

Allocation, Allocation, Allocation: Where are your location now?

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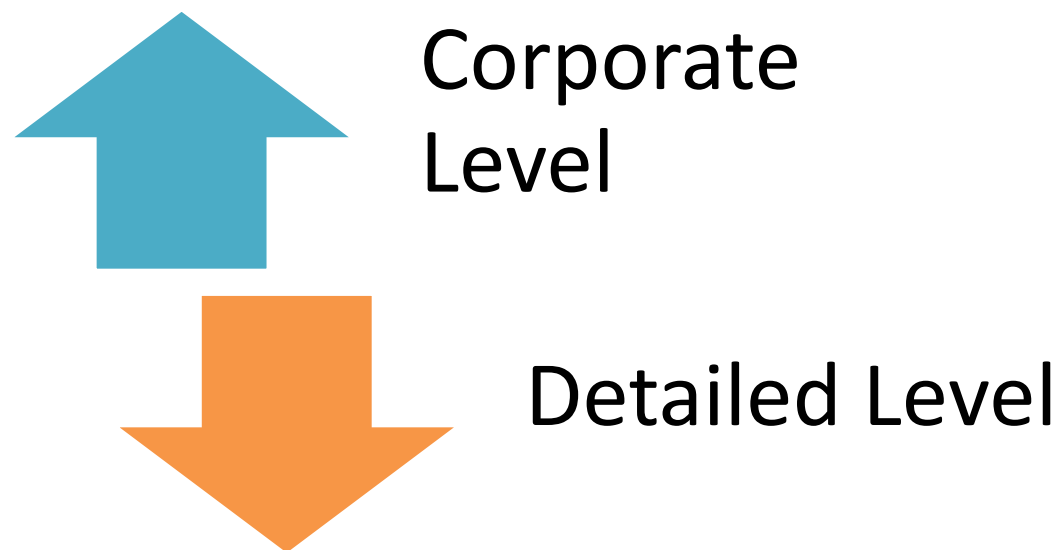
Why we Need Allocation?

Allocation of IBNR

Allocation of Reinsurance Costs

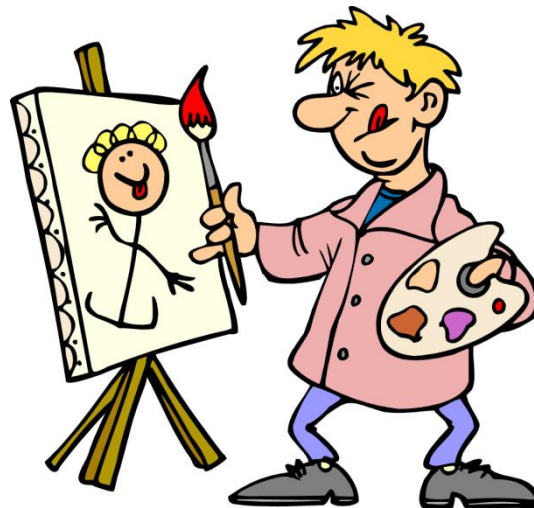
Allocation of Capital

- ☉ Some works are done at the corporate level, but many analyses will usually be performed at a lower level of detail.
 - For instance, the IBNR or capital analysis is completed in the headquarter. However, Switzerland Branch wants to know what its IBNR or capital consumption is.



- ☉ Many items at the corporate level need to be allocated to a detailed level, including but not limited to IBNR, capital requirement, cost of reinsurance.

Can Artists do better than Actuaries?



Why we Need Allocation?

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Allocation of Reserves

☞ Allocation of Reserves is an old question for actuaries, which has been raised for a long time.

