# Reserve Risk Dependencies
## under Solvency II and IFRS 4 perspective

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CAE: Zurich, Oct 2014

(Last update: 17 October 2014)

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### SST and Solvency II

**prior year risk (PY risk) or reserving risk:**
The risk of a huge negative claims development result in the next year-end closing. Or with other words, the risk of much too few claim reserves.
**current year risk (CY risk) or premium risk:**
The risk of having much more current year incurred losses than premium earned.

### IFRS 4

**risk margin:**
Should cover the uncertainty within the (discounted) future cash flows corresponding to insurance liabilities (includes claims that already happened as well as future claims of already written contracts).

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**Types of insurance risks**

SST and Solvency II

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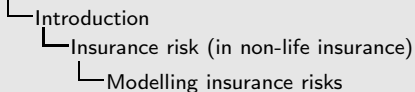
## Modelling insurance risks

A rough sketch of the classical way

1. Split up the total business into homogeneous portfolios.
2. Model each risk for each portfolio.
3. Aggregate the results via a correlation matrix.

Problem: How can we estimate such a correlation matrix?

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**Modelling insurance risks**

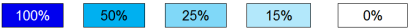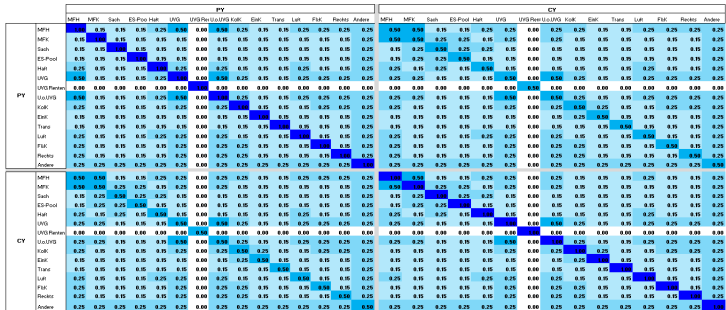A rough sketch of the classical way
1. Split up the total business into homogeneous portfolios.
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3. Aggregate the results via a correlation matrix.

Problem: How can we estimate such a correlation matrix?

- There are many other problems in modelling each risk for each portfolio.
- In order to measure the risk, Solvency II uses quantiles (99.5 %) and SST uses the expected shortfall (99 %). Under IFRS 4 Phase II (draft version) the company can choose their own risk measure.
- Often a parametric approach is taken to model each risk, that means:
  1. We choose a distribution family for the risk. Lognormal, Gamma, Normal and Pareto distributions are very popular. Usually the choice results from an expert opinion (or often called "industry standard") based on heuristic arguments.
  2. We estimate the corresponding parameters based on historic observations.
  3. We use the resulting probability distribution in order to calculate the risk.
- Often the risk mentioned will be further subdivided. For instance, the CY risk is usually split into risks of large and small claims.
- Correlation matrices only reflect linear dependencies. At least in the solvency context, where we are interested at very rare events, we should ask ourselves if an aggregation by correlation matrices is adequate.
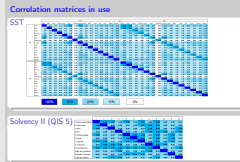
# Correlation matrices in use

## SST



## Solvency II (QIS 5)

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└─Introduction
  └─Correlation of insurance risks
    └─Correlation matrices in use

- The aggregation order of the SST and Solvency II is slightly different:
  - SST: Model PY and CY risk for each portfolio and aggregate them all at once.
  - Solvency II: Model first PY and CY risk for each portfolio and aggregate them with correlation of 50 %, then aggregate all portfolios.
- For the correlations, only four (or two) different values are used.
- For the CY correlation, one could find at least heuristic arguments why some lines of business are more correlated than others. For instance:
  - A hailstorm affects motor own damage and property but not third party liability.
  - Mandatory (UVG) and non-mandatory (U.o.UVG) accident insurance are correlated because both insure accidents.
  - Medical expenses have a strong influence on motor liability and accident.
- For PY correlation, it is much more difficult to find such heuristic arguments. The reason for this is that in this case we look at the correlation of change in estimates that are determined by actuaries, who should take such dependencies into account when setting the reserves.
- Inflation is an overall driver for correlation.
- Correlations in the case of SST have been set in dependence of the number of "drivers for correlation". Therefore, it is at least 15 %, except for annuities in mandatory accident (UVG-Renten), for which the risks are modelled like normal annuities of life-insurers.

