Weighting (Reserve) Indications from Multiple Methods

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1 Current Approach

What would you do?

Estimated ultimate claims at 12 months

Assume the following

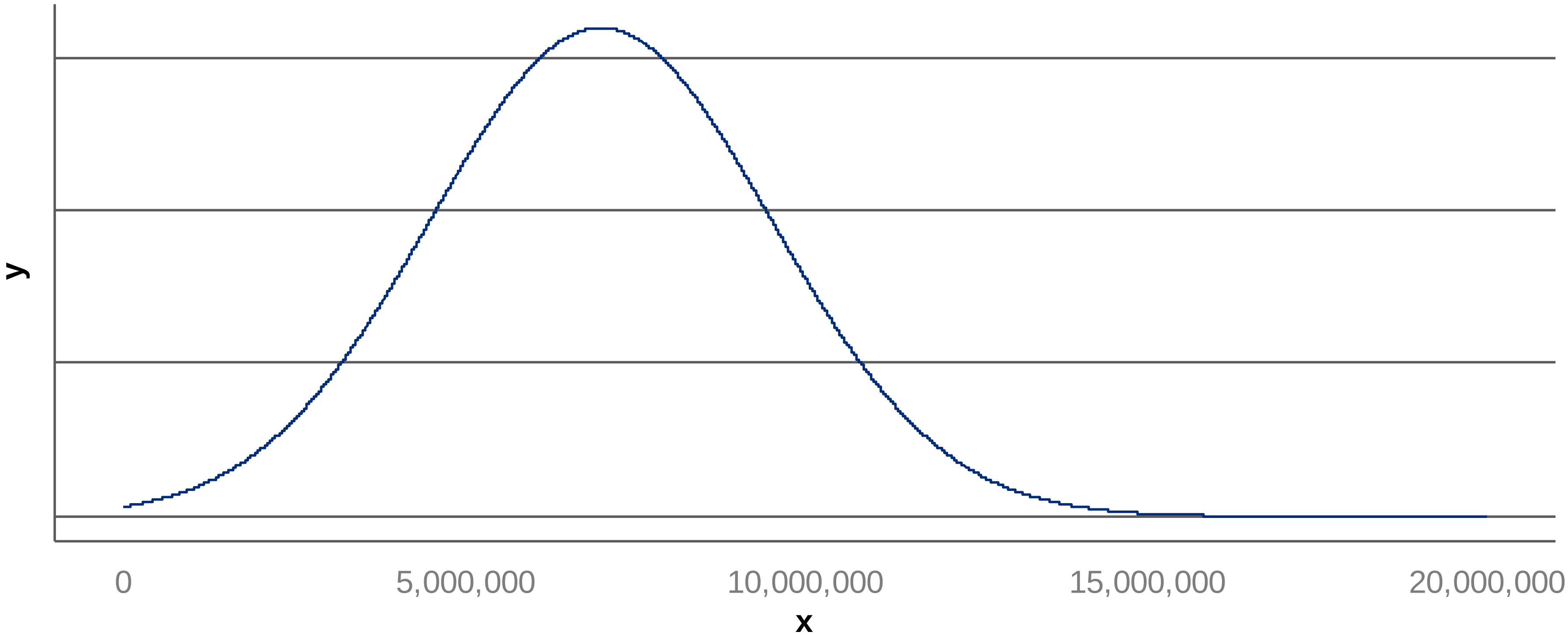
- Premium = $10 million
- Expected loss ratio = 70%
- Reported claims at 12 months = $4 million
- Paid claims at 12 months = $1.5 million
- Reported development factor = 2.00
- Paid development factor = 3.00

Indications

- Loss Ratio Method = $7.0 million
- Reported Development = $8.0 million
- Paid Development = $4.5 million
- Reported B-F = $7.5 million
- Paid B-F = $6.2 million

What would you do?