Premium Reserves



Done at the policy level

Very easy to allocate 

Loss Reserves

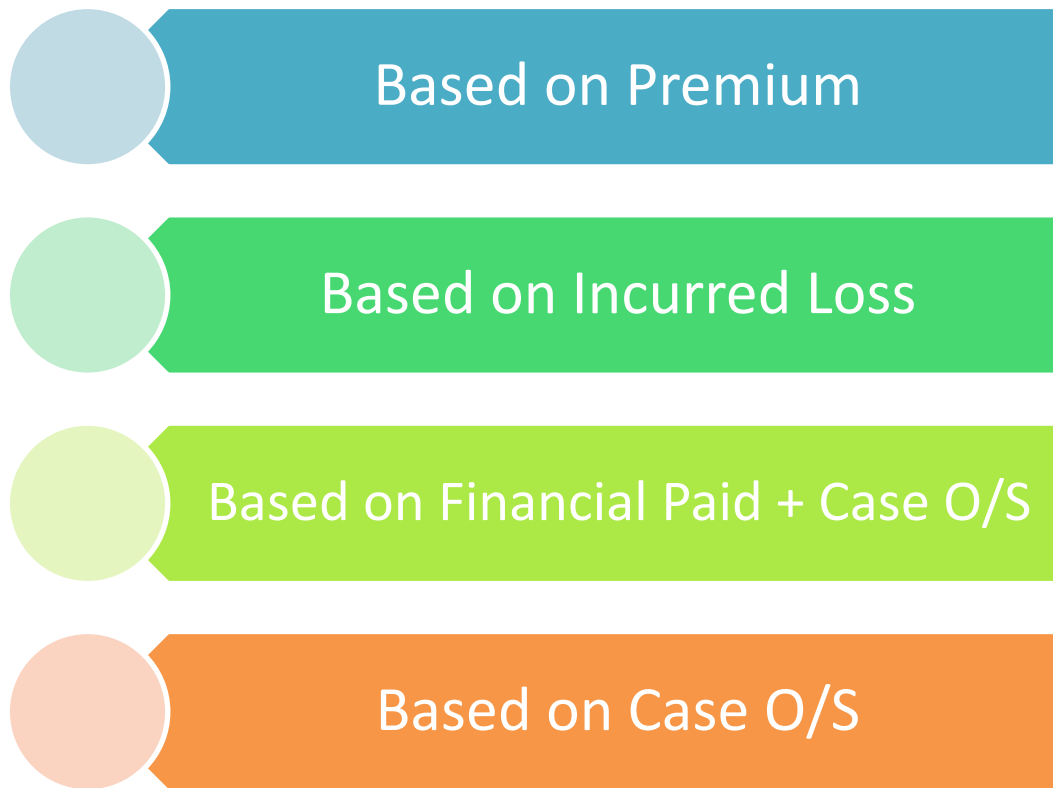


Reported Case Reserves 

IBNR 

Methods of IBNR Allocation

☞ There are many methods to allocate IBNR in the actuarial world.



- ☯ Here we will test which method of IBNR allocation is the most appropriate one.
- ☯ In order to evaluate which method is the best, a criterion must be established first. Here, Mean Squared Error is chosen.
 - Why not choose Chi-square? Because the scale should not be ignored.

$$\text{MSE} = \sum_{i=1}^n (\text{Allocated} - \text{Actual})^2$$

- ☯ How much is the actual IBNR for a lower level?
 - It is assumed that the regional actuary can do the IBNR reserving work for the lower level and that IBNR results are the actual ones.

The Reserving of Headquarter

- ☉ A company has two branches.
 - All the triangles are the results of combination of two branches.
- ☉ The headquarter reserving actuary does his work as shown below.

