

Intermediate GLMs

Central States Actuarial Forum

Emily Stoll, ACAS, MAAA Consultant EMB America

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PURPOSE: To discuss modeling strategy for building appropriate GLMs

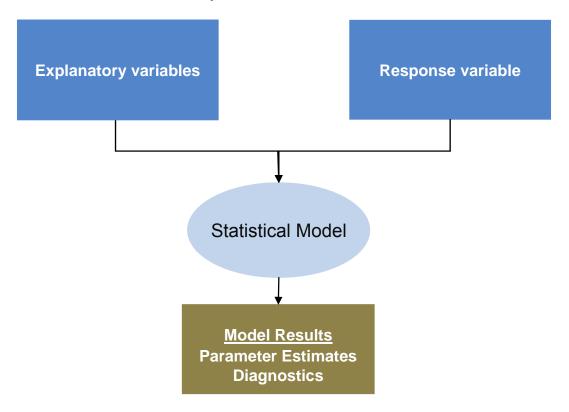
OUTLINE

- Background of GLMs
- What response variable should I use for modeling claims costs?
- What is my goal when iterating models?
- How do I know if my models are good?
- How should I combine component models and how should I incorporate constraints?
- Summary



Purpose of Predictive Modeling

To statistically measure the effect a series of explanatory variables has on an observed item, or response variable





Background of Generalized Linear Models (GLMs)

Link function Model Structure

$$(g=h^{-1})$$
 Structure

 $Y = h(X\beta + \xi) + \varepsilon$

Y = h(Linear Combination of Factors) + Error

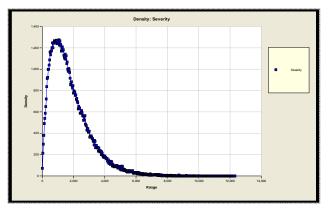
GLM Building Blocks

Error Structure

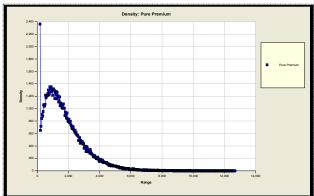


y = h(Linear Combination of Rating Factors) + Error

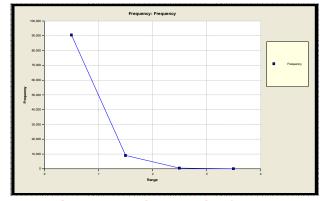
Reflects the variability of the underlying process and can be any distribution within the exponential family, for example:



 Gamma consistent with severity modeling, may want to try Inverse Gaussian



Tweedie consistent with pure premium modeling



Poisson consistent with frequency modeling

GLM Building Blocks

Model Structure



y = h(Linear Combination of Rating Factors) + Error

- Include variables that are predictive, exclude those that are not
- Simplify factors if appropriate
 - Groupings
 - Variates
- Complicate model by adding interactions if appropriate

GLM Building Blocks

Link Function



y = h(Linear Combination of Rating Factors) + Error

▶ Link function (g=h⁻¹) chosen based on how the variables relate to one another to produce the best signal:

➤ Log: variables relate multiplicatively (e.g., risk modeling)

Identity: variables relate additively (e.g., risk modeling)

Log it: retention or risk modelling



Important Modeling Questions



- What response variable should I use when modeling claim costs?
 - Loss ratios or loss costs?
 - Loss costs or frequency and severity components?
 - Aggregated claims data or individual claim types?
- What is my goal when iterating models?
- ➤ How do I know if my models are good?
- How should I combine component models and how should I incorporate constraints?







- Why some companies model loss ratios
 - Difficult to obtain exposures
 - Only want to analyze some rating variables and assume use of loss ratios will adjust for excluded variables
 - Habit
- Theoretical and practical disadvantages to loss ratio modeling
 - On-level calculations
 - No defined error distribution
 - Difficult to distinguish noise from pattern
 - Re-usability

On-Level Calculations

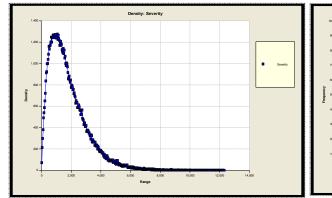


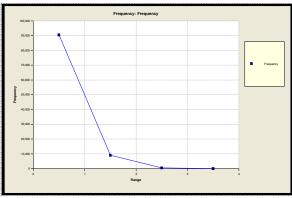
- When modeling loss ratios, premiums need to be put on-level
 - Depending on magnitude of historical changes, not doing so can result in serious under- and over-predictions
- Not sufficient to use an average on-level approach (e.g., parallelogram method) when changes impact classes differently
 - On-level at the granular level (e.g., extension of exposures)
 - Can be time consuming and data may not be available
- Pure premiums use exposures so this is a non-issue

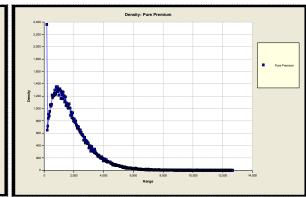
Defined Error Structure



When modeling loss costs, there are generally accepted loss distributions







Gamma considered a standard for severity modeling

Poisson considered a standard for frequency modeling

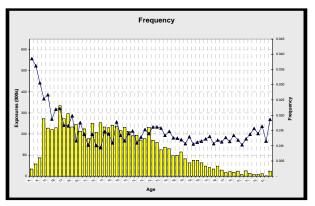
Tweedie considered a standard for raw pure premium modeling

- What is the typical distribution for loss ratios?
 - There is no generally accepted standard
 - ➤ The distribution will vary by company, line, and over time

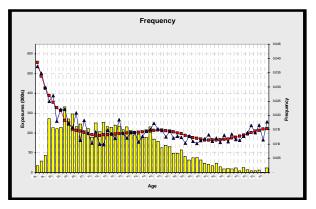
Distinguishing Patterns



When viewing frequency and severity data separately, easy to discern patterns from the noise

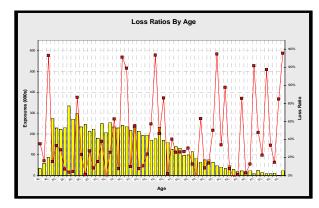


Raw Frequency by Age of Driver



Smoothed Frequency by Age of Driver

> When modeling loss ratios, difficult or impossible to discern pattern from noise



Raw Loss Ratio by Age of Driver





- Loss ratio modeling
 - Imperative that premiums be put on-level for each analysis
 - Rate changes will cause loss ratios and indicated differentials to change
 - Models built in last review will be inappropriate
- Pure premium modeling
 - Not necessary to put premiums on-level
 - Rate changes will not cause loss costs and indicated differentials to change
 - Models built in last review may still be appropriate



Granular or Combined Modeling?

