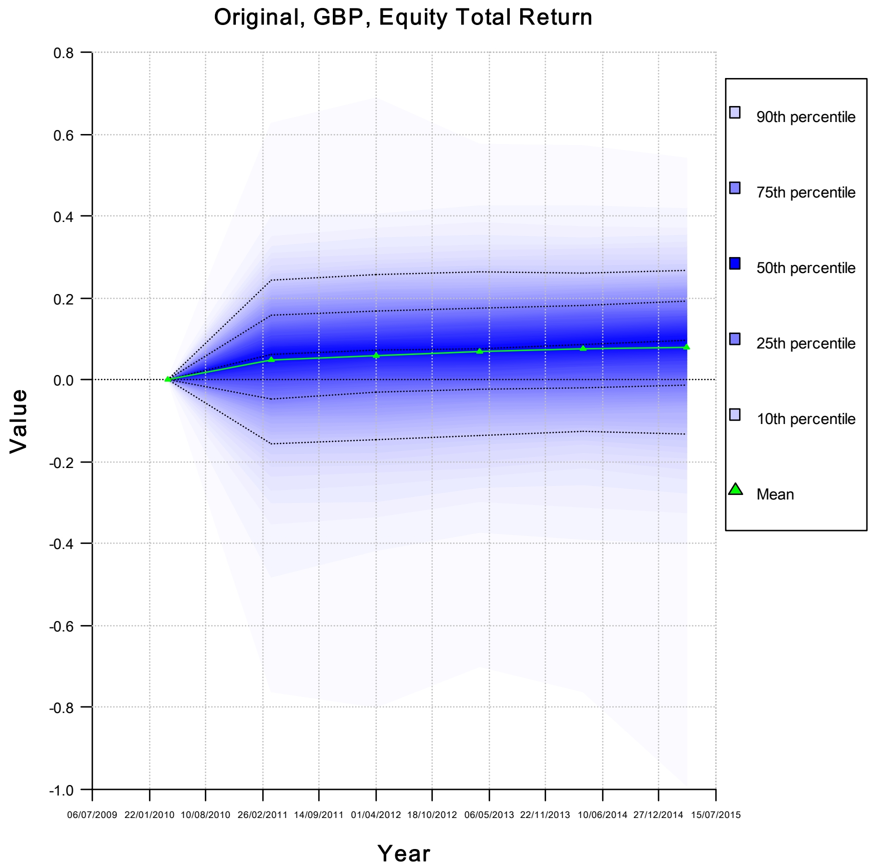
- The Principle of Maximum Entropy allows us to quantify the effect of introducing information – and hence design a scheme for adjusting probabilities as information is added.

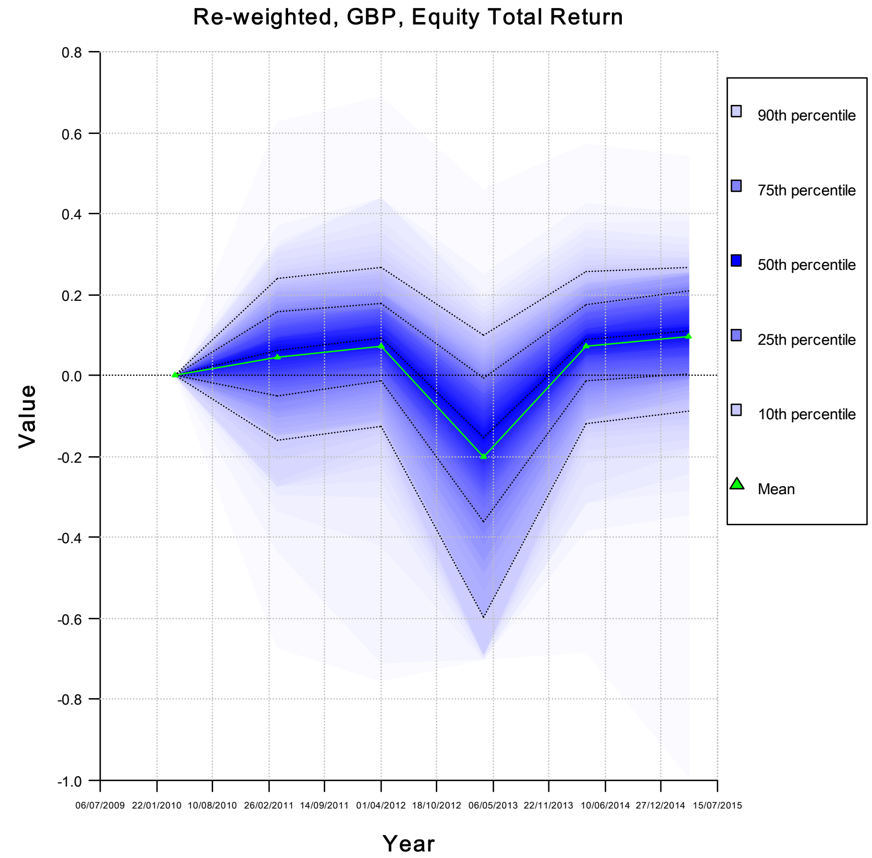
Given we now have a measure for the "Amount of Uncertainty" in a distribution, we can use standard numerical techniques that help us to "blend" in external information so as to minimize the "information" disturbance in the new distribution.

