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CAE-Meeting
From deterministic to stochastic Reserving – an overview
November 2009

PRICEWATERHOUSECOOPERS

Agenda

Deterministic methods
Statistical methods
Stochastic methods
Why stochastic reserving?

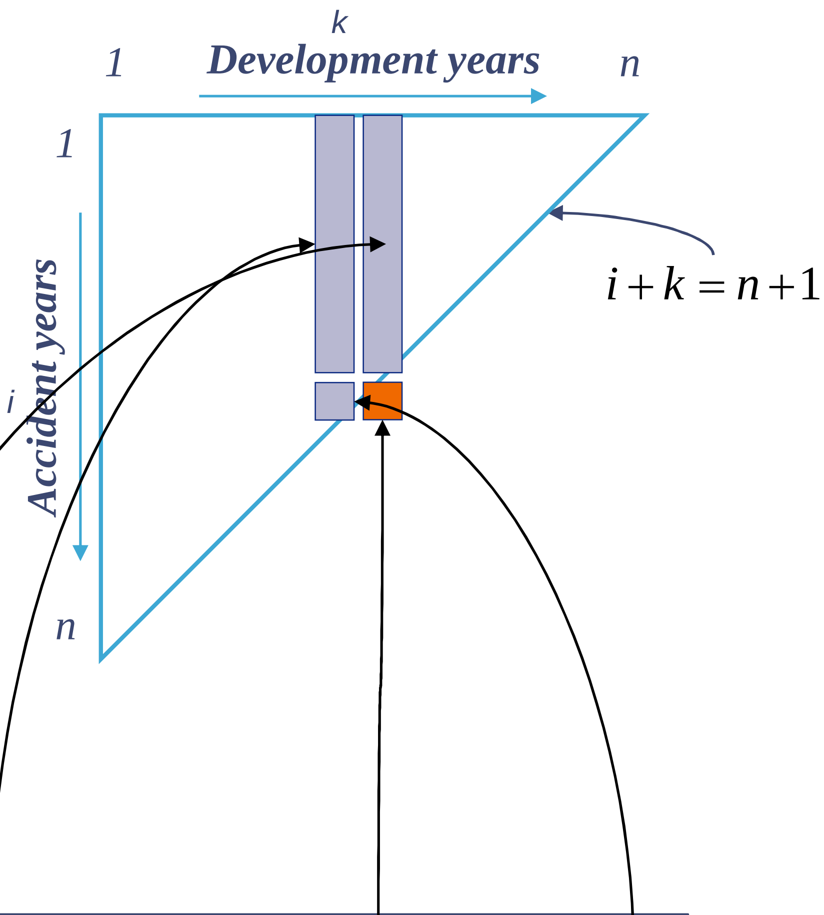
Focus on usability and ideas behind the concepts!

Deterministic Methods

Characteristics:

- Exactly reproducible
- Yields only an expected value
- Used since: ? (for a long time…)

Chain Ladder



Notation:

i = accident year

k = development year

$D_{i,k}$ = incremental claim amounts

$C_{i,k}$ = cumulative claim amounts

Algorithm:

$$f_k = \sum_{j=1}^{n-k} C_{j,k+1} \Big/ \sum_{j=1}^{n-k} C_{j,k} \qquad \hat{C}_{i,k+1} = f_k \cdot C_{i,k}$$

Chain Ladder

Pros:

- The whole historical triangle is taken into account
- Intuitive and easy to implement
- Can easily be adjusted by changing weights of prior years
- No further external data is needed

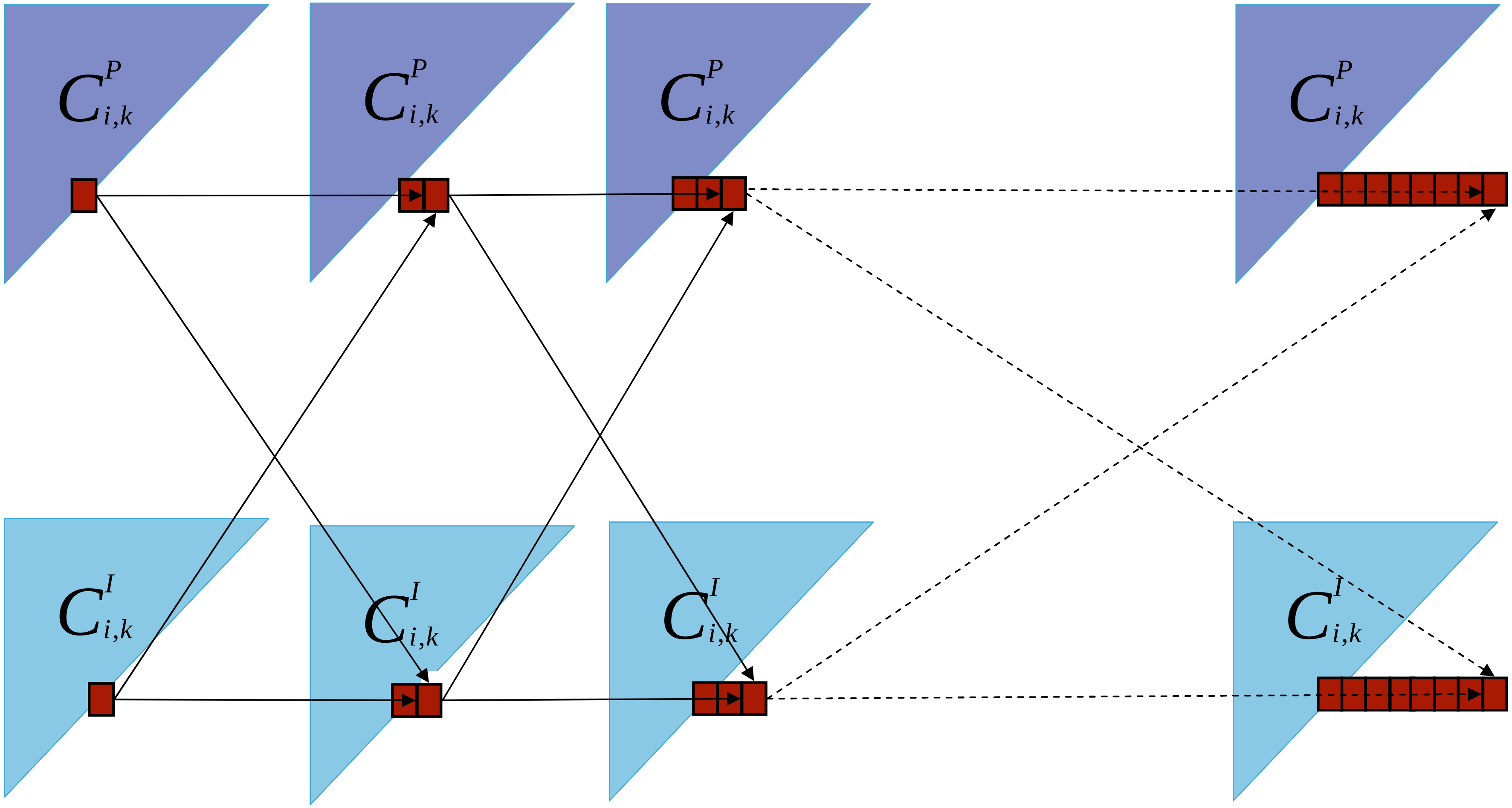
Cons:

- Late development periods and recent accident years: little data available
- No evaluation of the quality of the reserve estimate

Munich Chain Ladder (MCL)

- Problem: Either data from paid or from incurred triangle can be used as an input for most of the reserving methods.
- Ultimate shouldn’t depend on whether you reserve on your paid or on your incurred triangle.
- Goal of MCL: Close gap between paid and incurred ultimate.
- MCL doesn’t force the ultimates to be equal!
- Different approach from René Dahms (“A Loss Reserving method for incomplete data”): projected ultimates from paid and incurred are forced to be equal.

Munich Chain Ladder



Munich Chain Ladder

Pros:

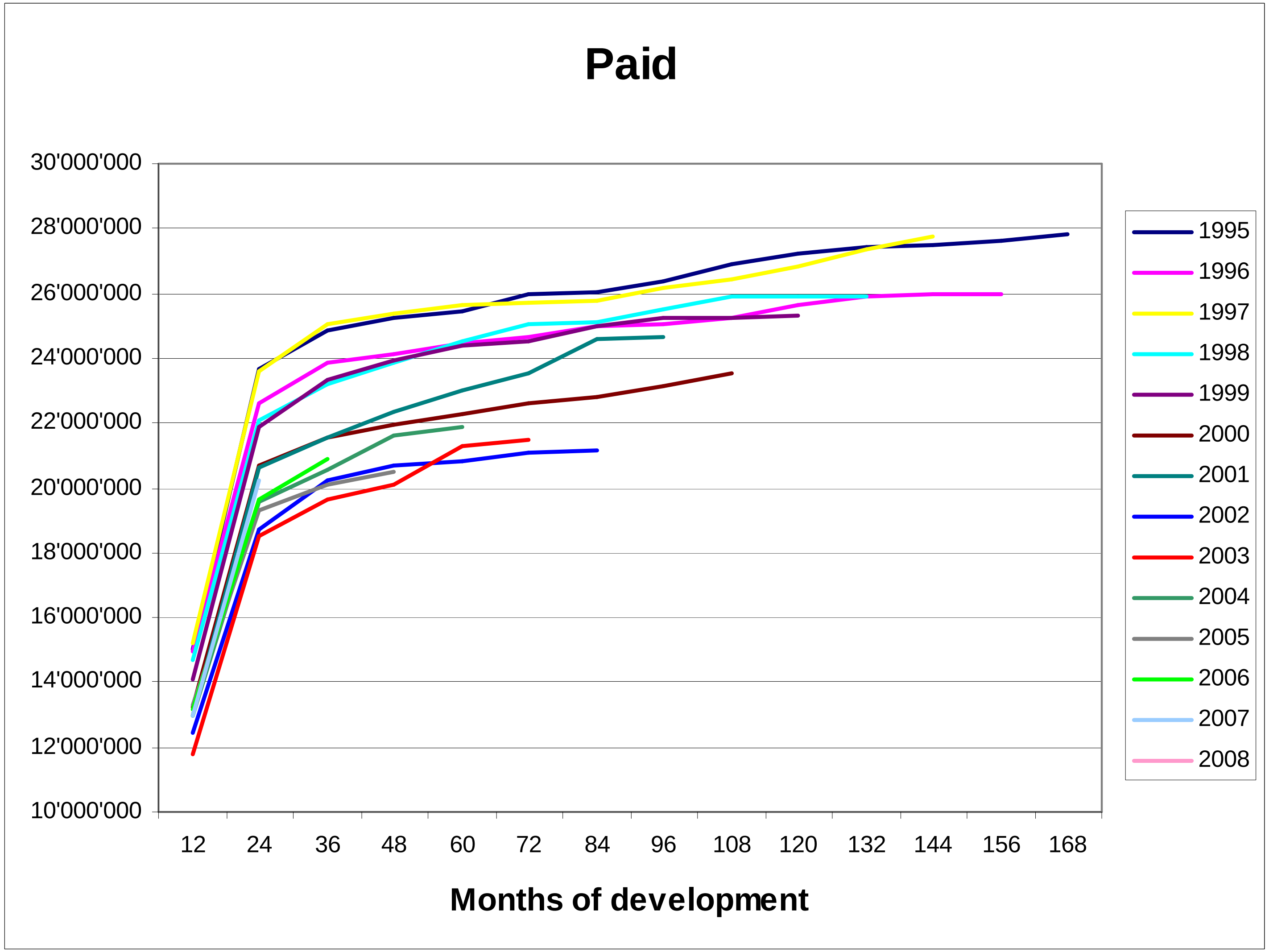
- Ultimates based on paid and incurred data
- Gap between paid & incurred ultimates becomes closer

Cons:

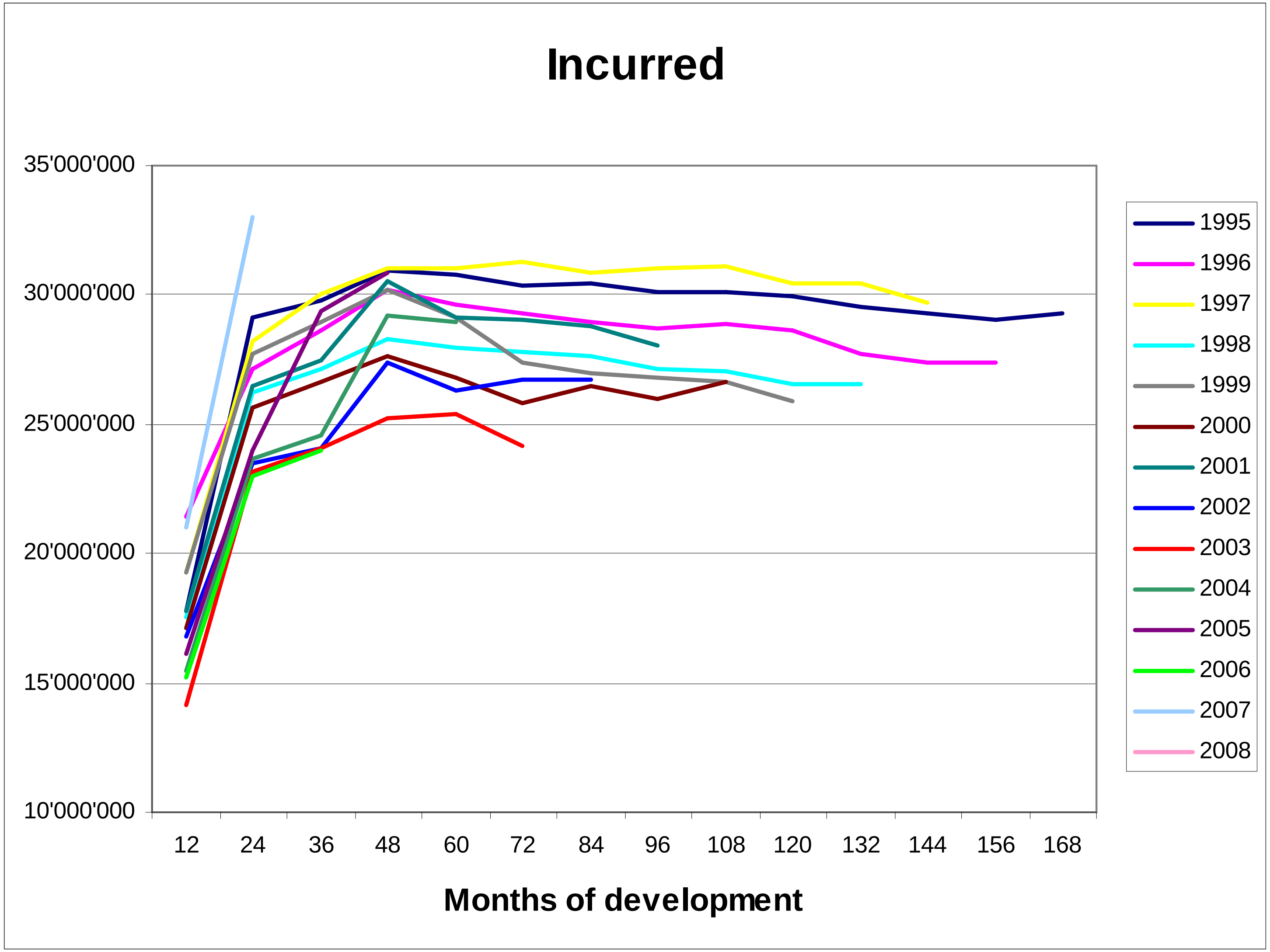
- Implementation of MCL more difficult than CL
- No evaluation of the quality of the calculated reserves

Mack & Quarg (2004): Munich Chain Ladder - A reserving method that reduces the gap between IBNR projections based on paid losses and IBNR projections based on incurred losses

Numerical example: Paid triangle



Numerical example: Incurred triangle



Ultimates for different deterministic models

In CHF 1000	CLpaid	CL inc.	B-F paid	B-F inc.	CC paid	CC inc.	MCL paid	MCL inc.
1995	27'831	29'297	27‘831	29'297	27'831	29'297	27'831	29'297
1996	26'190	27'615	26'200	27'614	26'199	27'625	26'195	27'614
⋮	⋮		⋮		⋮		⋮	
2007	24'198	34'183	25'716	34'159	24'281	34'030	27'653	31'314
2008	25'690	29'555	27'432	29'542	25'459	29'823	26'722	28'693
Total	**350'106**	**382'128**	**355'530**	**381'391**	**350'781**	**381'897**	**355'953**	**377'448**

- “Nice” portfolio: Similar results for all methods
- MCL minimizes the gap between paid and incurred

Statistical methods

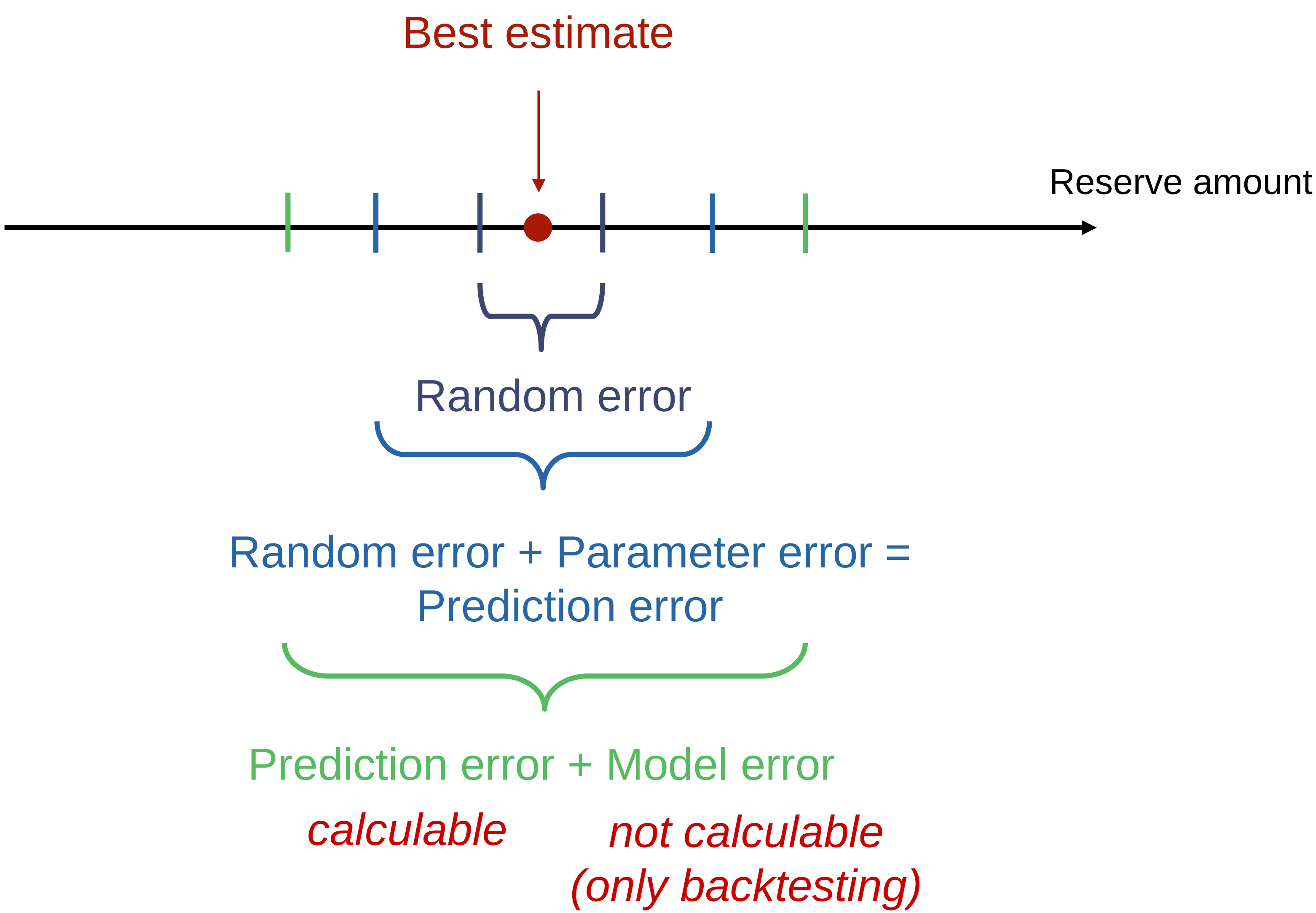
Characteristics:

- Exactly reproducible
- Yields expected value of ultimate = first moment of distribution
- Yields prediction variance of ultimate = second moment of distribution
- Since 1990

Statistical methods: an overview

- Deterministic models enriched with an underlying stochastic model
  - Statistical Chain Ladder (Mack)
  - Statistical Bornhuetter Ferguson
  - Statistical Munich Chain Ladder
- Log Regression Models and GLM
  - Zenwirth Models (see presentation by Spencer Gluck)
- Bayesian Models (Benktander, Cape Cod, Credibility models)
- Special case: Distributional models (Log-normal model -> SST)

Different kind of errors



Mack Chain Ladder

<u>Assumptions:</u>

$C_{i,k}$ are independent for different accident years i

$$E\left[C_{i,k+1} \middle| C_{i,1} \cdots C_{i,k}\right] = f_k \cdot C_{i,k}$$

$$Var\left[C_{i,k+1} \middle| C_{i,1} \cdots C_{i,k}\right] = \sigma_\kappa^2 \cdot C_{i,k}$$

Variance is proportional to the expectation with the same factors for all accident years

Development factors are the same for all accident years

Future development depends only on the diagonal value

Accident year i is independent of accident year j

Error calculation for Mack Chain Ladder

It is straightforward to find unbiased estimators for the parameters f_k and σ_k:

$$\widehat{f_k} = \frac{\sum_{j=1}^{n-k} C_{j,k+1}}{\sum_{j=1}^{n-k} C_{j,k}}$$

$$\widehat{\sigma}_k = \frac{1}{n-k-1} \sum_{i=1}^{n-k} C_{i,k} \left(\frac{C_{i,k+1}}{C_{i,k}} - \widehat{f_k} \right)^2$$

Error calculations for stochastic Chain Ladder

Notation:

- Estimated ultimate for accident year i (at time n): $\hat{C}_{i,n}$
- Theoretical best estimate based on the stochastic model: $C_{i,n}$
- Sigma-Algebra with information to date: $\mathscr{D}$

Mean Square error of prediction of the ultimate :

$$\begin{aligned}\mathrm{msep}_{C_{i,n}|\mathscr{D}}(\widehat{C}_{i,n}) &= E\left[\left(\widehat{C}_{i,n} - C_{i,n}\right)^2 \middle| \mathscr{D}\right] \\ &= \mathrm{Var}\left(C_{i,n} \middle| \mathscr{D}\right) + E\left[\left(\widehat{C}_{i,n} - E\left[C_{i,n} \middle| \mathscr{D}\right]\right)^2\right]\end{aligned}$$

Prediction error = Random error + Parameter error

Error calculations for stochastic Chain Ladder

Based on the estimators $\widehat{f}_k$ and $\widehat{\sigma}_k$ one can give an estimator of the mean square error of prediction:

$$\widehat{\mathrm{msep}}_{C_{i,n}|\mathscr{D}}\left(\widehat{C}_{i,n}\right) = \widehat{C}_{i,n}^2 \sum_{k=n+1-i}^{n-1} \frac{\widehat{\sigma}_k^2}{\widehat{f}_k^2} \frac{1}{\widehat{C}_{i,k}} + \widehat{C}_{i,n}^2 \sum_{k=n+1-i}^{n-1} \frac{\widehat{\sigma}_k^2}{\widehat{f}_k^2} \frac{1}{\sum_{j=1}^{n-k} C_{j,k}}$$

Prediction error = Random error + Parameter error

This formula can be implemented and calculated while running Chain Ladder => we have a measure of uncertainty for our ultimate estimation!

Mack Chain Ladder

Pros:

- Can easily be adjusted by changing weights of prior years
- No external data is needed
- Uncertainty of ultimate can be estimated by error calculations
- Symmetric prediction intervals can be determined

Cons:

- Error estimations require further implementation work
- Can only calculate a second moment => quantiles can’t be determined

Mack (1993): Distribution-Free Calculation of the Standard Error of Chain Ladder Reserve Estimates

Mack chain ladder – Error calculations for paid data

In CHF 1000	Ultimate	Pred. error	in % of Ult.	Estim. error	in % of Ult.	Rand. error	in % of Ult.
1996	26'190	57	0.22%	40	0.15%	41	0.16%
1997	28'027	126	0.45%	77	0.28%	100	0.36%
⋮	⋮	⋮		⋮		⋮	
2007	24'198	836	3.46%	283	1.17%	787	3.25%
2008	25'689	1'121	4.36%	353	1.37%	1'064	4.14%
Total	**350'105**	**2'898**	**0.83%**	**2'136**	**0.61%**	**1'957**	**0.56%**