- Some actuaries are tempted to model loss costs or combined coverages/perils, presumably to save time
- As with traditional analysis (e.g., selecting loss trends), preferable to analyze at the granular level

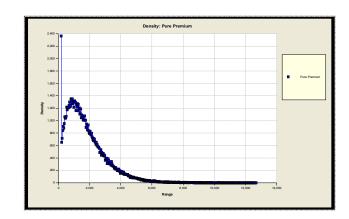
Freq/Severity or Pure Premium	By-Peril or All Perils
Severity trends mask frequency signal	Highly variable perils mask stable perils
Predictors impact frequency and severity differently (e.g., limit)	Predictors affect perils differently (e.g., theft device)
Frequency and severity have defined error structures	Perils have different size of loss distributions
Different frequency and severity trends can mask results	Different loss trends by peril can mask results

If necessary, use the Tweedie distribution for pure premium modeling



Tweedie Distribution

- Incurred losses have a point mass at 0 and then a continuous distribution
- Poisson and gamma not suited to this
- Tweedie distribution has
 - Point mass at 0
 - A parameter that changes the shape > 0

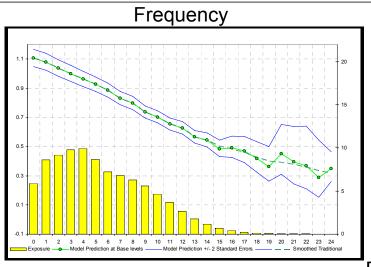


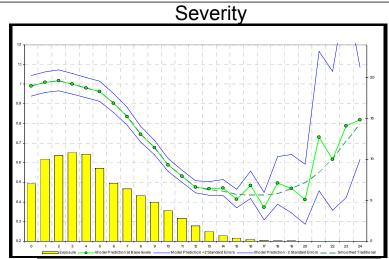
Observed Response	Most Appropriate Error Structure	Variance Function
Claim Frequency	Poisson	μ^1
Claim Severity	Gamma	μ^2
Raw Pure Premium	Tweedie	μ^{T}

Typically, T ≈ 1.5 for incurred losses

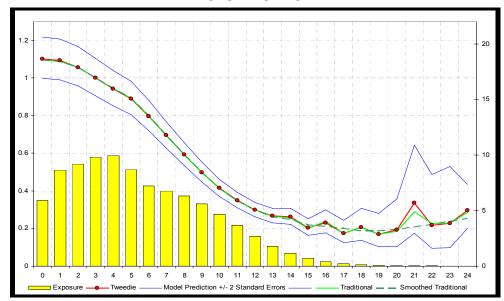


Example 1 – Vehicle Age MSOffice4









MSOffice4 might be too much red on this series of slides

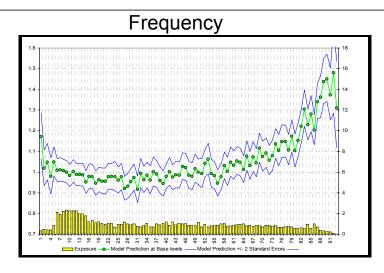
I think the series represents the following examples (tell me if you agree):

- -slide 16 Trad'l vs Tweedie give same result but Trad'l shows you that the blip in age 21 is coming from severity
- -slide 17 not entirely sure other than noise + noise = noise (and easier to wrap your head around and smooth the component results before you add them)
- -slides 18 & 19: freq up (pretty consistently) and sev down (pretty consistently) two offset to zero (but wouldn't have underlying info if hadn't modeled components

Interesting that we don't have one that shows different results b/w Tweedie and Trad'l (I guess the last one does slightly) , 2/16/2009

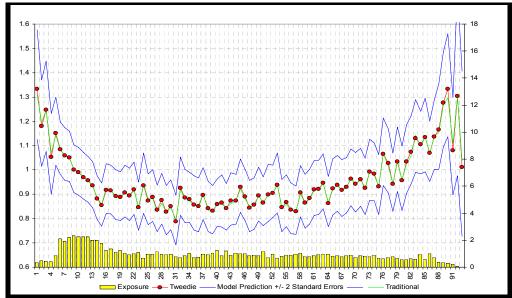


Example 2 – Urban Density



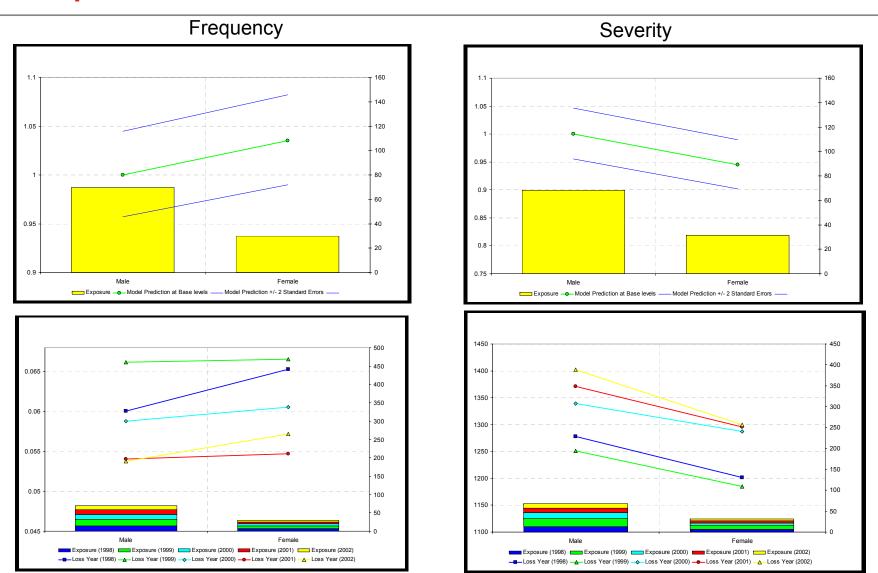
Severity 14 12 10

Pure Premium





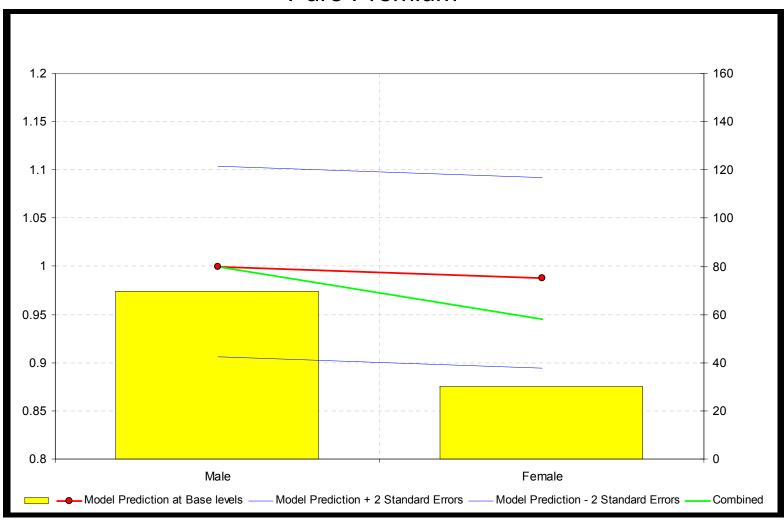
Example 3 – Gender MSOffice6



MSOffice6 confused by why consistency w/ time is being shown here - in the Tweedie vs trad'l section , 2/16/2009



