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Rating without data

How to estimate the loss frequency of loss-free risks

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Agenda

- Problem
- Proposed solution
- Example
- Statistical properties

The loss-free rating situation

Assess the loss frequency of a particular risk, e.g. a non-proportional reinsurance treaty

- Data from other risks is not representative, thus market experience cannot help much
- Own data from past years is (in principle) representative, i.e. no structural changes
- Loss record:

no losses in past 7 years

What loss frequency?

- The traditional estimators yield zero
- This value is undesirable

Solution: construct an appropriate estimator

Key idea: absence of losses **is** an item of information

The ideal rating method

- covers all situations
- “converges“ to the empirical loss frequency
- always > 0
- Bias nonnegative but small
- monotonic
- smooth renewal: regard premium for next year
  - <u>if</u> year was loss-free <u>then</u> no increase
  - <u>if</u> new loss <u>then</u> increase but not too much

Ansatz: ASM (amended sample mean)

Sample mean: N / k

N = # losses in observation period

k = # observed years (maybe volume-weighted)

Define an **amending function** $g(n)$, $n = 0, 1, 2, \ldots$
and set

$$ASM \; := \; g(N) / k$$

Good amending functions

- always work

  $g(n)$ must be defined for all $n = 0, 1, 2, \ldots$

Good amending functions

- in case of many losses are close to the sample mean

$g(n) \to n$ or in particular $g(n) = n$ for $n \geq d$

Good amending functions

- never equal zero

$$g(n) > 0$$

Good amending functions

- have bias > 0 but small

$$g(n) \geq n, \quad \text{not} \gg$$

Good amending functions

- are (strictly) increasing

$$g(n+1) > g(n)$$

Good amending functions

- facilitate a smooth renewal

$$g(n+1)/g(n) \text{ “reasonable“}$$

Smoothness

- Premium should increase roughly like the loss record

$$g(n+1)/g(n) \approx (n+1)/n \quad \text{for } n>0$$

Smoothness

- The more losses, the less the impact of a new loss should be

$$g(n+2)/g(n+1) \leq g(n+1)/g(n)$$

Smoothness

- Premium should be less volatile than the loss experience

$$g(n+1)/g(n) \leq (n+1)/n \quad \text{for } n>0$$

Smoothness

- Premium should at the utmost double after a new loss

$$g(1)/g(0) \leq 2$$

Synopsis

- $g(n)$ defined for all $n = 0, 1, 2, ...$

- $g(n) \rightarrow n$ or in particular $g(n) = n$ for $n \geq d$
- $g(n) > 0$
- $g(n) \geq n$, not $>>$
- $g(n+1) > g(n)$