REVO: Practical Examples

REVO Results: Economic Crash View

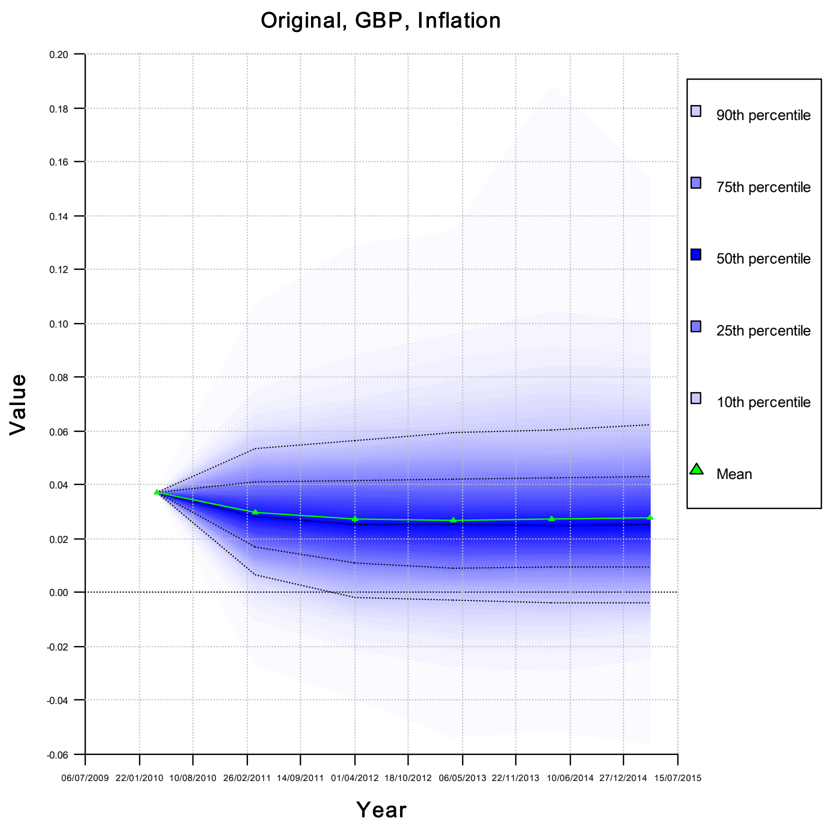
- Here, we have specified a view of an equity total return of -20% in March 2013.

