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Loss Reserve Variability

CAS Working Party Report

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What's the Question?

- Given any value (estimate of future payments) and our current state of knowledge, what is the probability that the final payments will be no larger than the given value?
- So target is distribution of runoff (future payments)
- Not "range of reasonable estimates"

Section 1: Types of Uncertainty

- Process uncertainty
  - Random fluctuation when distributions have been specified
- Parameter uncertainty
  - Additional uncertainty due to estimation and projection of model parameters
- Model uncertainty
  - Might not have the right model

Section 2: Definitions and Notation

- 28 definitions
- 19 abbreviations
- Key notation example:
- $q(w, d + 1) = c(w, d)f(d) + e(w, d)c(d,w)^i$
  - Incremental losses for accident year w from delay d to d+1 = cumulative at d times a factor plus mean 0 error times a power of cumulative

Section 3: Principles of Model Evaluation and Variability Estimation

- Principles - need to determine:
  - How well the model measures and reflects the uncertainty inherent in the data
  - If the model captures and replicates the statistical features in the data
- Estimating runoff variability from model
  - Analytical
  - Bootstrap
  - Bayesian

Principles

- Practicality
- Reasonability
- Statistical validity

Practicality

- Data availability
- Software needed
- Time needed for analysis
- Etc.

Reasonability

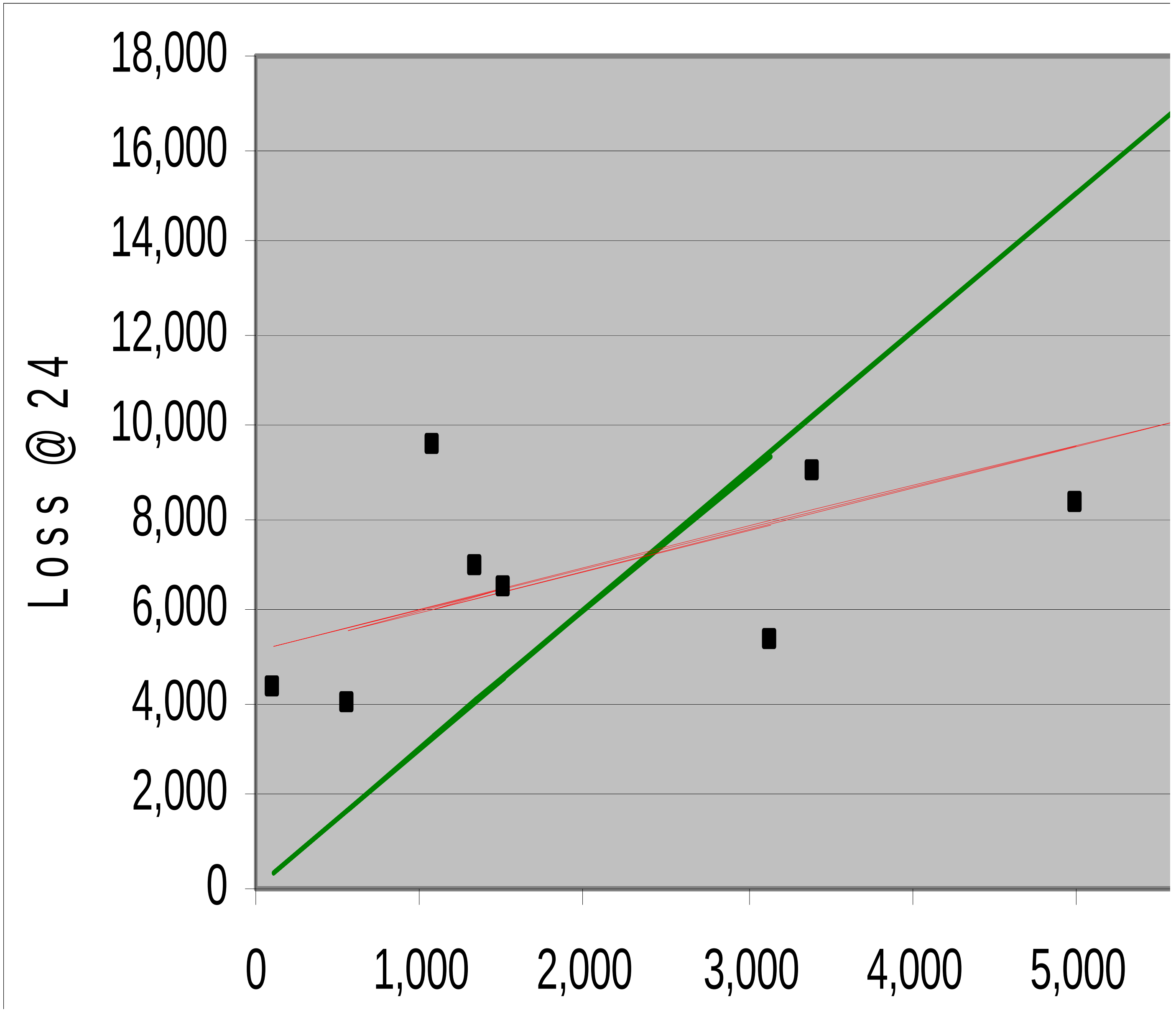
- Standard errors higher for more recent years but higher as a percent of losses for older years
- Parameters should have reasonable interpretations and explainable behavior
- Etc.

