Problems in Current Financial Theory

(and how to solve them)

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Casualty Actuaries of the Southeast

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Outline

- Three Problems, One Mistake
  - Risk-Adjusted Discounting
  - Capital Allocation
  - Market “Realism”
- The Mistake
  - Uncritical acceptance of banking/accounting notions
  - Confusing risk with time value
  - Insurance is betting, not banking
  - The “Market” is not suprapersonal; hysteron-proton fallacy
- Stochastic Cash-Flow (SCF) Theory
  - Present Value as a factor, not as a rate of return
  - Present Value as a random variable
  - Utility Theory: how much θ should one buy at price q?
  - A market solves the q at which net buying is zero.

Risk-Adjusted Discounting

- A standard RAD statement

  "*To calculate present value, we discount expected payoffs by the rate of return offered by equivalent investment alternatives in the capital market. The rate of return is often referred to as the discount rate, hurdle rate, or opportunity cost of capital.*" [Brealey and Myers, 2002, p. 15]

- Statement is nonsensical. MBAs (and accountants, lawyers, politicians, etc) are not philosophers (but they should be).
  - Deterministic: only need "expected" payoffs
  - What is "rate of return?" Rate of return (% per annum) $\neq$ return
  - What are "equivalent investments?" Stacking the deck for the CAPM beta
  - Begs the question. How do "capital markets" price? Can a market somehow determine a price unless individuals scratch their heads over the problem?

- Present-Value Function
  - How much would I pay now, $v(t,x)$, in order to receive x dollars at time t?
  - Force of interest δ not essential, as if $v(t,x) = x \cdot e^{-\delta t}$
  - Not necessarily scalable, as if $v(t,x) = x \cdot v(t,1)$
  - Individual can accept Treasury pv (passive), or use own function (active).
  - See PV.xls for Treasury pv function as of 31Aug2012.

Properties of a Present-Value Function

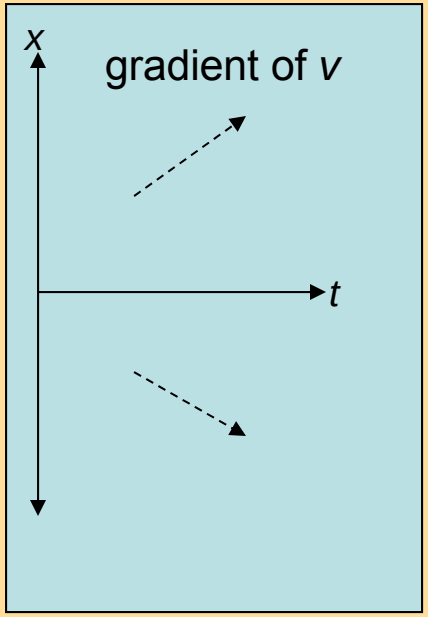
- $v(t,x)$ is a pv function, if it has these four properties for $t \geq 0$, $x \in \Re$:

$$1.\ v(0,x) = x$$

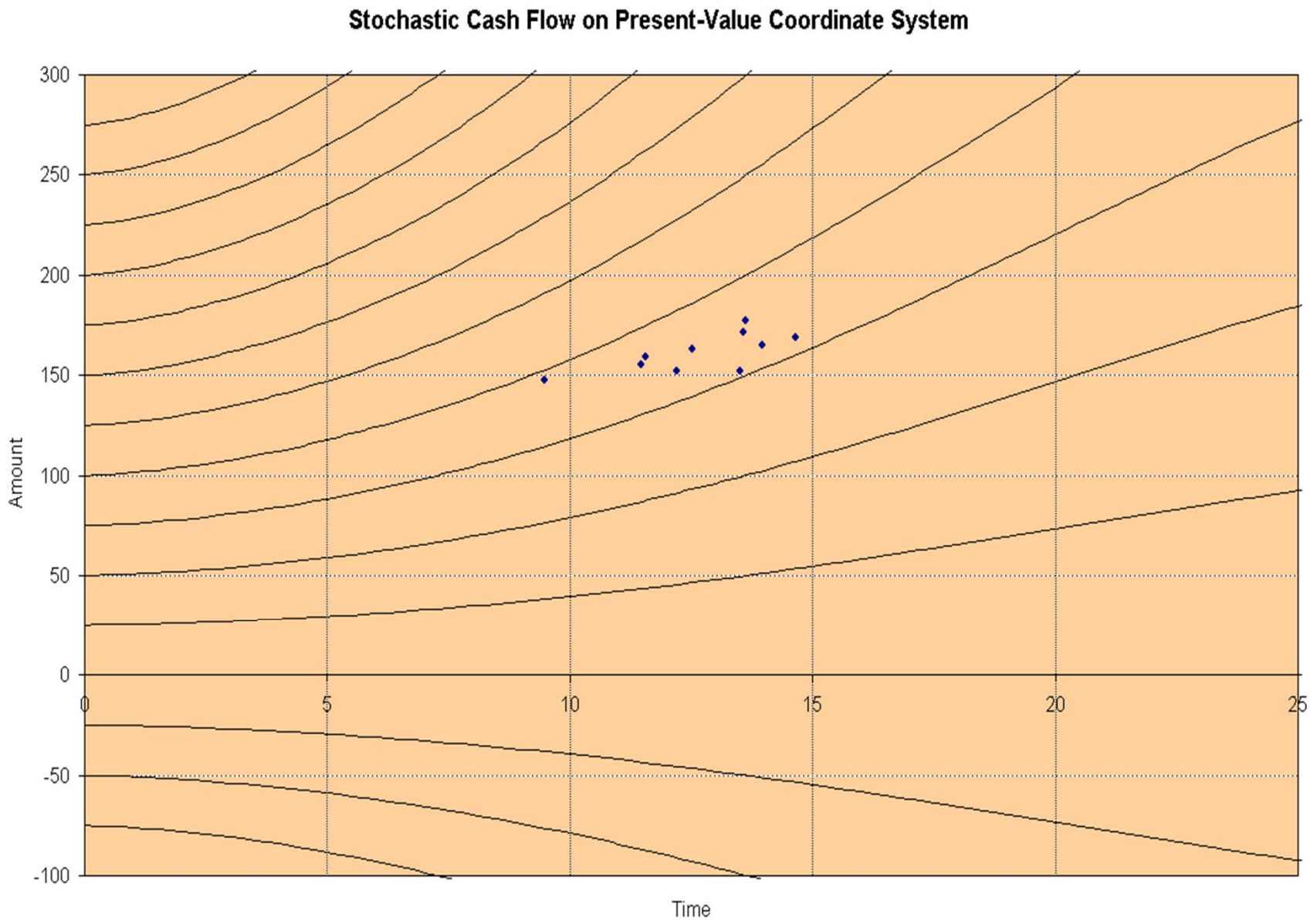
$$2.\ v(t,0) = 0$$

$$3.\ \frac{\partial v}{\partial x} > 0$$

$$4.\ \frac{\partial v}{\partial t} \begin{cases} < 0 & \text{for } x > 0 \\ = 0 & \text{for } x = 0 \\ > 0 & \text{for } x < 0 \end{cases}$$



- Verify that $v(t,x) = x \cdot e^{-\delta t}, \delta > 0,$ is a pv function.


Stochastic Cash Flow on Present-Value Coordinate System
300
250
200
150
100
50
0
-50
-100
Amount
0
5
10
15
20
25
Time

Implications of Present-Value Coordinates

- Present value is a random variable. Exhibit 2 (VSCF, 40) graphs the CDF of the PV of this stochastic cash flow.
- The value of this stochastic cash flow should lie between 75 and 100. If all points lay on the same isobar, the value would be that of the isobar. Risk-adjusted discounting can break out of the envelope.
  - Objection: What if the coordinate system changes? False dynamism
- If two PV random variables are (almost surely) equal, then they must have the same value. In symbols:

$$\text{Prob}\{PV[X] = PV[Y]\} = 1 \quad \Rightarrow \quad Value[X] = Value[Y]$$

- PVs of outcomes are like sufficient statistics. Negative PVs as legitimate as positive; solvency and bankruptcy are constraints, not criteria (VSCF, footnotes 25 and 26).

How to Nullify any Risk-Adjusted Discount

- Bifurcate the cash flow by time: $CF(t) = CF_1(t) + CF_2(t)$
  $CF_1(t) = E[CF(t)]$, the expected amount by time.
  $CF_2(t) = CF(t) - CF_1(t)$, the residual.

- $CF_1(t)$ is certain; hence to be discounted at a "risk-free" rate.
  $CF_2(t)$ is stochastic; but $E[CF_2(t)] = E[CF(t) - CF_1(t)] = 0$.

- Discounting the "expected payoff" of CF_2 at *any* discount rate gives zero. Go figure, Brealey-Myers!