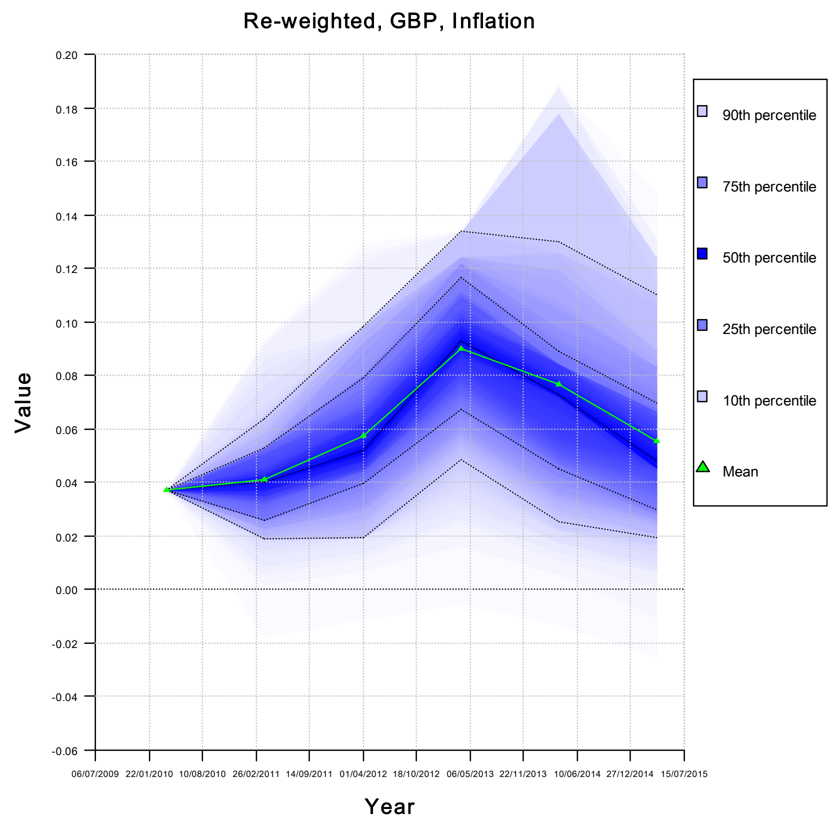




REVO Results: Hyperinflation View

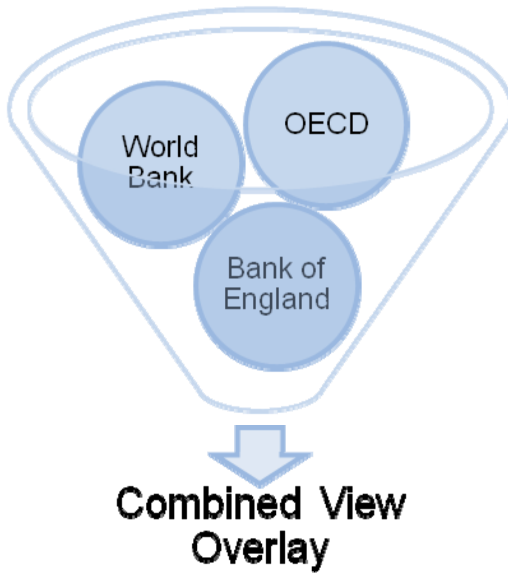
- Here, we have specified a view on inflation of 9% in March 2013.





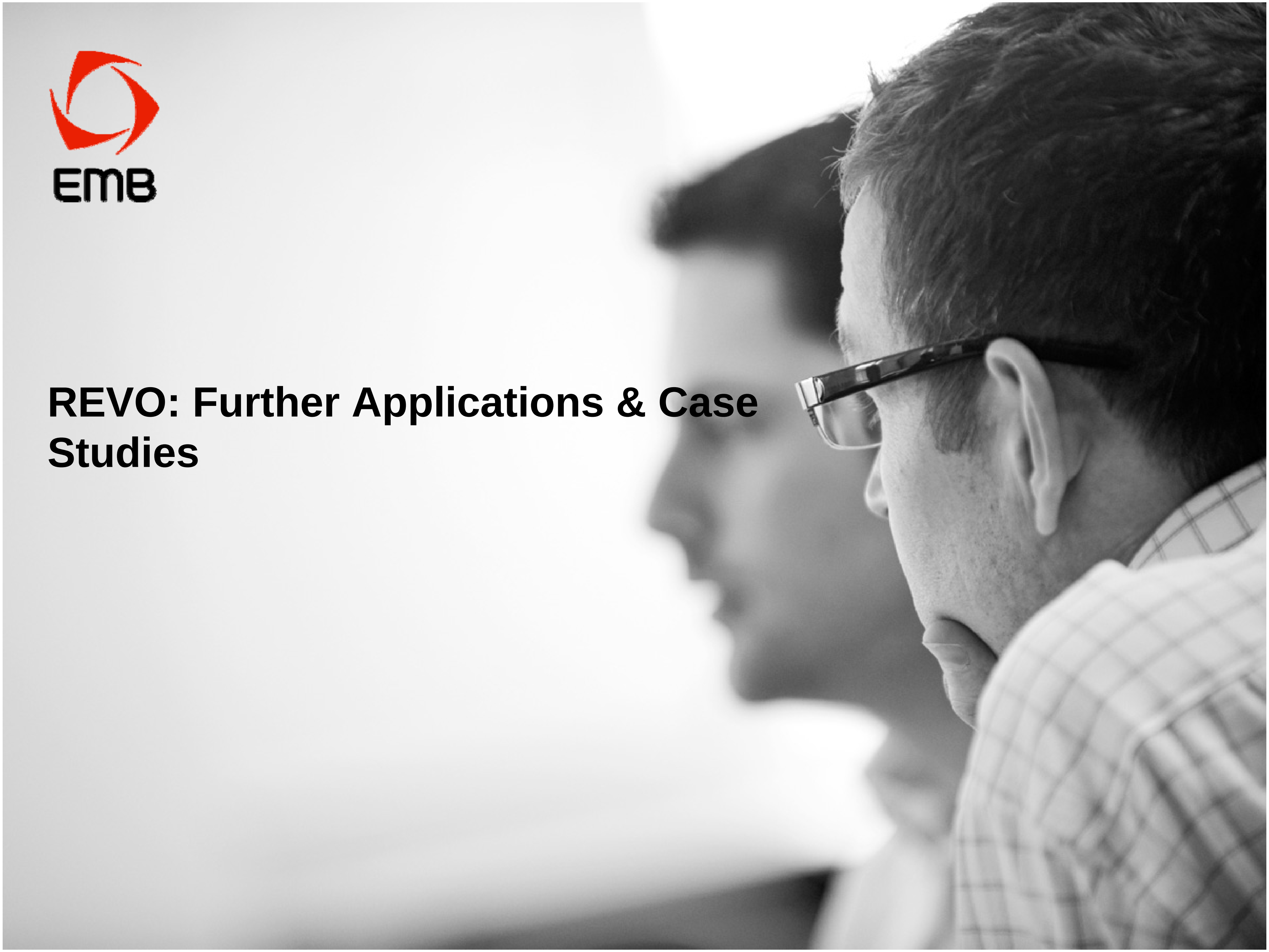
A Note on Confidence Measures

- Confidence measures give us a way to "scale" the effect of our judgement overlay.
- We can assign a confidence to "multiple" providers of overlay, and separately ascribe a confidence to each element of data the provider supplies.