Headquarter	Rptd Incurred Loss							
Acc Year	0	1	2	3	Ult Loss	Earned Prem	Loss Ratio	IBNR
2011	300	350	382.6	396.67	396.67	628.72	63.09%	-
2012	360	420	459.12		476.00	754.46	63.09%	16.8840
2013	420	490			555.34	880.20	63.09%	65.3380
2014	480				634.67	1,005.95	63.09%	154.6720
LDFs:	1.166667	1.093143	1.03677	1				236.8940

Headquarter	Paid Loss			
Acc Year	0	1	2	3
2011	190	333	371.86	396.67
2012	228	399.6	446.232	
2013	266	466.2		
2014	304			

Headquarter	Case O/S			
Acc Year	0	1	2	3
2011	110	17	10.74	0
2012	132	20.4	12.888	
2013	154	23.8		
2014	176			

If there is a Branch Actuary

☉ Branch A's reserving actuary will do the reserving work as shown below.

➤ Note: The BF IBNR uses the Headquarter loss ratio (63.09%) rather than its own loss ratio (70%) as the initial.

Branch A	Rptd Incurred Loss								
Acc Year	0	1	2	3	Ult Loss	Earned Prem	Loss Ratio	CL IBNR	BF IBNR
2011	100	120	129.6	136.08	136.08	194.40	70%	-	-
2012	120	144	155.52		163.30	233.28	70%	7.7760	5.2206
2013	140	168			190.51	272.16	70%	22.5120	20.2026
2014	160				217.73	311.04	70%	57.7280	47.8248
LDFs:	1.2	1.08	1.05	1				88.0160	73.2479

Branch A	Paid Loss			
Acc Year	0	1	2	3
2011	50	112	125.76	136.08
2012	60	134.4	150.912	
2013	70	156.8		
2014	80			

Branch A	Case O/S			
Acc Year	0	1	2	3
2011	50	8	3.84	0
2012	60	9.6	4.608	
2013	70	11.2		
2014	80			

If there is a Branch Actuary

☉ Branch B's reserving actuary will do the reserving work as shown below.

➤ Note: The BF IBNR uses the Headquarter loss ratio (63.09%) rather than its own loss ratio (60%) as the initial.

Branch B	Rptd Incurred Loss								
Acc Year	0	1	2	3	Ult Loss	Earned Prem	Loss Ratio	CL IBNR	BF IBNR
2011	200	230	253	260.59	260.59	434.32	60%	-	-
2012	240	276	303.6		312.71	521.18	60%	9.1080	11.6634
2013	280	322			364.83	608.04	60%	42.8260	45.1354
2014	320				416.94	694.91	60%	96.9440	106.8472
LDFs:	1.15	1.1	1.03	1				148.8780	163.6461

Branch B	Paid Loss			
Acc Year	0	1	2	3
2011	140	221	246.1	260.59
2012	168	265.2	295.32	
2013	196	309.4		
2014	224			

Branch B	Case O/S			
Acc Year	0	1	2	3
2011	60	9	6.9	0
2012	72	10.8	8.28	
2013	84	12.6		
2014	96			

- ☉ The CL IBNR in each branch is regarded as the “actual” IBNR. Then, the MSEs of all the methods could be found as below.
- The results based on Ult Loss seems to be better, as the impact of premium adequacy among different branches has been adjusted.
 - Following that are IL basis method, EP basis method, with their combined method.
 - The results based on Case O/S seems to be the worst.

Method	MSE
Based on Incurred Loss	139.04
Based on EP	436.19
Weighted of Two*	181.41
Based on UL	91.07
Based on Case O/S	727.60
Based on Financial Paid+Case O/S*	2.16

☯ What is the Weighted method?

- It is a method weighting average of the IL basis method and the EP basis method.
- It is called “Modified B-F method of IBNR Allocation” by some people.
- The weight given to the EP basis results for each accident year is the unreported loss percentage in the Headquarter triangle.

$$w = \frac{Ult LDF - 1}{Ult LDF}$$

- ⑧ What is the method based on “Financial Paid + Case O/S” here?
 - It is a method that is stipulated in a guideline on IBNR allocation issued by the insurance regulator in China.
 - The method does not do the jobs on accident years, but on financial years.
 - Steps:
 - The sum of the paid loss amount during the past 12 months and the latest case O/S amount is calculated for each branch.
 - The allocation percentage for each branch is proportionally based on the sum amount.