- In the search for a "natural" decomposition MBAs (CROs and ERM-ers) have augmented cash flows with allocated capital: $CF^*(t) = CF(t) + AC(t)$. Two bad ideas don't make a good one.

Just Say “No!” to Capital Allocation

- Capital (or money) does not work, despite advertising slogans.
- Would capital allocated to a one-year FL hurricane treaty work off season (Dec-May)? Can we make capital moonlight? Some try to have it both ways.
- Capital allocation inevitably confuses rate of return (% per year) with return (%).
- Example of short-lived exposure:

  25% per year is 0.5% per week

RAD & CapAlloc at a Gambling Casino

- A gambler usually says, “I won (lost) $x at the casino.”
  Because (within limits) bets are scalable, he might say, “I made (lost) x% on my money at the casino.

  The time it took is irrelevant. No one says, “I made a rate of return of x% per hour (day) at the casino.” Why not, if rate of return is essential? Try gambling with “allocated capital.”

- Do casinos allocate capital? Is allocating capital to an insurance policy more meaningful than allocating it to a slot machine?

  ‘0’ and ’00’ in Roulette are wins for the casino. Might a casino CRO argue: “Our stock beta indicates that our investors are demanding a higher rate of return. We should add ‘000’ to the wheel.” Why not?

- Insurance is like betting, not like banking! (But even banks bet, and need a theory for it other than that of RAD and CapAlloc.)

Markets

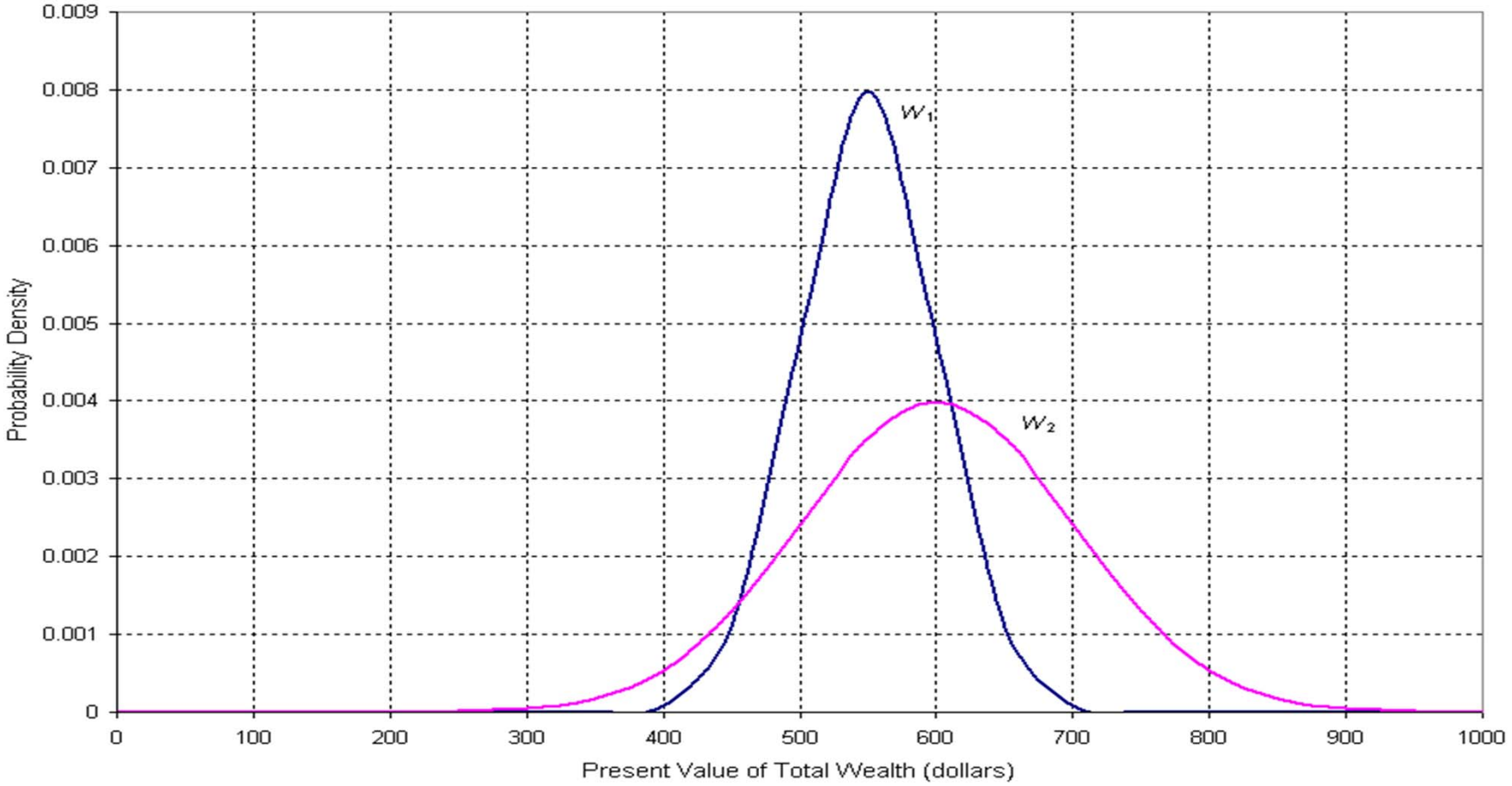
- Philosophy: Are markets universals? Are they <. =, or > than the sum of their participants? Do markets have opinions? Like a Delphi project, are their opinions more reliable than individual opinions?