How do Confidence Measures Work?

- Essentially, we linearly blend in the "rescaled" simulation weights.
- A view with "Zero" confidence is effectively the original distribution, whereas a view with "Full" confidence, is effectively fully resampled.



REVO: Further Applications & Case Studies

Application 1: Alignment to Corporate Strategy

- This approach can be used to ensure that the economic and internal modelling framework is consistent with any beliefs held by the corporate planning function (CFO, CIO etc). e.g. forecasted levels of inflation, retail property, unemployment, GDP.
- Business hence gains “trust” in the modelling framework as it “makes sense”.

Application 2: Stress Testing

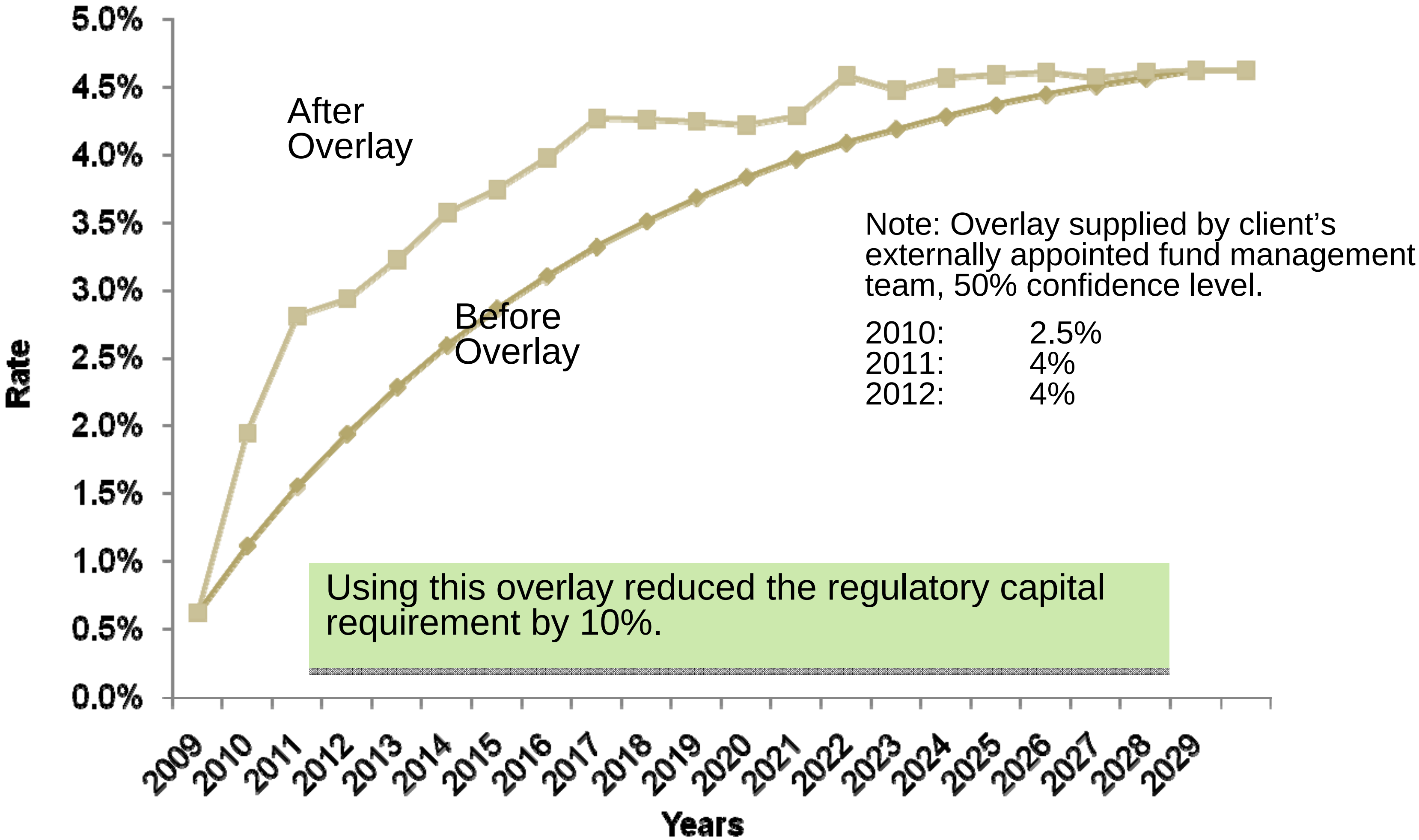
- Assuming a business is subject to financial sensitivities driven by unemployment and property prices, the rescaling approach can be used to stochastically stress test both factors, ensuring that all economic factors adjust consistently to reflect the stressing assumptions.
- E.g. What happens if the 10th percentile downside scenario has property prices falling -20% whereas history suggests only -10%. In addition, we also “expect” that, on average, property prices will actually grow.
- What if we now also wish to see the effect of solvency if unemployment also increases above expectations …..
- There are many (many) possibilities.

