Emerging Practices in Reinsurance Analytics

CAMAR

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May 30, 2013

TOWERS WATSON

Emerging Practices in Reinsurance Analytics

I. Evolution of Broker Analytics

II. Evaluation of Reinsurance as a Form of Capital

III. Optimization Modeling in Practice

IV. Collective Risk Model for Simulating Insurance Losses

Evolution of Broker Analytics

The reinsurance placement process has become an increasingly technical exercise

The type of analytics supporting the reinsurance placement has evolved over time.

I. Contract Experience
II. Contract Pricing
III. Underwriting Distributions
IV. Advanced Value-Added Modeling

“Keeping score” of contract experience has always been a part of the discussion

I. Contract Experience

<u>Examples:</u>

- “Reinsurance Bank”: calculate cumulative contract experience
- “As-is” review: how contract would have performed based on current terms
- “As-if” review: how contract would have performed under alternative terms

<u>Comments:</u>

- May include some basic adjustments for loss trend, development, exposure trend, etc.
- Actuarial skillset and tools not necessarily required
- Does the bank matter anymore?

II. Contract Pricing
III. Underwriting Distributions
IV. Advanced Value-Added Modeling

The estimation of reinsurance market pricing has become a routine practice

I. Contract Experience

II. Contract Pricing

<u>Objectives:</u>

- Evaluate expected loss to contract (and its distribution)
- Convert loss cost to market price
- Evaluate impact of reinstatement structure, aggregate limits, etc.

<u>Tools:</u>

- Excel models
- Stochastic models such as IGLOO

III. Underwriting Distributions
IV. Advanced Value-Added Modeling

There are various techniques for estimating market prices

- Traditional Loss Loading: Expected ceded losses are loaded by a fixed expense and profit load factor to convert to reinsurance rate
- Standard Deviation Loading: Reinsurance premium estimated as the sum of the expected loss cost plus the product of the loss cost standard deviation multiplied by a load factor (30%)
- Reinsurer Return on Capital: Reinsurance premium calculated so that the contract provides a target return on capital to the reinsurer (e.g. 10%). A reinsurer's capital amount supporting the contract is estimated based on the downside risk (e.g. 99th percentile).
- Minimum Capacity / Clash Charges: Reinsurance premium is not based on technical pricing measures; reinsurers may require a minimum premium charge to support the capacity they are providing and/or compensate for the clash occurrences or other tail events that may not be properly reflected in modeling

Methodology	1,000,000 xs 1,000,000	3,000,000 xs 2,000,000	5,000,000 xs 5,000,000	9,000,000 xs 1,000,000
Expense / Profit Load	1,747,892	565,768	18,056	
Loss Cost Plus 35% SD Load	1,669,713	736,921	132,168	
10% Reinsurer Return on Capital	1,638,335	782,280	196,800	
Min Capacity/Clash Charge • Minimum Layer ROL • Minimum Layer Premium			 8.0% 400,000	
Final Estimated Market Premium	**1,747,892**	**600,000**	**400,000**	**2,747,892**

Understanding the distribution of outcomes is important

Reinsurance Contract: 1M xs 1M

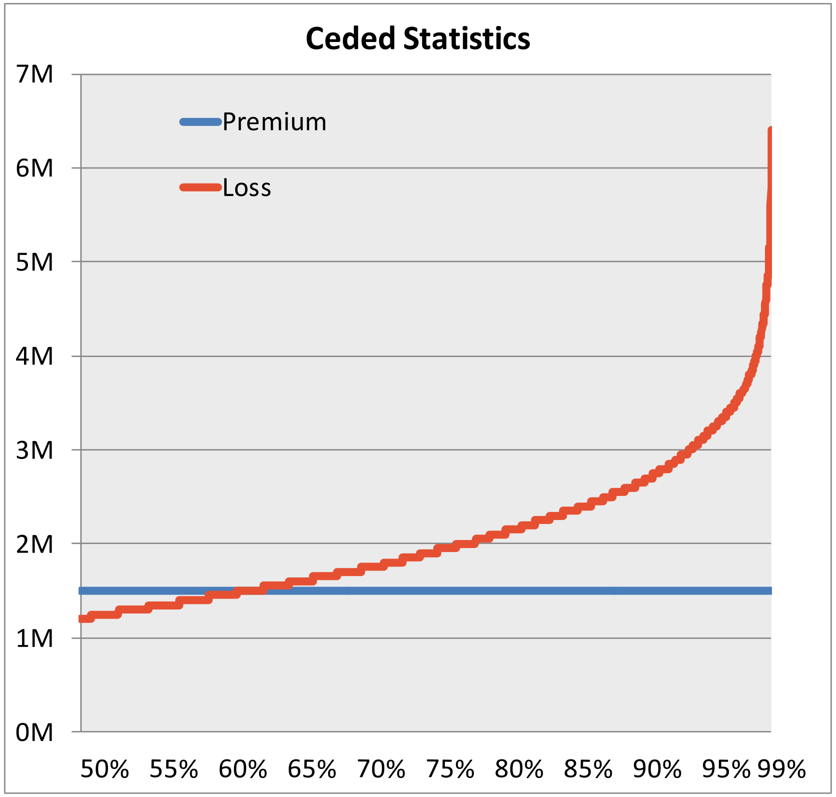
100% Basis

Pricing Summary	Current Review	Prior Bound
1M xs 1M Unlimited - Experience Indication	0.84%	
1M xs 1M Unlimited - Exposure Indication	1.89%	
1M xs 1M Unlimited - Selected	1.00%	
1M xs 1M - Final Selected Rate	1.00%	0.90%
1M xs 1M - Final Selected Amount	1,747,892	1,498,000