Statistical Validity

- Simulated data from the fitted model should be statistically indistinguishable from the original data
- Any link ratios in the models are statistically significant when viewed as no-constant regressions of incrementals against previous cumulatives

Link Ratios as No-Constant Regressions

- Best regression here not through <0,0>
- Additive constant is significant
- Often seen for 12 to 24
- Use it when significant



Standardized Residuals

- Residual divided by its estimated standard deviation
- Check for distribution of residuals and any relationship of residuals to independent variables
- They should have no dependence on independent variables, lag, accident year, calendar year or fitted value

Other Statistical Tests

- Test parameters on holdout sample
- Goodness of fit
  - AIC = SSE (or -loglikelihood) plus penalty = number of parameters = ln(e) times number of parameters
  - BIC penalty = ln(sqrt(sample size)) times number of parameters
  - HQIC penalty = ln(ln(sample size)) times number of parameters

Estimating runoff variability from model

- Analytical
  - Use residuals of model and estimated parameter uncertainty to get distribution
- Bootstrap
  - Resample from triangle or simulate from model
- Bayesian
  - Start with a prior distribution of parameters and estimate posterior and predictive

Analytical Method

- Distribution of residuals (process variance) comes from fit
- If using MLE (which is least squares if using normal distribution) parameter uncertainty comes from the information matrix
  - See Part 4 of CAS Exams
- Can combine by simulation
- Or add parameter and process variances to get total variance and assume lognormal (Murphy PCAS 1994 can give some insights)

Bootstrap

- Sample from development factors to simulate many lower triangles and use the resulting distribution of future payments
- Or sample from residuals of fitted model to simulate many upper triangles and refit the model to each, calculating the future payment distributions for each analytically and combining for total risk

Bayesian Approach

- Use expert experience to devise a prior distribution of the parameters of the model you want to use
- Use Bayesian methods to get a predictive distribution (see CAS Part 4 again)
- This requires multi-dimensional numerical integration
- Techniques for that are outlined

Section 4: Methods and Models

- Method is an algorithm – a series of steps to follow
  - Chain ladder, BF, Cape Cod, separation etc.
  - Calculate variances within each column of factors and assume lognormal (ER method: too low in theory, too high in practice)
- Model: hypothesize an underlying mechanism which generates the observed data
  - Part of a model is the deterministic generation of means for the observations
  - Another part is the random generation of the actual observations given the means
  - From that you can get a distribution of the runoff
  - Some research has been to find model assumptions that support traditional methods

Classification of Reserve Models

- Models of individual claim histories
- Models of triangles
  - Simultaneous models of several triangles
  - Models of single triangles
    - Conditional models – data from the triangle is part of model, and model is conditional on that data
      - Link ratios: next observation is factor times last
    - Unconditional models – all observations generated from parameters
      - BF: losses at every observation a factor times ultimate
  - Parameters for diagonal terms or not
  - Fixed or varying parameters
  - Error terms parametric or not

Types of Models

- Triangle Based Models vs. Individual Claim Models
  - Single Triangle Models vs. Multiple Triangle Models
    - Conditional Models vs. Unconditional Models
    - Parametric Models vs. Non-Parametric Models
    - Diagonal Term vs. No Diagonal Term
    - Fixed Parameters vs. Variable Parameters

Conditional Models

- $q(w, d+1) = c(w, d)f(d) + e(w, d+1)c(d,w)^{i/2}$
  - For accident year w, incremental losses at delay d+1 are a factor f(d) specific to lag d times the previous cumulative losses
  - plus a random error which is proportional to a power of the previous cumulative (maybe i=0)
  - Var[$e(w, d+1)$] is a function of d but not of w
  - Accident years are independent of each other
  - Mack and Murphy model capturing chain ladder assumptions
- Dividing by $c(d,w)^{i/2}$ gives constant variance so can use regression to estimate f(d) for each d
- Residual analysis may help pick i
- Becomes parametric if you assume a distribution for *e*.
- With diagonal factors (w+d constant on each diagonal):
  - $q(w, d+1) = c(w, d)f(d)h(w+d+1) + e(w, d+1)c(w, d)^{i/2}$

Unconditional Models

- $q(w, d) = G(w)f(d) + e(w, d)[G(w)f(d)]^{i/2}$
  - G(w) is a parameter for the w accident year, like estimated loss
  - Or estimate G(w) and f(d) from the data
    - Can use iterative procedure from Bailey minimum bias
    - Fix G's, q linear in f's; then fix f's, q linear in G's, etc.
  - BF is this model with G(w) selected by analyst
    - I call formal model stochastic BF
- $q(w, d) = Gf(d) + e(w, d)$. Again e can be parametric
  - When G does not vary by accident year, called Cape Cod
  - Increment is constant for each column, and can set $G = 1$
- $q(w, d) = Gf(d)h(w+d) + e(w, d)$ includes a diagonal term
  - Called separation model (Taylor), or Cape Cod with super-imposed inflation

Unconditional Multiplicative Errors

- $q(w, d) = G(w)f(d)\exp[e(w, d)]$
  - $\ln q(w, d) = \ln G(w) + \ln f(d) + e(w, d)$
  - Or $\ln q(w, d) = \ln G(w) + \ln f(d) + e(w, d)[G(w)f(d)]^{i/2}$
  - Can now fit by regression
- Parametric: $\ln q(w, d) = \ln G(w) + \ln f(d) + e(w, d)$, e normal($0,\sigma^2$)
  - Proposed by Kremer (1982)
- With diagonal terms
  - $\ln q(w, d) = \ln G(w) + \ln h(w+d) + \ln f(d) + e(w, d)$