Application 3: Investment Strategy

- Use stochastic stress testing to review risks on both sides of the balance sheet
  - E.g. What happens if we expect equities fall a further 20%?
- Asset Liability Modelling, Portfolio Construction (and Business Strategy) aligned to corporate views (or views provided by asset management providers)
- Bond ALM with Overlay – Strategic allocation able to reflect strategic market views
- Risk Budgeting
- Setting Investment Guidelines & Benchmarks

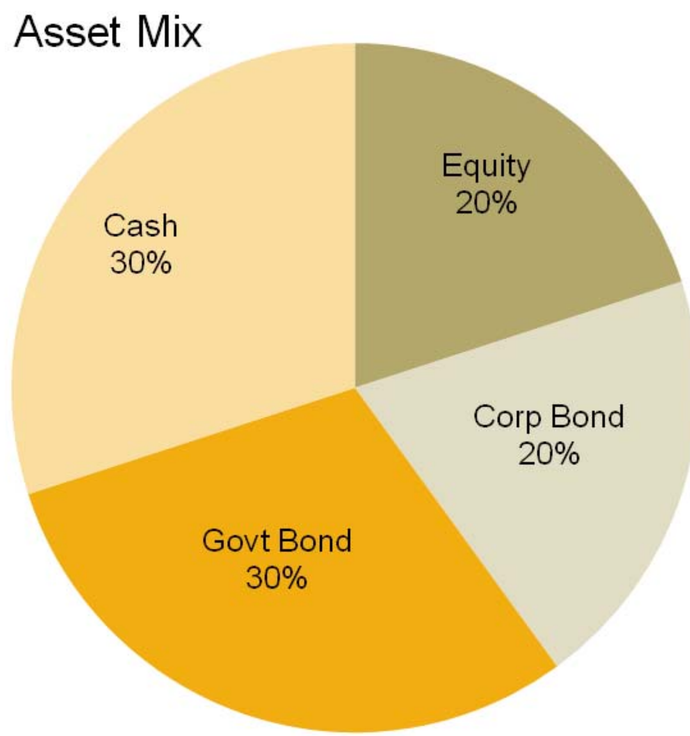
Case Study 1: Short Rate Overlay for a "live" 2009 ICA

GBP Short Rate Projections (3m UK Treasuries)



Case Study 2

- Multi-line insurance writer
- ~£200m GWP
- ~£500m reserves
- Mature book
- Well diversified – low insurance risks
- Tough market conditions: ~underwriting break even, but investments produce a net profit



Case Study 2: Stressed Views to Overlay

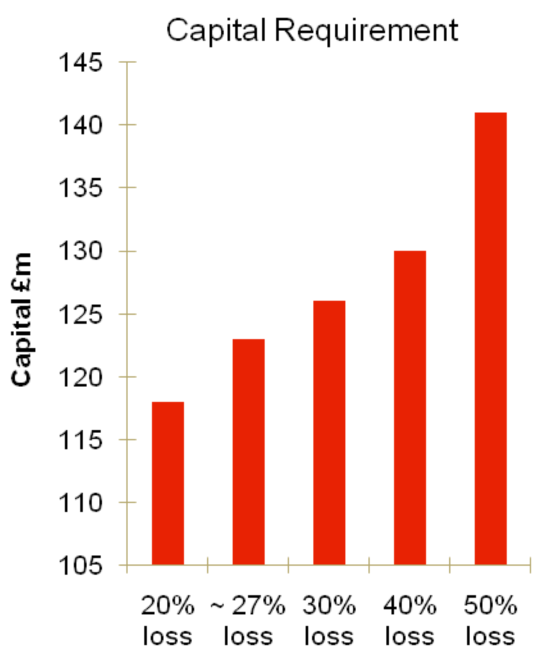
- We consider a "central overlay" set of views and a "stressed" set of views.

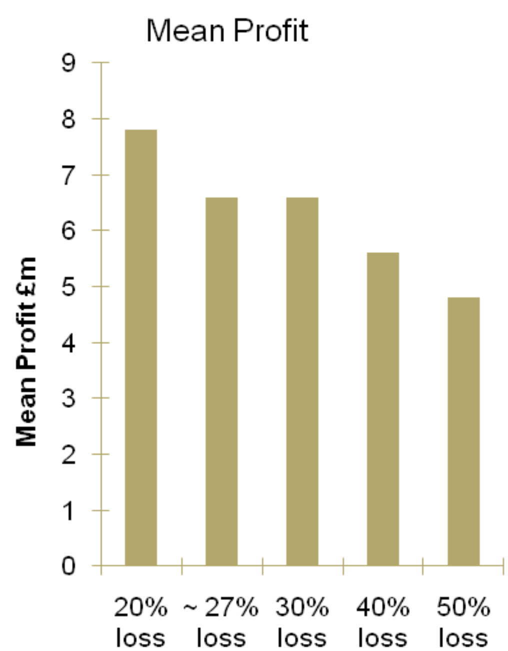
Variable	Year	Original	Central View	View	Confidence
Cash	2009	*1.42%*	*1.00%*	0.75%	80%
Cash	2010	*1.85%*	*1.50%*	1.25%	70%
Cash	2011	*2.23%*	*3.00%*	2.00%	50%
Cash	2012	*2.55%*	*3.50%*	3.00%	50%
Equity	2009	*3.48%*	*0.00%*	-10.00%	80%
Equity	2010	*3.92%*	*4.00%*	0.00%	70%