- Are stock markets efficient? If so, can a stock be over(under)-valued? Can one take advantage of the market? What about being contrarian? Most stock theory confuses fundamental and technical analysis. Much advice is just "Buy low; sell high" in fancy dress.

- What information can a stock market provide a company's management? Stocks are ownership in companies, ownership in the business decisions of their managers. Shareholders evaluate those decisions (Mgmt $\rightarrow$ Shldr). Is there a feedback Shldr $\rightarrow$ Mgmt? Which causes which: faster horse or better odds?

- Why buy stocks? To outfox others? ("Money flows from weak hands to strong.") To reap the profit(loss) of business decisions? Both?

Question: Which Wealth is Better, \$550 ± 50 or \$600 ± 100?



Answer: Which is greater, $E[u(W_1)]$ or $E[u(W_2)]$?

Ordering Projects by Expected Utility

- Current stochastic wealth W
  Project has present value X and cost q.
  Wealth with project is $W+X-q$.
  Compare $E[u(W+X-q)]$ with $E[u(W)]$.

- Borch's insurance formulation, $W-L+p$, equivalent (VSCF, 16)

- Instantaneous formulation Value$=H[X]$ frequent in actuarial literature. (e.g., Bühlmann, Gerber)

  Some are so future-oriented that they must displace instantaneous results to the end of an accounting period (with interest)! Why is a future time better than the present? Is an instantaneous problem illegitimate?

New Perspective:
How much to buy at a given price?

- Comparing $E[u(W+X-q)]$ with $E[u(W)]$ can give you only a ceiling price (for one unit of X), not the appropriate price.
- An economic agent should be free to choose how much of X to purchase at unit price q. Cash flows are scaleable.
- New objective: Maximize $f(\theta)=E[u(W+\theta X-\theta q)]$. State-price form: $q = E[\Psi X]$.
- "Fundamental Theorem" of Appendix B: For a given q, $f(\theta)$ has one and only one maximum. The curve looks like an upside-down parabola. (cf. VSCF, Exhibit 5, p. 43)
- Two or more agents together find the unique price q at which each maximizes its expected utility and all of X clears. This is a Pareto optimum.
- The agents constitute a market, but each agent is entitled to its own beliefs. The market is an epiphenomenon. Don't put the cart before the horse.

Exponential Utility

$$u(x) = \left(1 - e^{-ax}\right)/a, \quad u'(x) = e^{-ax}$$

- Just about the only game in town, as argued by Hans Gerber and in VSCF, Appendix C. Has all the desirable properties, including absolute risk aversion (ARA)
- Some argue for power-curve utility and relative risk aversion (RRA).
  - RRA is appropriate for bundles of physical goods, e.g, apples and oranges, which come in non-negative amounts.
  - With SCFs we are dealing with one unit (dollars) in random outcomes that can be positive or negative.
  - Only exponential utility is defined for all real numbers.
  - Only exponential utility allows an *X* independent of *W* to be valued by itself. Otherwise, one might have to know everything in order to value anything.

Notes on Excel Examples

- Ex 1: Normal approximations (analytic solution) are very good.

  Overall $a = 3.33E{-}06$ is harmonic sum of individuals. Market persona exists because everyone has the same risk assessment, but market derives from individuals

  Interpretation of a "zero-β" stochastic cash flow

- Ex 2: Counter intuitively, all risk is insured for E[*X*].