Restricting Parameter Variation

- Set a lot of factors equal, e.g., f(11) = f(12) = ... = f(19)
  - Maybe none are significant individually but a single one is significant for all as a group
- Postulate a relationship among parameters
  - $f(12) = \frac{1}{2}[f(11) + f(13)]$
  - h(w+d) = b(d)w (a different trend factor for each column)
- Smoothing the parameters
  - $G(w+1) = zG(w) + (1 - z)G^*(w+1)$, where G^* is the unsmoothed estimate of the accident year parameter
- Trends model form: $\ln q(w,d) = G(w) + \sum_{i=1}^{d} f(i) + \sum_{j=2}^{w+d} h(j) + e(w,d)$
  - Impose constraints on G's, f's, h's to specify a specific model

Multiple Triangle Models

- Simultaneous modeling of paid and incurred (chain ladder example):

$$q_I(w,d+1) = f_{II}(d)c_I(w,d) + f_{PI}(d)c_P(w,d) + e_I(w,d+1) \text{ ; and}$$

$$q_P(w,d+1) = f_{IP}(d)c_I(w,d) + f_{PP}(d)c_P(w,d) + e_P(w,d+1) \text{ .}$$

  - Factors applied to both previous cumulative paid and incurred losses in estimating each
- Correlated lines could be done similarly

Individual Claim Development

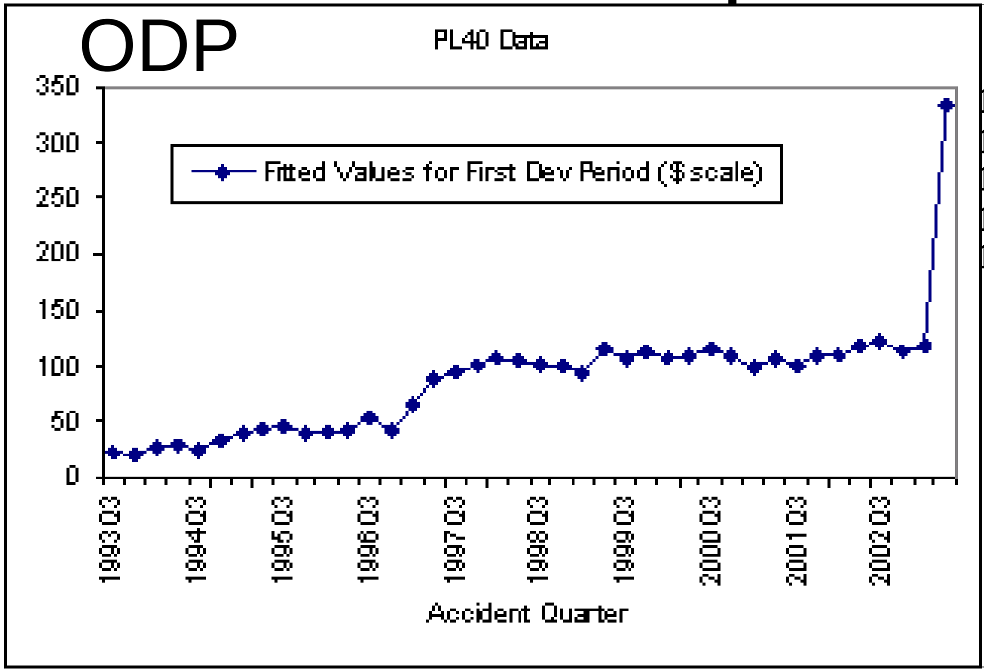
- Transition matrix method
  - Claims bucketed into categories
    - Size ranges, reported or not, open or closed, % paid, etc.
  - Probabilities for movement from one category to another calculated at each age
    - Put those into a matrix – called transition probability matrix
  - Multiply matrix by vector of claims by category to get expected mix of claims by category at next age
- Conditional development distribution
  - Fit a distribution to the development factors from age m to age u by open claim
  - Can spread claims to a range of possible outcomes before fitting a severity distribution