- ☯ In our example, the Financial Paid plus Case O/S method fits the actual numbers extraordinarily well, even better than the Ult Loss basis method.
- ☯ Will it be the champion all the time?
- ☯ Anyway, many insurers in Chinese market utilise the “Financial Paid + Case O/S” method, following the regulatory guideline.



What if the “actual” changed

- ☉ If the BF IBNR in each branch is regarded as the “actual” IBNR, then what will the MSEs be changed?
 - The deviation is zero for the EP basis method.
 - That means, **IBNR allocation based on EP is completely consistent with the BF IBNR for each branch.**
 - The results based on Case O/S is still the worst.
 - The “Financial Paid+ Case O/S” method does not fit very well under this circumstance, as the “actual” branch IBNR is changed.

Method	MSE-CL	MSE-BF
Based on Incurred Loss	139.04	82.70
Based on EP	436.19	-
Weighted of Two*	181.41	55.00
Based on UL	91.07	128.64
Based on Case O/S	727.60	2,290.50
Based on Financial Paid+Case O/S*	2.16	376.98

Why we Need Allocation?

Allocation of IBNR

Allocation of Reinsurance Costs

Allocation of Capital

- ☯ The headquarter often purchases a reinsurance cover for the whole company, protecting all of its branches.
- ☯ How should the mother allocate the costs to her children fairly?
- ☯ In practice, there are several approaches to deal with that question:



An Illustration

- ☉ The headquarter purchased an XL reinsurance cover of \$700M xs \$300M, with a reinsurance premium of \$200M.
- ☉ It has only two branches, Germany and France.
- ☉ Two branches face the loss distributions as below, and the two distributions are independent.

Branch	Probability	Loss	Premium	ELR
Germany	50%	400	400	50%
	50%	0		
France	20%	800	200	80%
	80%	0		

☞ This method only takes into account the premium of each branch.

➤ The reinsurance cost allocated to Germany Branch

$$=200M*400/(400+200)=133.33M$$

➤ The reinsurance cost allocated to France Branch

$$=200M*200/(400+200)=66.67M$$

- ☉ Germany Branch may say it is unfair to allocate the cost based on premium, as its premium adequacy is much better than France Branch.
- ☉ In order to make adjustment to premium rate adequacy, an alternative approach is brought up. In fact, the allocation is based on **expected loss** rather than simple premium.
 - The expected losses for Germany and France are 200M and 160M respectively.
 - Therefore, the reinsurance cost allocated to Germany Branch
 $=200M * 200 / (200 + 160) = 111.11M$
And the reinsurance cost allocated to France Branch
 $=200M * 160 / (200 + 160) = 88.89M$



- ☉ The “Stand-alone” Allocation approach calculates what the cost is when a branch uses the XL reinsurance alone, and then allocates the reinsurance cost based on that.

Branch	Probability	Loss	XS Loss Cost
Germany	50%	400	$(400-300)*50\%=50$
	50%	0	
France	20%	800	$(800-300)*20\%=100$
	80%	0	

- Therefore, the reinsurance cost allocated to Germany Branch

$$=200M*50/(50+100)=66.67M$$

And the reinsurance cost allocated to France Branch

$$=200M*100/(50+100)=133.33M$$

- ☞ If Germany Branch and France Branch would never trigger the XL cover alone but would do jointly (e.g. $200+300 >$ the attachment point 300), how should we allocate?

Branch	Probability	Loss
Germany	50%	200
	50%	0
France	20%	300
	80%	0

- ☞ Or if Germany Branch could never trigger the XL cover alone but would do jointly with France, is it fair to allocate all the cost to France Branch?

Branch	Probability	Loss
Germany	50%	200
	50%	0
France	20%	800
	80%	0

- ☉ The Marginal Allocation approach calculates what the cost is added when a branch comes into the XL cover, and then allocates the reinsurance cost based on that.