Stochastic methods

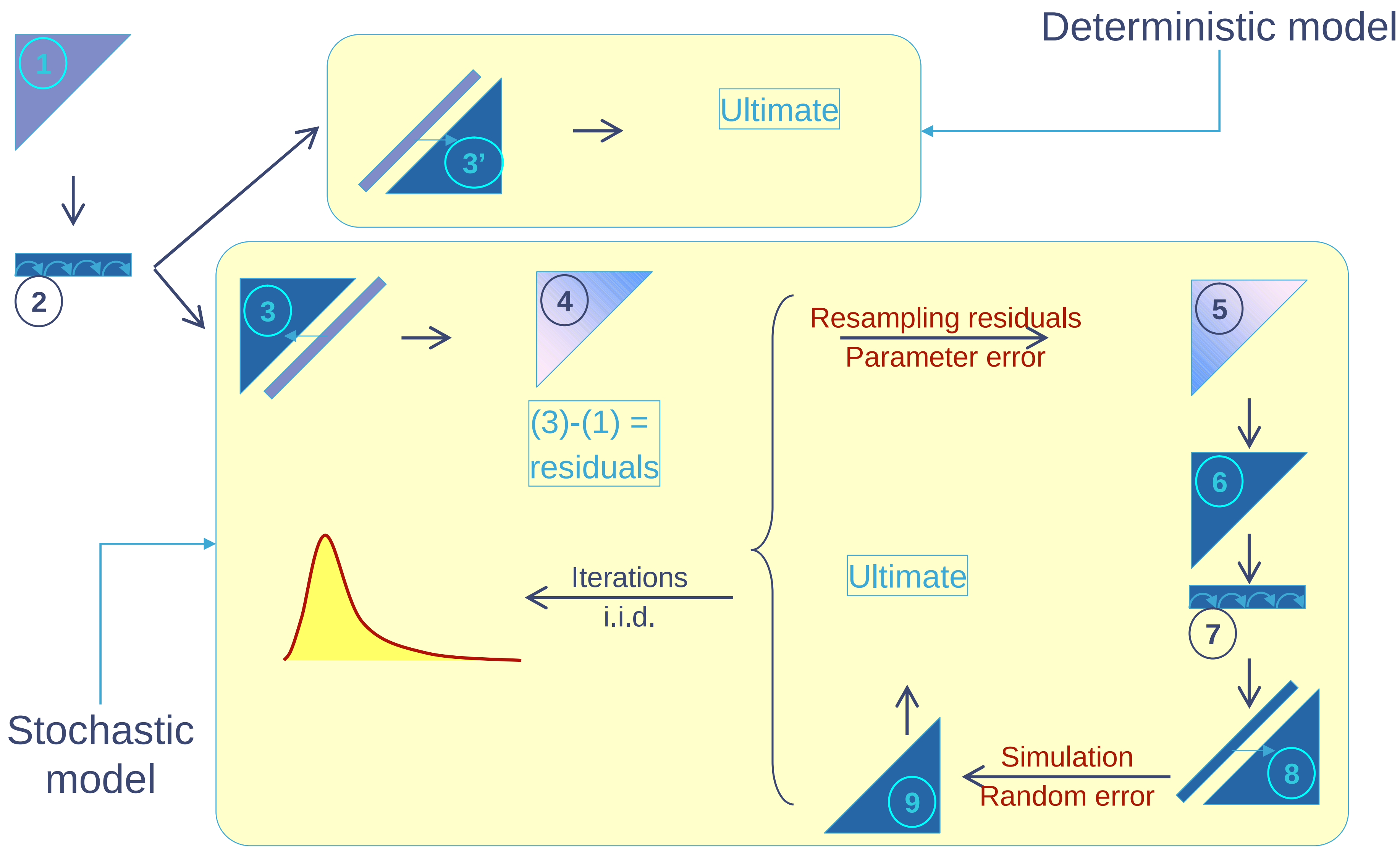
Characteristics:

- Monte Carlo simulation
- not reproducible (only asymptotically)
- yields full distribution of ultimate
- First appeared in 1979, main improvements since 1999

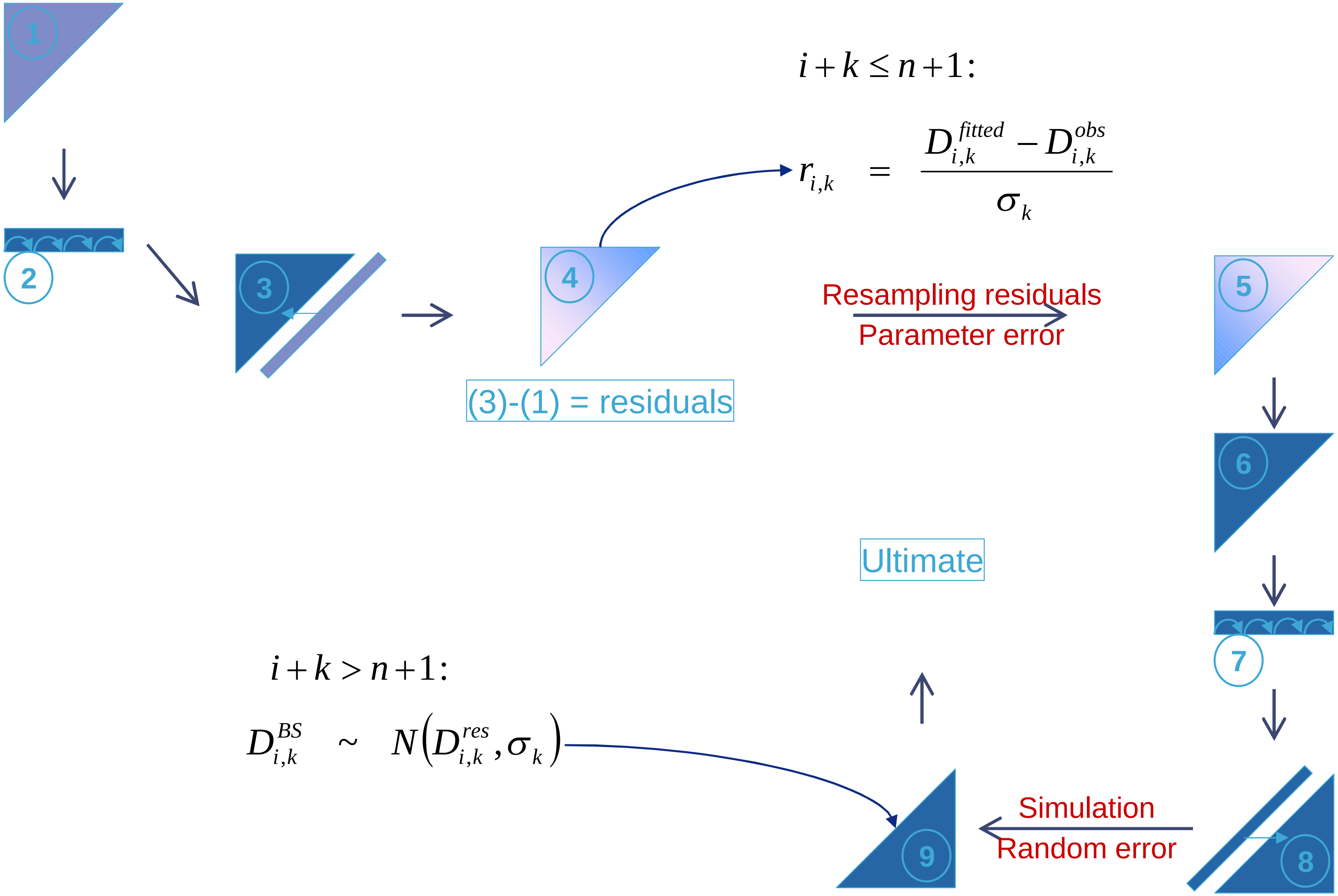
Stochastic methods: an overview

- Bootstrapping methods
  - Chain Ladder Bootstrap
  - Non-parametric bootstrap
  - Parametric bootstrap
- Markov Chain Monte Carlo methods
  - Metropolis Hastings
  - Gibbs
- Scenario generation