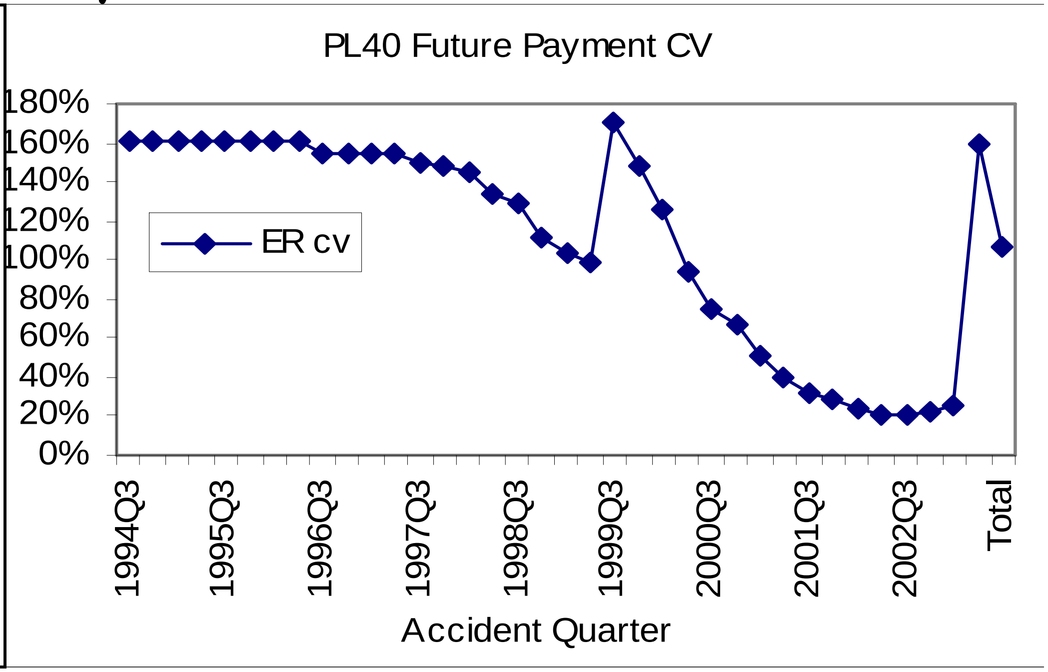
Section 5: Examples

- Apply principles from section 3 and models from section 4 to some data
- Use ER method (variance of factors in each column), Murphy's chain ladder, and ODP:
  - $q(w,d) = G(w)f(d)e^{e(w,d)}$, called over-dispersed Poisson if Var q > Eq, even if not Poisson
  - Murphy$_2$: $q(w,d+1) = c(w, d)f(d) + e(w,d+1)c(d,w)$
- Two data sets
  - 40 quarters of incurred losses: IL40
  - 40 quarters of paid losses: PD40
    - If a bit rusty, try WD40

Practicality and Reasonability

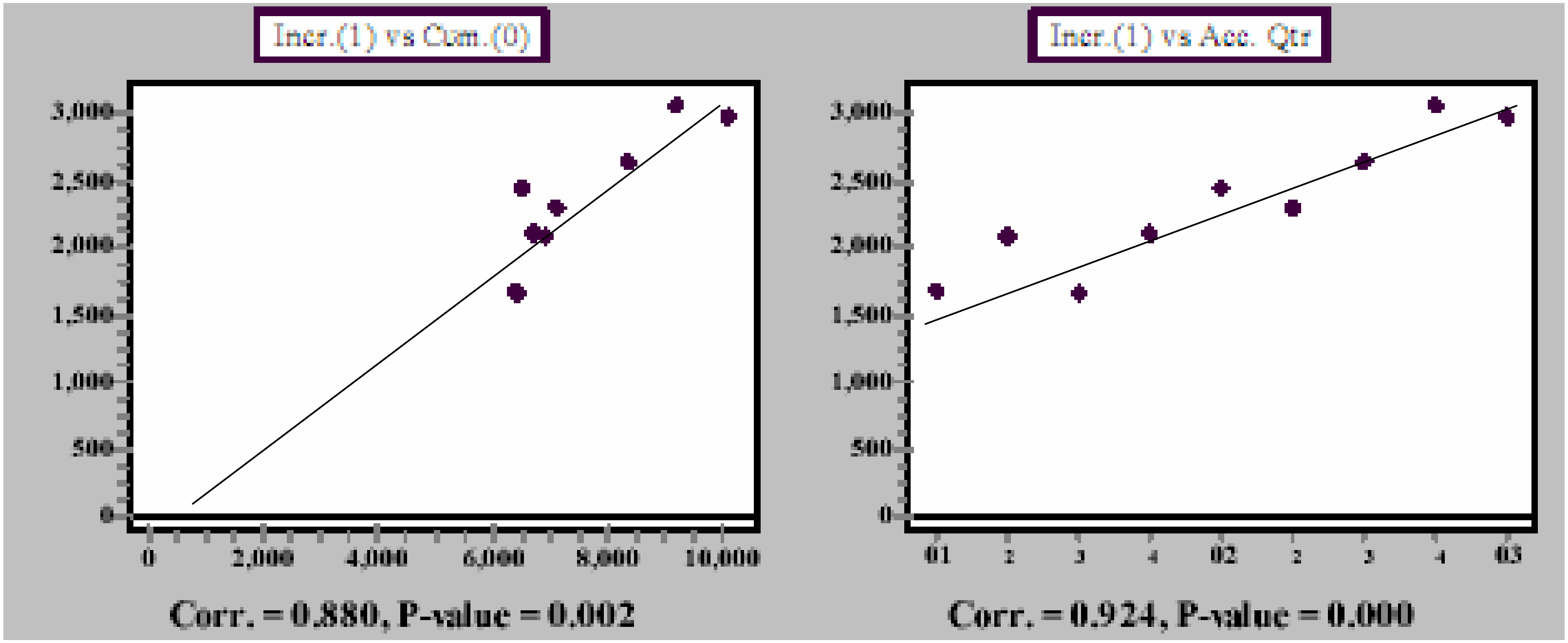
- ER easier than ODP but not as supported by any theory
- Both look suspicious under reasonability tests of CV patterns, etc.