### Definition 2.1 ($\sigma$-algebras)
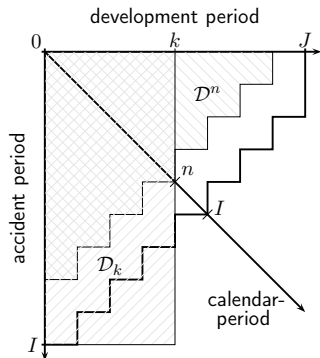


- $\mathcal{B}_{i,k}$ is the $\sigma$-algebra of all information of accident period $i$ up to development period $k$:
$$\mathcal{B}_{i,k} := \sigma\left(S_{i,j} \colon 0 \leq j \leq k\right) = \sigma\left(C_{i,j} \colon 0 \leq j \leq k\right)$$

- $\mathcal{D}^n$ is the $\sigma$-algebra of all information up to calender period $n$:
$$\begin{aligned}
\mathcal{D}^n := & \,\sigma\left(S_{i,k} \colon 0 \leq i \leq I,\, 0 \leq k \leq J \wedge (n-i)\right) \\
= & \,\sigma\left(C_{i,k} \colon 0 \leq i \leq I,\, 0 \leq k \leq J \wedge (n-i)\right) \\
= & \,\sigma\left(\bigcup_{i=0}^{I} \bigcup_{k=0}^{J \wedge (n-i)} \mathcal{B}_{i,k}\right)
\end{aligned}$$

- $\mathcal{D}_k$ is the $\sigma$-algebra of all information up to development period $k$:
$$\begin{aligned}
\mathcal{D}_k := & \,\sigma\left(S_{i,j} \colon 0 \leq i \leq I,\, 0 \leq j \leq k\right) \\
= & \,\sigma\left(C_{i,j} \colon 0 \leq i \leq I,\, 0 \leq j \leq k\right) \\
= & \,\sigma\left(\bigcup_{i=0}^{I} \mathcal{B}_{i,k}\right)
\end{aligned}$$

- $\mathcal{D}_k^n := \sigma\left(\mathcal{D}^n \cup \mathcal{D}_k\right)$

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Reserve risk dependencies
└─Modelling reserve risks
  └─Classical reserving methods based on triangles
    └─$\sigma$-algebras

- Classical triangle based reserving methods group the already observed data into a triangles (or trapezoids if $I > J$) and try to complete them to a rectangles.

## Chain Ladder Method (CLM)

Actuaries often use the Chain Ladder Method for reserving. This means they believe in

i)$^{\text{CLM}}$ $\mathsf{E}\Big[C_{i,k+1}|\mathcal{D}_k^{i+k}\Big] = f_k C_{i,k}$,

ii)$^{\text{CLM}}$ $\mathsf{Var}\Big[C_{i,k+1}|\mathcal{D}_k^{i+k}\Big] = \sigma_k^2 C_{i,k}$ and

iii)$^{\text{CLM}}$ accident periods are independent.

Moreover, in order to quantify the corresponding risk,

- the approach of T. Mack, see [4], is often used for the ultimate risk and
- the approach of M. Merz and M. Wüthrich, see [5], is often used for the solvency risk.

## Idea

Why not take several triangles $C_{i,k}^m$, $0 \le m \le M$ and couple them via

$$\mathsf{Cov}\Big[C_{i,k+1}^{m_1}, C_{i,k+1}^{m_2}\Big|\mathcal{D}_k^{i+k}\Big] = \sigma_k^{m_1,m_2}\sqrt{C_{i,k}^{m_1}C_{i,k}^{m_2}} \ ?$$

