A Bayesian State-Space Model of Loss Development

Frank A. Schmid
Director and Senior Economist

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Thanks to Jon Evans for comments.

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Outline of the Presentation

- Dimensions of Loss Development
- Loss Development as a Time Series Problem
- Modeling Loss Development in State Space
- Bayesian Model of Loss Development
- Application (1): Automobile Insurance
- Application (2): The AFG Data Set
- Application (3): Tail Estimation in NCCI Ratemaking (Test Phase)
- Conclusion

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Dimensions of Loss Development

- Estimating the runoff triangle ("squaring the triangle")—shaded yellow
- Estimating the tail—shaded pink
- Forecasting future accident (policy/injury) years—shaded aquamarine

	Development Year								
Accident Year	1	2	3	4	5	6	7	8	9
1	x	x	x	x	x				
2	x	x	x	x					
3	x	x	x						
4	x	x							
5	x								
6									
7									

x: observed value

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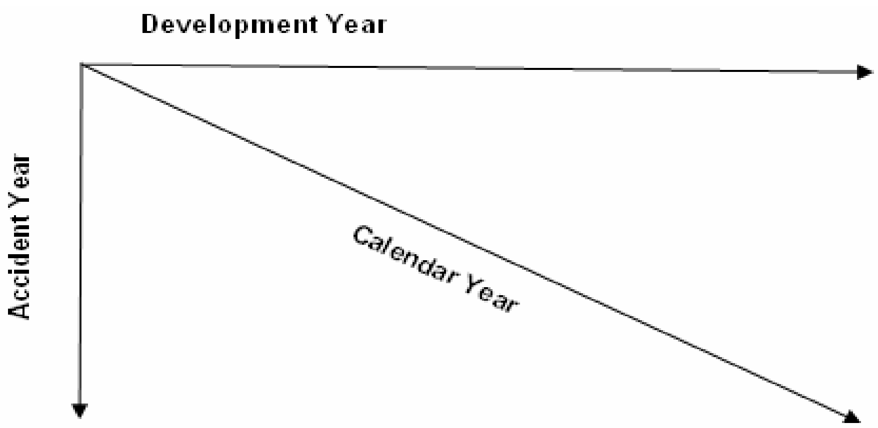
Loss Development as a Time Series Problem

- Time permeates loss development
  - All data are observed on the time axis, at low (annual) frequency
    - There is development time
    - There is calendar year time
    - There is accident (policy/injury) year time



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Modeling Loss Development in State Space

- The state-space framework is a powerful tool for modeling time series

$$x_{t+1} \sim N(x_t, \sigma^2)$$

  - The future value of a variable, x_{t+1}, is distributed around its current value, x_t, with an innovation variance σ^2
  - Such a random walk specification is suitable for *describing* low-frequency time processes (of which there are three in loss triangles), even if the variables in question do not follow random walks
    - For meaningful *extrapolation*, the variable in question must indeed follow a random walk

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Bayesian Model of Loss Development

- The model is written in state space
- The model is estimated using the Metropolis-Hastings algorithm
  - The posterior distributions are arrived at by means of Markov-Chain Monte-Carlo (MCMC) simulation
- The model employs conjugate priors
  - Posteriors and priors are of the same family of distributions
- The model allows for arbitrarily close fitting to the observed incremental and cumulative payments
  - The researcher is able to control the degree of measurement noise
- All anti-log transformations are carried out “draw by draw” (instead of on the expected value)
  - There is no issue with bias adjustment

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Bayesian Model of Loss Development

- State-space models consist of measurement equations and transition equations
  - Measurement equations fit the model to the observations
    - Here, there are two measurement equations, which form a hierarchy (as the second measurement equation employs the predicted values of the first measurement equation)
  - Transition equations describe the trajectories (path in time) of the state variables (which are the variables that adopt a new value [state] in each period)
    - There are three transition equations, one for each of the three mentioned processes

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Bayesian Model of Loss Development

- Measurement equations
  - The (natural) logarithms of the incremental payments are assumed independently and identically normally distributed (first measurement equation)

$$y_{i,j} \sim \mathrm{N}(b_{i,j}, \sigma_y^2) \text{ , } i = 1, \ldots, N, \; j = 1, \ldots, N + 1 - i$$

  - What follows are the predicted values that result from the first measurement equation:

$$\hat{y}_{i,j} \sim \mathrm{N}(b_{i,j}, \sigma_y^2) \text{ , } i, j = 1, \ldots, N$$

    - These predicted values feed into the second measurement equation

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Bayesian Model of Loss Development

- Measurement equations, *cont'd*
  - The series of the cumulative sum of errors between the *predicted* log incremental payments and the actual log incremental payments for any given accident year are multivariate normal (second measurement equation)

$$\hat{m}_{i,j} = \log\left(\sum_{k=1}^{j} \exp(\hat{y}_{i,k})\right), \; i, j = 1, \ldots, N$$

$$\hat{\mathbf{m}}_i' - \mathbf{z}_i' \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\Sigma}), \; i = 1, \ldots, N$$

- This second measurement equation makes use of the "cusum chart technique," which is used in engineering for process control
- As the model develops the incremental payments for a given accident year, the cusum chart technique ensures that the cumulative sum of errors stays close to zero

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Bayesian Model of Loss Development

- Transition equations
  - There are three time processes, that is, three (logarithmic) rates of growth, to be modeled
    - Exposure growth

$$\eta_{i,1} \sim N(\eta_{i-1,1}, \sigma_{\eta}^{2}) \ , \ i = 3, \ldots, N$$

    - Growth in development (“Decay”)

$$\delta_{j} \sim N(\delta_{j-1}, \sigma_{\delta}^{2}) \ , \ j = 3, \ldots, N$$

    - Calendar year effect

$$\kappa_{i+j} \sim N(\kappa_{i+j-1}, \sigma_{\kappa}^{2}) \ , \ i + j = 4, \ldots, N + 1$$