Pure Premium



MSOffice7 can you re-label red to say Tweedie and Green to say Traditional (like other graphs)?

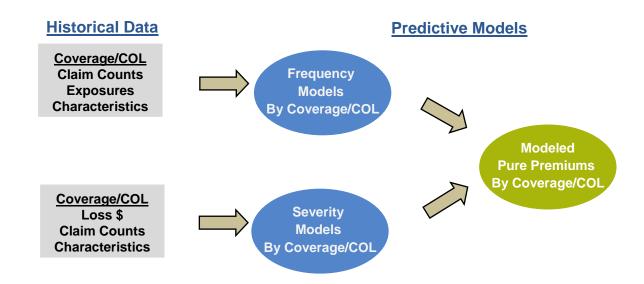
Tweedie GLMs



- Helpful when it's important to fit to incurred costs directly
- Similar results to frequency/severity traditional approach if frequency and loss effects are significant
- Distorted by large parameter estimates with wide standard errors
- Removes understanding of what is driving results
- Smoothing harder



Predictive Modeling Overall Strategy



- Build frequency and severity models by coverage/cause of loss
 - Or use the Tweedie distribution to model raw pure premium if necessary
- Avoid modeling loss ratios



Important Modeling Questions

- What response variable should I use when modeling claims?
- What is my goal when iterating models?
 - > Find the signal, remove noise
 - > Use all available data
- How do I know if my models are good?
- How should I combine component models and how should I incorporate constraints?







Theoretical

Statistical Modeling

- Find signal using all available information
- Remove the noise from the underlying data

When building initial component models, this is our focus

Practical

Constrained Modeling

- Incorporate real world constraints
- Transform the theoretical results into usable pricing information

Theoretical Modeling

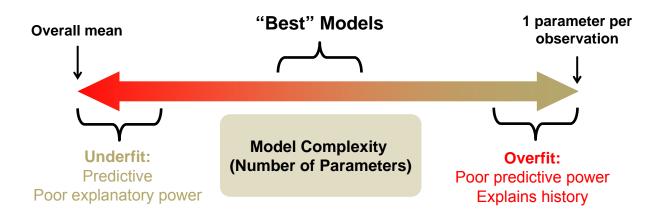
Designing model structure



The goal is to produce a sensible model that explains recent historical experience and is likely to be predictive of future experience

 1. Separate the random components from the systematic components of the estimator

 2. Balance predictive power and explanatory effects

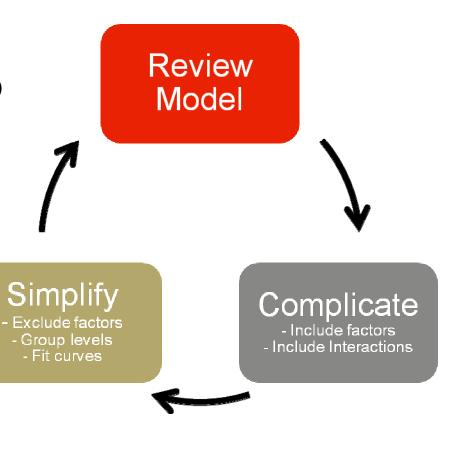


Iterative Modeling



Modeling is an iterative process

- How does the analyst decide the "best" model?
 - Parameters/standard errors
 - ➤ Type III statistical tests (e.g., X² tests)
 - Consistency of patterns over time or random data sets
 - Judgment (e.g., do the patterns make sense)



Include/Exclude Factors



Parameter estimates (PEs) and standard errors (SEs) indicate strength and confidence in estimates

➤ If all PEs are roughly the same and/or have large SEs, the variable may not be predictive

1.15		130%
1.10		- 120%
		- 110% Med
1.05	<u> </u>	- 100%
1.00		90% Mod Penal 281 Ento
		- 80%
0.95		— 70% — Mod Pale State
0.90		- 60%
		- 50%
0.85		- 40%
0.80		30%
		- 20%
0.75		- 10%
0.70		0%

Name	Value	Standard Error	Standard Error (%)	Exp(Value)
Any	0.0174	0.04183	240.8	1.0175
Any>25	0.0212	0.04349	205.4	1.0214
Named >50	-0.0961	0.08120	84.5	0.9084
Named 25-50	0.0357	0.02194	61.4	1.0364
Insured Only				
Insured & Spouse	0.0255	0.01272	49.8	1.0259
Named <25	-0.0446	0.02663	59.7	0.9564

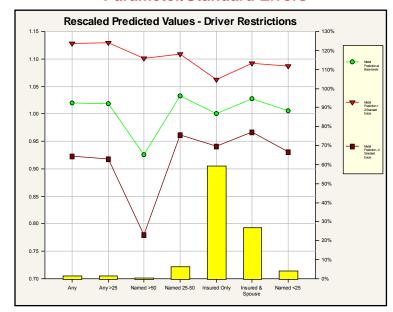
Graph of PEs and SEs and "horizontal line test" identifies importance of a variable

Include/Exclude Factors



Examine consistency over time or over random subsets

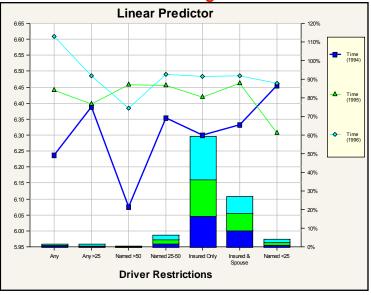
Parameter/Standard Errors



➤ By testing the pattern over time can see if the same thing happens each year

Main effects graph may show a questionable pattern

Time Testing



Include/Exclude Factors



- > Statistical tests (e.g., X^2 or F-tests) can be used to determine the significance of a factor
 - ➤ Null hypothesis: models with and without a factor have the same statistical significance (alternative hypothesis suggests more complex model is better)