Test Fit of Link Ratios

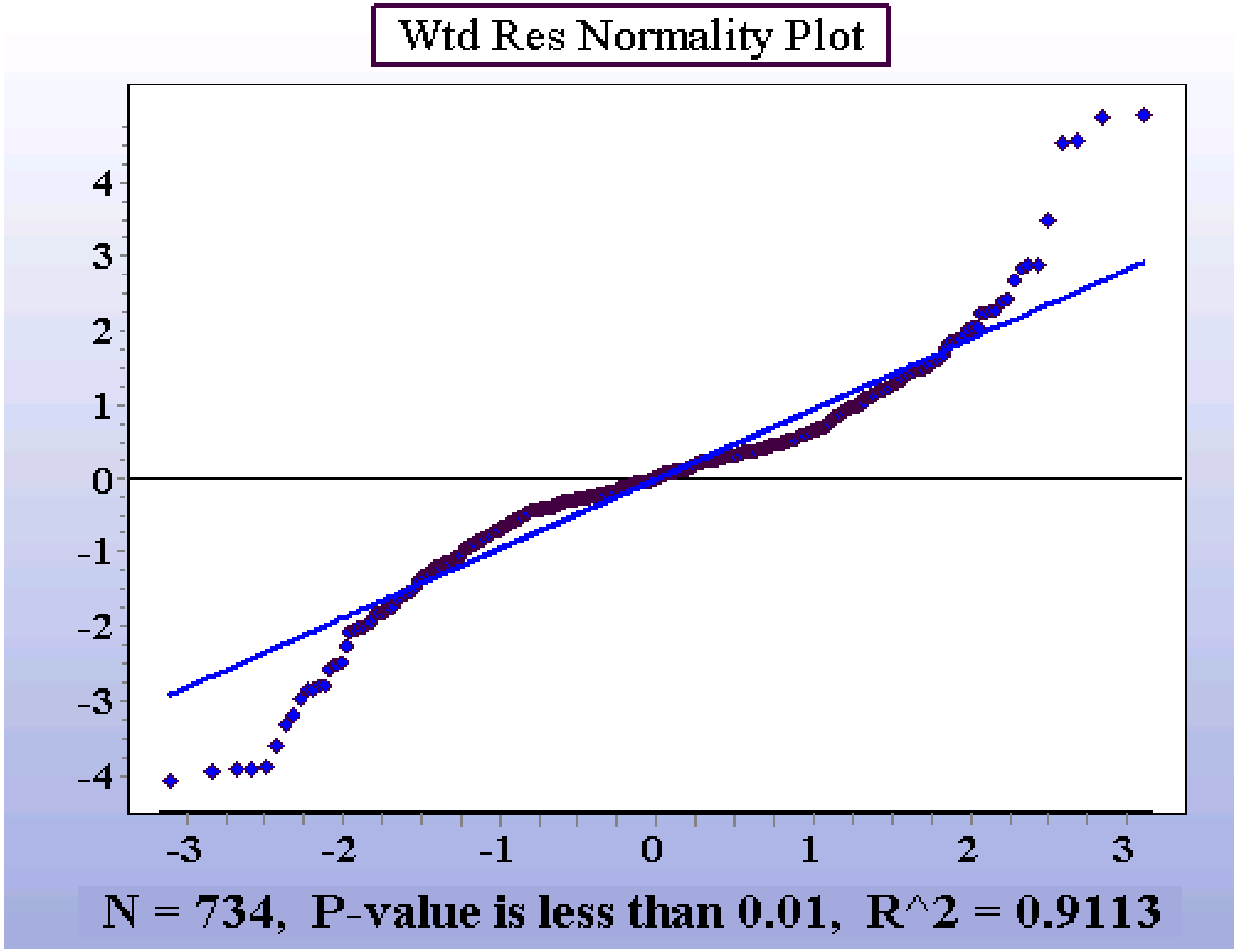
- IL10 2nd incrementals vs. 1st cumulative and accident year