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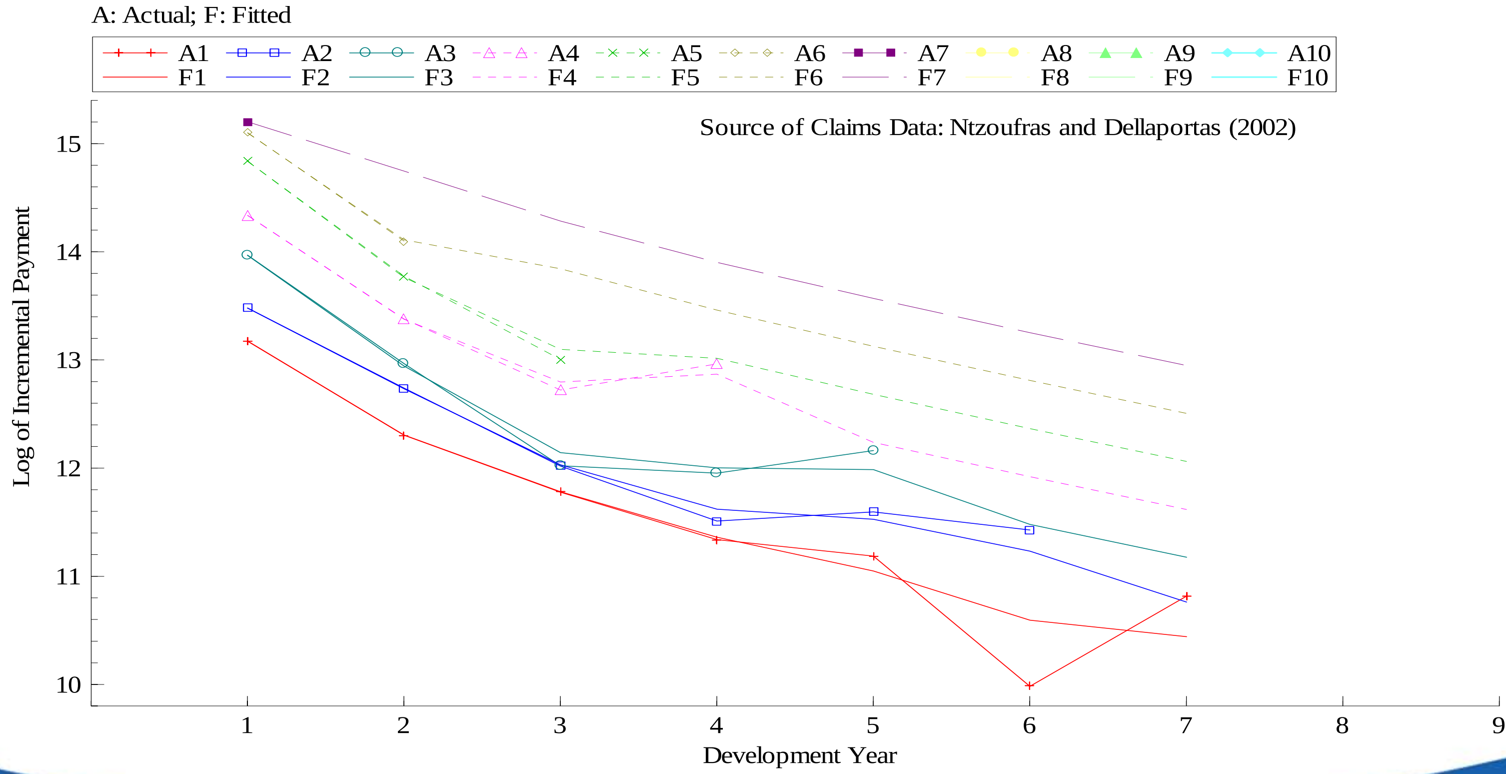
Application (1): Automobile Insurance

- There is in the public domain an automobile insurance claims data set from Greece that has been analyzed repeatedly in professional journals



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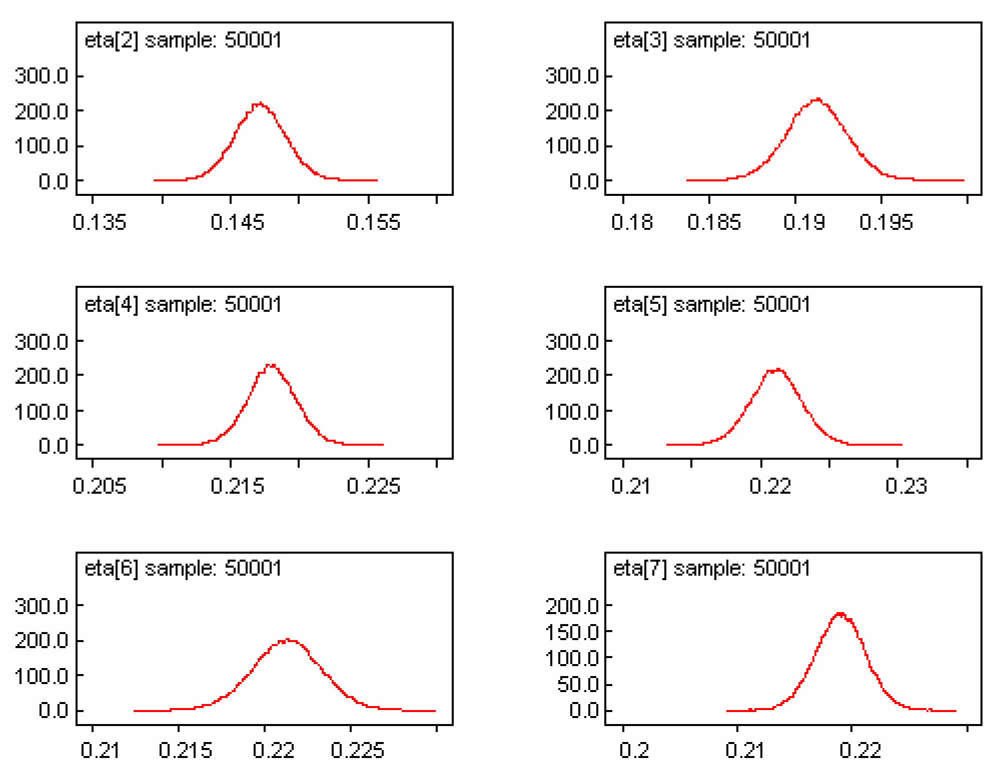