Chain Ladder bootstrap



Chain Ladder bootstrap



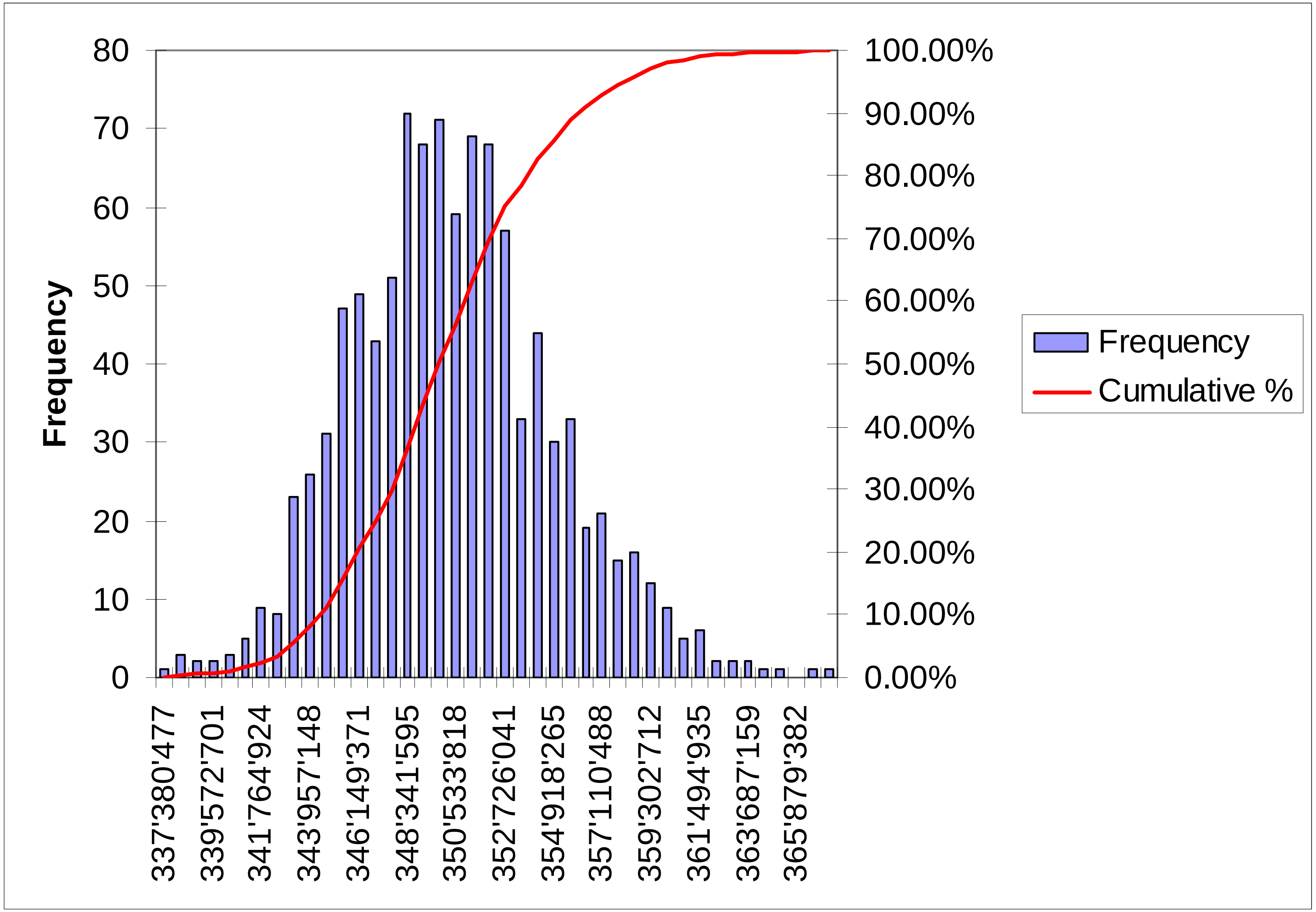
$$i+k \le n+1:$$

$$r_{i,k} = \frac{D_{i,k}^{fitted} - D_{i,k}^{obs}}{\sigma_k}$$

$$i+k > n+1:$$

$$D_{i,k}^{BS} \sim N\left(D_{i,k}^{res}, \sigma_k\right)$$

Chain Ladder Bootstrap



Mack chain ladder – Chain Ladder Bootstrap

Paid triangle	Mack Ult.	Pred. error	in % of Ult.	BS Ult.	Pred. error	in % of Ult.
1996	26'189	57	0.22%	26'164	1'116	4.27%
1997	28'027	126	0.45%	28'015	1'148	4.10%
⋮		⋮			⋮	
2007	24'198	836	3.46%	24'106	1'254	5.20%
2008	25'689	1'121	4.36%	25'742	1'856	7.21%
Total	**350'105**	**2'898**	**0.83%**	**350'010**	**4'551**	**1.30%**

Chain Ladder Bootstrap

Pros:

- We obtain the empirical distribution of the ultimate
- We can calculate average, median and all other quantiles
- Asymmetric prediction intervals can be determined
- Easy to implement
- Yields empirical distribution for other figures (e.g. claim development)

Cons:

- The results strongly depend on how you define your residuals. There are a lot of different approaches – which one to choose?

England & Verrall (1999): Analytic and bootstrap estimates of prediction errors in claims reserving.

Prediction intervals for the total Ultimate

In general: 95% prediction interval is of interest.