- Link ratio not bad but acc yr trend better

Test of Normal Residuals

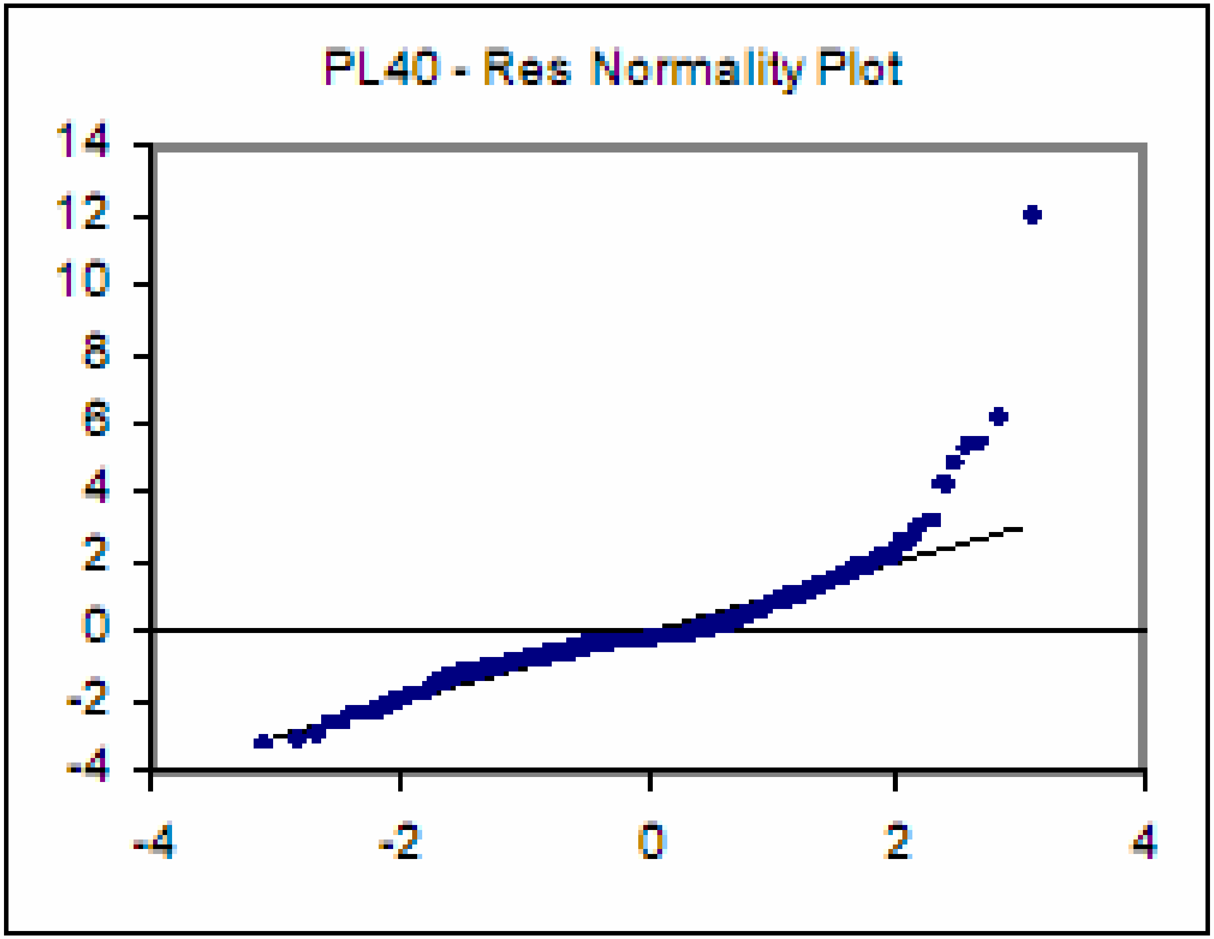
Murphy on IL40



Normality of Residuals

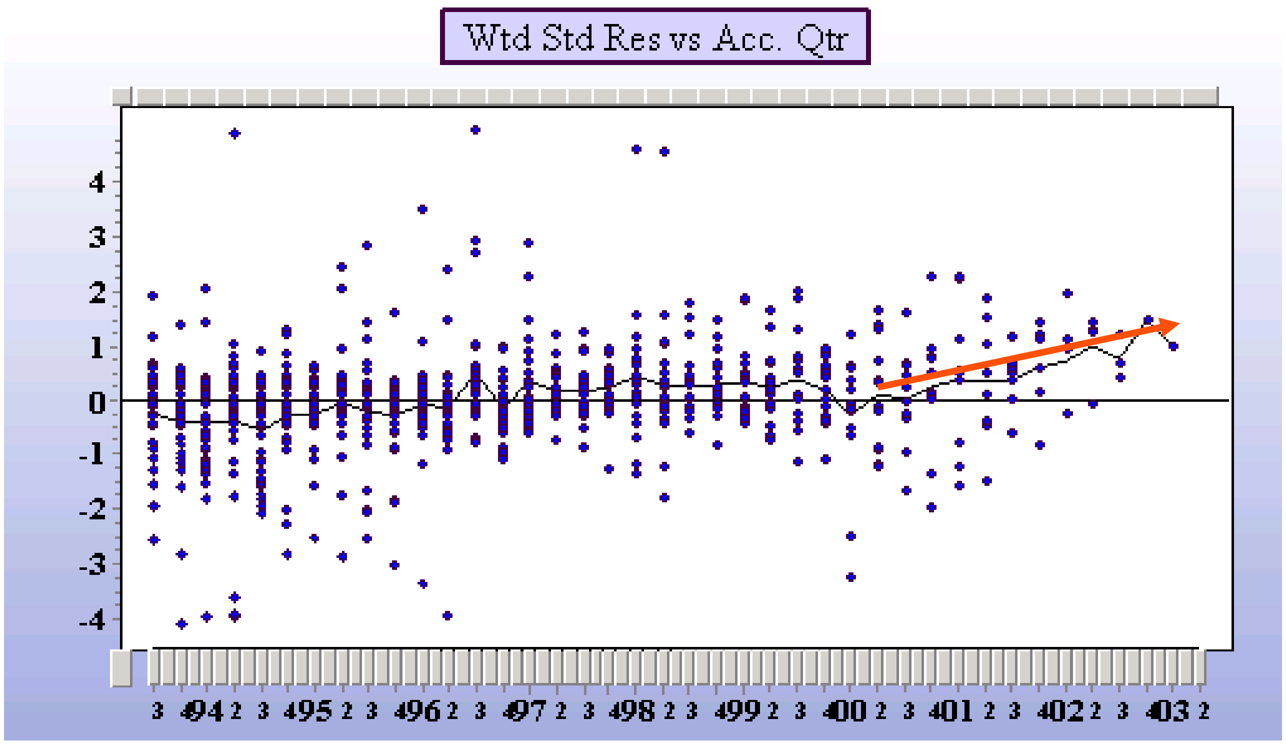
- Heavy tails in residuals likely to give a poor estimate of the mean
- ER: residuals not defined
- Murphy on IL40 and ODP on PL40: poor fit to normal