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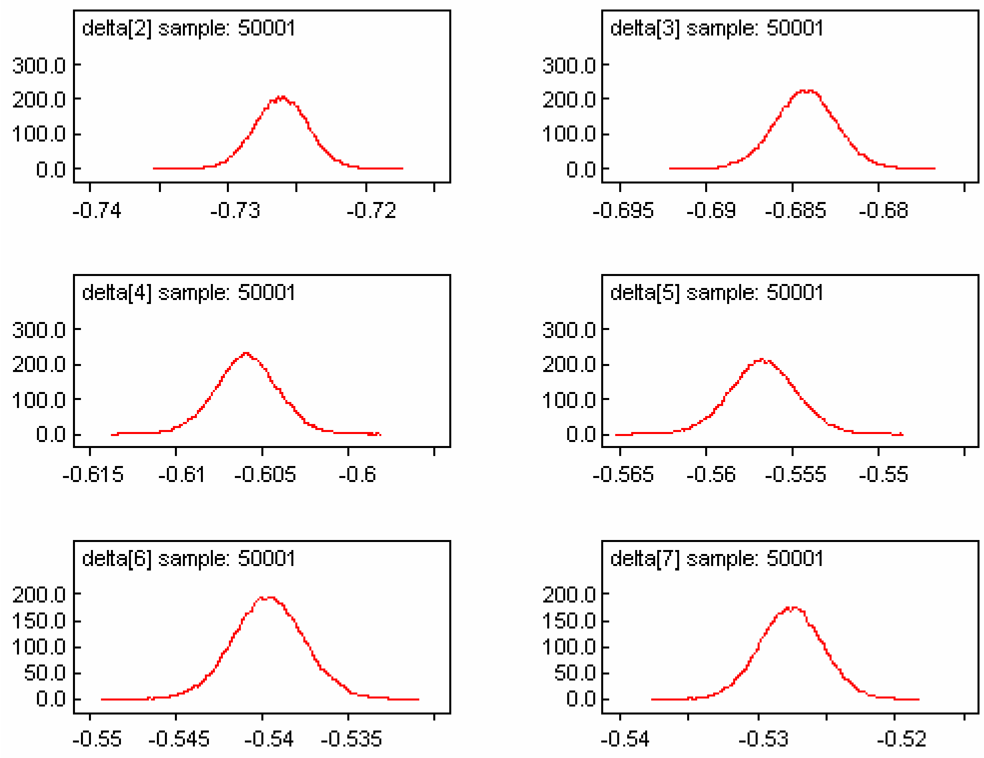

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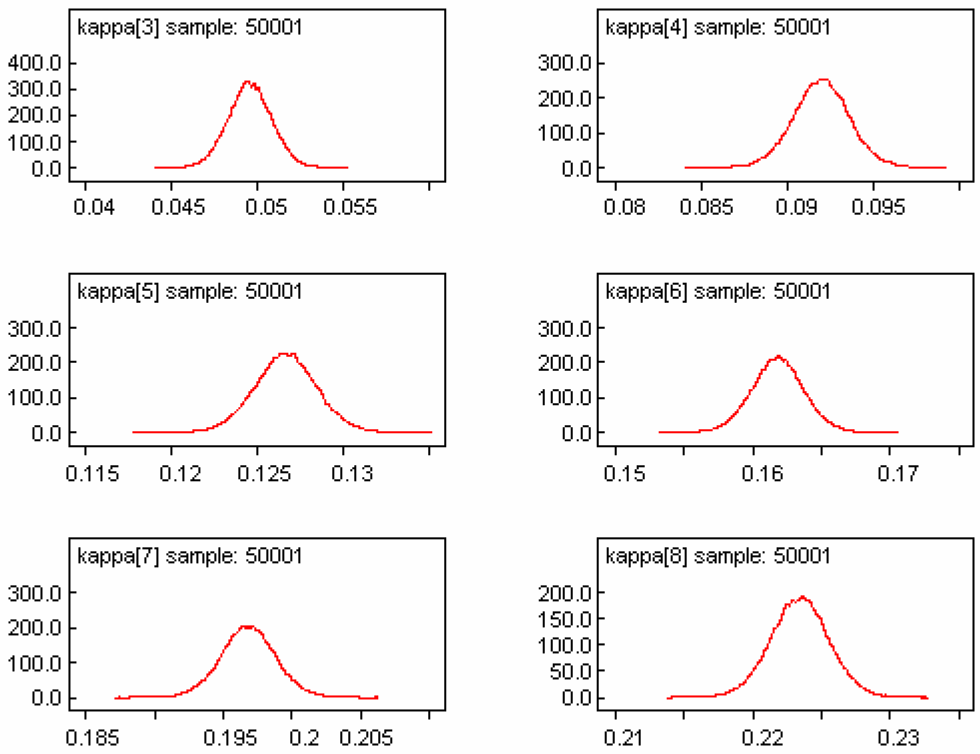
Application (1): Automobile Insurance