Chi-Squared

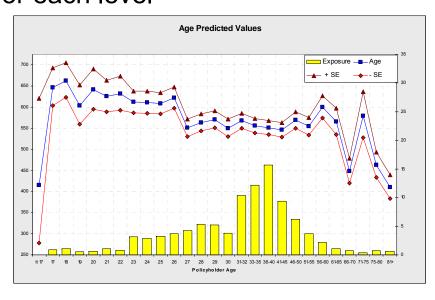
Model 💌	With 💌	Without 💌
Deviance	8,906.4414	8,909.6226
Degrees of Freedom	18,469	18,475
Scale Parameter	0.4822	0.4823
Chi Square Test		78.6%

Test result	H _o	Indicated Model
<5%	Reject	More Complex Model (i.e., include factor)
5%-30%	???	???
>30%	Accept	Simpler Model (i.e, exclude factor)

Group Factor Levels



Parameters/standard errors tell importance of varying estimates for each level



- Group levels with
 - Base level
 - Neighboring classes

- Similar parameters or "plateaus" indicate potential groups
- Look for low volume

Name	Value	Standard Error	Standard Error (%)	Weight	E(Value)
Lt 17	-0.2872	0.40047	139.4	3	0.7504
17	0.1597	0.06488	40.6	162	1.1731
18	0.1838	0.05642	30.7	211	1.2018
19	0.0915	0.07222	78.9	106	1.0958
20	0.1506	0.07009	46.6	111	1.1625
21	0.1254	0.05478	43.7	195	1.1336
22	0.1364	0.05916	43.4	156	1.1462
23	0.1038	0.03476	33.5	587	1.1094
24	0.1022	0.03559	34.8	539	1.1076
25	0.0979	0.03288	33.6	602	1.1029
26	0.1207	0.03098	25.7	700	1.1283
27	-0.0015	0.02947	1,929.7	795	0.9985
28	0.0221	0.02635	119.0	1,004	1.0224
29	0.0345	0.02611	75.7	983	1.0351
30	-0.0021	0.02925	1,396.1	711	0.9979
31-32	0.0291	0.02059	70.8	1,952	1.0295
33-35	0.0079	0.01941	244.6	2,294	1.0080
36-40				2,953	
41-45	-0.0103	0.02110	204.5	1,769	0.9897

Group Factor Levels



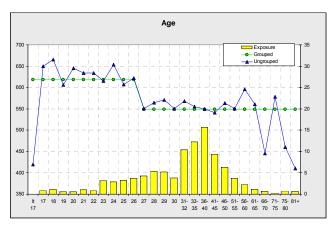
- Standard errors discussed earlier identify levels that should be grouped with the base class
- Standard error of the parameter differences identifies non-base levels that may be grouped

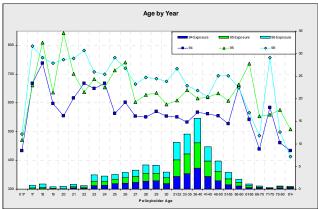
	Lt 17	17	18	19	20	21	22
Lt 17							
17	90.4						
18	85.6	308.9					
19	107.2	132.7	91.2				
20	92.7	995.9	255.1	161.6			
21	97.8	236.1	127.0	254.7	332.7		
22	95.4	362.2	163.9	199.5	620.3	685.0	
23	102.6	124.2	76.9	618.2	158.1	273.1	193.0
24	103.1	122.4	76.6	719.3	154.6	259.0	186.9
25	104.2	112.5	71.7	1,182.8	140.8	217.5	165.4
26	98.4	176.5	96.1	258.8	246.0	1,250.8	399.8
27	140.4	42.3	32.4	80.8	48.0	45.9	45.2
28	129.6	48.8	36.4	106.9	56.1	55.3	53.7
29	124.6	53.7	39.5	130.3	62.0	62.9	60.3
30	140.7	42.4	32.5	80.6	48.0	46.1	45.5
31-32	126.6	50.0	36.8	116.4	58.0	57.3	55.5
33-35	135.7	43.0	32.3	86.7	49.3	46.9	46.3
36-40	139.4	40.6	30.7	78.9	46.6	43.7	43.4

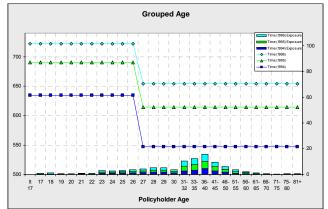
Group Factor Levels



Explore if proposed groupings are consistent over time or random subsets of the data







Consistency without groupings

➤ Consistency with groupings

Group Factor Levels



- Statistical tests (e.g., X² or F-tests) can be used to determine the statistical significance of a re-grouped variable
 - ➤ Null hypothesis is that the original model and model with factor re-grouped have the same statistical significance

Score	H _o	Indicated Model
<5%	Reject	More Complex: Without Grouping
5%-30%	???	???
>30%	Accept	Simpler: With Grouping

Incorporate Variates



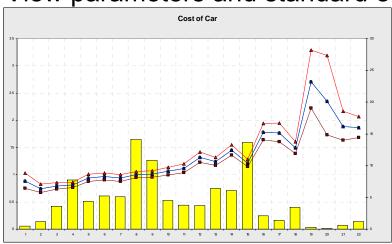
- Curves can be fit to continuous variables, but not discrete (a.k.a. categorical) variables
 - Levels of a continuous variable have a natural, numerical relationship

	Categorical	Continuous
Homeowners	Type of HO Alarm	Amount of Insurance
Auto	Vehicle Usage	Age of Driver
Commercial Lines	Occupation	Revenue
Retention	Gender	Premium change
Geography	Territory	Latitude/longitude

Incorporate Variates



View parameters and standard errors for sensibility of variate



Variates can be very helpful at smoothing out nonsensible results

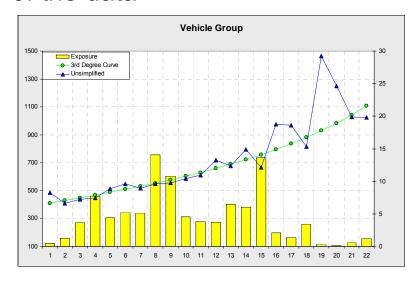
Standard errors of parameter differences can identify smooth progression of parameters