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i) $^{CLM}$ $E[C_{i,k+1}|\mathcal{D}_k^{i+k}] = f_k C_{i,k}$,

ii) $^{CLM}$ $Var[C_{i,k+1}|\mathcal{D}_k^{i+k}] = \sigma_k^2 C_{i,k}$ and

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Why not take several triangles $C_{i,k}^m$, $0 \leq m \leq M$ and couple them via

$$Cov\left[C_{i,k+1}^m, C_{i,k+1}^{m_2} \Big| \mathcal{D}_k^{i+k}\right] = \sigma_k^{m_1, m_2} \sqrt{C_{i,k}^{m_1} C_{i,k}^{m_2}} ?$$

- Classically the assumptions are stated with the smaller $\sigma$-algebras $\mathcal{B}_{i,k}$ instead of $\mathcal{D}_k^{i+k}$.
- The third assumption is not necessary (if we use the $\sigma$-algebras $\mathcal{D}_k^{i+k}$).
- The first assumption of the Chain Ladder Method means that we believe in the cumulative values to be good exposures for the development of the next period.

## Extended Complementary Loss Ratio Method (ECLRM)

We take incremental payments $S_{i,k}^0$, changes in reported amounts $S_{i,k}^1$, case reserves $R_{i,k}$ and assume that

i)$^{\text{ECLRM}}$  $\mathsf{E}\left[S_{i,k+1}^m\Big|\mathcal{D}_k^{i+k}\right] = f_k^m R_{i,k}$ and

ii)$^{\text{ECLRM}}$  $\mathsf{Cov}\left[S_{i,k+1}^{m_1}, S_{i,k+1}^{m_2}\Big|\mathcal{D}_k^{i+k}\right] = \sigma_k^{m_1,m_2} R_{i,k}$.

Moreover, in order to quantify the corresponding risk we use

- the approach of R. D., see [1], for the ultimate risk.
- the approach of M. Wüthrich and R. D., see [3], for the solvency risk.

## Idea

Why not take several portfolios and couple them via

$$\mathsf{Cov}\left[S_{i,k+1}^{m_1}, S_{i,k+1}^{m_2}\Big|\mathcal{D}_k^{i+k}\right] = \sigma_k^{m_1,m_2}\sqrt{R_{i,k}^{m_1}R_{i,k}^{m_2}} \ ?$$

**Extended Complementary Loss Ratio Method (ECLRM)**

We take incremental payments $S_{i,k}^p$, changes in reported amounts $S_{i,k}^i$, case reserves $R_{i,k}$ and assume that

$$i)^{ECLRM} \quad E\left[S_{i,k+1}^p\middle|\mathcal{D}_k^{i+k}\right] = l_k^{p,e} R_{i,k} \text{ and}$$
$$ii)^{ECLRM} \quad Cov\left[S_{i,k+1}^p, S_{i,k+1}^i\middle|\mathcal{D}_k^{i+k}\right] = \sigma_k^{p_e, i_e} R_{i,k}.$$

Moreover, in order to quantify the corresponding risk we use
- the approach of R. D., see [1], for the ultimate risk.
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**Idea**

Why not take several portfolios and couple them via

$$Cov\left[S_{i,k+1}^{p,a}, S_{i,k+1}^{p,b}\middle|\mathcal{D}_k^{i+k}\right] = \sigma_k^{p_a, p_b} \sqrt{R_{i,k}^{a,a} R_{i,k}^{b,b}} \text{ ?}$$

- Taking the ECLRM means that we believe in the case reserves to be good exposures for the next year's payments and the next year's changes in the reported amounts.
- The case reserves fulfil the assumptions of the CLM.

### Other examples

Similar statements can be formulated for

- the Bornhuetter Ferguson Method,
- the Complementary Loss Ratio Method,
- the Cape Cod Method,
- the Benktander Hovinen Method
- . . .

### What do those methods have in common?

- The expectation of next year's development is proportional to some exposure, which is a linear combination of the past developments.
- Covariances are proportional to some exposures, which depend only on the past developments.

Reserve risk dependencies

└─ Modelling reserve risks

    └─ Classical reserving methods based on triangles

**Other examples**

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**What do those methods have in common?**

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- Covariances are proportional to some exposures, which depend only on the past developments.