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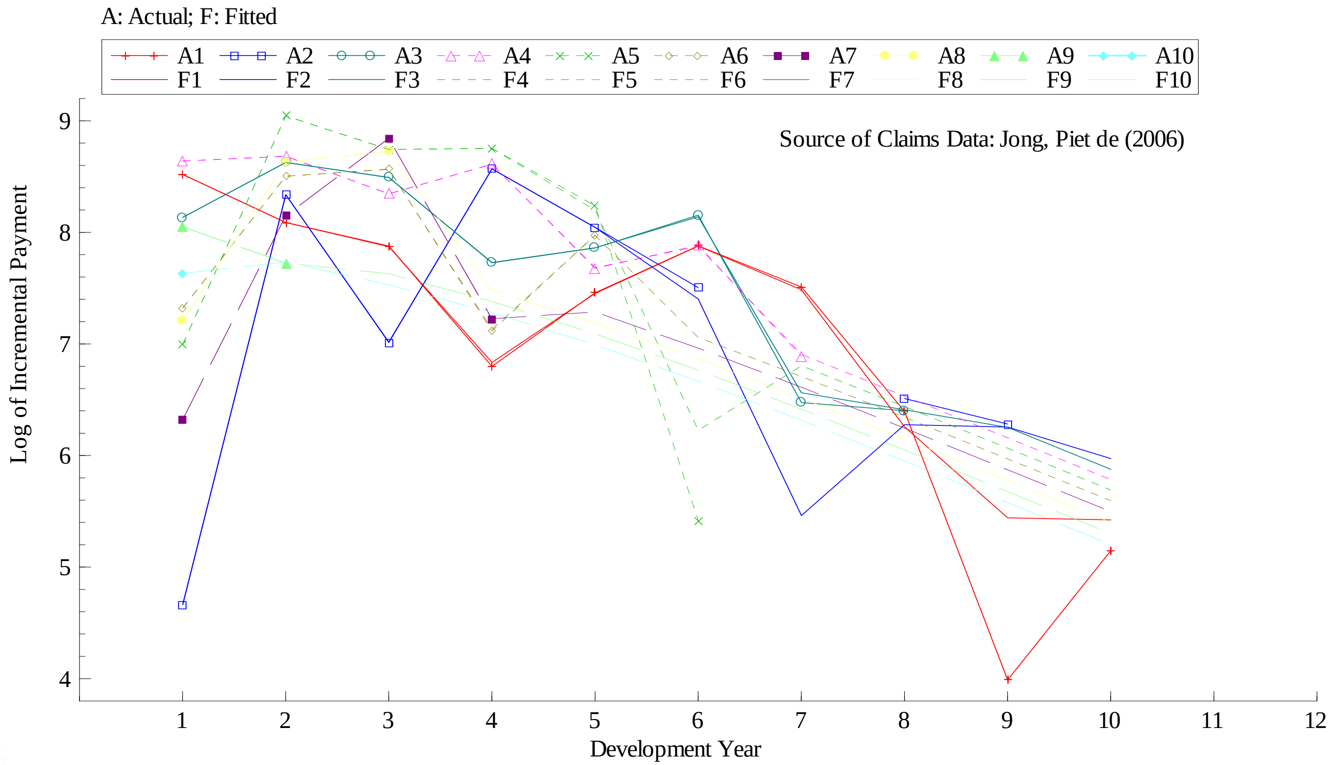
Application (2): The AFG Data Set

- There is in the public domain an Automatic Facultative General Liability (ex asbestos) data set from Australia that has been analyzed repeatedly in professional journals



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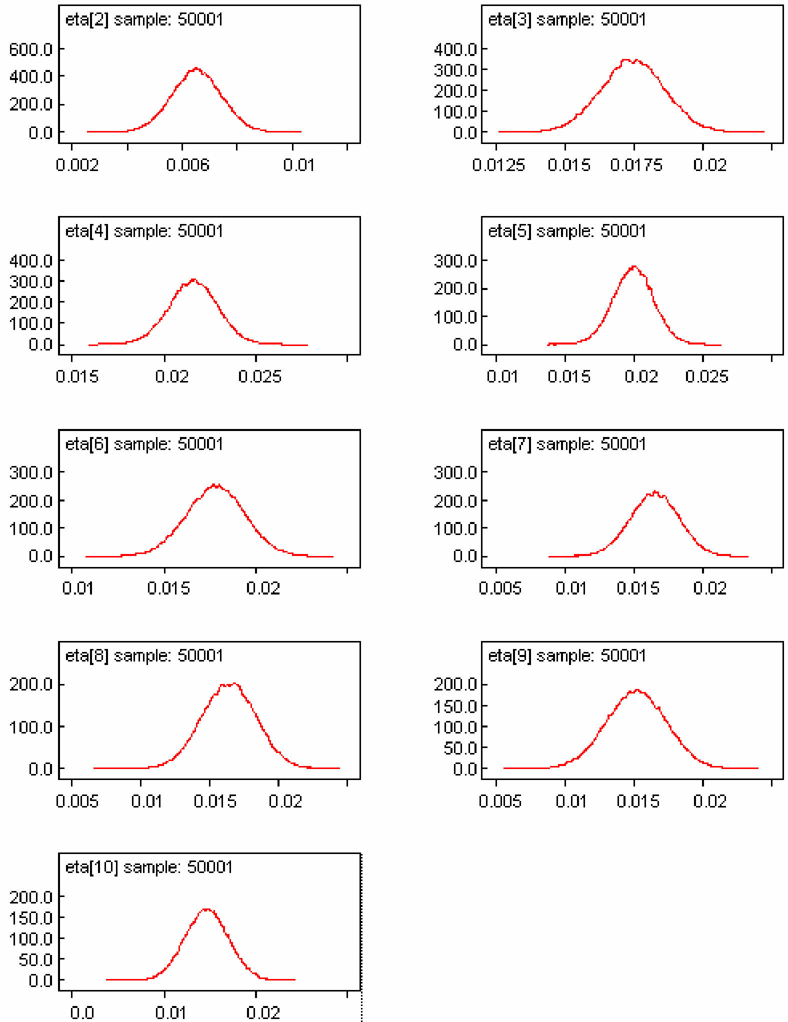