Estimated ultimate claims at 12 months



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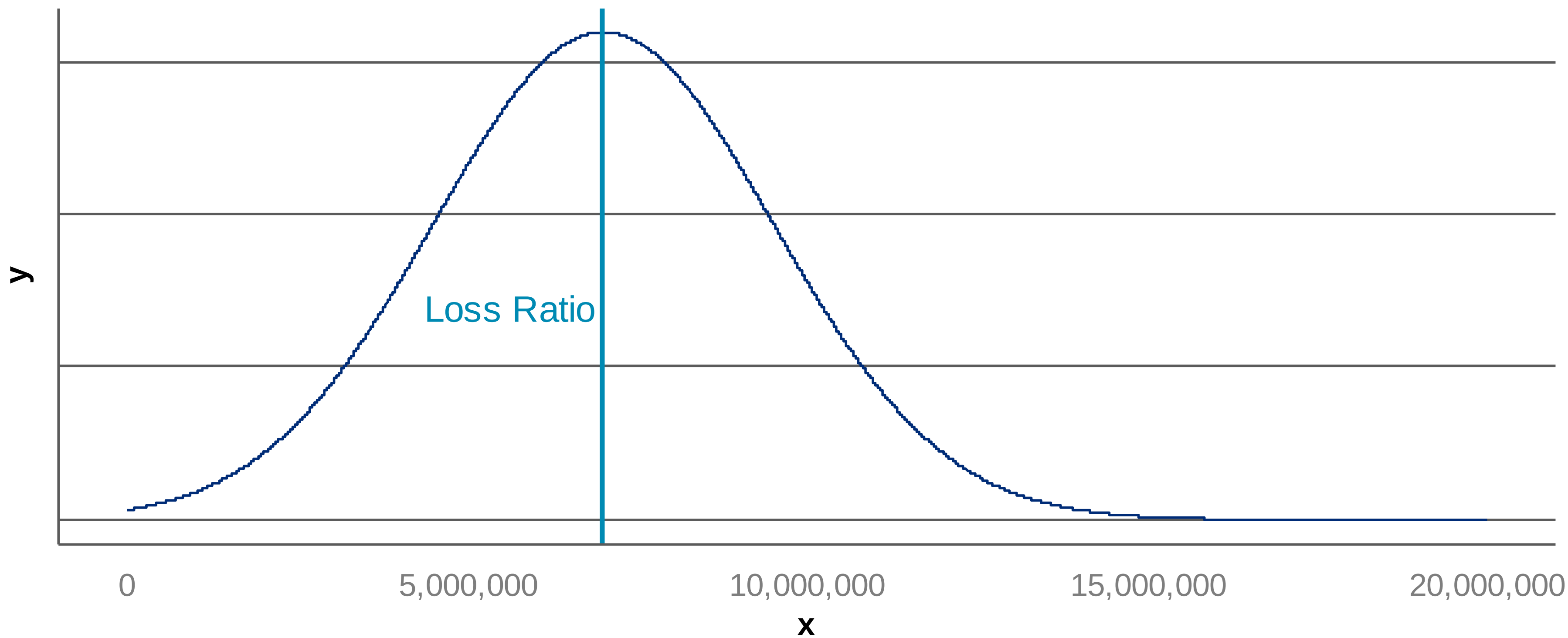