							Vehicle Group			Vehicle Group
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	Group (8)	(9)	(10)
Vehicle Group (1)										
Vehicle Group (2)	52.9									
Vehicle Group (3)	74.8	88.5								
Vehicle Group (4)	93.6	59.0	133.8							
Vehicle Group (5)	123.8	22.4	210	20.6						
Vehicle Group (6)	86.9	19.8	17.5	16.5	123.1					
Vehicle Group (7)	129.3	22.4	20.8	20.0	1,051.2	105.6				
Vehicle Group (8)	61.8	16.5	13.0	10.9	46.2	76.9	411			
Vehicle Group (9)	56.6	16.0	12.8	10.9	39.9	59.0	35.9	170.1		
Vehicle Group (10)	42.4	14.7	122	111	27.6	33.6	25.8	43.3	55.5	
Vehicle Group (11)	34.3	13.2	110	10.0	210	23.9	19.9	26.9	311	76.6
Vehicle Group (12)	20.1	9.4	7.5	6.7	10.7	112	10.2	10.8	116	16.7
Vehicle Group (13)	23.0	9.9	7.5	6.5	114	12.0	10.8	11.3	12.5	20.3
Vehicle Group (14)	15.9	7.7	5.7	4.8	7.5	7.5	7.0	6.7	72	10.2
Vehicle Group (15)	24.3	10.0	7.3	5.9	113	11.8	10.5	10.4	11.7	212

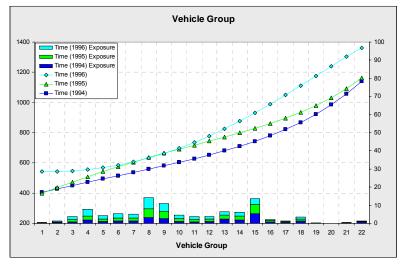
Incorporate Variates



Check consistency of curve over time or random subsets of the data



Check to see the consistency of that curve fit to different parts of the data ➤ After choosing the curve



Incorporate Variates



- ➤ Statistical tests (e.g., e.g., X² or F-tests) can be used to determine the appropriateness of a variate
 - Null hypothesis is that the models with and without the variate are the same

Chi-Squared

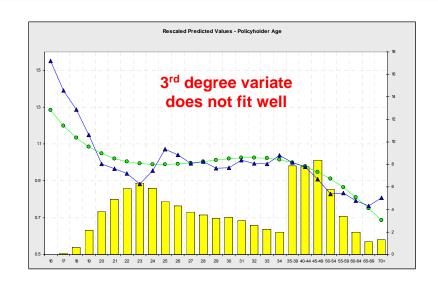
Model	No Curve	Curve
Deviance	8,906.4460	9,020.2270
Degrees of Freedom	18,469	18,487
Scale Parameter	0.4822	0.4879
		·
Chi Square Test		0.0%

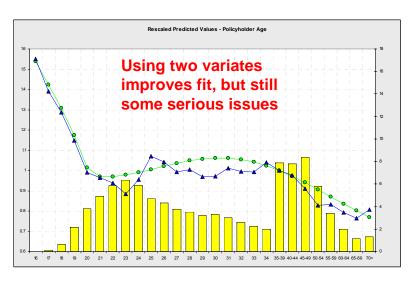
Score	H_0	Indicated Model
<5%	Reject	More Complex: No Curve
5%-30%	???	???
>30%	Accept	Simpler: With Curve

Incorporate Variates



- Variates tend not to perform as well with regards to Type III testing (as compared to groups)
- If variates are not fitting the data well, the modeler can increase the responsiveness
 - Increase the power of the polynomial
 - Create multiple variates
 - Use combination of groupings and variates
 - Fit splines

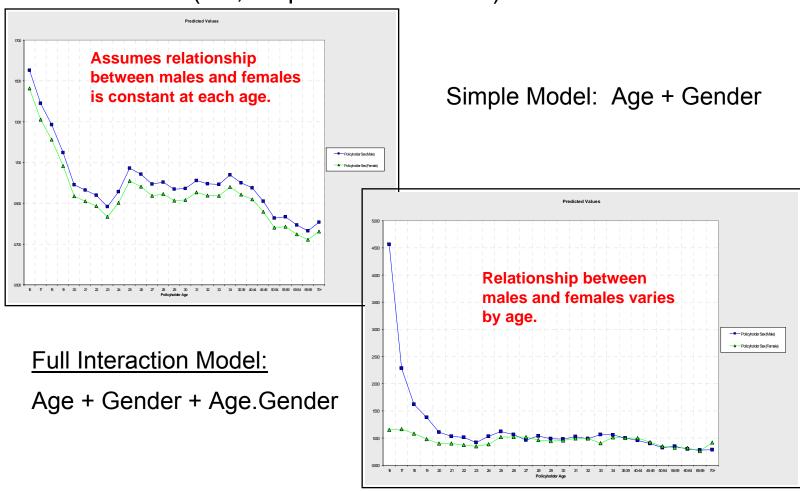




Include Interactions



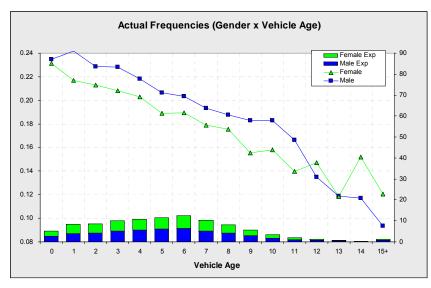
Relationship between levels of one variable may vary by levels of another variable (i.e., response correlation)



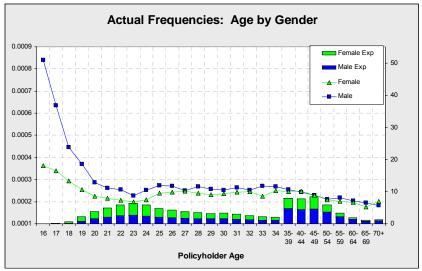
Identify Potential Interactions



Patterns of actual results highlight potential interactions



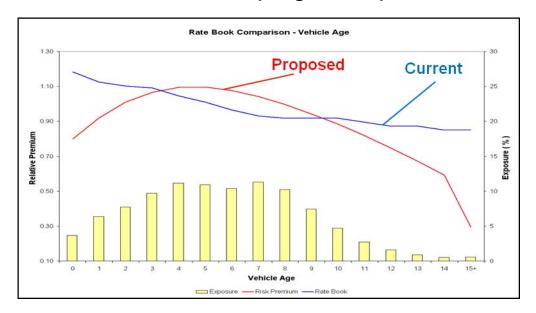
Actual frequencies show relationship between male and female is very different for youth and adults ➤ Actual frequencies support relationship between male and female is basically constant for each vehicle age





Use of Judgment

- The following output shows a comparison of current vs. indicated factors for vehicle age
 - Pattern was not downward sloping as expected