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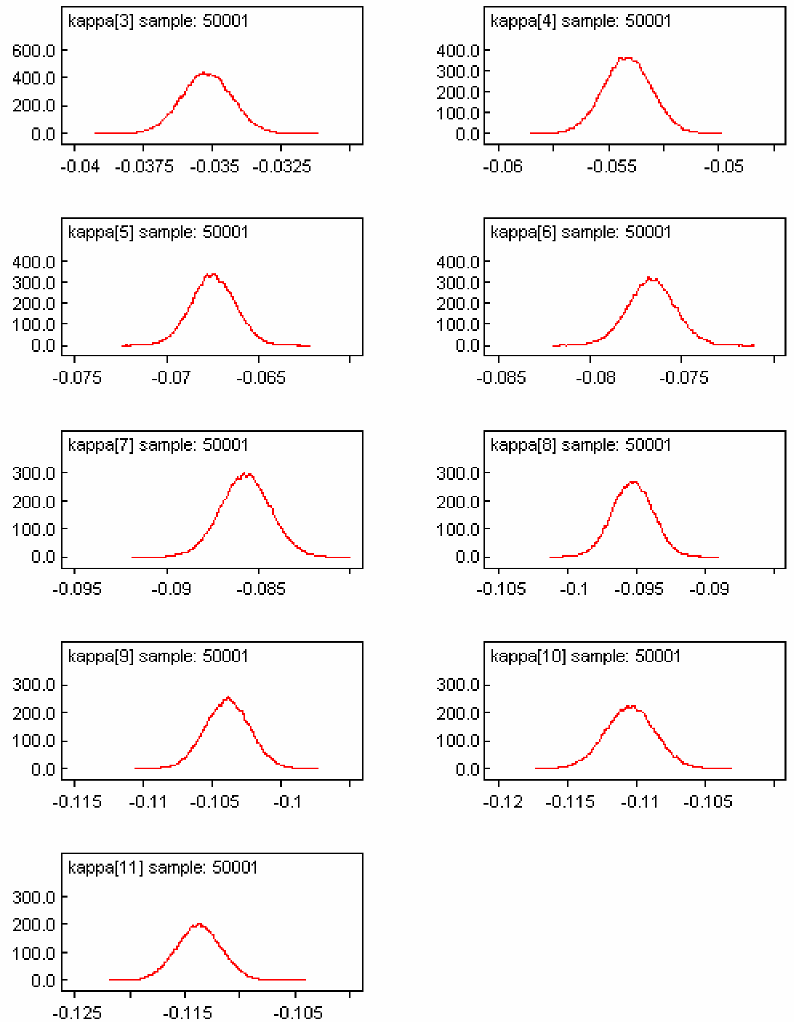

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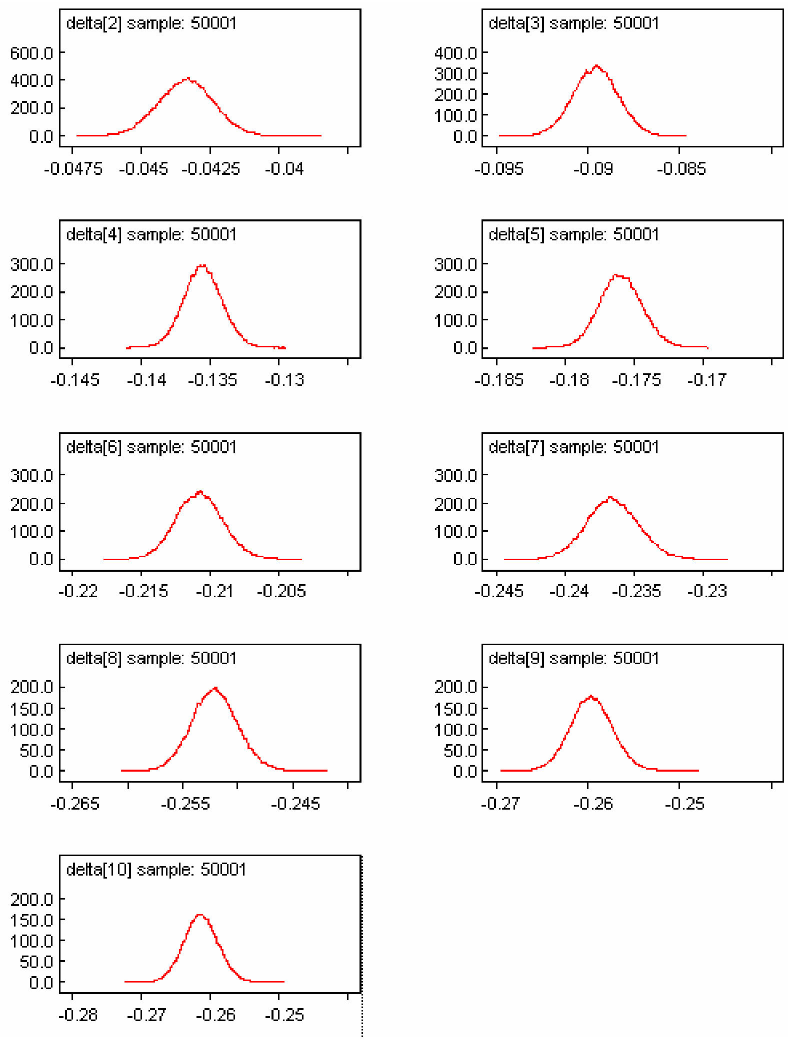
Application (2): The AFG Data Set



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Application (3): Tail Estimation in NCCI Ratemaking (Test Phase)

- For tail estimation, it is critical not to fit too closely to the observed incremental payments but instead allow for an appropriate degree of measurement noise
  - Overfitting shifts measurement noise to the state variables η, δ, and κ, which, via the *final states* of δ and κ, bears on the tail factor
    - The term *final state* refers to the estimated value of the state variable at the end of the time axis of the respective trajectory
    - Note that the final state of η (rate of exposure growth) is irrelevant to the tail factor

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Application (3): Tail Estimation in NCCI Ratemaking (Test Phase)

- Further, for tail estimation, it is necessary to extrapolate the final estimates of κ (calendar effect)
  - In workers' compensation, the calendar effect (the rate of benefits inflation) is in the neighborhood of the respective official rate of inflation
    - For indemnity, this rate of inflation is the average weekly wage
    - For medical, this rate of inflation is the Medical Care component of the CPI (Consumer Price Index)
  - There are also one-off effects in κ, as caused by regulatory reforms
  - To the extent that there are no one-off effects, the calendar year effect follows a random walk
    - In this case, the forecast for any future value of κ is the final state