Synopsis smoothness

- $g(n+1)/g(n) \approx (n+1)/n$ for $n>0$

- $g(n+2)/g(n+1) \leq g(n+1)/g(n)$

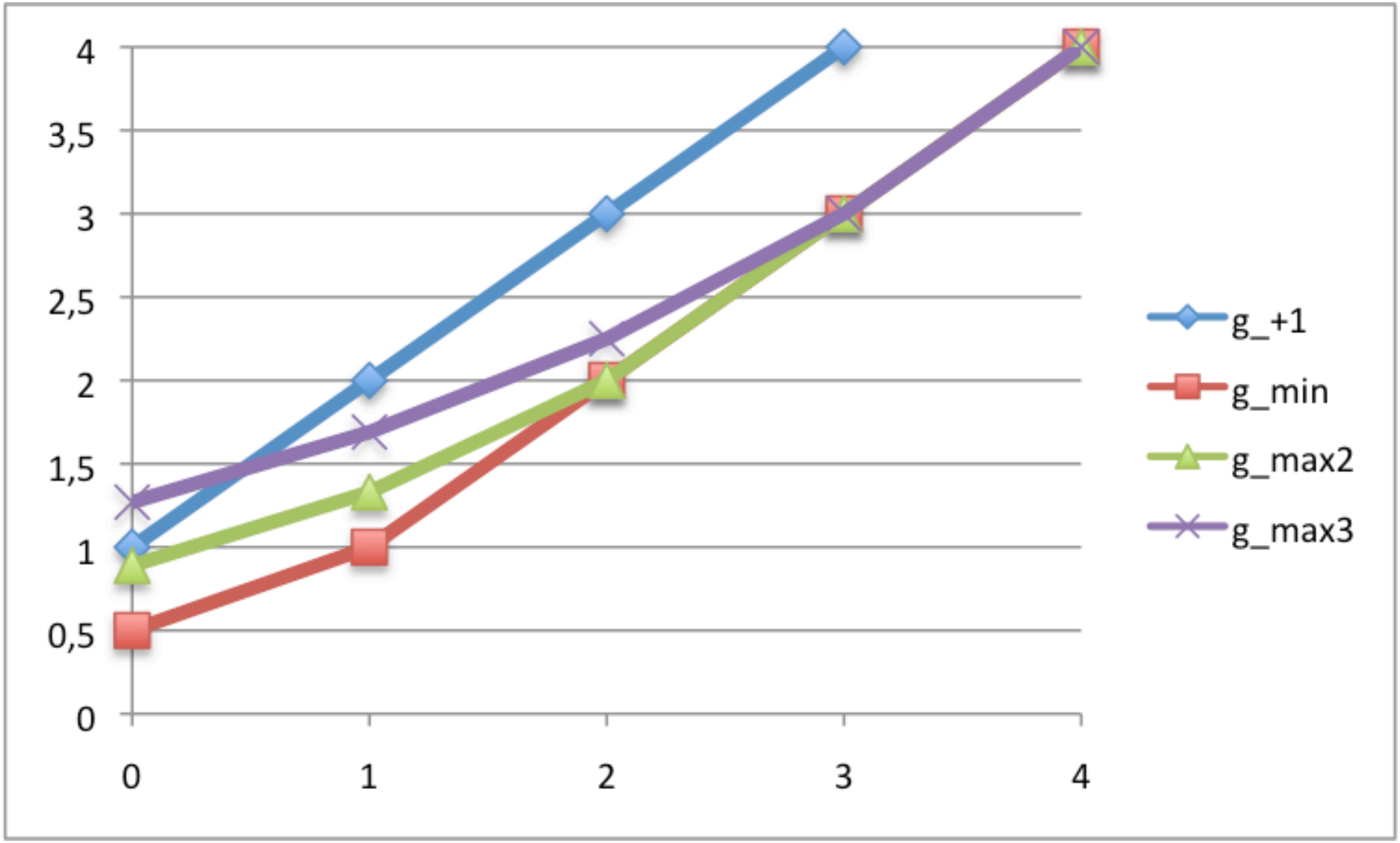
- $g(n+1)/g(n) \leq (n+1)/n$ for $n>0$
- $g(1)/g(0) \leq 2$

Candidates

n	**0**	**1**	**2**	**3**	**4**	**5**	**6**
$g_{+1}(n)$	1	2	3	4	5	6	7
$g_{min}(n)$	0.5	1	2	3	4	5	6
$g_{max2}(n)$	0.89	1.33	2	3	4	5	6
$g_{max3}(n)$	1.27	1.69	2.25	3	4	5	6

...

Candidates



Further optional constraints

Although estimators are mathematical things, ASMs may incorporate some strategy:

- minimum level: $g(n) \geq a$
- maximum percentage increase: $g(n+1)/g(n) \leq b$

You trade off small increases against a low minimum

Rating procedure

Step 1: Rate the frequency with your preferred ASM.

Step 2: Get the average loss from somewhere.

- Not easy for particular risks, but often much less uncertain than the frequency
- Well-tried approach for reinsurance layers: (European) Pareto with parameter alpha taken from market experience

Example: Nat Cat XL

Property CXL 100 xs 50 (say mln Euro)

10 years clean (notably using as-if corrected losses)

- $k = 10$
- $n = 0$
- $g_{max2}(n) = 0,89$

Frequency estimate: $g(n) / k = 8,9\%$

Rating

- alpha = 0,8   (very prudent)

- average loss: 61,3‘‘ Euro

Risk premium: 5,5‘‘ Euro   (Rate on Line 5,5%)

Renewal

A loss occurs, say 80 Mio Euro (or 55, or 130, or ...)

As we just know this loss, its size must be random:

Stay with the market alpha and only update the assessment of the frequency.