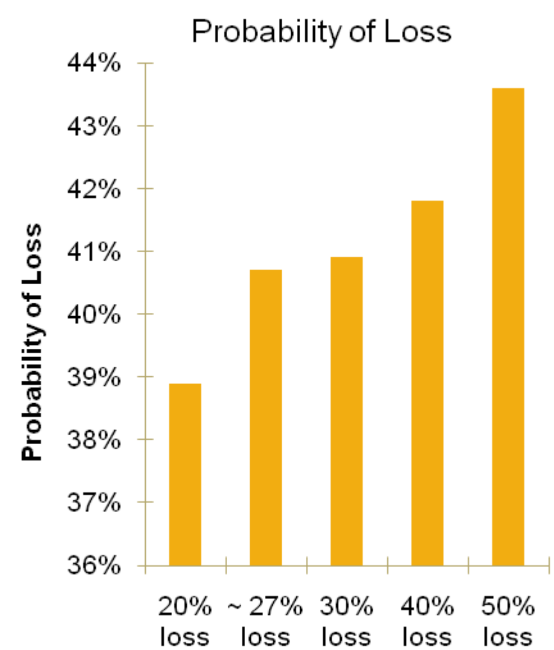
Basis	Capital Requirement	Mean Profit	Probability of Loss
Original	£124m	£5.7m	41.4%
Overlay	£127m	£5.6m	41.8%
Stressed	£132m	£0.3m	49.0%

Case Study 2: Downside risk stressing

- Use the overlay to fix downside risk metrics
- Let's target "Equity Return 1in10 VaR (3 year cumulative return)" as our downside risk measure







Case Study 2: Re-optimisation

- If we believe that the Equity Return 1in10 VaR (3 year cumulative return) is 50% => 43.6% chance of loss
- Our risk appetite maybe for the overall business to be limited to a 40% chance in any one year
- Re-optimise portfolio to come into line (Use test)

Basis	Capital Requirement	Mean Profit	Probability of Loss
20% equity	£141m	£4.8m	43.6%
17.5% equity	£135m	£5.3m	42.4%
15% equity	£128m	£5.7m	41.5%
12.5% equity	£123m	£6.2m	40.2%
10% equity	£118m	£6.7m	39.5%

REVO: Summary

What are we trying to do?

- Given an initially calibrated ESG (based on some pre-specified calibration window), we wish to overlay **additional information** in such a way that the entropy of the new distribution is reduced by the smallest possible amount.
- The Economic Scenario Generator (ESG) produces a range of scenarios that project a possible evolution of the global economy.
- Each scenario is represented by a single Monte Carlo simulation, and hence is deemed equally likely to all other scenarios. To reiterate, the model "structure" is contained in the values of each variable for each simulation – but…. **the simulations themselves are all (initially) equally likely.**

1 Obtain a calibrated ESG given some historic window of data (e.g. last 20 years)

2 Determine which financial metric(s) (the target(s)) we wish to stress test

3 Bias the Monte Carlo process to overweight" those simulations where the target is in line with our bias|

HENCE: In using a ME Monte-Carlo engine, each simulation of the global economy is assumed to occur with NON EQUAL probability – This is achieved by "Re-Sampling".

What does REVO add over other approaches?

- **"Statistically Robust":** The REVO technique uses a statistically robust, proven, methodology of blending "information" into an historically calibrated ESG.
- **"Simple":** The REVO approach requires intuitive inputs – i.e. "The forecasted level of price inflation in 2013", not the specific parameters underlying the stochastic models themselves. These more intuitive items are understandable by the board, and readily available from a multitude of financial sources.
- **"Limited Practical Alternative":** It is often extremely difficult to attempt to re-parameterise individual models in a consistent manner. Indeed many approaches are forced to greatly simplify the dependencies that any single econometric forecast may introduce.
- **"Fast":** The REVO technique is able to re-scale a complete multi factor ESG (6 countries, 30 years, 100,000 simulations) in a minute or so.
- **"Flexible":** The REVO technique is able to re-scale an ESG to ensure that the ESG is aligned with a range of alternative objectives, not necessarily just a mean value. For example, it is possible to rescale the ESG so that following a 1 in 200 year event for equities , they fall 30% rather than fall 20% as history may suggest.