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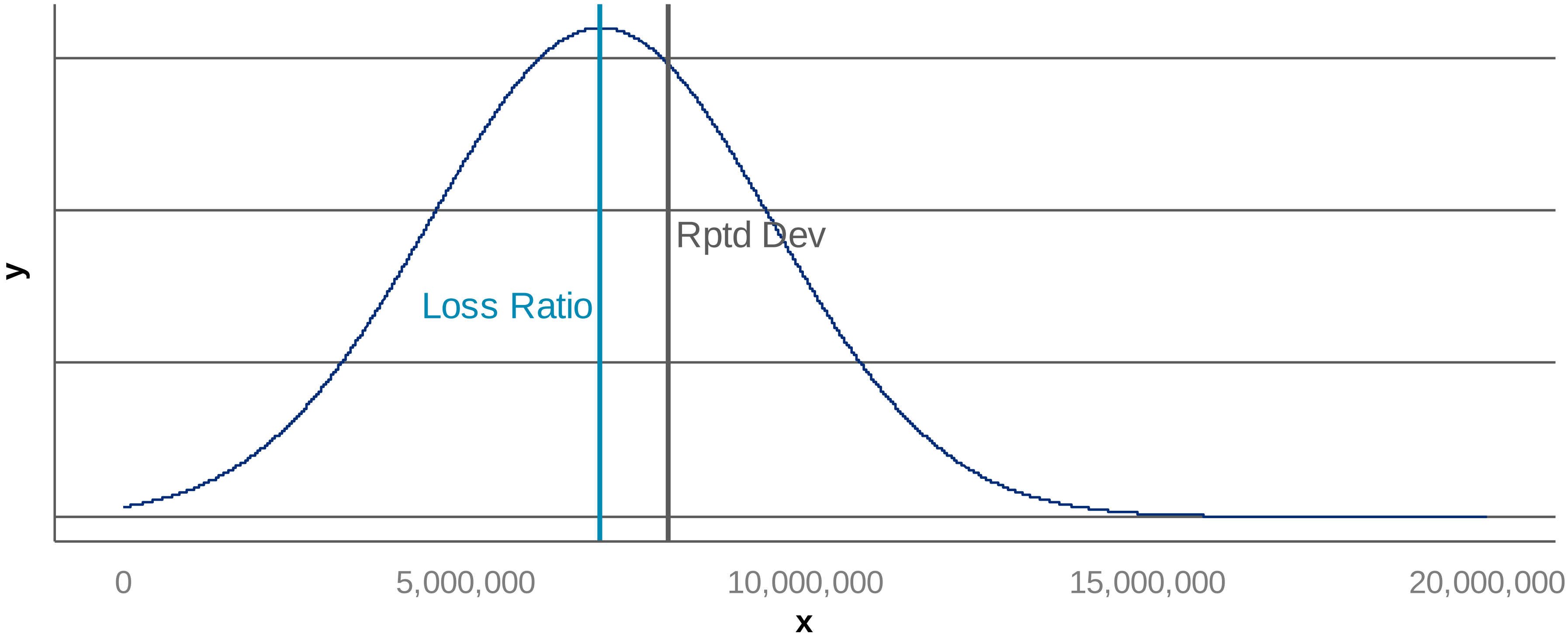

