Casualty Actuaries of the South East

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ASOP 36
Expected Value Estimate

- In evaluating the reasonableness of reserves, the actuary should consider one or more expected value estimates of the reserves except when such estimates cannot be made based on available data and reasonable assumptions.

ASOP 36
Expected Value Estimate

- The actuary may use various methods or assumptions to arrive at expected value estimates. In arriving at such expected value estimates, it is not necessary to estimate or determine the range of all possible values, nor the probabilities associated with any particular values.

ASOP 36
Range of Reasonable Estimates

- The actuary may determine a range of reasonable reserve estimates that reflects the uncertainties associated with analyzing the reserves. A range of reasonable estimates is a range of estimates that could be produced by appropriate actuarial methods or alternative sets of assumptions that the actuary judges to be reasonable.

Ranges vs. Distributions

- All the materials presented in the "Ranges vs. Distribution presentation are based on the CAS Reserve Variability Webinar presented by Mark Shapland, FCAS and Louise Francis, FCAS.

Definition of Terms

- Process Risk – the randomness of future outcomes given a known distribution of possible outcomes.
- Parameter Risk – the potential error in the estimated parameters used to describe the distribution of possible outcomes, assuming the process generating the outcomes is known.
- Model Risk – the chance that the model (process) used to estimate the distribution of possible outcomes is incorrect or incomplete.

A “Range” is not the same as a “Distribution”

- A Range of Reasonable Estimates is a range of estimates that could be produced by appropriate actuarial methods or alternative sets of assumptions that the actuary judges to be reasonable.
- A distribution is a statistical function that attempts to quantify probabilities of all possible outcomes.

A range, by itself, creates problems:

- A range can be misleading to the layperson – it can give the impression that any number in that range is equally likely.
- A range can give the impression that as long as the carried reserve is within the range anything is reasonable.

A range, by itself, creates problems:

- There is currently no specific guidance on how to consistently determine a range within the actuarial community.
- A range, in and of itself, needs some other context to help define it. e.g. How do you calculate a risk margin?

Ranges vs. Distributions

- A distribution provides:
- Information about all possible outcomes
- Context for defining a variety of other measures (e.g. risk margin, materiality, risk based capital, etc.)

A probability distribution has several advantages

- The risk in the data defines the range.
- Adds context to other statistical measures.
- A reserve margin can be defined more precisely.
- Can be related to risk of insolvency and materiality issues.
- Others can define what is reasonable for them.

Case Against Stochastic Reserving

- Does not measure model risk or systemic risk
- The underlying process isn't necessarily random. We have insufficient information concerning the event's deterministic nature.
- Stating, e.g. a 75th percentile implies a level of precision that doesn't exist and results in less perceived credibility of the actuary.

Case For Stochastic Reserving

- “Essentially, all models are wrong, some are useful.”
- Box && Draper, Empirical Model-Building and Response Surfaces, pg 424

England and Verrall Bootstrap

- Samples from residuals as opposed to sampling from the sample.
- Uses weighted loss development factors
- Incorporates process uncertainty by sampling incremental payments from a gamma distribution.
- Methodology described by G. Kirschner et al in Variance Spring 2008.

Simple Lognormal sampling

- Compute lognormal of loss development factors.
- Calculate mean and standard deviation for each column.
- Select lognormal mean and standard deviation for each column.
- Create simulated factors for bottom half of loss development triangle.
- Coefficient of variation should increase as age increases

Methods vs. Models

- A Method is an algorithm or recipe – a series of steps that are followed to give an estimate of future payments.
- The well known chain ladder (CL) and Bornhuetter-Ferguson (BF) methods are examples.
- The search for the "best" pattern

Methods vs. Models

- A Model specifies specific statistical assumptions about the loss process, usually leaving some parameters to be estimated.
- Estimating the parameters gives an estimate of the ultimate losses and some statistical properties of that estimate.
- The search for the "Best" distribution

Model Reasonability Checks

- Criterion 5: Coefficient of Variation by Year
- Should be largest for oldest year.
- Criterion 6: Standard Error by year
- Should be smallest for the oldest year on a dollar scale.

Model Reasonability Checks

- Criterion 7: Overall Coefficient of Variation
- Should be smaller for all years combined than any individual year.
- Criterion 8: Overall Standard Error
- Should be larger for all years combined than any individual year.

Model Reasonability Checks

- Criterion 9:Correlated standard error and coefficient of variation
- Should both be smaller for all LOBs combined than the sum of individual LOBs.

Model Reasonability Checks

- Criterion 10: Reasonability of Model parameters and Development Patterns
- Is loss development pattern implied by model reasonable?

Testing Age to Age Assumptions

- Gary Venter's PCAS paper, "Testing the Assumptions of Age-to-Age Factors" provides the actuary with a well written documentation of the issues to be considered in deciding on a loss reserving methodology for a data set under study.