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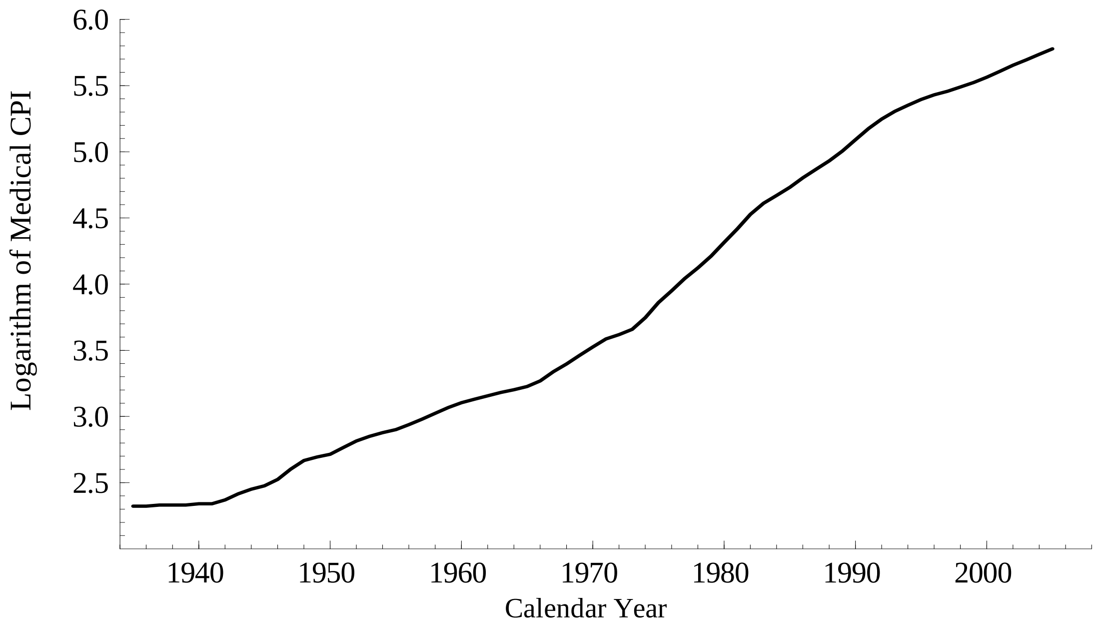
Application (3): Tail Estimation in NCCI Ratemaking (Test Phase)

- To demonstrate the random walk property of the calendar year effect, let us look at the Medical Care component of the CPI (“Medical CPI”)



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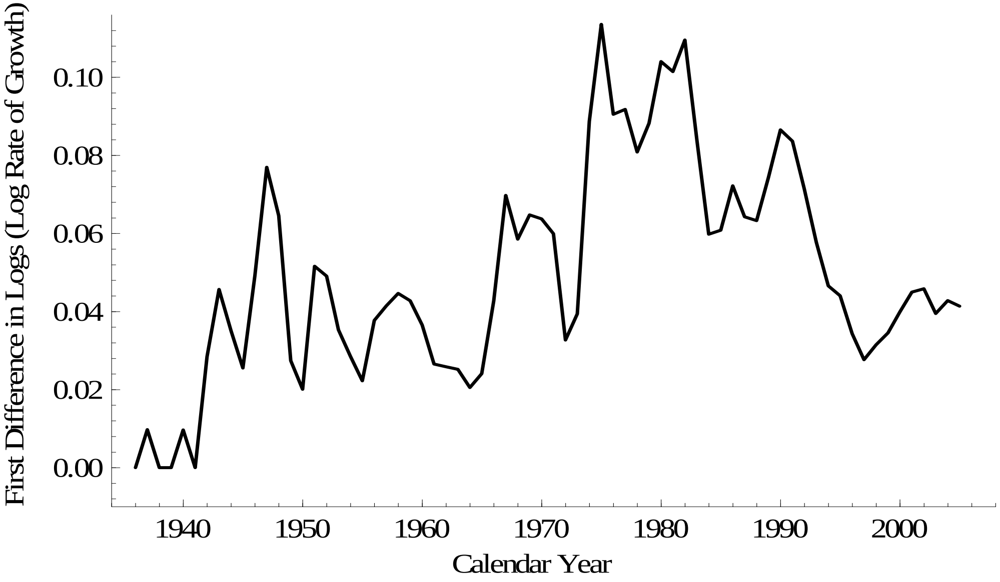
Application (3): Tail Estimation in NCCI Ratemaking (Test Phase)

- Take the first difference of the log Medical CPI
  - This series follows a random walk
    - All innovations (changes to the series) are independent draws from the normal distribution—independence implies that these innovations are permanent



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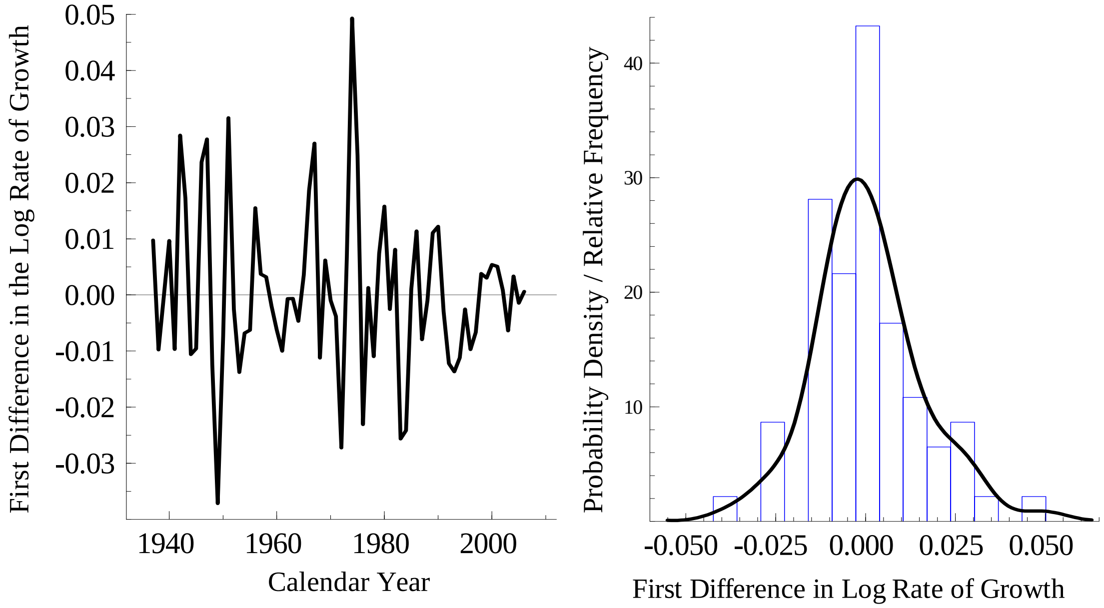
Application (3): Tail Estimation in NCCI Ratemaking (Test Phase)