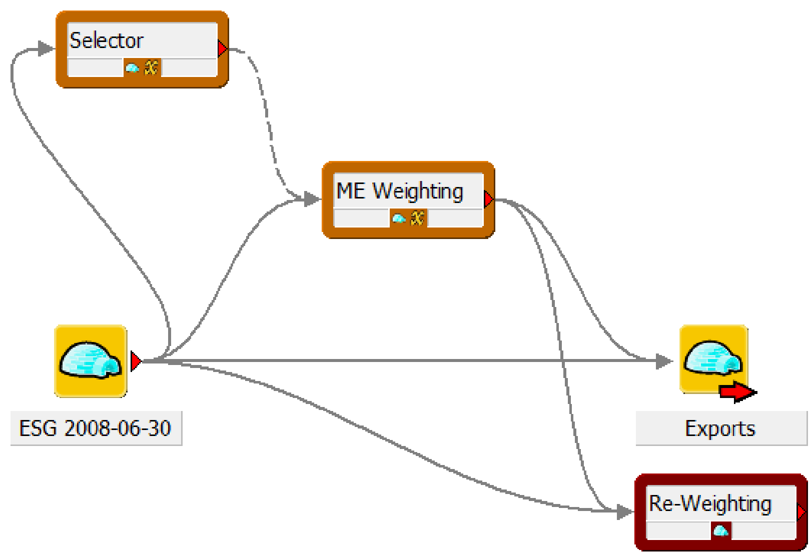
The REVO technique can also be used to rescale other financial observations, including market volatility and correlation, which leads to applications in other areas (e.g. the assessment of collateralised securities to name but one example.)

ME in Practice

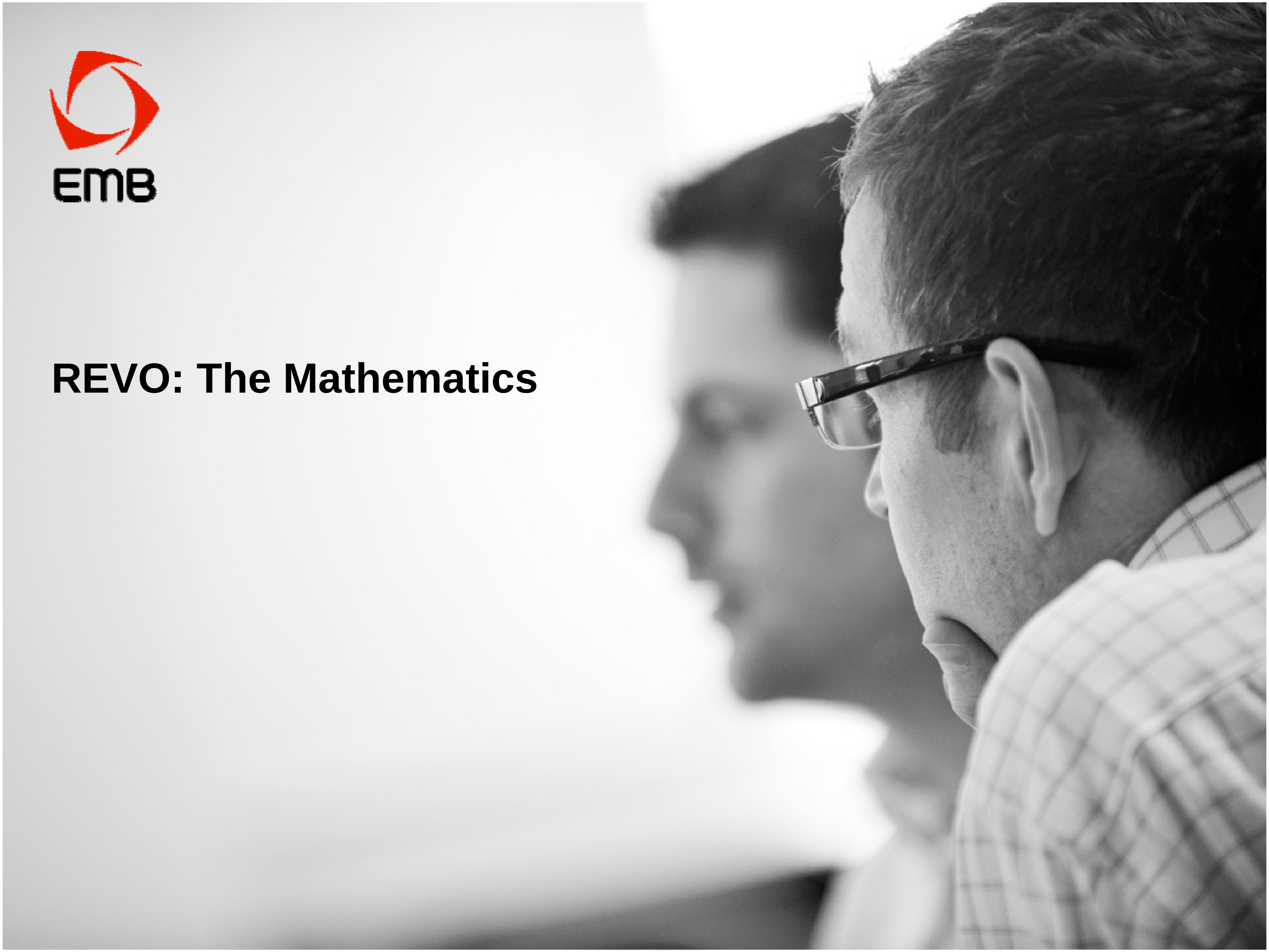
- We "simply" wish to modify the probabilities ascribed to the occurrence of each simulation.
- It is worth noting that each simulation represents one projection of the fully calibrated and integrated global economy – i.e. for all currencies, series, and time steps
- By rescaling one simulation, we modify in equal likelihood, all its constituent components, and hence, we implicitly retain any "inter" series dependencies that may exist.