## Linear Stochastic Reserving Methods (LSRMs)

We have several claim properties (triangles) $S_{i,k}^m$ and assume that:

i)$^{\text{LSRM}}$ There exist exposures $R_{i,k}^m \in \mathcal{D}^{i+k} \cap \mathcal{D}_k$, which depend linearly on claim properties $\mathbf{S}$, such that

$$\mathsf{E}\Big[S_{i,k+1}^m \big| \mathcal{D}_k^{i+k}\Big] = f_k^m R_{i,k}^m := f_k^m \Gamma_{i,k}^m \mathbf{S}.$$

ii)$^{\text{LSRM}}$ There exist exposures $R_{i,k}^{m_1,m_2} \in \mathcal{D}^{i+k} \cap \mathcal{D}_k$ such that

$$\mathsf{Cov}\Big[S_{i,k+1}^{m_1}, S_{i,k+1}^{m_2} \big| \mathcal{D}_k^{i+k}\Big] = \sigma_k^{m_1,m_2} R_{i,k}^{m_1,m_2}.$$

An updated version of the original paper, see [2], can be obtained from the author.

Linear Stochastic Reserving Methods (LSRMs)

We have several claim properties (triangles) $S_{i,k}^m$ and assume that:

i) $^{\text{LSRM}}$ There exist exposures $R_{i,k}^m \in \mathcal{D}^{>k} \cap \mathcal{D}_k$, which depend linearly on claim properties $\mathbf{S}$, such that

$$E\left[ S_{i,k+1}^m \middle| \mathcal{D}_k^{i+k} \right] = f_k^m R_{i,k}^m := f_k^m \Gamma_{i,k}^m \mathbf{S}.$$

ii) $^{\text{LSRM}}$ There exist exposures $R_{i,k}^{m_1,m_2} \in \mathcal{D}^{>k} \cap \mathcal{D}_k$ such that

$$\text{Cov}\left[ S_{i,k+1}^{m_1}, S_{i,k+1}^{m_2} \middle| \mathcal{D}_k^{i+k} \right] = \sigma_k^{m_1,m_2} R_{i,k}^{m_1,m_2}.$$

An updated version of the original paper, see [2], can be obtained from the author.

- There are classical reserving methods which do not belong to LSRMs. For instance, Munich Chain Ladder Method, Overdispersed Poisson Model, GLMs ...
- In the original paper it was assumed that the exposures $R_{i,k}^{m_1,m_2}$ depend linearly on claim properties, too, but as long as we are only interested in the solvency risk this assumption is not necessary. Moreover, even for the ultimate risk, one could skip this linearity assumption at the cost of a more rough estimation of the process variance as part of the ultimate risk.
- Note, we do not need an assumption about the independence of accident periods.
- The exposures $R_{i,k}^{m_1,m_2}$ cannot be chosen arbitrarily, because the matrices

$$\left( \sigma_k^{m_1,m_2} R_{i,k}^{m_1,m_2} \right)^{0 \leq m_1, m_2 \leq M}$$

have to be symmetric and positive semi-definite for all $i$ and $k$.

- A natural choice for exposures $R_{i,k}^{m_1,m_2}$ for different portfolios $m_1, m_2$ is $\sqrt{R_{i,k}^{m_1} R_{i,k}^{m_2}}$.