Ceded Statistics	Premium	Loss	RI UW Profit
Mean	**1,498,000**	**1,343,135**	**154,865**
Standard Deviation	0	933,080	933,080
Minimum	1,498,000	0	(4,902,000)
Maximum	1,498,000	6,400,000	1,498,000
0.5th percentile	1,498,000	0	1,498,000
1st percentile	1,498,000	0	1,498,000
2nd percentile	1,498,000	0	1,498,000
5th percentile	1,498,000	150,000	1,348,000
10th percentile	1,498,000	300,000	1,198,000
20th percentile	1,498,000	500,000	998,000
30th percentile	1,498,000	700,000	798,000
40th percentile	1,498,000	1,000,000	498,000
50th percentile	1,498,000	1,200,000	298,000
60th percentile	1,498,000	1,450,000	48,000
70th percentile	1,498,000	1,700,000	(202,000)
80th percentile	1,498,000	2,100,000	(602,000)
90th percentile	1,498,000	2,600,000	(1,102,000)
95th percentile	1,498,000	3,100,000	(1,602,000)
98th percentile	1,498,000	3,650,000	(2,152,000)
99th percentile	1,498,000	4,050,000	(2,552,000)
99.5th percentile	1,498,000	4,450,000	(2,952,000)

Return on Alloc. Capital*	RI UW Profit	Capital	Return
90th percentile	154,865	1,102,000	14%
95th percentile	154,865	1,602,000	10%
98th percentile	154,865	2,152,000	7%
99th percentile	154,865	2,552,000	6%
99.5th percentile	154,865	2,952,000	5%

* Capital allocated based on stand-alone percentile underwriting loss for reinsurer



Exceedance Probabilities	Attach	Exhaust
1st Limit	97.8%	60.3%
2nd Limit	57.5%	22.8%
3rd Limit	21.4%	6.0%
4th Limit	5.7%	1.1%
5th Limit	1.0%	0.2%
6th Limit	0.2%	0.0%
7th Limit	0.0%	0.0%

Today reinsurance is evaluated on an underwriting portfolio basis

I. Contract Experience
II. Contract Pricing

III. Underwriting Distributions

Objectives:

- Incorporate attritional losses, large losses, and cat losses
- Evaluate the gross, ceded, and net underwriting distributions under alternative reinsurance structures
- Dynamic Reinsurance Management ("DRM") using, for example, 50,000 simulated trials
- Evaluate "trade-off" between ceded margin and gross/ceded/net risk

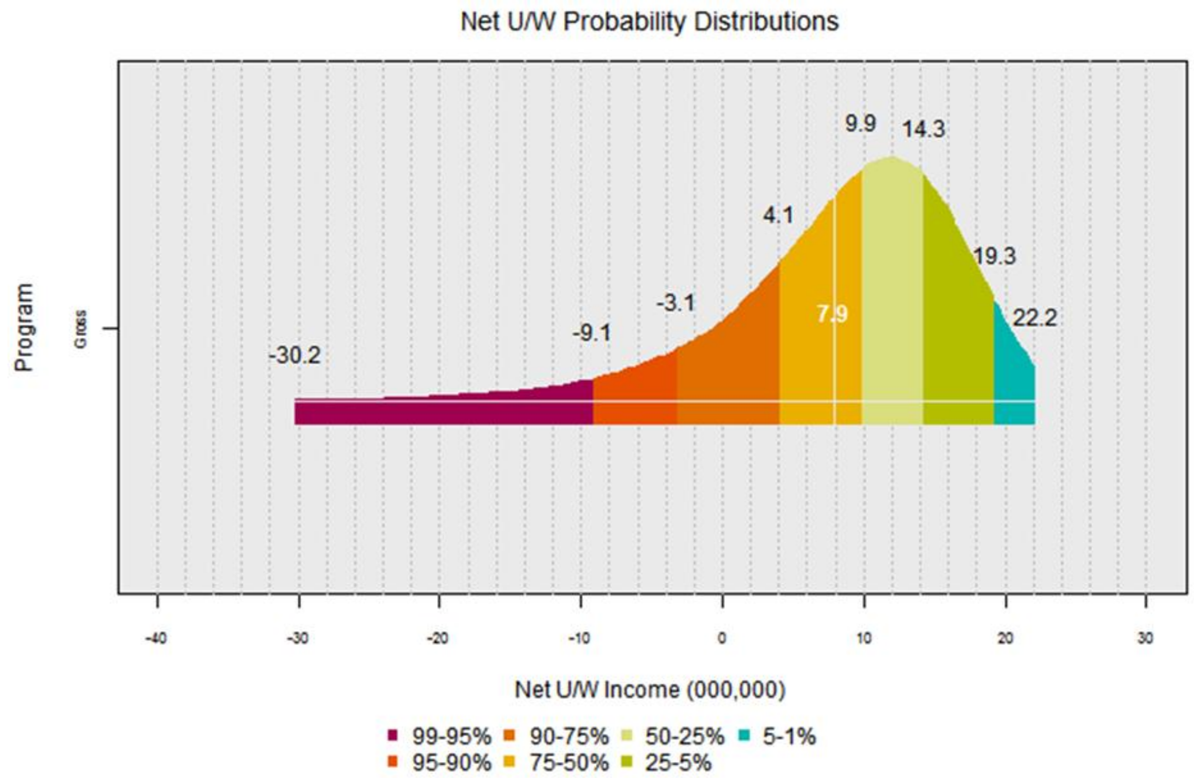
Comments:

- Can be presented in different ways such as tables and graphics
- Risk can be measured in different ways
- Ideal when client provides risk tolerance or "what they want to solve for"