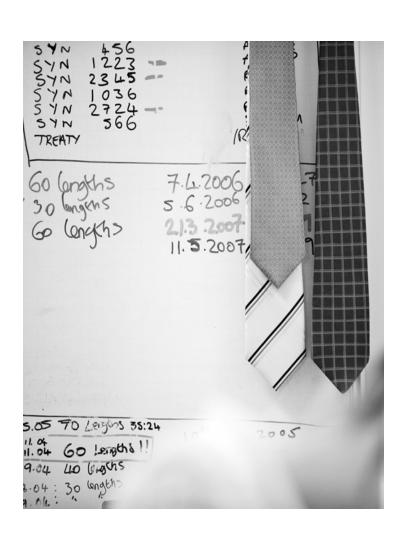


- As modeler used GLMs and understood this was a severity issue, contacted claims to brainstorm potential causes
 - Trend due to claims-leakage for middle age vehicles



Important Modeling Questions

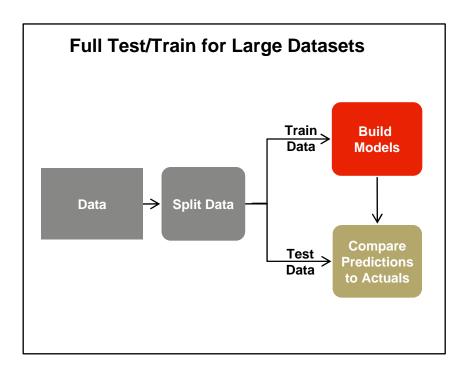
- What response variable should I use when modeling claims?
- What is my goal when iterating models?
- How do I know if my models are good?
 - Model validation
- How should I combine component models and how should I incorporate constraints?

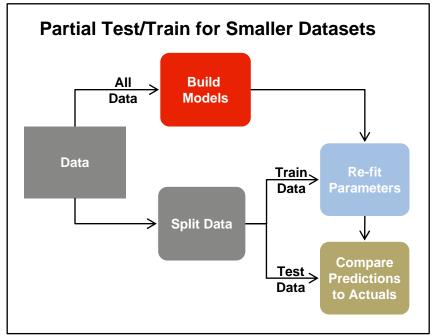


Hold-out Samples



- Hold-out samples are effective at validating model(s)
 - Determine parameter estimates based on part of dataset
 - ➤ Use estimates to predict outcomes on other part of dataset

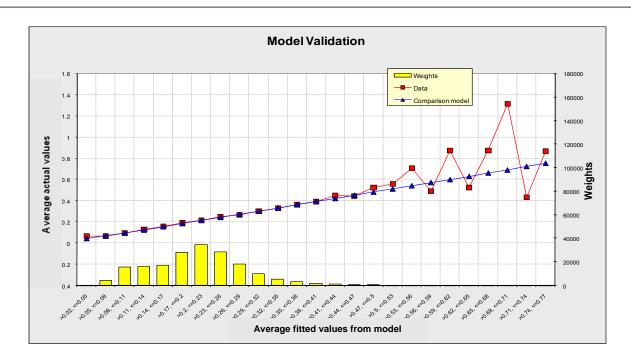




Predictions should be close to actual results for heavily populated cells



Fitted Values Compared to Actual Values – Aggregate



Populate fitted values from model onto a hold-out sample of data and compare these to the actual values

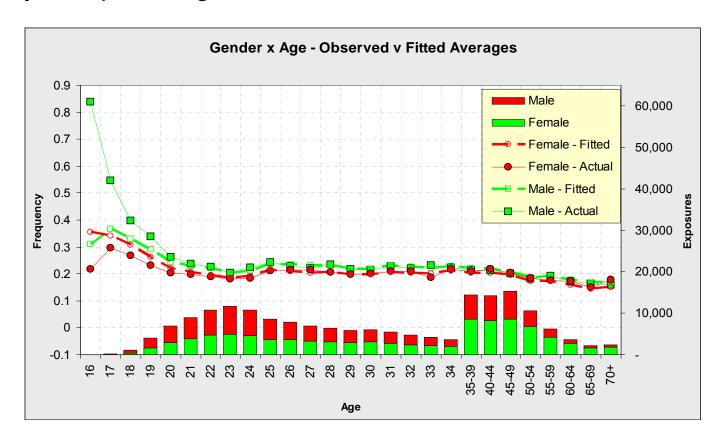
- The two lines should be very close where the volume of data is large
- If there is a systematic pattern (fitted values consistently above or below actual values), this indicates a poorly fitting model



Fitted Values Compared to Actual Values – By Segment

Look for large or systematic differences between fitted and actual values

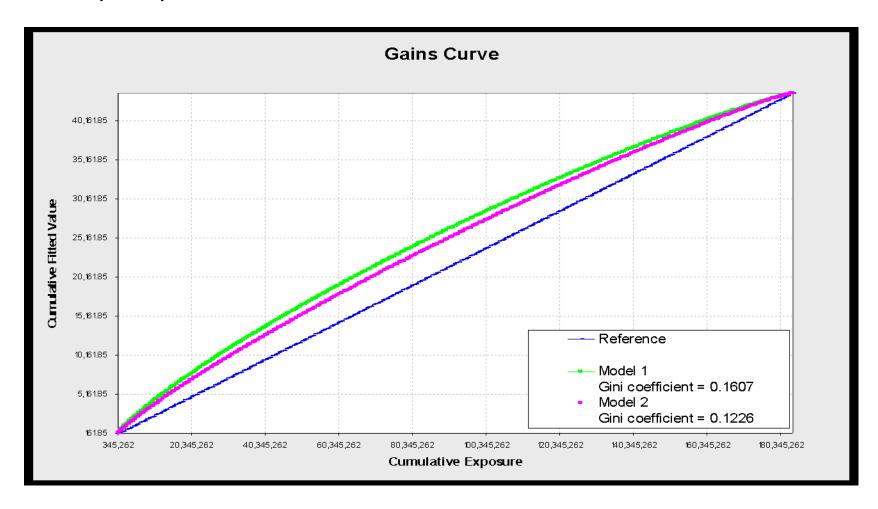
- Across levels of individual rating variables
- Split by multiple rating variables