- Take the first difference of the first difference (the "second difference") of the log Medical CPI
  - This series is stationary (of constant expected value) and normal, and centers on zero (mean: 0.0006)



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Application (3): Tail Estimation in NCCI Ratemaking (Test Phase)

- Further, for tail estimation, it is necessary to extrapolate the final estimates of δ (the rate of "decay" in development)
  - In workers' compensation, this rate of decay will eventually approach the rate of mortality
    - Note that the rate of decay, δ, is inflation-adjusted
    - It is straightforward to build mortality information into the loss development model
- Finally, because in workers' compensation the calendar effect can be expected to be in the neighborhood of the rate of average weekly wage and medical care inflation, respectively, the respective official rate of inflation may substitute for the random walk:

$$\kappa_{i+j} \sim N(\pi_{i+j}, \sigma_{\kappa}^{2}) \text{ , } i+j = 3,\ldots,N+1$$

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Application (3): Tail Estimation in NCCI Ratemaking (Test Phase)

- Medical worker’s compensation triangle (“paid”) of an anonymous state

Log of Incremental Payments																									
1981	16.92204153	15.07053206	13.96677919	13.57142942	13.1230928	13.08377431	13.87833599	13.07633873	12.95884395	13.0110267	12.82617185	12.41011933	12.66028005	11.96131533	13.35224044	13.29519688	12.00528553	12.51535718	11.82634347	11.94346726	#N/A	#N/A	#N/A	#N/A	1
1982	17.02523603	15.26497523	14.10483384	13.72236319	13.68292912	12.86324643	13.4813108	13.55325059	12.99340856	12.50634768	12.84939397	12.60559863	12.67967335	13.28379018	12.49285182	12.62733226	12.63348633	13.38496957	11.23623307	13.14977963	#N/A	#N/A	#N/A	#N/A	2
1983	17.20795485	15.71629997	14.45438413	13.88603804	13.35403742	13.0822287	13.27932952	13.16119418	13.36344799	14.28366593	13.54803278	12.93212975	13.01191206	13.1245543	12.86923321	12.92537368	12.37859792	12.43160677	12.63297214	12.91857336	#N/A	#N/A	#N/A	#N/A	3
1984	17.27968403	15.81866612	14.96259425	14.48617005	14.65374574	14.07784482	13.77602292	13.25510716	14.78480564	13.41280139	13.37725007	13.11100603	12.46889575	12.92183232	13.90289438	13.5389143	13.51939527	13.58211166	13.4558355	13.31231722	#N/A	#N/A	#N/A	#N/A	4
1985	17.41652404	15.94249077	15.03245771	14.39820817	13.45213752	13.54375528	13.29931752	12.71897892	12.71447392	13.38347414	12.88181703	12.32546657	12.66766267	13.43377007	12.55357419	10.70530751	11.78995627	11.97044437	11.81925746	11.93097782	#N/A	#N/A	#N/A	#N/A	5
1986	17.54760142	16.26386365	15.22816514	14.83982163	14.4518569	13.7761287	14.42142902	13.81567926	14.16998601	14.29525494	14.01720979	13.78381578	13.25166458	13.61314085	12.81206062	12.6401883	13.18107063	13.63034354	12.43094487	#N/A	#N/A	#N/A	#N/A	#N/A	6
1987	17.71946715	16.41013383	15.48594573	14.77967065	14.35582102	13.8136983	13.65621374	13.66519109	13.23668921	13.42632568	13.37651644	13.45459582	12.71390716	12.90505311	12.29900277	12.22737449	12.05438693	12.74412858	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	7
1988	17.8061269	16.46944031	15.61616457	14.99288748	14.17452278	13.33829926	13.17194594	13.44094604	12.96478194	12.24394898	12.49814535	10.40548451	12.95359748	12.71997913	12.66870823	13.07006546	12.71477874	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	8
1989	17.91284601	16.69425183	15.60859128	14.67195852	14.11479479	14.960515	14.35444421	13.42052313	13.79557257	12.59869161	13.04598344	13.20149233	12.24069932	12.61136838	12.46989479	12.08283705	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	9
1990	17.93001002	16.55858801	15.70888634	14.75692979	14.31376708	13.97814877	13.57322552	13.19107538	13.00852942	12.9852802	12.99812508	13.0801155	13.25534226	13.19857072	12.94206095	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	10
1991	17.76403913	16.28498973	15.23844744	15.06654098	14.31604207	13.58295884	13.42727985	12.43236698	12.54741097	12.72163476	13.16964601	13.30393276	11.06357994	12.51538252	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	11
1992	17.44722255	16.0936053	15.24873877	14.58951266	13.61817273	13.52548654	13.71770713	13.14032303	12.11581555	12.19764809	12.24609827	11.76010311	12.0823336	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	12
1993	17.13546571	15.72781177	14.73392818	13.88559494	13.99303429	13.53732137	13.17464555	12.30696769	12.57884056	11.78091378	10.86884003	10.36517608	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	13
1994	17.24023757	15.86957198	14.66116561	14.15479499	13.38068471	13.07940828	13.53687309	13.18129959	13.06316351	13.03393414	13.05445373	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	14
1995	17.1799325	15.88695849	14.91373161	14.01069805	14.1064252	13.73741947	12.81766462	13.27388135	13.66845693	13.41672075	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	15
1996	17.44426709	16.1351616	15.24197311	14.14400582	14.21863818	13.80852084	12.86282191	13.10605504	12.98239755	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	16
1997	17.26994057	15.937254	15.05414775	13.81842988	13.65511334	12.88822779	13.03768688	12.74579542	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	17
1998	17.31198288	16.17968292	15.32213599	14.48957439	14.21933885	13.99916075	14.14606766	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	18
1999	17.39038132	16.2175486	15.37916488	14.79984574	14.32148195	14.11317409	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	19
2000	17.23431725	16.40449498	15.41949243	14.58976198	14.05320732	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	20
2001	17.33297107	16.21723793	15.3436098	14.99969045	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	21
2002	17.23992958	16.13654707	15.30252575	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	22
2003	17.37395194	16.39848313	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	23
2004	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	#N/A	24
Payment No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	