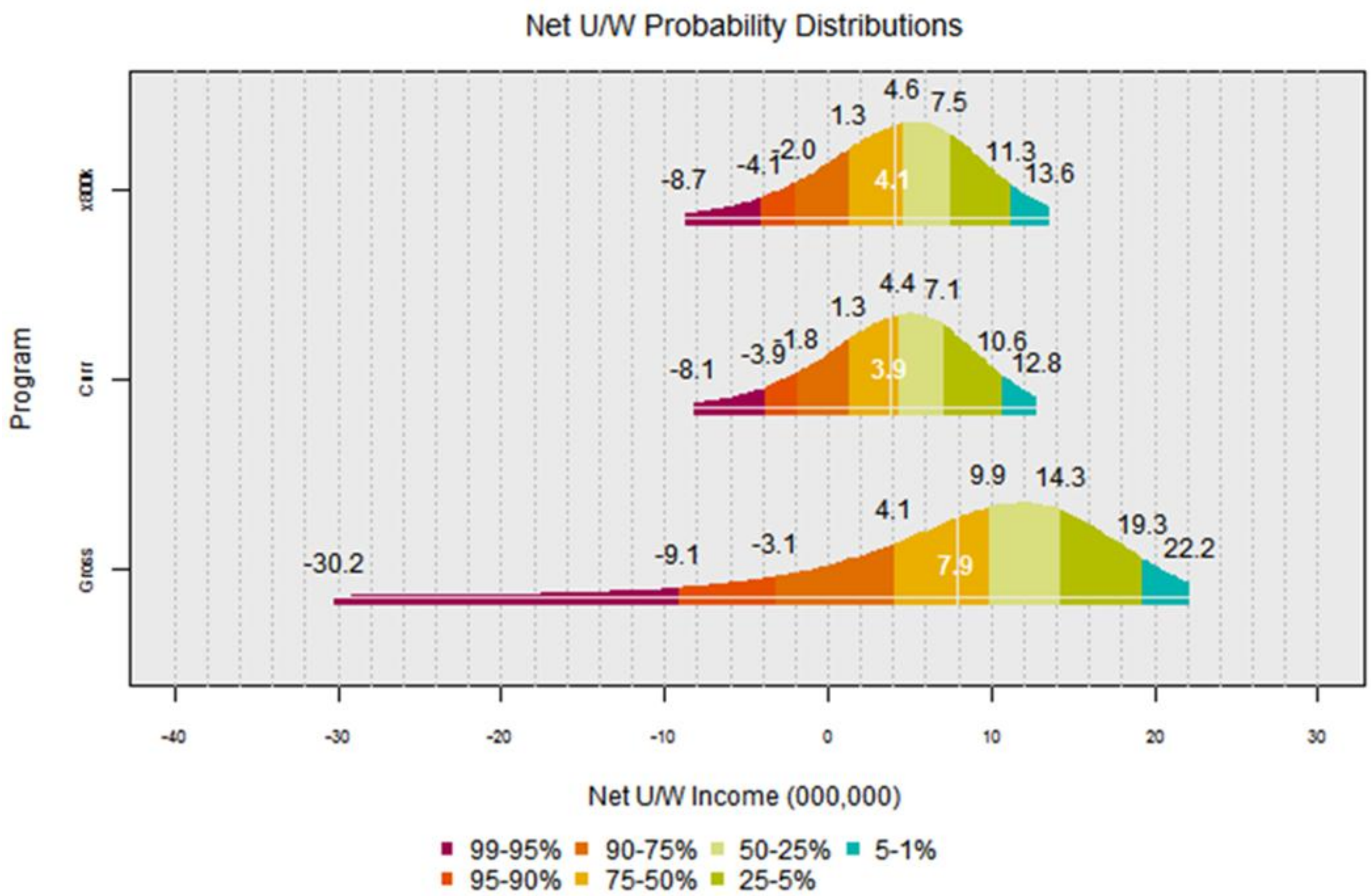
IV. Advanced Value-Added Modeling

The subject business is modeled to determine the gross underwriting distribution

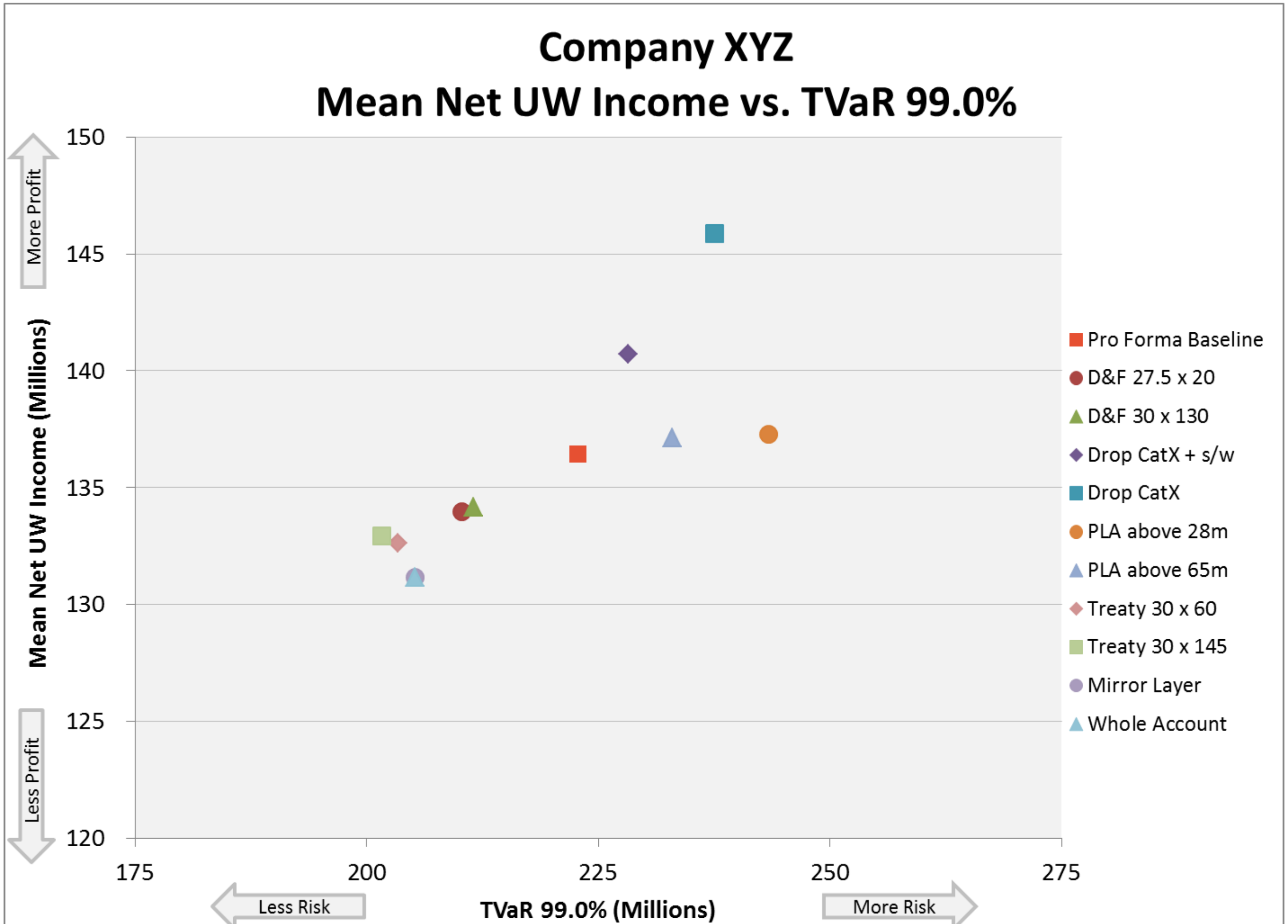
	Structure
	Gross
99%	(30,222,024)
95%	(9,096,486)
75%	4,147,932
50%	9,850,633
25%	14,290,838
5%	19,315,557
1%	22,155,321
Mean	7,857,367
Std Dev	12,397,829