ODP on P40



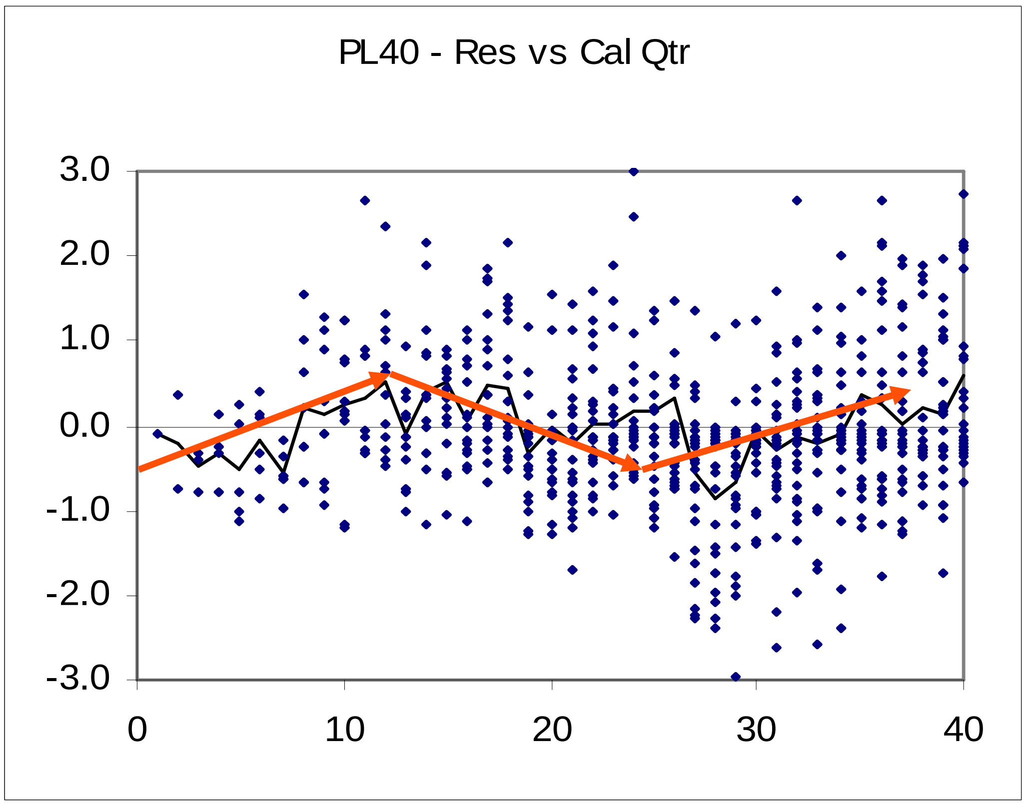
Patterns in Residuals – Murphy

- Murphy on IL40: residuals trend up in later accident periods, forecast means likely to be too low



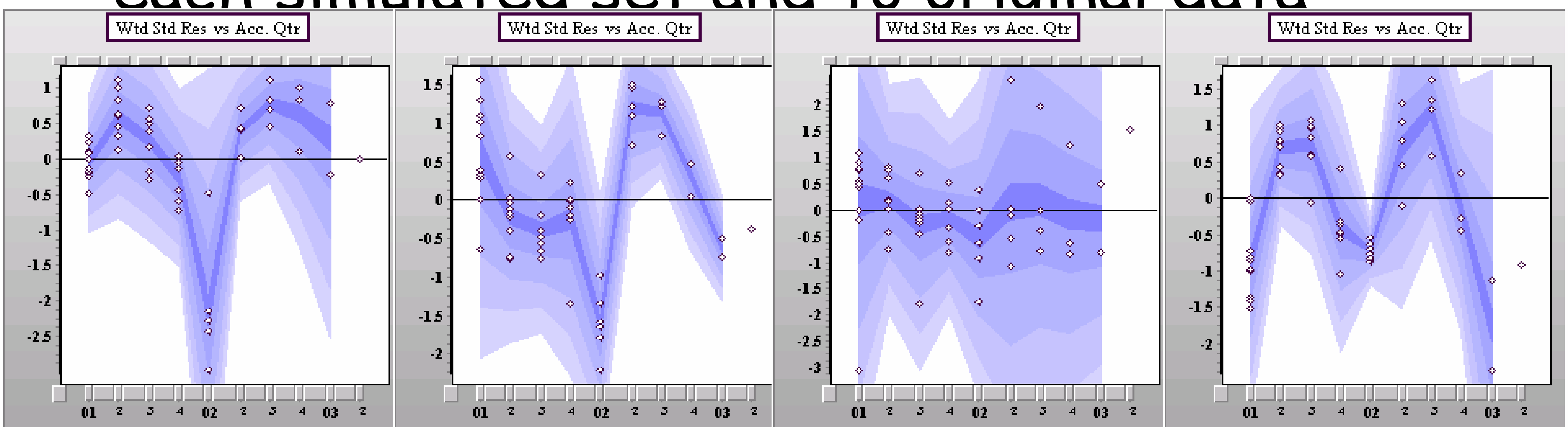
Patterns in Residuals – ODP

- ODP on PL40: residuals trend up and down over calendar periods, forecast means might be high or low