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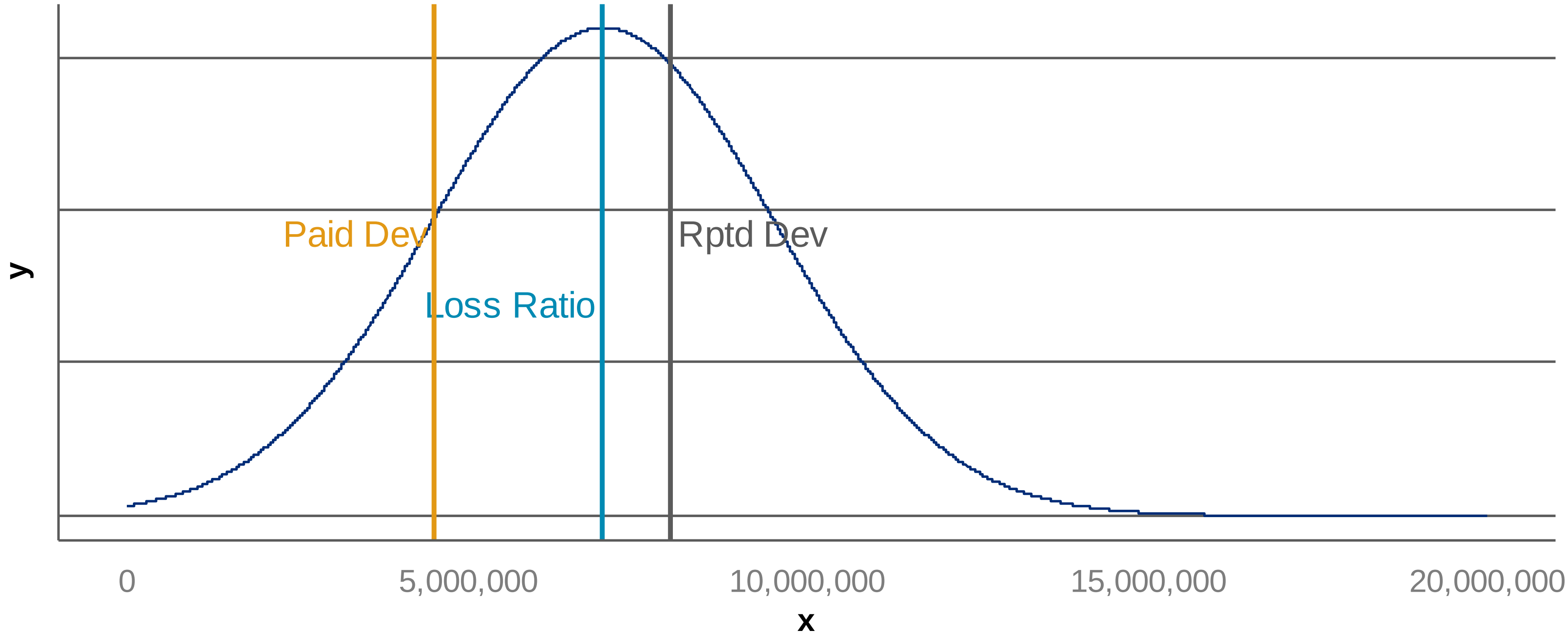