The overlay of alternative structures illustrates the costs and benefits of reinsurance



Reinsurance structures can be compared based on the risk/reward trade-off



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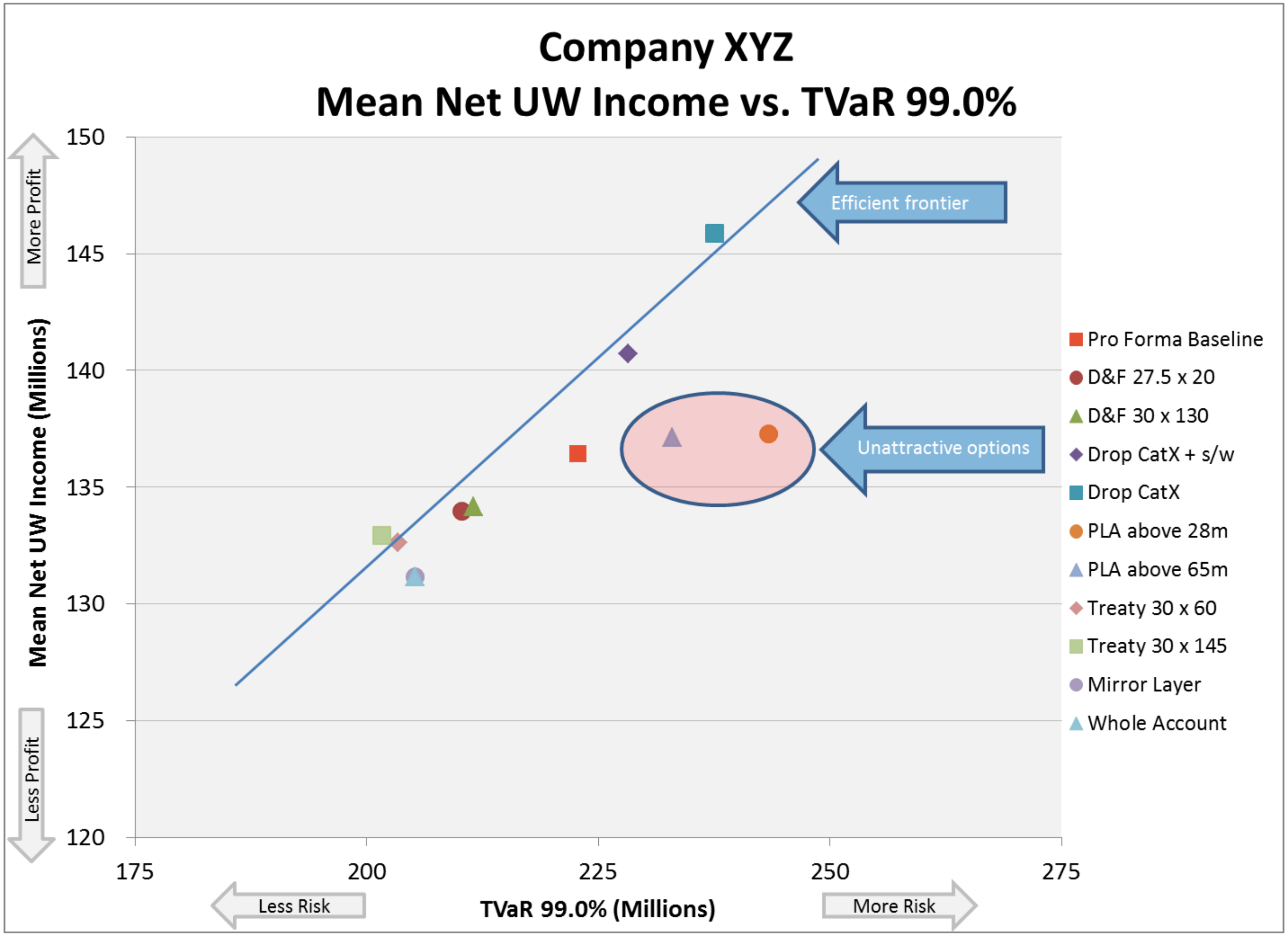


Evolution of Broker Analytics – Value-Added Modeling

I. Contract Experience
II. Contract Pricing
III. Underwriting Distributions

IV. Advanced Value-Added Modeling

Objectives:

- Expand beyond underwriting distributions to larger operational/capital context

Answer questions:

- What is the impact on rating agency capital (A.M. Best BCAR)?
- What is the impact on regulatory capital (ORSA)?
- What is the impact on economic capital?

A.M. Best announced plans to update its BCAR capital adequacy model

- Incorporate stochastic simulations into the calculation of BCAR risk factors
- Incorporate company-specific risk profile into the calculation of BCAR risk factors
- Consistently tie insurers' probability of default to the determination of capital required to support individual rating levels within the assessment of balance sheet strength

Stochastic modeling may become a key component to the NAIC Own Risk and Solvency Assessment (ORSA)

ORSA is an internal assessment of the risks associated with an insurer's current business plan, and the sufficiency of capital resources to support those risks

- ❑ Section 1 – Description of the Insurer's Risk Management Framework
- ❑ Section 2 – Insurer's Assessment of Risk Exposure
- ❑ Section 3 – Group Risk Capital and Prospective Solvency Assessment

From NAIC Own Risk and Solvency Assessment (ORSA) Guidance Manual

- ❑ " . . .should consider a range of outcomes . .. "
- ❑ ". . .should document the quantitative and/or qualitative assessments of risk exposure in both normal and stressed environments . . ."
- ❑ "Methods for determining the impact on future financial position may include simple stress tests or more complex stochastic analyses."