- Example of simulation rescaling.

Simulation Number	Original Weight	Rescaled Weight
1	100.0%	361.6%
2	100.0%	90.5%
3	100.0%	96.7%
4	100.0%	50.2%
5	100.0%	56.3%
…	…	…



REVO: The Mathematics



Define the Maximum Entropy Optimization Problem

We wish to maximize Entropy, H(p)

$$H(p) = -\sum_{i=1}^{n} p_i \ln p_i$$

Where **p** is the vector of all probabilities and n is the length of **p**.

Subject to the following constraints

$$C_1(p) = \sum_{i=1}^{n} p_i = 1$$

Sum of all probabilities equals 1.

$$C_2(p) = \sum_{i=1}^{n} p_i x_i = M$$

The “average” of the target measure is M

IMPORTANT: This is saying that we are constraining the solution so that after rescaling the simulations, the mean of the target measure is equal to M

The Principle of Maximum Entropy (1)

- Let X be a random variable, with range $(x_1,x_2,\ldots,x_n)$ and unknown law $(p_1,p_2,\ldots,p_n)$
- Suppose we have some information, , about X (in this case just the expectation), i.e. [X]=M (some constant)
- Which law should we use to define the unknown probability law, i.e. the p's
- We use the technique of Lagrangian Optimisation to maximize the Entropy, H(X) subject to the two following constraints…

$$\sum_{i=1}^{n} p_i = 1 \quad \text{and} \quad \sum_{i=1}^{n} x_i p_i = M$$

- Hence, we wish to solve the Lagrangian

$$L(p_1, p_2, \ldots, p_n; \lambda_1, \lambda_2) = -\sum_{i=1}^{n} p_i \ln p_i + \lambda_1 \left(\sum_{i=1}^{n} p_i - 1 \right) + \lambda_2 \left(\sum_{i=1}^{n} x_i p_i - M \right)$$

where λ_1 and λ_2 are the Lagrangian multipliers.

- The differentials of the Lagrangian (below) and the 2 constraints leads to n+2 equations

$$\frac{\partial L}{\partial p_i} = -\left(\ln p_i + 1\right) + \lambda_1 + \lambda_2 x_i = 0 \quad \text{for } 1 \leq i \leq n$$

The Principle of Maximum Entropy (2)

- We have n+2 equations in n+2 unknowns (n equations in the p's, and 2 constraints).
- Solving these n+2 simultaneous equations, we obtain

$$p_j = e^{(\lambda_1 + \lambda_2 x_j - 1)}$$

and given $\sum_{j=1}^{n} p_j = 1$

$$\lambda_1 = 1 - \ln \sum_{j=1}^{n} e^{-\lambda_2 x_j}$$

Substituting λ_1 into $p_j = e^{(\lambda_1 + \lambda_2 x_j - 1)}$ gives

$$p_j = \frac{e^{\lambda_2 x_j}}{\sum e^{\lambda_2 x_j}}$$

Given $\sum_{j=1}^{n} x_j p_j = M$, and the equation for p_j above, then λ_2 can be determined.

$\sum e^{\lambda_2 x_j}$ is known as the "Partition" function, and $p_j = \frac{e^{\lambda_2 x_j}}{\sum e^{\lambda_2 x_j}}$ as the "Gibbs Distribution"

> Note 1: The Gibbs distribution is the "natural alternative" to the uniform distribution when we are ignorant of all but the mean of our random variable.
>
> Note 2: The p_j's represent the weighting we apply to each "simulation".

The Principle of Maximum Entropy (3)

- Finally, we need to formulate an “objective” to enable a numerical solver to find the value of λ_2.

  Given the constraint

  $$\sum_{i=1}^{n} p_i x_i = \mu$$

  We substitute for p_i and find

  $$\frac{\sum_i x_i e^{\lambda_2 x_i}}{\sum_i e^{\lambda_2 x_i}} = \mu$$

  Re-writing

  $$\sum_i (x_i - \mu)\, e^{\lambda_2 x_i} = 0$$

- Hence, we aim to solve for λ_2 to minimize the difference between the observed and target metrics.

Q & A

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Legal Notice: Patent Application

On May 29th 2009, EMB applied for a patent on the REVO technique. Please refer to the patent application number 0909282.6 for further details.

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