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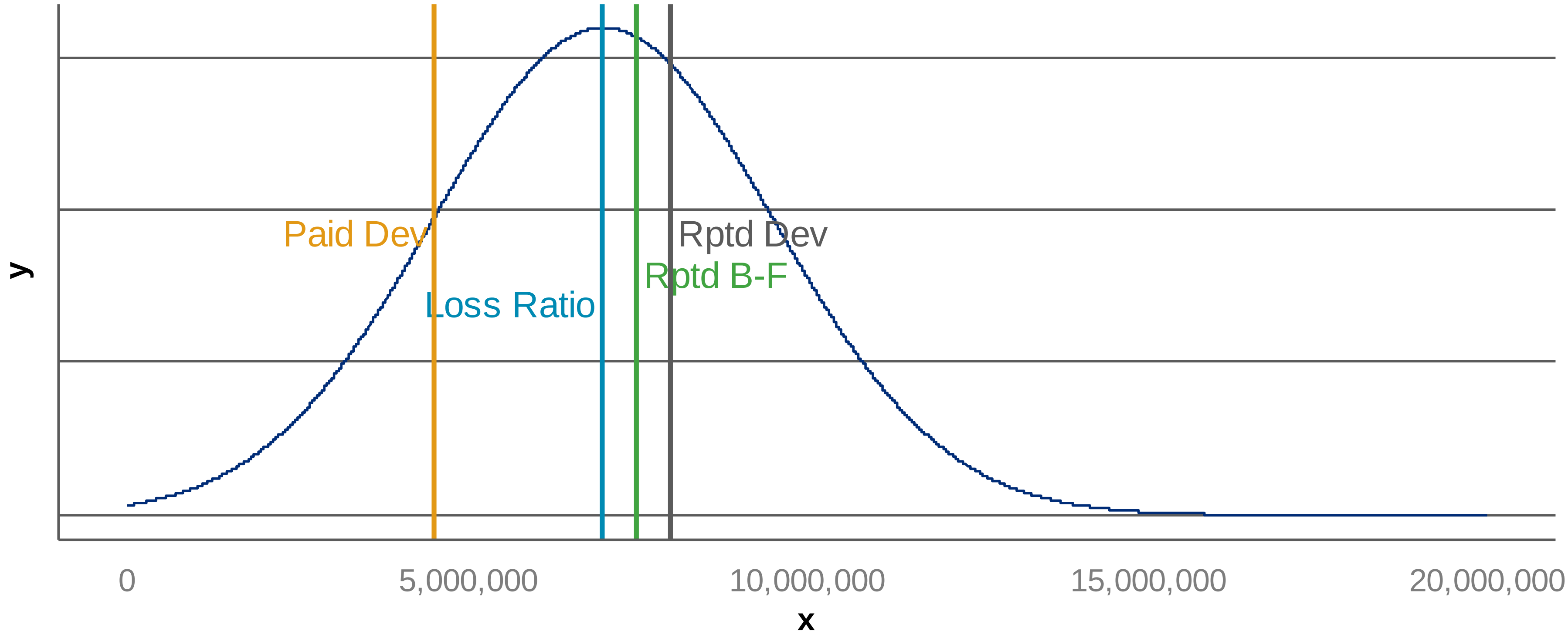