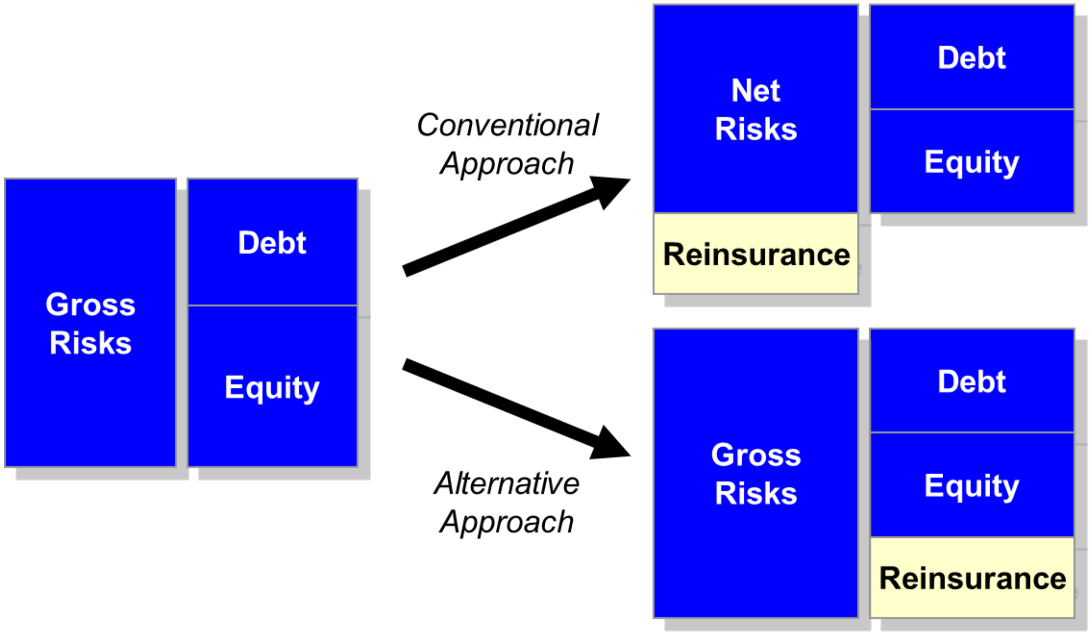
Reinsurance modeling is a key component of economic capital modeling and ERM

Dynamic Reinsurance Modeling	DFA and Economic Capital Modeling	Enterprise Risk Management
• Isolate risk exposure in liabilities • Focus on frequency and severity of large losses • Quantify impact of alternative reinsurance structures and market prices • Use probability models and scenario testing to illustrate risk/reward tradeoffs	• Reinsurance Strategy Fully Integrated • Capital Management • Asset Allocation • Growth Strategies • Reserve Risk • Credit Risk • Other	• **Financial** • Hazard • Human Assets • Legal Liability • Market • Operational • Political • Regulatory

Reinsurance as a Form of Capital

How do we analyze and measure the “capital” benefit of reinsurance?

Reinsurance is increasingly viewed in the context of a company’s overall capital strategy



The ceded underwriting distribution describes the specific results transferred to the reinsurance market

Is this the appropriate representation of the capital benefit to the cedent?

Ceded Statistics	Ceded UW Profit
Mean	**3,698,259**
Standard Deviation	14,088,183
1st percentile	8,728,100
5th percentile	8,568,100
10th percentile	8,448,100
20th percentile	8,208,100
30th percentile	7,968,100
40th percentile	7,648,100
50th percentile	7,238,100
60th percentile	6,648,100
70th percentile	5,808,100
80th percentile	4,308,100
90th percentile	142,272
95th percentile	(10,813,622)
97.5th percentile	(30,264,272)
99th percentile	(78,522,883)

Cost:	**3,698,259**
Capital Benefit 99th percentile:	**78,522,883**
Cost of Reinsurance Capital:	**4.7%**

The difference between the gross and net results illustrates the impact on the cedent

Is this the appropriate representation of the capital benefit to the cedent?

	Gross UW Result	Net UW Result	Difference
Mean	**14,667,822**	**10,969,564**	**3,698,259**
Standard Deviation	25,451,858	16,841,227	
1st percentile	49,318,185	41,500,359	7,817,825
5th percentile	40,901,034	33,610,233	7,290,801
10th percentile	36,219,421	29,234,290	6,985,130
20th percentile	30,347,054	23,569,365	6,777,688
30th percentile	25,800,564	19,323,812	6,476,752
40th percentile	21,792,288	15,686,624	6,105,664
50th percentile	18,023,667	12,188,990	5,834,677
60th percentile	13,943,137	8,474,643	5,468,494
70th percentile	9,423,217	4,477,986	4,945,231
80th percentile	3,420,583	(542,052)	3,962,635
90th percentile	(6,342,141)	(7,742,353)	1,400,211
95th percentile	(17,450,182)	(14,178,249)	(3,271,933)
97.5th percentile	(31,809,781)	(20,360,859)	(11,448,922)
99th percentile	(72,934,694)	(28,689,567)	(44,245,126)

Cost:	**3,698,259**
Capital Benefit 99th percentile:	**44,245,126**
Cost of Reinsurance Capital:	**8.4%**

There are challenges in using the cost of reinsurance capital as a means to compare to other forms of capital

Issues:

- Is it realistic for all companies to have a working economic capital model?
- On a fully diversified basis, will cost of reinsurance capital appear high?
- Can we really evaluate reinsurance by focusing exclusively on the tail?
- Can cost of reinsurance capital be an effective ranking mechanism?

