The HCL method: A loss reserving method with weighted data- and expert-reliance

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www.RiskLab.ch/hclmethod

Outline

Notation

Problem setting

The hybrid chain ladder method

Parameter estimation and model selection

Claims prediction, MSEP, CDR, uncertainty in the μ_i

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AY	$C_{i,j}$	$j=0$	$j=1$	$j=2$	$j=3$	$j=4$	$j=J=5$	μ_i
2005	$i=1$	1001	1855	2423	2988	3335	3483	3517
2006	$i=2$	1113	2103	2774	3422	3844		3981
2007	$i=3$	1265	2433	3233	3977			4598
2008	$i=4$	1490	2873	3880				5658
2009	$i=5$	1725	3261			$C_{i,j}$		6214
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	γ_j	28%	25%	17%	16%	10%	4%	
	β_j	28%	53%	70%	86%	96%	100%	

Basic Methods

Chain Ladder (CL):

$$\mathbb{E}[C_{i,j}|C_{i,j-1}] = f_j C_{i,j-1}, \quad \left(f_j = \frac{\beta_j}{\beta_{j-1}} = 1 + \frac{\gamma_j}{\beta_{j-1}} \right)$$

$$\widehat{C}_{i,J} = C_{i,I-i} \widehat{f}_{I-i+1} \cdots \widehat{f}_{I-1} = \frac{C_{i,I-i}}{\widehat{\beta}_{I-i}}$$

- Sensitive to outliers on the last diagonal
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Bornhuetter-Ferguson (BF):

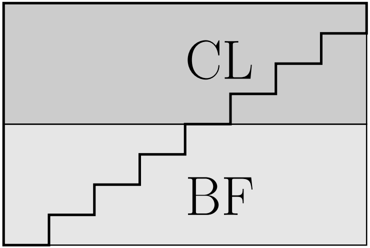
$$\mathbb{E}[C_{i,j}|C_{i,j-1}] = C_{i,j-1} + \gamma_j \mu_i$$

$$\widehat{C}_{i,J} = C_{i,I-i} + (\widehat{\gamma}_{I-i+1} + \cdots + \widehat{\gamma}_{I-1}) \mu_i$$

- Robust w.r.t. outliers.
- Estimation error in the μ_i.

Idea/Problem: combination of CL and BF

We would like to use CL in the upper part and BF in the lower part of the triangle.



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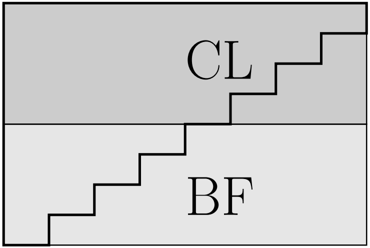

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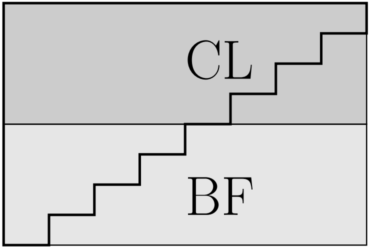

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But: these assumptions *differ*!

- CL: increments are strongly correlated
- BF: increments are independent
- One *cannot* apply CL and BF simultaneously without breaking assumptions. **Consequence: no error estimates.**

The Hybrid Chain Ladder method (HCL)

CL:

$$\mathbb{E}[C_{i,j}|C_{i,j-1}] = C_{i,j-1} + \gamma_j \frac{C_{i,j-1}}{\beta_{j-1}}$$

BF:

$$\mathbb{E}[C_{i,j}|C_{i,j-1}] = C_{i,j-1} + \gamma_j \mu_i$$

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HCL:

$$\mathbb{E}[C_{i,j}|C_{i,j-1}] = C_{i,j-1} + \gamma_j \left(\alpha_{i,j} \frac{C_{i,j-1}}{\beta_{j-1}} + (1 - \alpha_{i,j})\mu_i \right)$$

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Parameter $\alpha_{i,j} \in [0,1]$ controls the behaviour of the HCL method:

- $\alpha_{i,j} \in [0,1]$ can be chosen differently for each i and j
- $\alpha_{i,j} \approx 1$: High data reliance $\rightarrow$ CL
- $\alpha_{i,j} \approx 0$: High reliance on μ_i (expert) $\rightarrow$ BF

HCL model assumptions

HCL model: There exist parameters

- γ_j and $\sigma_j^2 > 0$ for $j = 0, \dots, J$,
- $\beta_j > 0$ for $j = 0, \dots, J-1$,
- $\alpha_{i,j} \in [0, 1]$ for $i = 1, \dots, I$ and $j = 1, \dots, J$,
- $\mu_i > 0$ for $i = 1, \dots, I$,

such that $(C_{1,j})_{j=0,\dots,J}, \dots, (C_{I,j})_{j=0,\dots,J}$ are independent Markov processes with $\mathbb{E}[C_{i,0}] = \gamma_0 \mu_i$,

$$\mathbb{E}[C_{i,j} | C_{i,j-1}] = C_{i,j-1} + \gamma_j \left(\alpha_{i,j} \frac{C_{i,j-1}}{\beta_{j-1}} + (1 - \alpha_{i,j}) \mu_i \right) \qquad (j > 0)$$

and

$$\mathrm{var}(C_{i,j} | C_{i,j-1}) = \sigma_j^2 \mu_i.$$

Mean and Variance

Define

$$\xi_{i,j} = 1 + \alpha_{i,j} \frac{\gamma_j}{\beta_{j-1}},$$

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Theorem: For $0 \le k \le j \le J$ we have

$$\mathbb{E}[C_{i,j}|C_{i,k}] = C_{i,k}\prod_{k<m\le j}\xi_{i,m} + \mu_i \sum_{k<n\le j}\left((1-\alpha_{i,n})\gamma_n \prod_{n<m\le j}\xi_{i,m}\right).$$

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Remark.

- if all $\alpha_{i,j} = 1$: $\mathbb{E}[C_{i,J}|C_{i,I-i}] = C_{i,I-i} \prod_{I-i<j\leq J} (1 + \gamma_j/\beta_{j-1})$
- if all $\alpha_{i,j} = 0$: $\mathbb{E}[C_{i,J}|C_{i,I-i}] = C_{i,I-i} + \mu_i \sum_{I-i<j\leq J} \gamma_j$

We can find an expression for the conditional variance.

Parameter estimation and model selection

- γ_j, σ_j^2 and β_j: Estimated from *data* $\mathcal{D}_I$

- $\alpha_{i,j}$: selected by the *reserving actuary* (model choice!)

- μ_i: *expert estimate* of ultimate claim
  (e.g. *expected loss ratio* $\times$ *premium volume*)

Estimation of γ_j

Recall

$$\mathbb{E}[C_{i,j}|C_{i,j-1}] = C_{i,j-1} + \gamma_j \left(\alpha_{i,j} \frac{C_{i,j-1}}{\beta_{j-1}} + (1-\alpha_{i,j})\mu_i \right).$$

For $i + j \leq I$ define

$$\Gamma_{i,j} = \frac{C_{i,j} - C_{i,j-1}}{\alpha_{i,j} \frac{C_{i,j-1}}{\beta_{j-1}} + (1-\alpha_{i,j})\,\mu_i}.$$

Note that $\mathbb{E}[\Gamma_{i,j}] = \gamma_j$. Thus, define

$$\widehat{\gamma}_j = \sum_{i=1}^{I-j} \widetilde{\omega}_{i,j} \Gamma_{i,j},$$

for some optimal weights $\widetilde{\omega}_{i,j}$.

For claims prediction: replace all unknown parameters by estimates.