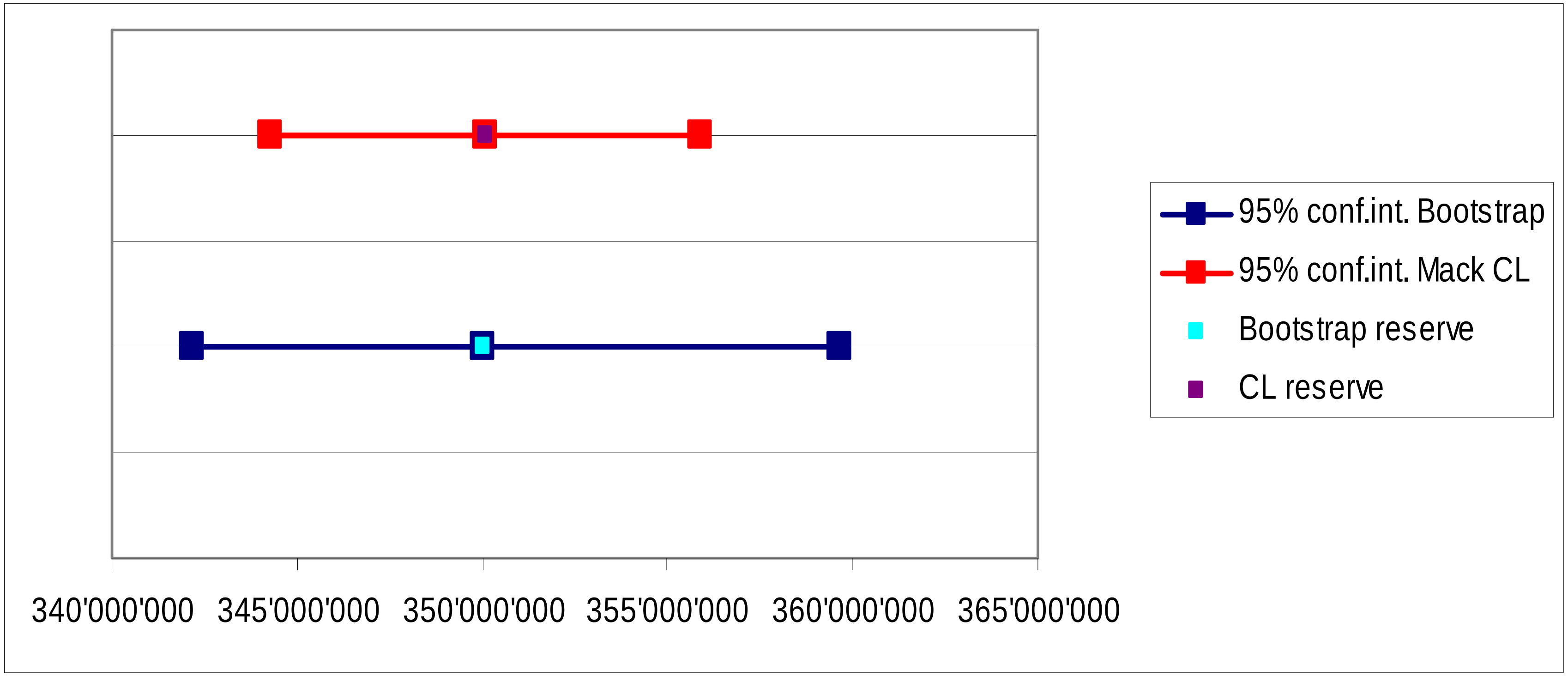
➢ *Statistical models:* Standard error of prediction can be used. For Mack Chain Ladder we use the following approach:

$$C_{total,n} \sim \mathcal{N}(\widehat{C}_{total,n}, \mathrm{msep(total)})$$
$$\Rightarrow$$
$$\mathrm{conf.int.}(0.95) \approx \left[\widehat{C}_{total,n} - 2\sqrt{msep(tot)}, \widehat{C}_{total,n} + 2\sqrt{msep(total)}\right]$$

➢ *Stochastic models:* The empirical distribution of the total ultimate can be used. For the Chain Ladder Bootstrap we would have:

$$C_{total,n} \sim F_{BS}^{emp.}$$
$$\Rightarrow$$
$$\mathrm{conf.int.}(0.95) = \left[(F_{BS}^{emp.})^{-1}(0.025), (F_{BS}^{emp.})^{-1}(0.975)\right]$$

Prediction Intervals - comparison



More sophisticated stochastic methods I

- *Multiline Bootstrap*
  - So far: Only the reserve for one line of business calculated
  - Goal: Empirical aggregate reserve distribution of several LoBs
  - Multiline Bootstrap accounts for dependencies between different LoBs
  - Idea: Simultaneous resampling of residuals

Kirschner, Kerley & Isaacs (2002): Two approaches to calculating correlated reserve indications across multiple lines of business.

More sophisticated stochastic methods II

- *Scenario generation*

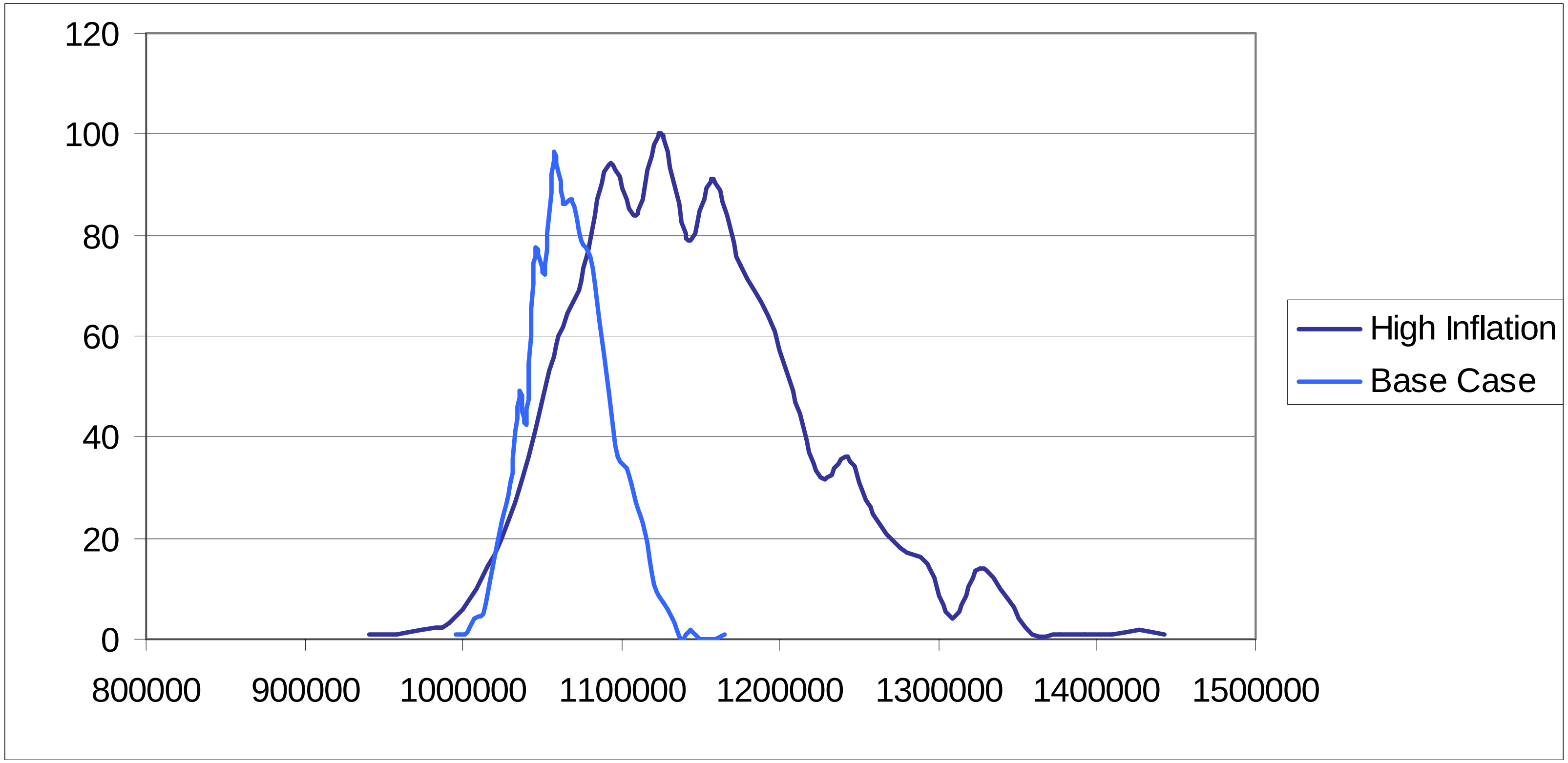
  - So far: Assumption that inflation behaves in the future as in the past. *What if this assumption is wrong?*