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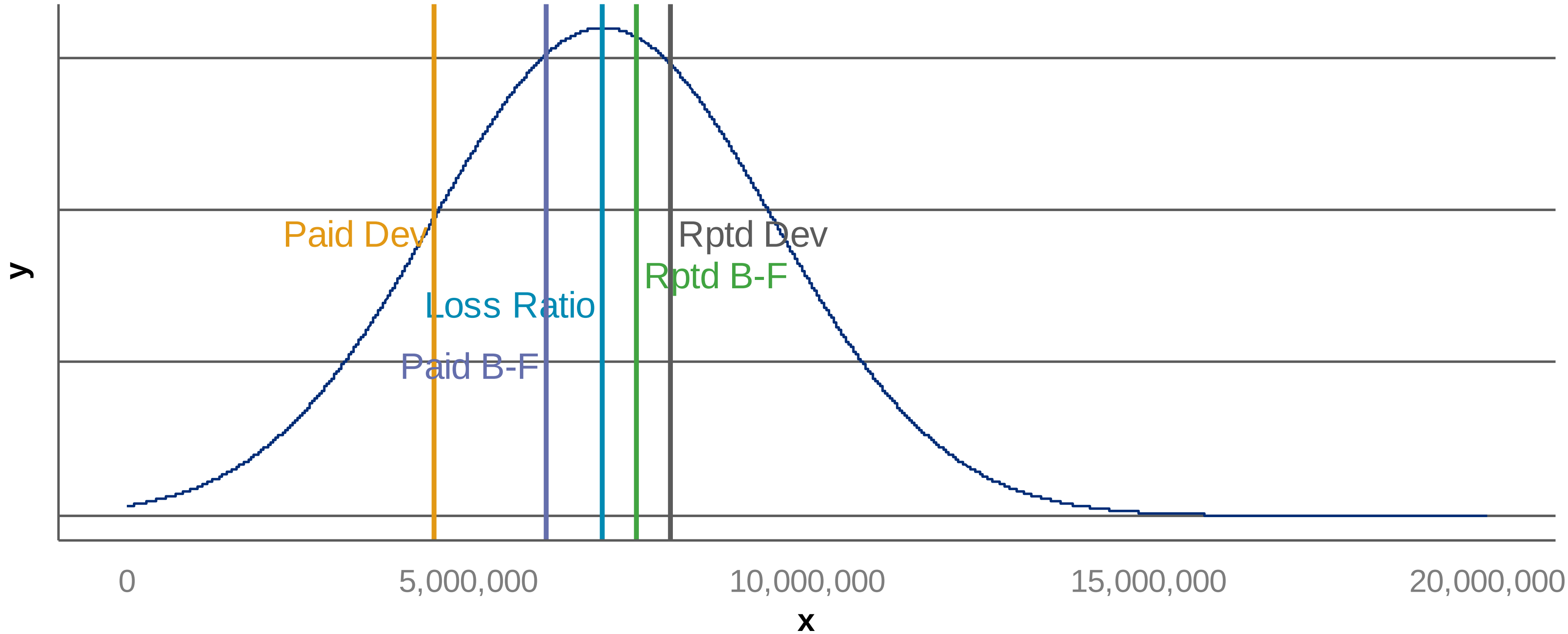
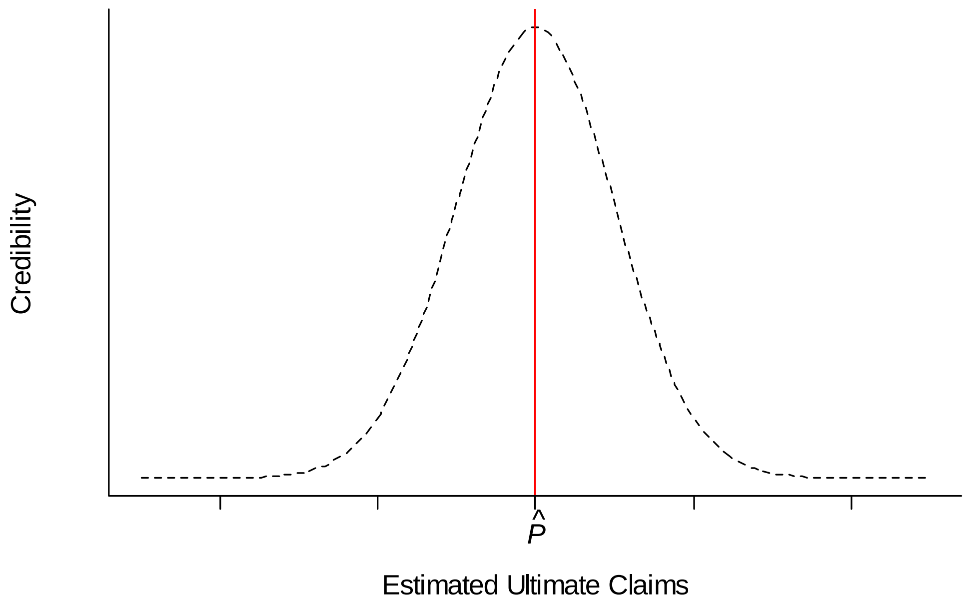