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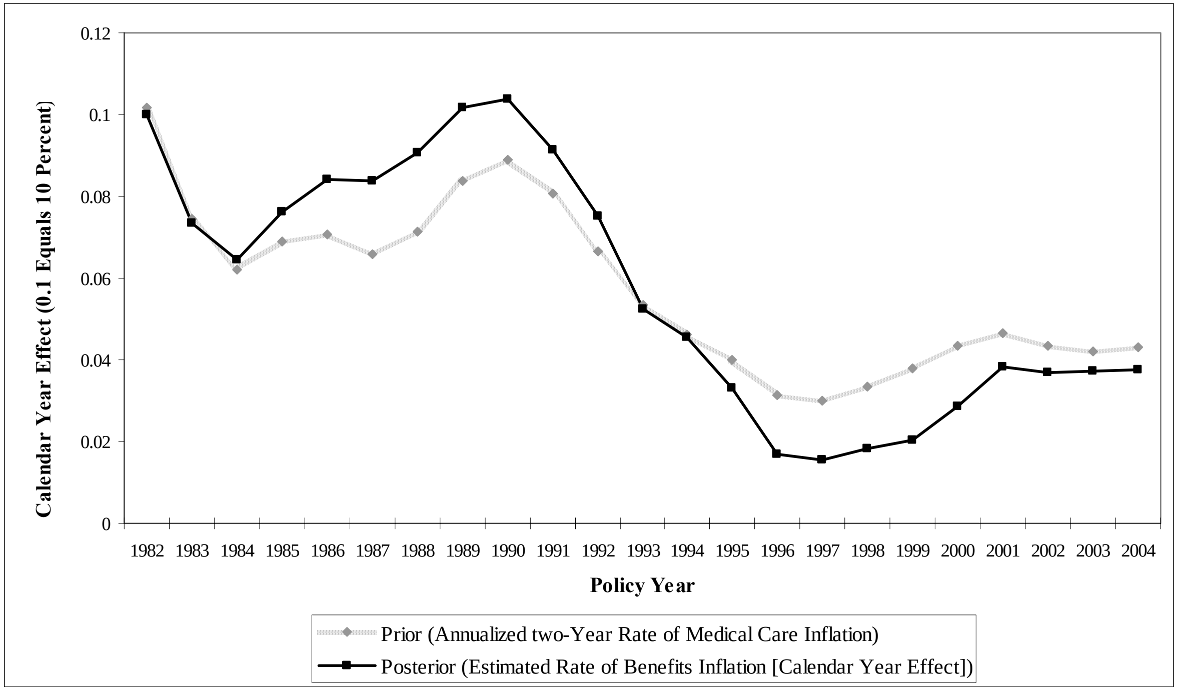
Application (3): Tail Estimation in NCCI Ratemaking (Test Phase)

- The calendar year effect for medical of the given anonymous state
  - The gray line indicates the prior, which is the official, published rate of Medical CPI



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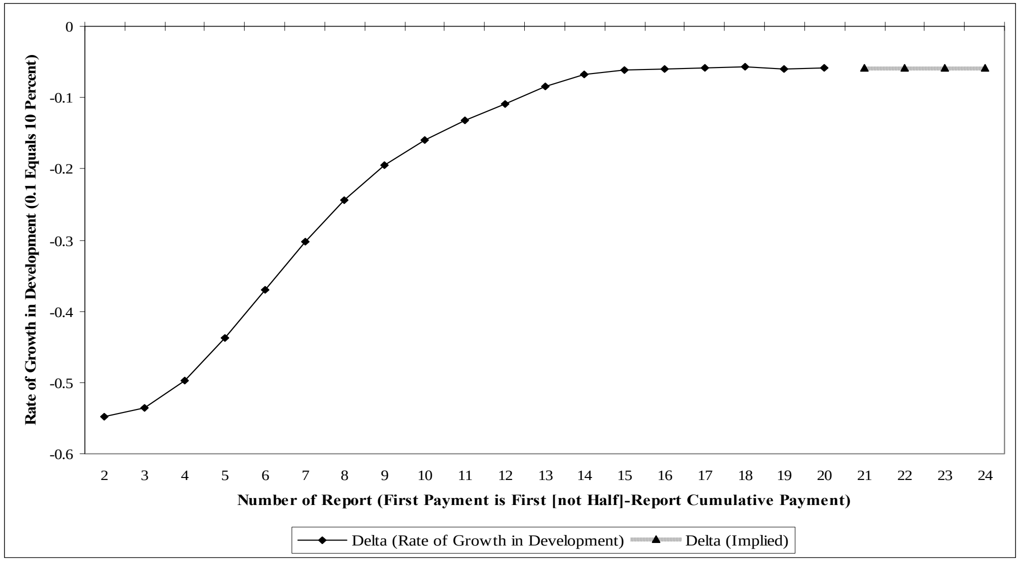
Application (3): Tail Estimation in NCCI Ratemaking (Test Phase)

- The decay in development for medical of the given anonymous state
  - Shown are only the development years for which data are available
    - Mortality information factors in beyond the data points shown here



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Conclusion

- At NCCI, there is available for research and ratemaking purposes a state-of-the-art statistical model of loss development
  - The model makes use of recent statistical advances such as state-space modeling and the Metropolis-Hastings algorithm
- The loss development model meets tomorrow's standards of actuarial practice
  - The exposure draft "Property/Casualty Unpaid Claim and Claim Adjustment Expense Estimates" by the Actuarial Standards Board from February 2006 emphasizes the probabilistic nature of stated developed losses
    - Because the model is Bayesian, it offers a complete characterization of the underlying probability distribution

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