  - In models based on scenario generation, different economic scenarios (regulars and extremes) are considered

Stephen D'Arcy, Alfred Au, Liang Zhang (2007): Property-Liability Insurance Loss Reserve Ranges Based on Economic Value (online-calculation available)

Scenario generation

Base Case: Current inflation rate 3.5%, inflation volatility 2%
High Inflation: Current inflation rate 8%, inflation volatility 5%

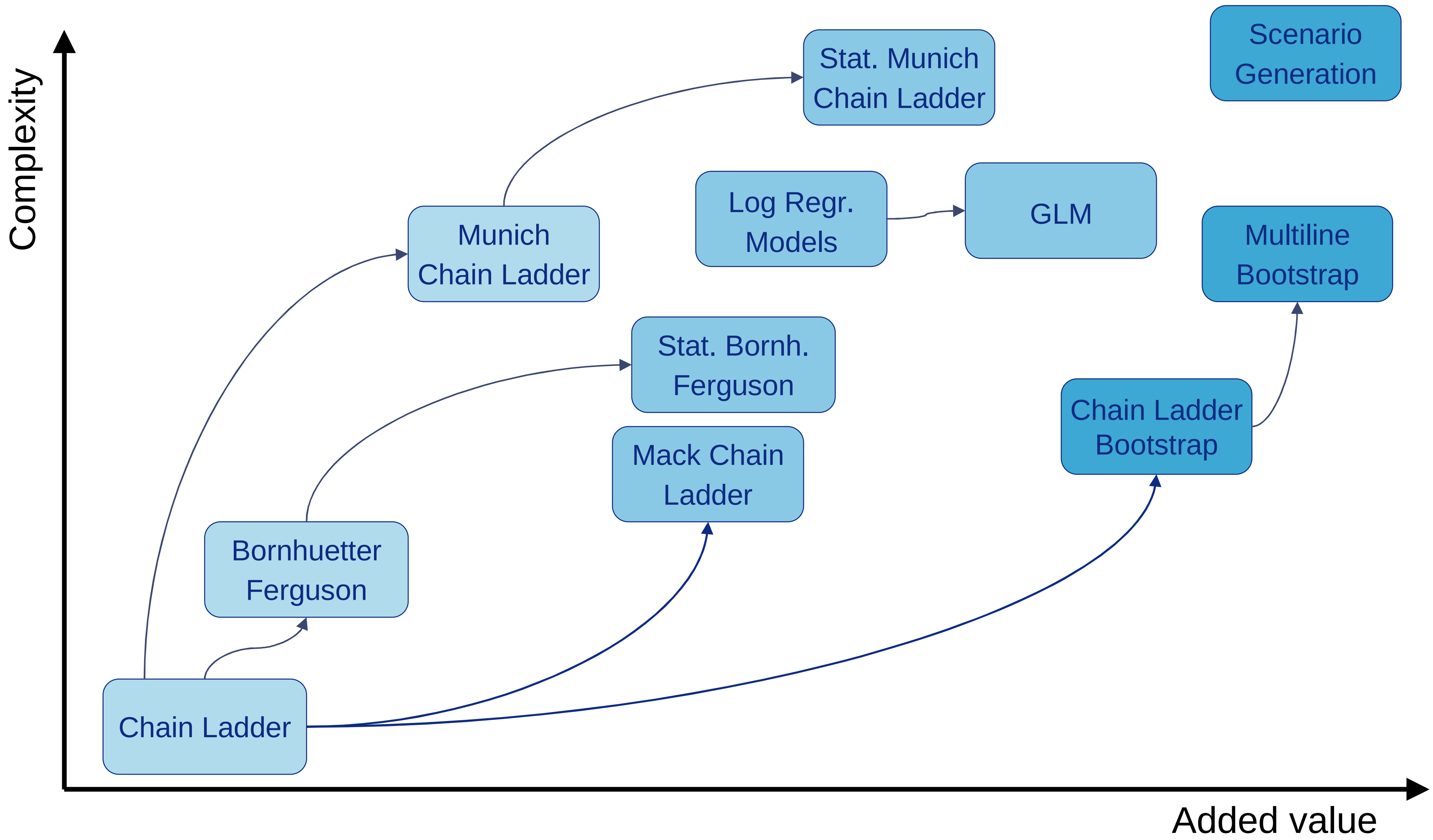


Mean 1’064’400, Stdev 25’300
Mean 1’137’900, Stdev 76’100

Why stochastic Reserving?

- Solvency II
- IFRS 4 Phase 2
- Pricing
- Analysts
- Rating Agencies
- M&A

Reserving methods – a landscape



Stochastic Reserving is not more precise,

but it determines its own precision.

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