  Insurer disagrees over lawsuit probability; yet Pareto optimum achieved.

  Try the example with risk-neutral insurer.

  Is the pooling theory of insurance valid?

- Ex 3: Insurer will pay any price at which reinsurers will sign for 100%.

  Reins A and B agree on E[*X*]; but B has correlated exposure. Rein C is pessimistic about the risk, estimating twice the pure premium.

  Part A: A and B rashly assume 100% for $5.1 million; C on sidelines at zero.

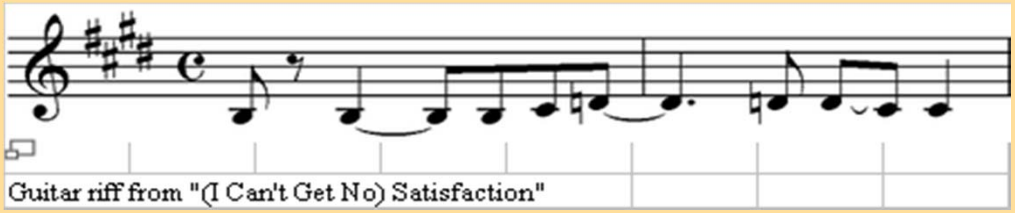
  Part B: C allowed a short position, and 100% clears for $6.1 million. Market stabilizes; everyone happy except for the insurer.

Quotable Quotes

- [This theory] sets the agents to the virtuous task of extracting value from projects, rather than from one another. ("Valuation of Stochastic Cash Flows," 2)
- Risk-adjusted discounting has misled many to elevate [solvency] from the status of a constraint to that of a valuation method. (VSCF, 31)
- One should make sound economic decisions and let the accounting chips fall where they may. (33, footnote 18)
- The business of insurance should be to underwrite well, not to underwrite to generate funds to invest well. … In companies that understand this theory and the near idealness of its application to insurance chief actuaries will be kings. (34)
- "Asking a valuation formula to depend on [wealth level] is like asking a shopkeeper to charge lower prices to the poor than to the rich." (63)

- To him whose only tool is a hammer everything looks like a nail. (anonymous)
- If I have seen farther, it is by standing on the shoulders of giants. (Isaac Newton)

Mick Jagged and the Roiling Scones



I can’t get no satisfaction;
I can’t get no client action.
And I tried (4x) …
I can’t get no!

When I read *Contingencies*,
And a CRO presumes to tell me,
How to allocate equity –
Well he can’t be for real,
‘Cause he doesn’t know
Diddly-squat about NPV!
I can’t get no … oh no no no!
Hey, hey, hey; that’s what I say!

Bibliography

- Borch, Karl H., "The Utility Concept Applied to the Theory of Insurance," *ASTIN Bulletin*, 1(5), 1961, 245-255.
- Brealey, Richard A., and Myers, Stewart, *Principles of Corporate Finance*, Seventh Edition, New York, McGraw-Hill, 2002.
- Bühlmann, Hans, "An Economic Premium Principle," *ASTIN Bulletin*, 11(1), 1980, 52-60.
- Debreu, Gerard, *Theory of Value: An Axiomatic Analysis of Economic Equilibrium*, New Haven, Yale, 1987.
- Gerber, Hans U., *An Introduction to Mathematical Risk Theory*, Philadelphia, S. S. Huebner Foundation, 1979.
- Gerber, Hans U., and Pafumi, Gérard, "Utility Functions: From Risk Theory To Finance," *North American Actuarial Journal*, Volume 2 (July 1998), 74-100.
- Halliwell, Leigh J. "The Valuation of Stochastic Cash Flows," *CAS Forum* (Reinsurance Call Papers, Spring 2003), 1-68.
  - "Valuing Stochastic Cash Flows: A Thought Experiment," *CAS Forum* (Winter 2004), 291-294.
  - "The Conditional Validity of Risk-Adjusted Discounting," *CAS E-Forum* (Spring 2011).
- Panjer, Harry H., editor, *Financial Economics: with Applications to Investments, Insurance and Pensions*, Schaumburg, IL, Actuarial Foundation, 1998.
- Robichek, Alexander A., and Myers, Stewart C., "Conceptual Problems in the Use of Risk-Adjusted Discount Rates," *Journal of Finance*, Vol. 21, No. 4 (Dec 1966), 727-730.