The Selected Estimate

- Generally a weighted average of indications
  - Explicit weighting
  - Implicit weighting
- Weights and significant digits
  - 20% (Paid Development) / 80% (Reported Development) weighted average (for example)
  - 3×10^{-1} / 7×10^{-1} (one significant digit)
  - [2.5×10^{-1} – 3.5×10^{-1}] / [7.5×10^{-1} – 6.5×10^{-1}]
  - 20% * $4.5 million + 80% * $8.0 million = $7.3 million ($5.8 million unpaid)
  - One significant digit: $7.0 million ($5.5 million unpaid)
  - Additional Error = $0.3 million / $5.5 million = 5.4%
  - What if you advised management that the range was +/- 10%?

The Selected Estimate

- How do actuaries develop the weights?
  - – The actuarial judgment function



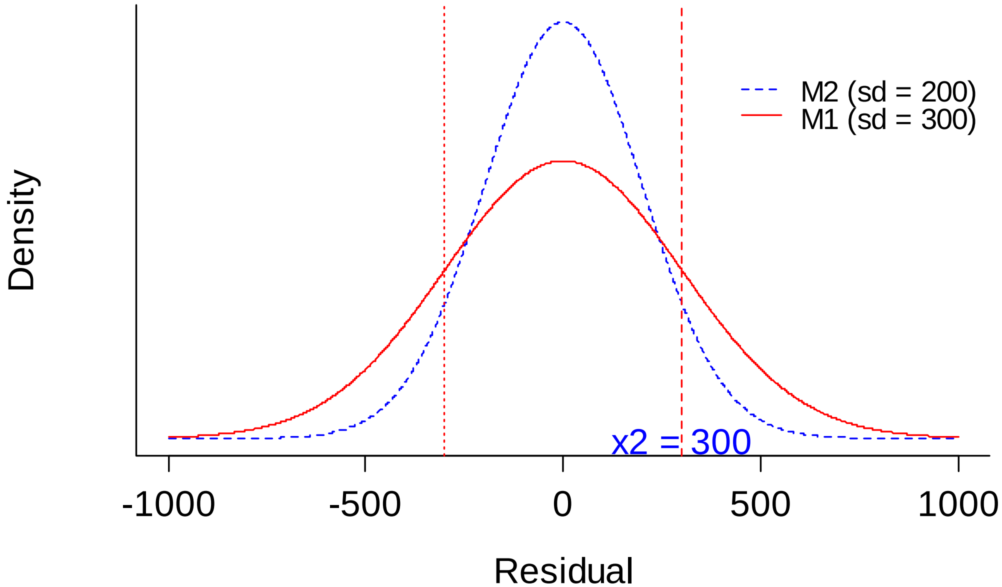
2 Proposed Approach

Is there another way to combine the estimates

- Each method represents a competing estimator
- Each estimator is (assumed) unbiased
- Credibility?
  - Terminology
  - Measurement
- Simplifying assumptions
  - Symmetric distribution centered around 0: for simplicity, we use only the positive domain of x and consider both tails of the distribution of x_1
  - F and f to represent the distribution and density functions, respectively, of the residuals

Two Method Example

- Paid chain-ladder (Method 1, M_1)
- Reported incurred chain-ladder (Method 2, M_2).
- The credibility of the reported incurred chain-ladder is the probability that:
  – the error of M_2 (random variable denoted X_2)
    is less than or equal to
  – the error of M_1 (random variable denoted X_1)
- So for any $X_2 = x_2$ (where x_2 is an observation of X_2), we have the following possibilities:
  1. $|X_1| < |x_2|$ (Credibility to Method 1)
  2. $|X_1| > |x_2|$ (Credibility to Method 2)



The Credibility Model

- Math Speak: $Z_2 \div 2 = \int_0^\infty 2[1 - F_1(x)]f_2(x)dx$
- English
  - – Over the domain of positive values of x: $\int_0^\infty$
  - – the credibility assigned to Method 2: Z_2
  - – is the probability that the error of Method 1 is greater than x: $\left(1 - F_1(x)\right)$
  - – or less than $-x$: $\left(1 - F_1(x)\right)$ by symmetry
  - – for all $X_2 = x$: $f_2(x)dx$
  - – The 2 inside the integral provides consideration for both:
    - values of $X_1 < -x_2$
    - values of $X_1 > +x_2$

    For example, if $x_2 = 100$, we would assign credibility to Method 2 for
    - $X_1 > 100$ and
    - $X_1 < -100$
  - – The 2 on the left-side is necessary as our limits of integration only consider one-half the domain of possible x values.

The Credibility Model

- Algebraic Simplification

$$Z_2 \div 2 = \int_0^\infty 2[1 - F_1(x)]f_2(x)dx$$

$$Z_2 = 2 \; - 4\int_0^\infty F_1(x)f_2(x)dx$$

- The Limiting Case: Method 1 has no error
- But how do we calculate this?
  - Option 1: Numerical Integration (examples provided with paper on CAS website)
  - Option 2: Simulation (R, @Risk) (sample R code provided in paper)
  - Option 3: Computational integration (R, SAS?) (sample R code provided in paper)

Assumptions and Generalization

- Assumptions / Implementation Issues
  - – Normality of Residuals: Rehman & Klugman; Central Limit Theorem
  - – Calculation of Errors: Look at history, testable relative uncertainty estimates
  - – Management's Recorded Estimate: Just another method
- Generalization for n methods

$$Z_2 \div 2 = \int_0^\infty 2[1 - F_1(x)]f_2(x)dx$$

$$Z_i = \int_0^\infty 2^n \begin{Bmatrix} [1 - F_1(x)] \cdots [1 - F_{i-1}(x)] \\ [1 - F_{i+1}(x)] \cdots [1 - F_n(x)] \end{Bmatrix} f_i(x)dx$$

- Remove / relax simplifying assumptions: See Appendix
  - – Symmetry
  - – Centered at 0

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