Testing the Assumptions of Age To Age Factors

- Regression Analysis with constant
- Regression Analysis without constant
- Constant
- Bornheutter/Ferguson
- Cape Cod
- BF/Cape Cod
- Berquist/Sherman

- PCAS paper by Gary Venter

Loss Development Methods

1. Three year average
2. Five year average
3. Five year average excluding the high and low
4. Four year weighted average
5. All years average
6. All years average excluding high and low
7. Five year excluding high
8. Five year excluding low
9. Selected loss development factor

Testing Age to Age Assumptions

- Regression analysis produces estimates for the standard deviation of each parameter estimated.
- Usually the absolute value of a factor is required to be at least twice its standard deviation for the factor to be regarded as significantly different from zero.
- This test is failed by many development triangles, which means that the chain ladder method is not optimal for these triangles.

Testing Age to Age Assumptions Objective

- The actuary has many data, resource and time constraints in performing loss reserving analyses.
- The objective of this presentation is to develop an approach to efficiently implement the theories in his paper to allow the actuary more time to evaluate the results.

Testing Age to Age Assumptions

- The ***Stanard*** loss generation scheme illustrates how far off reserves can be when one reserving technique is applied to losses that have an emergence process different from the one underlying the technique.
- "A simulation Test of Prediction Errors of Loss Reserve Estimation Techniques," PCAS LXXII

Testing Age to Age Assumptions

- Thomas Mack showed that some specific assumptions on the process of loss generation are needed for the chain ladder method to be optimal. Thus if actuaries find themselves in disagreement with one or another of these assumptions, they should look for some other method of development that is more in harmony with their intuition about the loss generation process.

Testing Age to Age Assumptions

- In this exercise, we will test the assumption that the expected emergence in the next period is proportional to the losses emerged to date. To test this assumption against its alternatives, the development method that leads from each alternative needs to be fit, and then a goodness-of-fit measure applied.

Chain Ladder Assumptions for Optimality

1. The expected value of the incremental losses to emerge in the next period is proportional to the total losses merged to date by accident year.
2. Except for the same accident year, the increments are independent.
3. The variance of the next increment observation is a function of the age and the cumulative losses to date

Test the following Assumptions

1. Expected losses emerge proportional to losses to date
2. Expected losses emerge proportional to losses to date plus a constant
3. Expected losses emerge proportional to ultimate losses
4. Expected losses emerge as a constant
5. Expected losses emerge as a constant with an adjustment

Testing Age to Age Assumptions

- If the chain ladder fails the assumption of least squares optimality, test the underlying assumptions for the Bornhuetter-Ferguson, Cape Cod, combination of Bornhuetter-Ferguson/Cape Cod or other creative approaches that the data might suggest.

Testing Age to Age Assumptions

- The results of the tests should increase the actuary's confidence in the hypothesis, still recognizing that no hypothesis can ever be fully verified.

Testing Loss Development Methods

Analysis	Sum Squared Error	IBNR
Selected	155,716	$20,696
Three year average	176,379	$17,722
Five year average	167,589	$21,959
Five year excl high/low	157,118	$23,894
Four year weighted	156,613	$20,481
All year excl high/low	212,266	$28,423
Five year excl high	157,093	$16,359

Testing loss development methods

Analysis	Sum Square Error	IBNR
Regression w/constant	152,270	$22,717
Regression w/o constant	110,296	$21,201
Constant	86,364	$13,322
Bornheutter-Ferguson	268,478	$22,290
Cape Cod	86,364	$13,322
BF/Cape Cod	116,672	$22,290

Testing Age to Age Assumptions

- This is a work in progress. It is hoped that further research of Gary Venter's paper will foster the development of efficient testing of loss reserving approaches for the practicing actuary..

Line of Business Correlation Study Small Data Base

	HO	Fire	Com	Boat	Auto PD	Auto Liab	Other Liab
HO/FO	1.0						
Fire	.43	1.0					
Com	.02	.01	1.0				
Boat	.15	.46	.01	1.0			
Auto Pd	.46	.80	.01	.33	1.0		
Auto Liab	.33	.49	.01	.28	.45	1.0	

Line of Business Correlation Study Large Data Base

	Auto Liab	HO/FO/CMP	Other Liab	Work Comp
Auto Liab	1.0000			
HO/FO/CMP	.5671	1.0000		
Other Liab	.2139	.2262	1.0000	
Work Comp	.2059	.4030	.5224	1.0000

Line of Business Correlation Study Standard deviation

Correlation	Paid estimate	Incurred estimate
10%	15,591	6,747
25%	17,780	7,651
50%	20,417	8,956
75%	22,750	10,094
Simulated	18,013	7,805

Line of Business Correlation Study Ranges by line of business

Line of Bus	Mean	Std dev	Range (low)	Range (high)
Home	10,498	2,339	7,480	13,516
Auto liab	28,445	2,397	25,352	31,537
Boat	214	208	-55	482
Com MP	173	146	-15	362
Other Liab	1169	239	-477	139
Pers Property	558	191	312	805
Auto PD	17,46	581	996	2,496
Reins	43,026	5,014	36,558	49,494

Loss Adjusting Methods

- Salzman
- Historical triangle
- Optimize opening, maintaining and closing percentages (Conger et al)
- Loss Activity Method (Paul Deemer)
- Ultimate minus paid
- Expected unpaid
- Multiple of paid to date

CAS Limited Attendance Seminar on Reserve Variability
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