Reinsurance Optimization

Reinsurance optimization modeling is becoming increasingly common

Description: Stochastic model that will “optimize” a reinsurance structure

Required:

1. Subject loss model
2. Alternative reinsurance contracts with indicative pricing
3. Objective function (this is the value you want to optimize)
4. Constraints
5. A strong optimization tool

Result: Model will identify the reinsurance structure that optimizes (3) subject to (4)

A Well Defined Risk Appetite and Objective Function is Required for Optimization

Examples of Objective Functions:

- Maximize income subject to risk constraints
- Maximize economic value added
- Maximize return on equity
- Minimize a risk measure for a given amount of spend

VaR vs TVaR

As a practical matter, TVaR has some numerical advantages vs. VaR that can significantly improve the efficiency of the process

It is Important to Evaluate the Robustness of the Optimization Model

<u>Company Application</u>: the optimal reinsurance program is quite sensitive to the catastrophe "model miss" factor

Optimized Placement Percentages

	Cat Adjustment Factor						
Layer	**As modeled**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**
50 xs 50	0%	0%	0%	0%	0%	0%	90%
50 xs 100	0%	0%	0%	0%	0%	80%	100%
150 xs 150	0%	0%	0%	30%	70%	100%	100%
50 xs 300	0%	0%	50%	100%	100%	100%	100%
50 xs 350	0%	30%	100%	100%	100%	100%	100%
50 xs 400	0%	100%	100%	100%	100%	100%	100%
50 xs 450	0%	100%	100%	100%	100%	100%	100%

Objective = Maximize net underwriting income subject to:
Probability(Insolvency) < 0.5%
Probability (Inviability) < 2.0%

Reinsurance Optimization – Will the broker and broker actuary be replaced?

Challenges:

- Requires appropriate market pricing for all alternatives
- Cannot program all market behavior
- Limitations of a single objective function
- Sensitivity to modeling assumptions

Comments:

- Optimization has significant potential
- Judgment will never be eliminated
- Can provide strong "directional" indications for companies
- Optimization has significant potential and will become a valuable tool
- The broker will NOT be displaced

Parameterizing Loss Models for Multiple Correlated Lines of Business

Overview

- Use Collective Risk Model (CRM) for each Line of Business

- Well-Trodden Earth:
  - Wang
  - Mildenhall
  - Meyers and Collaborators
  - Homer-Rosengarten
  - Many Others

- Correlation part is a common shock method as found in several of the references above – with a twist.

- Along the way point out some underappreciated aspects of CRM.

- Actually parameterizing simulation *method* consistent with the *model*.

Overview

- Goal: Create simulation **method** to generate losses by line of business.
  - ERM/Planning model
  - Capital allocation
  - Reinsurance Options – will be focus here.

- Requirements:
  - **Efficient** as to runtime.
  - **Efficient** as to parameterization – relativity low number of parameters,
  - Captures broad **properties** of distribution – match first and second moments, and possibly third. (Internal company analyses may be a lot more granular).
  - Simulate **small and large** losses – and reflect the appropriate dependency. Generate individual large losses and small losses in the aggregate.
  - Reflect **correlation** between lines/years.
  - **Consistent** with underlying model.

CRM - Setup

- CV: For any random variable Y, the **coefficient of variation**, or CV is
  - $\boldsymbol{\nu}(Y) = \sqrt{Var(Y)}/E(Y)$

- CV is unit-less, makes for nice formulas.

- **Collective Risk Model**,
  - $Z = X_1 + \cdots + X_N$, X_i iid

- Where $Z =$ aggregate losses, $X =$ severity, and the random variable N is the claim count, or “frequency”. **Key assumption**: X and N are independent.

- Independence of X, N could be violated by inhomogeneous data.

- Large/Small Losses – Threshold T such that (severity) losses $\geq T$ are “large”, losses $< T$ are “small”.

CRM – Contagion Factor, Moments

- Induced CRMs
  - $Z_L = X_{1,L} + \cdots + X_{N,L}, Z_S = X_{1,S} + \cdots + X_{N,S}$

- Will try to avoid further use of subscripts

- Contagion Parameter – Set $c = \big(Var(N) - E(N)\big)/E^2(N)$. Then c is invariant in the sense $c = c_L = c_S$ (follows from independence if X, N)

- Assume $c > 0$ (positive contagion).

- Moments of CRM:
  - $E(Z) = E(N)E(X)$
  - $\boldsymbol{\nu}(Z) = \sqrt{(\boldsymbol{\nu}^2(X) + 1)/E(N) + c}$

- It follows that $\boldsymbol{\nu}(Z) \to \sqrt{c}$ as $E(N) \to \infty$ (and $\boldsymbol{\nu}^2(Z) - c \to 0 \propto 1/E(N)$)

CRM – Large, Small, Total Losses

- Correlation:

$$\boldsymbol{\rho}(Z_S, Z_L) = c/\left(\boldsymbol{\nu}(Z_S)\boldsymbol{\nu}(Z_L)\right)$$

(common shock based on identical mixing distributions)

- Total Variation:

$$E^2(Z)(\boldsymbol{\nu}^2(Z) - c) = E^2(Z_L)(\boldsymbol{\nu}^2(Z_L) - c) + E^2(Z_S)(\boldsymbol{\nu}^2(Z_S) - c).$$

- **Interval for $\boldsymbol{\nu}(\boldsymbol{Z})$:**

$$\sqrt{c + \frac{E^2(Z_L)}{E^2(Z)}(\boldsymbol{\nu}^2(Z_L) - c)} \leq \boldsymbol{\nu}(Z) \leq \sqrt{c + \frac{E^2(Z_L)}{E^2(Z)}(\boldsymbol{\nu}^2(Z_L) - c) + \frac{T}{E(Z)}\left(1 - \frac{E(Z_L)}{E(Z)}\right)} \quad \textbf{(*)}$$

$$\sqrt{c} \leq \boldsymbol{\nu}(Z_S) \leq \sqrt{c + \frac{T}{E(Z_S)}}$$

Mixed Poisson CRM

- We now assume that the claim count r.v N is of *mixed Poisson* type, meaning $N \sim Poisson[E(N)G]$, where G is a r.v with mean 1.

- To draw from N:
  - 1. Draw g from G.
  - 2. Draw from $Poisson[E(N)g]$.

- $Var(G) = c$. Will use the notation $G[c]$

- N_L, N_S are also mixed Poisson with the same *mixing distribution* G.

- Example: $G \sim gamma$. Then $N \sim Negative\ Binomial$.

- Fact ("Severity is Irrelevant"): $Z/E(Z) \xrightarrow{D} G\ as\ E(N) \rightarrow \infty$

- This concludes description/key properties of the model.