Branch	Probability	Loss	XS Loss Cost
Germany	50%	400	$(400-300)*50\%=50$
	50%	0	
France	20%	800	$(800-300)*20\%=100$
	80%	0	

- The loss cost of the XL cover is 160M, so the marginal cost of Germany Branch is $160-100=60$ M and that of France Branch is $160-50=110$ M.
- Therefore, the reinsurance cost allocated to Germany Branch
 $=200\text{M}*60/(60+110)=70.59\text{M}$

And the reinsurance cost allocated to France Branch

$$=200\text{M}*110/(60+110)=129.41\text{M}$$

- ☉ This approach takes into account all the possible scenarios before allocating the cost.

Scenario	Probability	Loss	XS Loss	Germany Cost	France Cost
Germany occurs, but France not	.5*.8=40%	400	100	100*40%=40	0
France occurs, but Germany not	.5*.2=10%	800	500	0	500*10%=50
Both occur	.5*.2=10%	1200	700	$700*10%*400/1200$ =23.33	$700*10%*800/1200$ =46.67
Neither occurs	.5*.8=40%	0	0	0	0
	100%		Subtotal:	63.33	96.67

➤ Therefore, the reinsurance cost allocated to Germany Branch

$$=200M*63.33/(63.33+96.67)=79.17M$$

And the reinsurance cost allocated to France Branch

$$=200M*96.67/(63.33+96.67)=120.83M$$

Summary of Results

- ☯ The results from the various approaches are summarised in the chart below.
- ☯ In theory, the joint basis approach should be more appropriate.
 - Consistent with the principle, “Who use, who pay”

Branch	on Premium	on Expected Loss	Stand-alone	Marginal Basis	Joint Basis
Germany	133.33	111.11	66.67	70.59	79.17
France	66.67	88.89	133.33	129.41	120.83

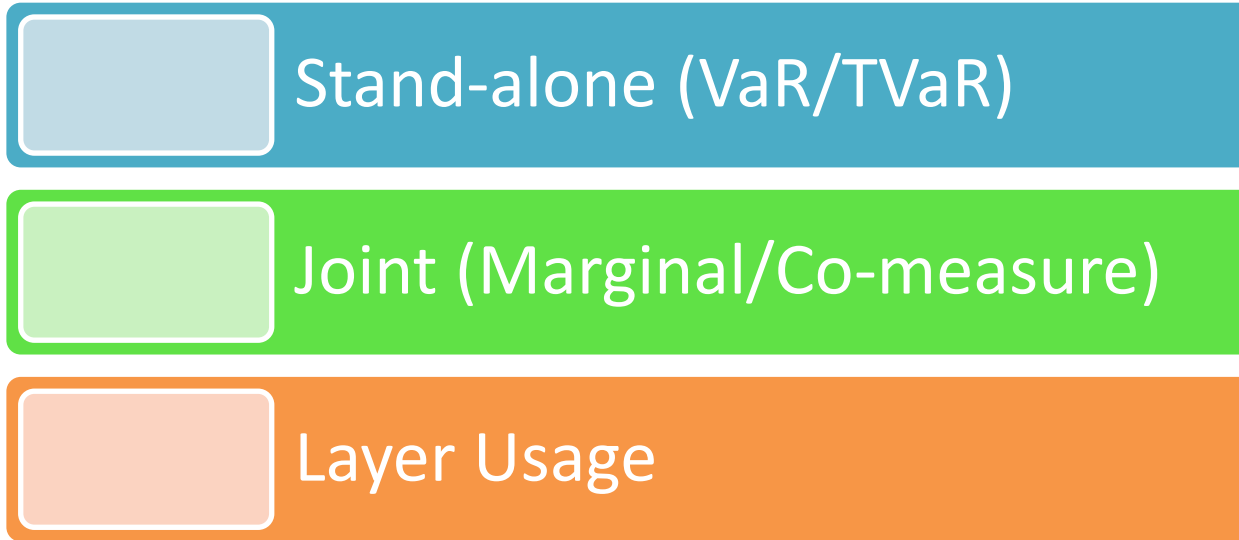
Why we Need Allocation?