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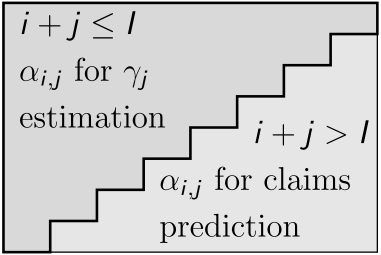
$$\begin{bmatrix} \alpha_{1,1} & & \cdots & & \alpha_{1,J} \\ & \ddots & & & \\ \vdots & & \alpha_{i,j} & & \vdots \\ & & & \ddots & \\ \alpha_{I,1} & & \cdots & & \alpha_{I,J} \end{bmatrix}$$



Our $\alpha_{i,j}$ selection proposal

- **Upper triangle**, $i + j \leq I$:
  $\alpha_{i,j}$: measure of predictive power of $C_{i,j-1}/\beta_{j-1}$ as an estimate of the ultimate claim $C_{i,J}$. Predictive power is high for late development years. *Proposal:*

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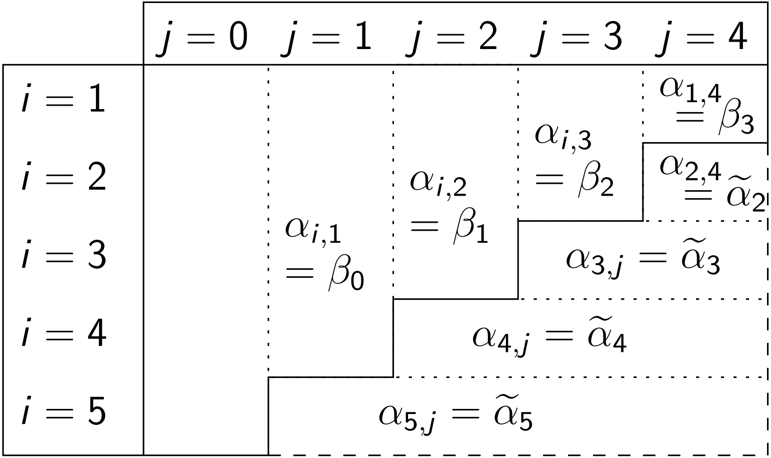
- **Lower triangle**, $i + j > I$:
  $\alpha_{i,j}$: determine the data-/expert- reliance of claims predictions. *Proposal:*

$$\alpha_{i,j} = \widetilde{\alpha}_i.$$

($\widetilde{\alpha}_i$ chosen by the reserving actuary)

Our $\alpha_{i,j}$ selection proposal - illustration



$\alpha_{i,j}$ selection proposal: consequence for reserve estimates

If $\alpha_{i,j}$ are chosen according to our proposal, i.e.

$$\alpha_{i,j} = \beta_{j-1} \quad \text{for } i + j \leq I,$$
$$\alpha_{i,j} = \widetilde{\alpha}_i \quad \text{for } i + j > I.$$

Then the reserves $R_i = \widehat{C}_{i,J} - C_{i,I-i}$ for accident year are approximately

$$R_i = \widehat{C}_{i,J} - C_{i,I-i} \approx \widetilde{\alpha}_i \left(\frac{C_{i,I-i}}{\beta_{I-i-1}} - C_{i,I-i} \right) + (1 - \widetilde{\alpha}_i) \left(\mu_i \sum_{I-i<j\leq J} \widehat{\gamma}_j \right).$$

- similar to the Benktander-Hovinen estimator
- $\widetilde{\alpha}_i \approx 1$: Reserve estimate similar to CL
- $\widetilde{\alpha}_i \approx 0$: Reserve estimate similar to BF

Mean square error of prediction (MSEP)

The aggregate **MSEP** is defined as

$$\text{msep}\left(\sum_{i=1}^{I}\widehat{C}_{i,J}\right) = \mathbb{E}\left[\left(\sum_{i=1}^{I}\left(\widehat{C}_{i,J} - C_{i,J}\right)\right)^2 \middle| \mathcal{D}_I\right]$$
$$= \sum_{i=1}^{I}\text{var}(C_{i,J}|\mathcal{D}_I) + \left(\sum_{i=1}^{I}\left(\widehat{C}_{i,J} - \mathbb{E}[C_{i,J}|\mathcal{D}_I]\right)\right)^2.$$

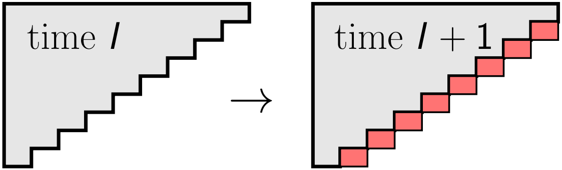
Both terms (*process variance* and *parameter estimation error*) can be estimated.