Criterion 11
Consistency with Simulation

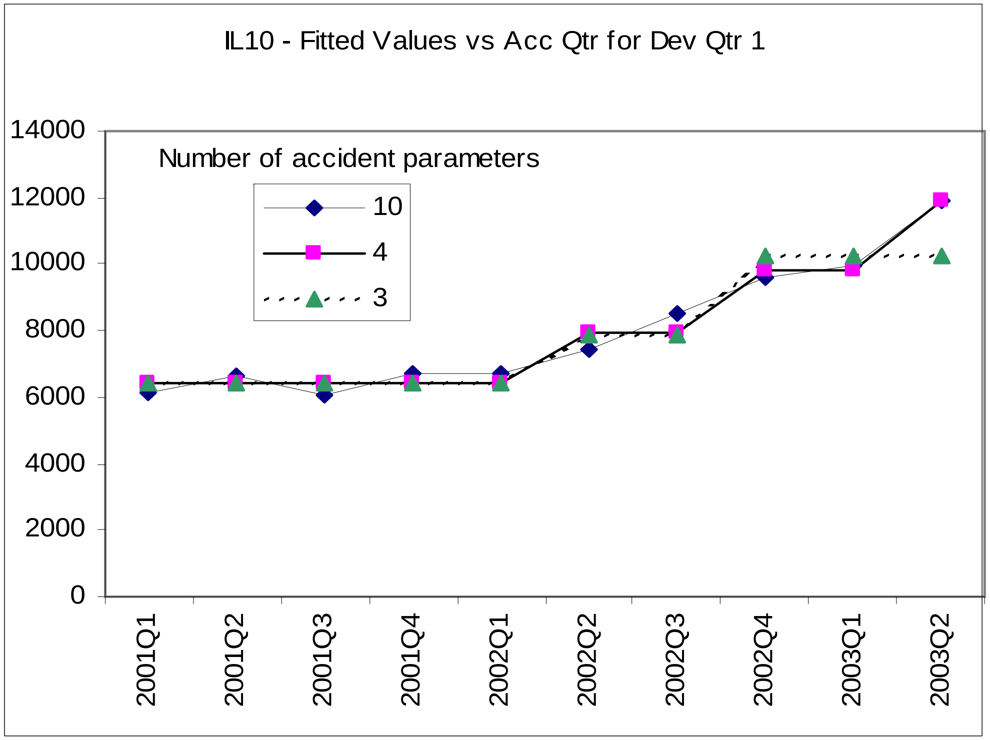
- Murphy on PL10: data simulated by applying development factors to losses at lag 1
- BF model plus calendar year trend fit to each simulated set and to original data

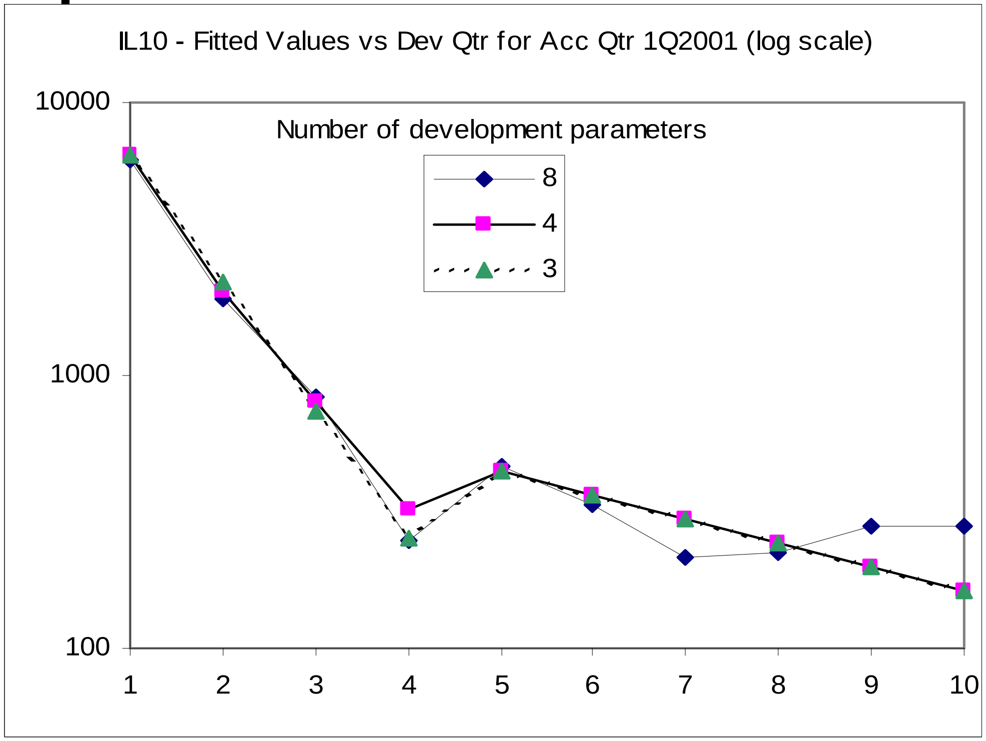


- Residuals graphed here: real data different

Criterion 18
Parsimony (Ockham's Razor)

- ODP on IL10: 18 parameters can be reduced to 6 with little loss of fit
- Parameter with largest ratio of SE to mean eliminated at each step





Fit For Purpose: Criterion 4 Cost/Benefit

- Caveats: small sample of data, personal opinion
- ER: low benefit
- ODP, Mack & Murphy: moderate benefit
- More parsimonious models: higher benefit
- But each data set could have its own issues – no single approach universally the best
- Moral: do the tests for each triangle

Rest of the Paper

- Section 6: The Future
  - Will resemble the past
- Section 7: Caveats
  - 8 of them
- 81 references
- 23 author bios
- 2 Appendices with 18 complicated formulas