## LSRM step by step
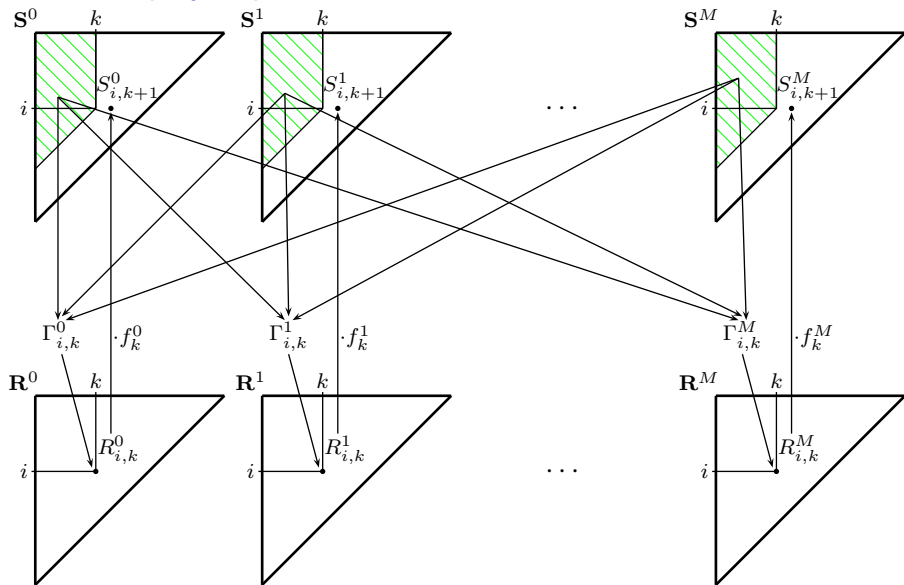
Reserve risk dependencies

  └─Modelling reserve risks

      └─Linear Stochastic Reserving Methods (LSRMs)
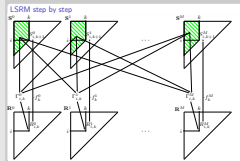
### Modelling all portfolios at once,

we can look at the total ultimate

$$\sum_{m=0}^{M} \sum_{i=0}^{I} \alpha_i^m \sum_{k=0}^{J} S_{i,k}^m,$$

where $\alpha_i^m$ are arbitrary real numbers (mixing weights).
Then the variance of the reserving risk can be estimated by the mean squared error of prediction (mse), which is of the form

$$\widehat{\mathrm{mse}} = \sum_{m_1,m_2=0}^{M} \sum_{i_1,i_2=0}^{I} \alpha_{i_1}^{m_1} \alpha_{i_2}^{m_2} \widehat{\beta}_{i_1,i_2}^{m_1,m_2}.$$

This is valid for the ultimate reserving risk as well as for the solvency reserving risk (with different $\beta$s of course).

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Reserve risk dependencies
└─LSRMs and reserving risk
 └─Derivation

Modelling all portfolios at once,
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Then the variance of the reserving risk can be estimated by the mean squared
error of prediction (mse), which is of the form

$$\widehat{mse} = \sum_{m_1,m_2=0}^{M}\sum_{i_1,i_2=0}^{I}\alpha_{i_1}^{m_1}\alpha_{i_2}^{m_2}\widehat{cr}_{i_1,i_2}^{m_1,m_2}.$$

This is valid for the ultimate reserving risk as well as for the solvency reserving
risk (with different $i$s of course).

- If we only look at triangles of payments of different portfolios, the mixing weights equal one.
- If we look at triangles of payments and reported amounts for each portfolio, the mixing weights represent the credibility we give to each projection.
- Using this approach, we do not need any correlation matrices to estimate the reserving risk of the total of all portfolios.

### Reverse engineering of a correlation matrix:

If we are required to use a correlation approach, we could use the components of overall $\widehat{\text{mse}}$ in order to define the correlation matrix, i.e. we could take

$$\left( \frac{\sum_{i_1,i_2=0}^{I} \alpha_{i_1}^{m_1} \alpha_{i_2}^{m_2} \widehat{\beta}_{i_1,i_2}^{m_1,m_2}}{\sqrt{\sum_{i=0}^{I} \alpha_i^{m_1} \alpha_i^{m_1} \widehat{\beta}_{i,i}^{m_1,m_1} \sum_{i=0}^{I} \alpha_i^{m_2} \alpha_i^{m_2} \widehat{\beta}_{i,i}^{m_2,m_2}}} \right)^{0 \leq m_1, m_2 \leq M}$$

as the correlation matrix.

### Model error

Since the real world does not entirely follow the assumptions of LSRMs, our estimations of the reserve risk should be increased by a model error.