Simulation Method - CAD Algorithm with Frequency, "Severity" and Serial Common Shock

- Ref:Homer-Rosengarten (2011), Meyers-Klinker-LaLonde (2003)

- **Full Info CAD** (Have N, X)
  - Draw from N (i.e. draw from G and then from $Poisson[E(N)G]$)
  - Draw N_L from $Bin(N,q)$, where $q = 1 - \mathrm{CDF}_X(T)$. $N_S = N - N_L$.
  - Draw $X_{1,L}, \ldots, X_{N,L}$ large losses. $Z_L = X_{1,L} + \cdots + X_{N,L}$
  - Draw $\widetilde{Z_S}$ from Conditional Aggregate Distribution (eg, lognormal) matching $k \geq 2$ moments of $Z_S|N_S$.
  - $\tilde{Z} = \widetilde{Z_S} + Z_L$

- H-R Paper: $\tilde{Z}, \widetilde{Z_S}\ (Z_L) \overset{\mathrm{D}}{\rightarrow} G$. This generalizes the "severity is irrelevant" result. Also, the method generates the correct dependence between large and small losses

Simulation Method

- **Limited Info CAD** (Don't have N, X)
  - Draw from G only.
  - Draw N_L from $Poisson[E(N_L)G]$
  - Draw large losses as previously.
  - Draw $\widetilde{Z_S}$ from CAD matching first **two** moments of $Z_S|G$

- **Minimum Parameterization**: $G[c], E(N_L), X_L, E(Z), \boldsymbol{\nu}(Z)$

- Can then eliminate severity, N_S from equations for first two moments of $Z_S|G$.

- To wit, $E(Z_S|G) = GE(Z_S)$, $\boldsymbol{\nu}(Z_S|G) = \sqrt{(\boldsymbol{\nu}^2(Z_S) - c)/G}$

- **But,** it is not automatic that this minimal parameterization is consistent with CRM

Simulation Method

- To address, suppose we have all the minimal parameters except $\boldsymbol{\nu}(Z)$. We can then evaluate the lhs and rhs of inequality (*)

$$\sqrt{c + \frac{E^2(Z_L)}{E^2(Z)}(\boldsymbol{\nu}^2(Z_L) - c)} \le \boldsymbol{\nu}(Z) \le \sqrt{c + \frac{E^2(Z_L)}{E^2(Z)}(\boldsymbol{\nu}^2(Z_L) - c) + \frac{T}{E(Z)}\left(1 - \frac{E(Z_L)}{E(Z)}\right)}$$

- Any choice for $\boldsymbol{\nu}(Z)$ within this interval is a) possible and b) consistent with MP CRM.

Common Shock Correlation

- Correlate LoBs modeled with MP CRM/CAD method.
- LoBs are organized into covariance groups. Only Lobs within the same covariance group co-vary with one another.
- Frequency, “severity”, and serial common shock.

Frequency Common Shock

- General Idea: Common draw from mixing distribution.
- Need to allow that LoBs might have different mixing distributions.
- Solution is draw common uniforms and use these to invert the mixing distributions ($g = F_G^{-1}(u)$).
- Remaining problem is that this will tend to generate very high correlation.
- Usual solution is to assume that G is an independent product, ie
  - $G[c] = G_1[c_1]G_2[c_2]$
  - Then apply common shock only to G_1.
  - Note that $c = c_1 + c_2 + c_1c_2$

Frequency Common Shock

- Variant is the "twisted product" $G[c] = G_1[c_1] \ltimes G_2\,[c_2]$ defined by $G = G_1 G_2[c_2/G_1]$.

- That is, to draw from G:
  - Draw g_1 from G_1.
  - Draw g_2 from $G_2[c_2/g_1]$.
  - $g = g_1 g_2$.

- Nice thing about twisted product is $c = c_1 + c_2$.

- **Parameter**: FrCoVarWt $= w, 0 \leq w \leq 1$. Varies by LoB.

- In twisted product set $c_1 = wc, c_2 = (1-w)c$ (where $G_i[0] \equiv 1$).

Serial Common Shock

- Bring in uniforms necessary to invert G_1's for frequency c.s. These vary by covariance group and year.
- Also bring in uniforms for G_2's – varying by LoB and year.
- Reason for G_2's is generate sufficient correlation between years but within LoB.
- Flip a weighted coin.
- For year $j, j \geq 2$, if coin flip comes up "heads" use the uniforms from year $j - 1$. Otherwise use year j.
- **Parameter** – FrSerialCoVarWt – the weight for the coin flip. Can vary by covariance group or LoB. Usually by covariance group.

Serial Common Shock

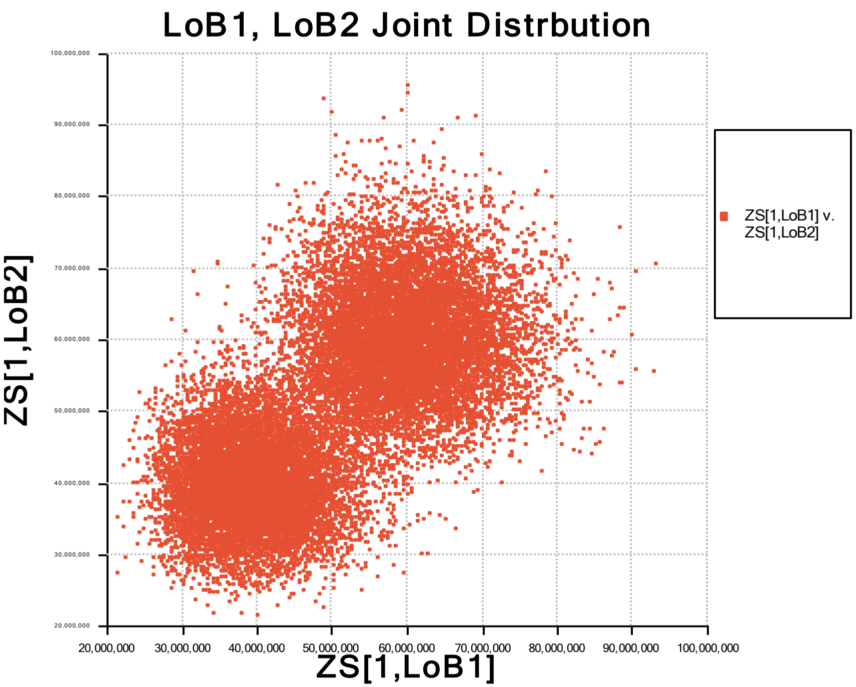
- Summary
  - G_1 correlates non-identical LoBs, both within-year and serially.
  - G_2 - serial correlation for identical LoBs.
  - Serial correlation decays by FrSerialCoVarWt.