Gains Curves



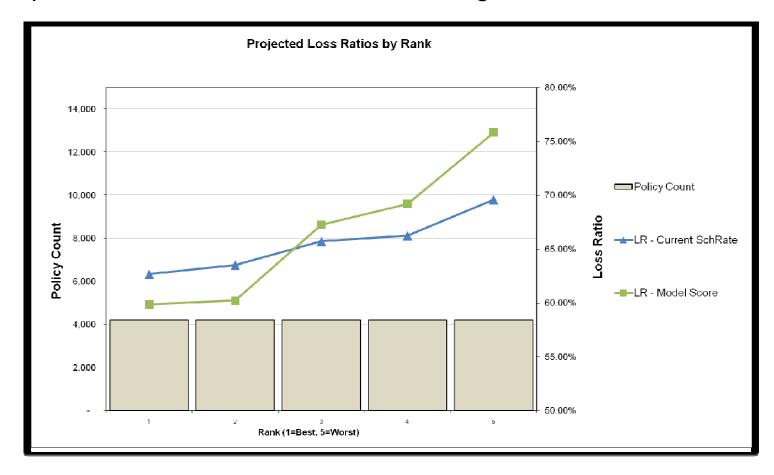
Compare predictiveness of different models



Lift Curves



Compare how well two different models segment the book





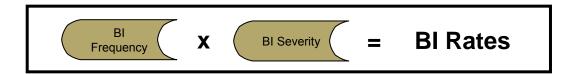
Important Modeling Questions

- What response variable should I use when modeling claims?
- What is my goal when iterating models?
- ➤ How do I know if my models are good?
- How should I combine component models and how should I incorporate constraints?
 - Model combining strategies
 - ➤ Ways to address constraints





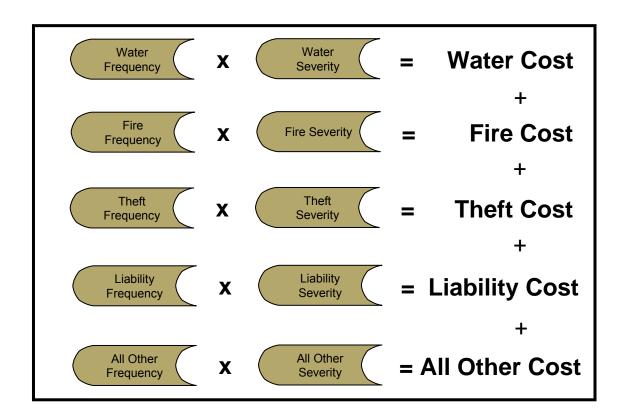
Standard combination 1: individual claim type, no constraints



- Multiply the predictions of the underlying models
 - Equivalent to adding parameter estimates in log space
 - Standard errors can be calculated as the square root of the sum of squared standard errors
- Total premium for a risk is the sum of the rates for each coverage



Standard combination 2: many claim types





- Build underlying component models for each peril
- For each record in your data, calculate expected frequencies and severities for each peril according to the models
 - You may want to use only a subset of your modeling data (e.g., most recent year) or a different dataset (e.g., current in-force policies)
- For each record, calculate expected overall cost of claims "C"
- Fit a GLM to "C" using all available factors
 - This model's standard errors are meaningless



Combining models

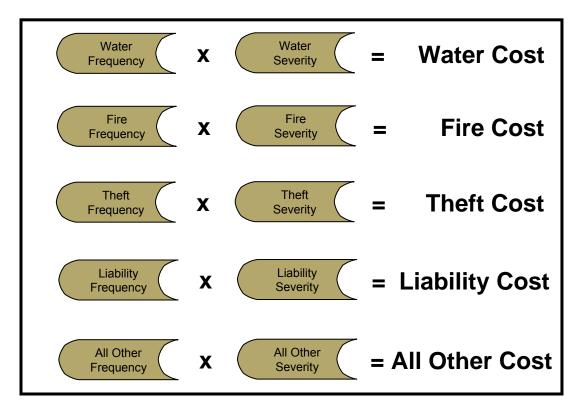


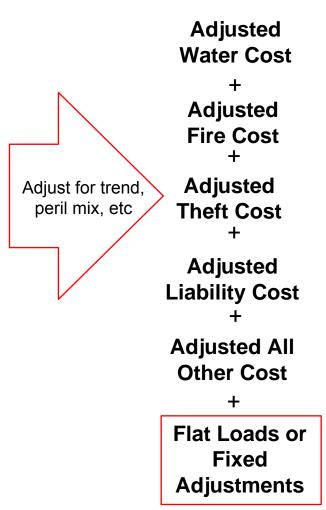
	Water Frequency	Water Severity	Fire Frequency	Fire Severity
Base	3.9%	\$4250	0.7%	\$8325
Single policy	1.00	1.00	1.00	1.00
Multi-policy	0.87	0.92	0.72	0.96
Prior Losses				
0	1.00	1.00	1.00	1.00
1+	1.68	1.00	1.35	1.08

Policy	Multi-policy	Prior Losses	Water Freq	Water Sev	Fire Freq	Fire Sev	Cost
62374	No	0	3.9%	\$4,250	0.7%	\$8,325	\$224.03
62375	No	1+	6.6%	\$4,250	0.9%	\$8,991	\$363.42
62376	Yes	0	3.4%	\$3,910	0.5%	\$7,992	\$172.95
62377	No	1+	6.6%	\$4,250	0.9%	\$8,991	\$363.42
	-						

Other Adjustments









Convert theoretical risk premium results into real world indications after consideration of internal and external constraints

- Not always possible or desirable to charge the fully indicated rates in the short run
 - Marketing decisions
 - Regulatory constraints
 - Systems constraints
- Build a risk premium model that is consistent with proposed rating structure
- Incorporate constraints
 - Eliminate variables not used
 - Group levels
 - Restrict relativities
 - The decision to offset vs make selections

Eliminating Variables Not Used



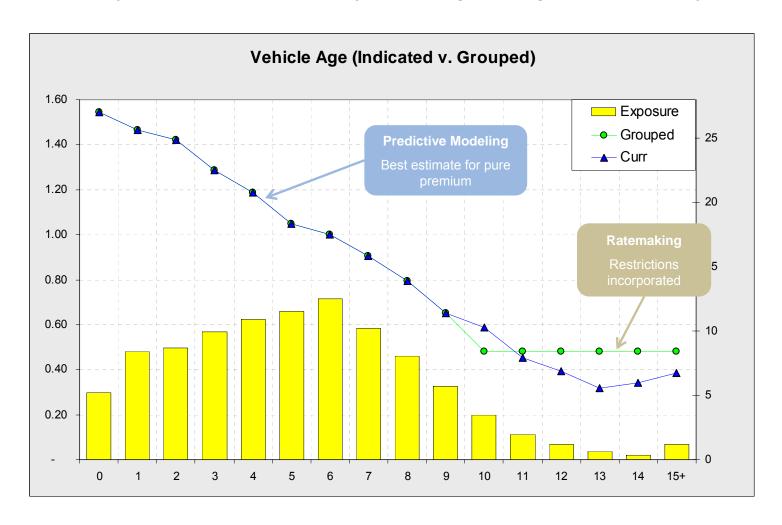
- Variable may be predictive, but cannot implement in rating algorithm at this time
 - Regulators may restrict use of variable (e.g., credit)
 - Cannot make systems change to implement new variable
- Include variable in predictive model to determine "correct" risk premiums, but exclude from final rating algorithm
 - Include variable as an UW characteristic
 - Eliminate the variable and have other variables compensate to the extent exposure correlations exist
 - Accept short run cross-subsidy and move toward future implementation