Claims development result (CDR)

Suppose we have a one year perspective (balance-sheet!).

Today:

- Estimate $\widehat{C}_{i,J}^{I}$ of ultimate claim $C_{i,J}$ based on triangle $\mathcal{D}_I$



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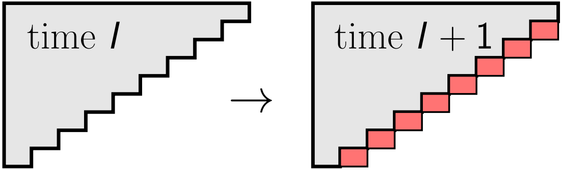

Next year:

- More data becomes available → additional diagonal
- Volatility realized on new diagonal → updated parameter estimates
- New predictions $\widehat{C}^{I+1}_{i,J}$ based on the enlarged triangle

Uncertainty in the CDR

The CDR of accident year i is defined by

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$$\mathbb{E}[CDR_i | \mathcal{D}_I] = 0.$$

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Solvency (eg. SST) regulations consider risk in a one-year perspective!

Estimate

$$\mathbb{E}[CDR_i^2 | \mathcal{D}_I] \qquad \text{and} \qquad \mathbb{E}\left[\left(\sum_{i=1}^{I} CDR_i \right)^2 \middle| \mathcal{D}_I \right].$$

- Can be estimated for the HCL method, but cumbersome expressions.

Uncertainty in the μ_i - Possible extension of the model

Up to now, the μ_i were deterministic.

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Idea: Use scenarios for μ_i!

Consequence: MSEP gets larger.

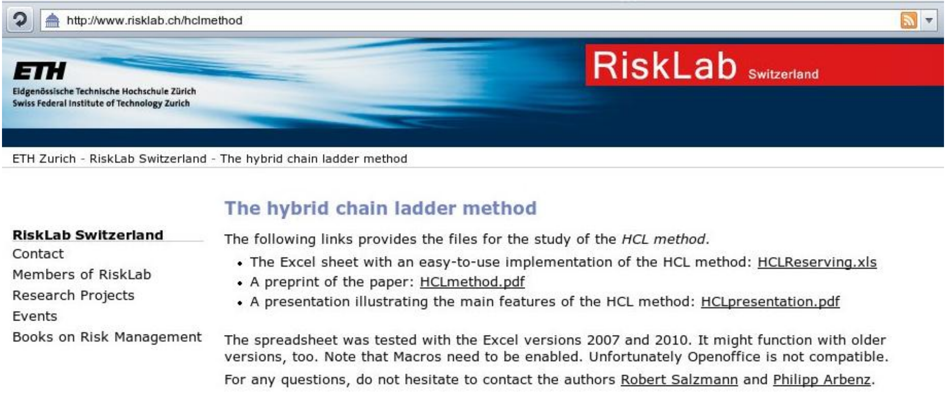
Excel spreadsheet

Discussion

Excel spreadsheet

`www.RiskLab.ch/hclmethod`

$\Downarrow$



http://www.risklab.ch/hclmethod

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The hybrid chain ladder method

The following links provides the files for the study of the *HCL method*.

- The Excel sheet with an easy-to-use implementation of the HCL method: HCLReserving.xls
- A preprint of the paper: HCLmethod.pdf
- A presentation illustrating the main features of the HCL method: HCLpresentation.pdf

The spreadsheet was tested with the Excel versions 2007 and 2010. It might function with older versions, too. Note that Macros need to be enabled. Unfortunately Openoffice is not compatible.

For any questions, do not hesitate to contact the authors Robert Salzmann and Philipp Arbenz.

References

- Arbenz, P. and Salzmann, R. (2010): *A robust distribution-free loss reserving method with weighted data- and expert-reliance*.
- Wüthrich, M.V. and Merz, M. (2008): *Stochastic Claims Reserving Methods in Insurance*. Wiley Finance: Chichester

Thank you for your attention!