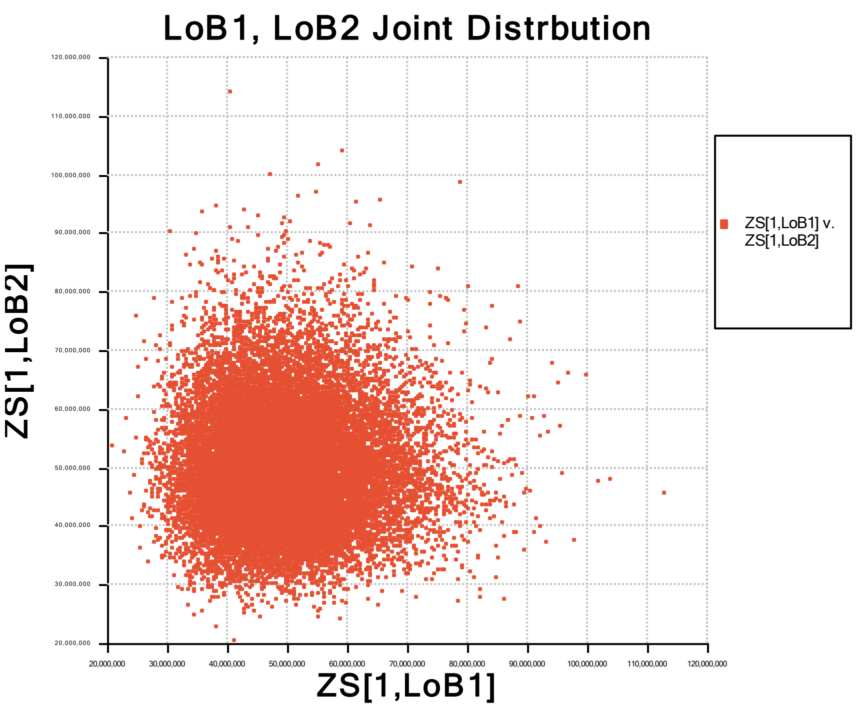
"Severity" Common Shock

- Really it's c.s. applied to the conditional aggregate distribution generating $\widetilde{Z_S}$.
- By H-R, the particular distribution family used doesn't matter.
- Assume lognormal, with $Mu, Sigma$ the conditional parameters.
- **Parameters**: ZSCoVarWt, ZSSerialCoVarWt.
- Express CAD as a product of Lognormals
- $CAD = \text{logn}\left[.5Mu, Sigma\sqrt{ZSCovarWt}\right]\text{logn}\left[.5Mu, Sigma\sqrt{1 - ZSCovarWt}\right]$
- Play same game as previously.

Why do we need ZSCoVarWt?

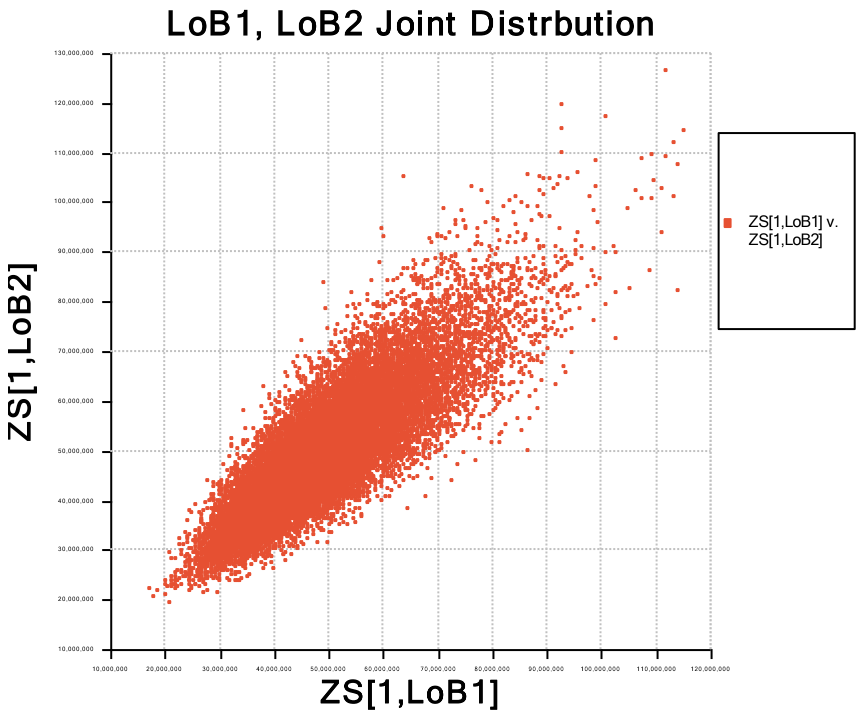
- Example: Identical LoBs LoB1, LoB2
- $FrCoVarWt = .85$, $ZSCoVarWt = 0$, $G_1 = 1 \pm \sqrt{c}$, with probabiltiy .5.
- $c = O(\boldsymbol{\nu}^2)$ - High Correlation $c = 0\ (\boldsymbol{\nu}^2 \gg c)$ - No Correlation





Why ZSCoVarWt?

- $FrCoVarWt = 0, ZSCoVarWt = .85, c = 0$



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Why ZSCoVarWt?

- For Identical LoBs:

	FrCoVarWt=1 ZSCoVarWt=0	FrCoVarWt=0 ZSCoVarWt=1
$\mathbf{v}^2 \to c$	$\boldsymbol{\rho} \to 1$	$\boldsymbol{\rho} \to 0$
$\mathbf{v}^2 \gg c$	$\boldsymbol{\rho} \to 0$	$\boldsymbol{\rho} \to E^2\left(\sqrt{G}\right)(\mathbf{v}^2 - c)/\mathbf{v}^2$