Group Levels of Variables



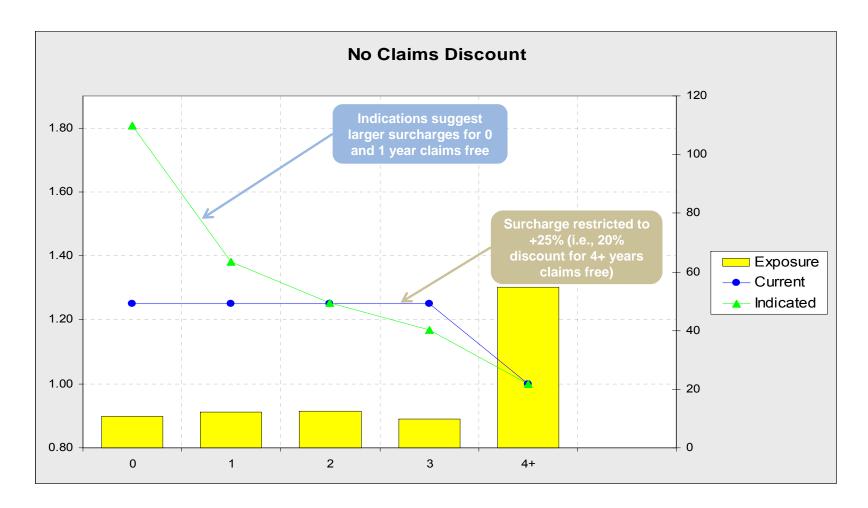
Example: systems constraints may require grouping vehicles 10+ years old



Restrict Relativities



Company may decide not to implement indicated relativities

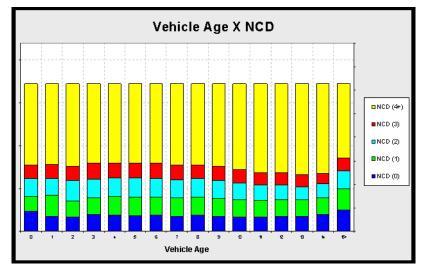


Offsetting – No Claims Discount

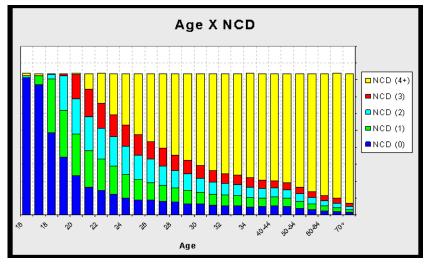


Cramer's V measures exposure correlation

F4 (#1	0 1	Rating	Vehicle	A	No Claims	Driving	Vehicle	1 V
Factor (#Levels)	Gender	Area	Category	Age	Discount	Restriction	Age	LossYear
Gender	1	-	-	-	-	-	_	_
Rating Area	0.017	-	-	-	High -	-	_	-
Vehicle Category	0.297	0.017	-	-	<u> </u>	-	=	=
Age	0.182	0.035	0.087		-	ow -	=	=
No Claims Discount	0.126	0.021	0.139	0.253) <u>-</u> '		=	=
Driving Restriction	0.076	0.034	0.088	0.224	0.112	_	=	=
Vehicle Age	0.044	0.016	0.068	0.025	0.025	0.041	-	=
LossYear	0.006	0.014	0.064	0.126	0. i24	0.055	0.049	-



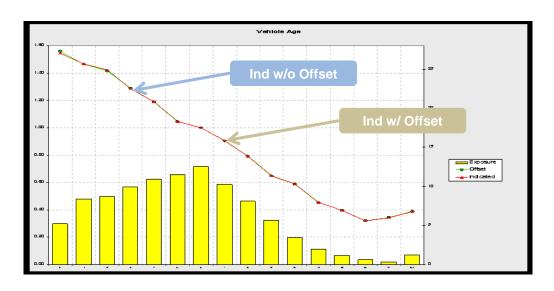
➤ 0.025 implies low correlation



➤ 0.253 implies high correlation

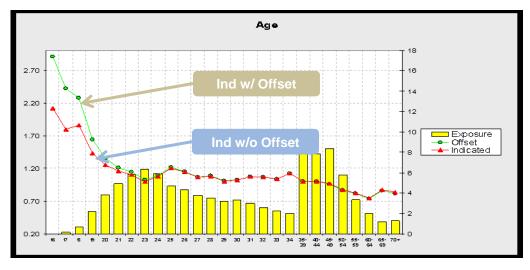
Offsetting – No Claims Discount Example





Cramer's V=.025 (Low)

No material difference between model with and without the offset for NCD



Cramer's V=.253 (High)

Youthful relativities increased to account for premium lost by dampening surcharge for policies with fewer than 4 years clean

Offsetting vs Selecting



- Offsetting one factor's parameters changes parameters of other correlated predictor(s) to compensate for the restriction
 - ➤ The stronger the exposure correlation, the more that can be "made up" through the other variable(s)
 - The more insureds in the class that need to "make up" the difference, the smaller the impact

ne impact	Desirable Subsidy	Undesirable Subsidy		
Example	Management wants to attract drivers 65+	Regulators force subsidy of drivers 65+		
Result of Offset	Correlated factors will adjust to m territories with retirement commun			
Recommendation	Do not offset	Offset		



Summary

- When modeling risk, it is ideal to
 - Model loss costs as opposed to loss ratios
 - Model frequency and severity separately
 - Model by coverage or cause of loss
- Regardless of what is being modeled, the goal is to remove the "noise" and find the "signal" in the data
- Validate your models at multiple steps of the process to ensure optimal results
- Combine your models appropriately and incorporate constraints in order to apply theoretical results to the real world

Contact us



EMB

12235 El Camino Real Suite 150 San Diego, California 92130

T +1 (858) 793-1425 F +1 (858) 793-1589

www.emb.com

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