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Reserve risk dependencies
└─ LSRMs and reserving risk
   └─ Derivation

**Reverse engineering of a correlation matrix:**
If we are required to use a correlation approach, we could use the components of overall risk in order to define the correlation matrix, i.e. we could take

$$\left( \frac{\sum_{l=1}^{I} \alpha_l^{m_1} \alpha_l^{m_2} \widehat{\beta}_{l,t}^{m_1,m_2}}{\sqrt{\sum_{l=1}^{I} \alpha_l^{m_1} \alpha_l^{m_1} \widehat{\beta}_{l,t}^{m_1,m_1} \sum_{l=1}^{I} \alpha_l^{m_2} \alpha_l^{m_2} \widehat{\beta}_{l,t}^{m_2,m_2}}} \right)^{0 \le m_1, m_2 \le M}$$

as the correlation matrix.

**Model error**
Since the real world does not entirely follow the assumptions of LSRMs, our estimations of the reserve risk should be increased by a model error.

- Since the choice of mixing weights was arbitrary, the correlation matrix with the true $\beta$s instead of their estimated values $\widehat{\beta}$ is positive semi-definite!
- Nevertheless, the estimated correlation matrix may not be positive semi-definite. If so, it usually goes along with some slightly negative eigenvalues and at least one large positive eigenvalue.

## Fire non commercial vs. motor own damage

| correlation | ultimate | solvency |
|---:|---|---|
| SST | | 15 % |
| QIS 5 | | 25 % |
| mixed CLM | 20 % | 25 % |
| CLM on paid | 25 % | 35 % |
| CLM on incurred | 20 % | 30 % |
| ECLRM | -5 % | -5 % |

## Motor TPL vs. motor own damage

| correlation | ultimate | solvency |
|---:|---|---|
| SST | | 15 % |
| QIS 5 | | 50 % |
| mixed CLM | 5 % | 5 % |
| CLM on paid | 0 % | 0 % |
| CLM on incurred | 10 % | 15 % |
| ECLRM | 10 % | 20 % |

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Reserve risk dependencies
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| Fine non commercial vs. motor own damage | | |
| --- | --- | --- |
| correlation | ultimate | solvency |
| SST | | 15 % |
| QIS 5 | | 25 % |
| mixed CLM | 20 % | 25 % |
| CLM on paid | 25 % | 35 % |
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| Motor TPL vs. motor own damage | | |
| --- | --- | --- |
| correlation | ultimate | solvency |
| SST | | 15 % |
| QIS 5 | | 50 % |
| mixed CLM | 5 % | 5 % |
| CLM on paid | 0 % | 0 % |
| CLM on incurred | 10 % | 15 % |
| ECRLM | 10 % | 20 % |

- The QIS 5 numbers are not completely comparable since they represent the combined correlation of premium and reserve risks. Therefore, they usually have to be higher.
- Since the correlation manifests itself mostly in early development periods we expect that the correlation under solvency perspective is higher than the correlation from the ultimate point of view (Corr $= \frac{\text{Cov}}{\sqrt{\text{VarVar}}}$ and the variance is much more influenced by later development periods than the covariance).
- When estimating correlations, systemic changes in the data, for instance a change in the case reserve methodology, have to be excluded as far as possible.
- The estimates depend on few data only. Therefore we cannot expect them to be statistically significant.
- We have to take some model errors into account, in particular if we are interested in very rare events like the once of the SST and Solvency II. One reason for this is that LSRMs (like the CLM) cannot deal with inflation or other "diagonal effects". Therefore we may add some percentages, for instance 15 % like in the SST standard model.
- It may be worth investigating how much of the correlation is caused by the development of claims already known and how much is caused by late claims. Therefore we could look at the paid and incurred triangles per reporting date as well as at the number of reported claims.

### Including the CY risk

A master student will start to investigate the possibilities of including the CY risk. The basic idea is:

- add a column to the left side of each triangle that corresponds to the estimated ultimate of the next period (CY ultimate).
- explore different exposures with respect to stability and comparability.

### LSRMTools

There is a runtime library that implements LSRMs under the public licence GPL 3. Moreover, it includes an Excel Add In that allows easy access to these futures. The examples of this presentation have been generated by using these tools. It (except for the original data of the examples) can all be obtained from the author or from

http://sourceforge.net/projects/lsrmtools/

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└─Outlook, tools and bibliography
  └─Outlook and tools

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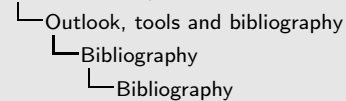
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