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Allocation of Reinsurance Costs

Allocation of Capital

- ☯ The headquarter has to hold some capital in order to meet the regulatory solvency requirement.
- ☯ Which child causes the mother to hold capital?
- ☯ There are several approaches that have been brought out to solve that question:



An Illustration

- ☉ The headquarter holds a capital requirement of USD 500 million, and it has two branches, Germany and France.
- ☉ Two branches face the distributions of net loss as below, and the two distributions are independent.
 - The capital allocation could be based on VaR 99.5% or Expected Loss.

Branch	Probability	Net Loss	VaR 99.5%	Expected Loss
Germany	10%	200	200	200*10%=20
	90%	0		
France	5%	500	500	500*5%=25
	95%	0		

Illustration of Capital Allocation

- ☯ The net loss distribution of the headquarter is as below.
- ☯ The marginal approach and the co-measure approach will not be explained here, but the focus will be put on the layer usage approach as it follows the principle, “Who use, who pay”.

Scenario	Probability	Net Loss	Layer 200 Allocated to Germany	Layer 200 Allocated to France	Layer 300 Allocated to Germany	Layer 300 Allocated to France
Neither occurs	$.9 \cdot .95 = 85.5\%$	0	0	0	0	0
Germany occurs, but France not	$.1 \cdot .95 = 9.5\%$	200	$200 \cdot 9.5\%$	0	0	0
France occurs, but Germany not	$.9 \cdot .05 = 4.5\%$	500	0	$200 \cdot 4.5\%$	0	$300 \cdot 4.5\%$
Both occur	$.1 \cdot .05 = 0.5\%$	700	$200 \cdot .5\% \cdot 2/7$	$200 \cdot .5\% \cdot 5/7$	$300 \cdot .5\% \cdot 2/7$	$300 \cdot .5\% \cdot 5/7$
	100%	Subtotal:	19.2857	9.7143	0.4286	14.5714
		Percentage:	66.50%	33.50%	2.857%	97.143%

Illustration of Capital Allocation

Scenario	Probability	Net Loss	Layer 200 Allocated to Germany	Layer 200 Allocated to France	Layer 300 Allocated to Germany	Layer 300 Allocated to France
Neither occurs	.9*.95=85.5%	0	0	0	0	0
Germany occurs, but France not	.1*.95=9.5%	200	200*9.5%	0	0	0
France occurs, but Germany not	.9*.05=4.5%	500	0	200*4.5%	0	300*4.5%
Both occur	.1*.05=0.5%	700	200*.5%*2/7	200*.5%*5/7	300*.5%*2/7	300*.5%*5/7
	100%	Subtotal:	19.2857	9.7143	0.4286	14.5714
		Percentage:	66.50%	33.50%	2.857%	97.143%

➤ The capital allocated to Germany Branch

$$=200*66.50\%+300*2.857\%=141.57$$

➤ The capital allocated to France Branch

$$=200*33.50\%+300*97.143\%=358.43$$

Summary of Results

- ☯ The results from the various approaches are summarised in the chart below.
- ☯ The Layer Usage approach is Consistent with the principle, **“Who use, who pay”**.

Branch	on VaR 99.5%	on Expected Loss	Layer Usage
Germany	142.86	222.22	141.57
France	357.14	277.78	358.43

- ☯ The problem of allocation maybe falls into the range of art, rather than that of science.
- ☯ Allocation, allocation, allocation—
 - Which method is the best one?
 - It depends on your own location.



Questions & Answer

Many Minds are better than Only One.



Thank you!



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