- $k = 11$
- $n = 1$
- $g_{max2}(n) = 1,33$

New rating

- Frequency estimate: g(n) / k = 12,1%
- Average loss unchanged: 61,3‘‘ Euro
- Risk premium: 7,5‘‘ Euro (R.o.L. 7,5%)

The risk premium increases by 36%

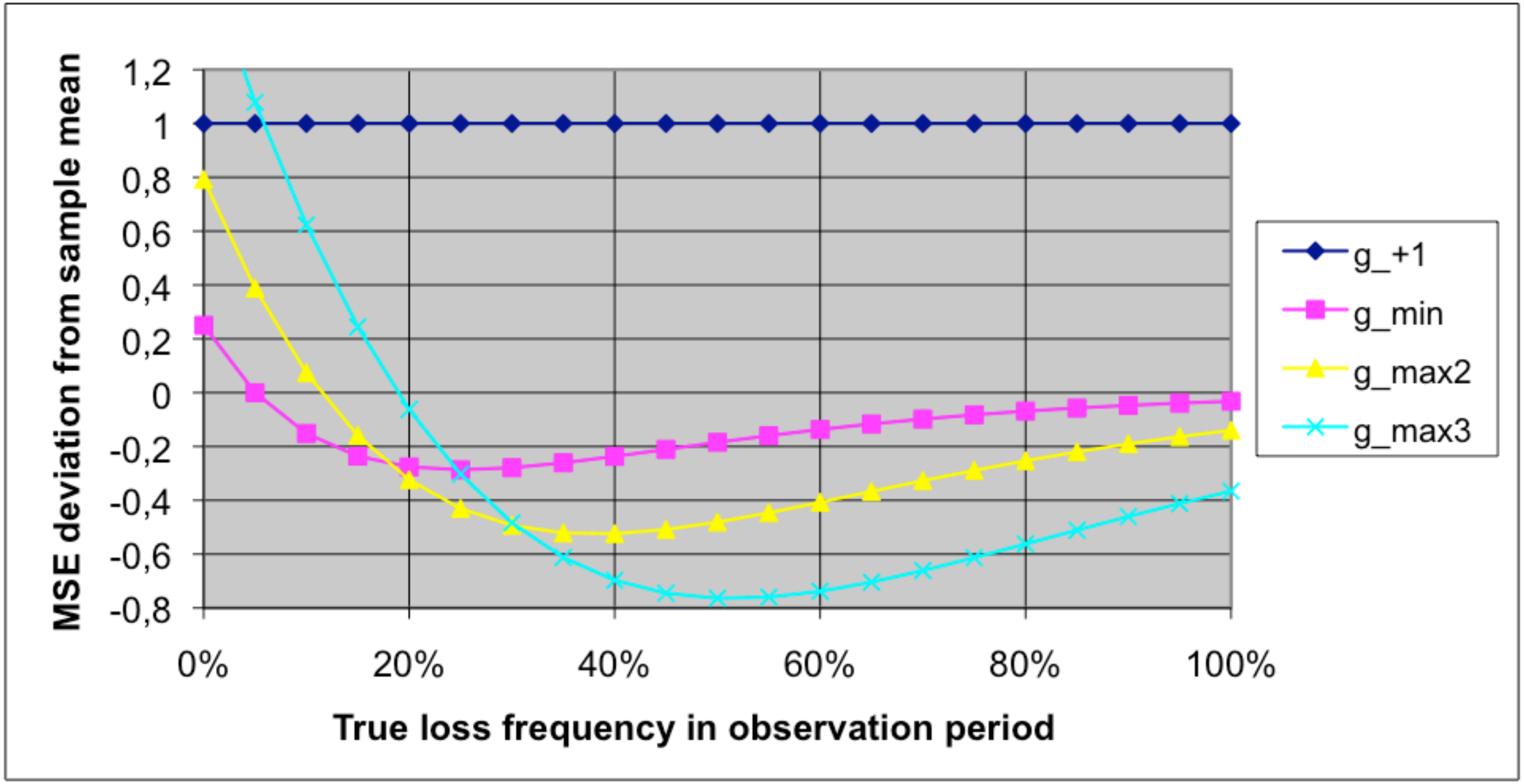
Statistical properties: Bias

- g_{+1} is too expensive

- g_{min} has the smallest bias (but is far from smooth)

All amending functions trade off smoothness against a small bias

Mean Squared Error: beats the sample mean!



Practical experience from XS reinsurance

Method has grown somewhat popular (although few admit that they come across such rating situations)

Results are often cheaper than

- pure “expert“ judgment (= educated guessing)
- workarounds used although clearly inadequate (e.g. exposure curves from totally different markets)
- premiums written

Conclusion: Use it!

- quick (but not dirty at all)
- systematic, not case by case
- always yields a result
- very easy to implement
- for much data same result as other methods
- mathematically consistent, statistically sound
- smooth renewal, according to choice of amending function

The End

Link to Paper: http://www.actuaries.org/ASTIN/Colloquia/Helsinki/Papers/S7_8_Fackler.pdf

Content:

- Construction of amending functions
- Rating examples
- Math for Poisson, Binomial, and NegBin case.

Thanks michael_fackler@web.de