GUY CARPENTER

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Hands on: How to Use Generalized Linear Models to Model Loss Triangles

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HOW TO: Generalized Linear Model

1. What is a GLM for Reserving
2. How to use a GLM in Reserving
3. Why use a GLM in Reserving

HOW TO: Generalized Linear Model

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What is a GLM?

Linear Model

$$E[y] = mx + b$$

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Definition:

the **generalized linear model** (**GLM**) is a flexible generalization of ordinary linear regression that allows for response variables that have other than a normal distribution. The GLM generalizes linear regression by allowing the linear model to be related to the response variable via a **link function** and by allowing the magnitude of the variance of each measurement to be a function of its predicted value.

What is a GLM?

Linear Link Function:

$$E[Y] = f(X_1) + g(X_2) + h(X_3) + \beta$$

What is a GLM?

Linear Link Function:

$$E[Y] = f(X_1) + g(X_2) + h(X_3) + \beta$$

Log Link Function:

$$E[Y] = e^{f(X_1)+g(X_2)+h(X_3)+\beta}$$

What is a GLM?

Linear Link Function:

$$E[Y] = f(X_1) + g(X_2) + h(X_3) + \beta$$

Log Link Function:

$$E[Y] = e^{f(X_1)+g(X_2)+h(X_3)+\beta}$$

Traditionally used in pricing:

$$X_1 = \textit{Credit Score}$$
$$X_2 = \textit{Zip Code}$$
$$X_3 = \textit{Make \& Model of vehicle}$$

What is a GLM for Reserving?

Accident Year (AY) 2002

	0	1	2	3	4	5	6	7	8	9
1998	4,645	4,927	3,016	1,485	1,172	806	594	438	316	316
1999	4,205	5,412	3,114	1,865	1,018	584	532	447	356	
2000	4,543	5,800	3,335	1,867	1,145	641	596	471		
2001	4,546	5,773	3,414	1,858	738	443	488			
2002	4,253	5,258	3,002	1,650	1,106	614				
2003	4,273	5,177	2,938	1,748	1,145					
2004	4,624	5,174	2,675	1,661						
2005	4,865	5,082	2,843							
2006	5,130	5,594								
2007	5,212									

Calendar Year of Payment (CY) 2007
(CY = AY + DY)

Development Year (DY) 1
(payments made in the year after the accident year)

What is a GLM for Reserving?

Incremental Paid Loss

104	123	81	61
106	136	79	
101	142		
116			

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- May increase / decrease by accident year
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- Inflation by calendar year

What is a GLM for Reserving?

Generalized Linear Model

a1	a1	a1	a1

- May increase / decrease by accident year

What is a GLM for Reserving?

Generalized Linear Model

a1	a1	a1	a1
a2	a2	a2	

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a1	a1	a1	a1
a2	a2	a2	
a3	a3		

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What is a GLM for Reserving?

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a1	a1	a1	a1
a2	a2	a2	
a3	a3		
a4			

- May increase / decrease by accident year

What is a GLM for Reserving?

Generalized Linear Model

a1	a1*b2	a1	a1
a2	a2*b2	a2	
a3	a3*b2		
a4			

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What is a GLM for Reserving?

Generalized Linear Model

a1	a1*b2	a1*b2*b3	a1
a2	a2*b2	a2*b2*b3	
a3	a3*b2		
a4			

- May increase / decrease by accident year
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What is a GLM for Reserving?

Generalized Linear Model

a1	a1*b2	a1*b2*b3	a1*b2*b3*b4
a2	a2*b2	a2*b2*b3	
a3	a3*b2		
a4			

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What is a GLM for Reserving?

Generalized Linear Model

a1	a1*b2*c2	a1*b2*b3	a1*b2*b3*b4
a2*c2	a2*b2	a2*b2*b3	
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a2*c2	a2*b2*c2*c3	a2*b2*b3	
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a2*c2	a2*b2*c2*c3	a2*b2*b3*c2*c3*c4	
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a4*c2*c3*c4			

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What is a GLM for Reserving?

Generalized Linear Model

a	a * b * c	a * b * b * c * c	a * b * b * b * c * c * c
a * c	a * b * c * c	a * b * b * c * c * c	
a * c * c	a * b * c * c * c		
a * c * c * c			

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GLM Theory for Reserving

1. Distribution of incremental losses: y
2. Linear Predictor:
3. Link Function:

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GLM Theory for Reserving

Predictor:

a	a * b * c	a * b * b * c * c	a * b * b * b * c * c * c
a * c	a * b * c * c	a * b * b * c * c * c	
a * c * c	a * b * c * c * c		
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GLM Theory for Reserving

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GLM Theory for Reserving

1. Distribution of incremental losses: y
   ~~Over dispersed~~ poisson $p(y) = \frac{e^{-\lambda}\lambda^y}{y!}$
2. Linear Predictor: $predictor = a * b * c$
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   $linear\ predictor = \alpha + \beta + \varsigma$
3. Link Function:

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3. Link Function: $E(y) = \exp(linear\ predictor)$

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   $E(y) = e^{\alpha} * e^{\beta} * e^{\varsigma}$

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   => Log Link

Finding a, b, c

Maximize likelihood

"…finding particular parametric values that make the observed results the most probable (given the model)."

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…find values for a, b, c that make the observed incremental losses (104, 106, 101, etc) most probable.

…find values for a, b, c that maximize:

$p(this\ triangle) = p(104) \times p(106) \times p(101) \times \ldots$

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$$p(this\ triangle) = p(104) \times p(106) \times p(101) \times \ldots$$

…find values for a, b, c that maximize:

$$\log(p(this\ triangle)) = \log(p(104)) + \log(p(106)) + log(p(101)) + \cdots$$

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What is $log(p(y))$?

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What is $log(p(y))$?

Each incremental loss, y, is an outcome from an Over-dispersed Poisson Distribution.

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$$p(y) = \frac{e^{-E(y)}E(y)^y}{y!}$$

Finding a, b, c

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$$p(y) = \frac{e^{-E(y)}E(y)^y}{y!}$$

$$log(p(y)) = -E(y) + y\,log(E(y)) - log(y!)$$

Finding a, b, c

$E(y)$

a	a * b * c	a * b * b * c * c	a * b * b * b * c * c * c
a * c	a * b * c * c	a * b * b * c * c * c	
a * c * c	a * b * c * c * c		
a * c * c * c			

y

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Finding a, b, c

Steps:

1. y = 123

Finding a, b, c

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2. Fill in dummy values for a, b and c

Finding a, b, c

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1. y = 123
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3. $linear\ predictor$ = ln(a) + ln(b) + ln(c)

Finding a, b, c

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1. y = 123
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3. $linear\ predictor$ = ln(a) + ln(b) + ln(c)
4. $E(y)$ = exp(linear predictor) = a * b * c

Finding a, b, c

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1. y = 123
2. Fill in dummy values for a, b and c
3. $linear\ predictor$ = ln(a) + ln(b) + ln(c)
4. $E(y)$ = exp(linear predictor) = a * b * c
5. $log(p(y)) = -E(y) + y log(E(y)) - log(y!)$

Finding a, b, c

Steps:

1. y = 123
2. Fill in dummy values for a, b and c
3. $linear\ predictor$ = ln(a) + ln(b) + ln(c)
4. $E(y)$ = exp(linear predictor) = a * b * c
5. $log(p(y)) = -E(y) + ylog(E(y)) - log(y!)$

$$= -(a*b*c) + 123 * log(a*b*c) - log\ (123!)$$

Finding a, b, c

Steps:

1. y = 123
2. Fill in dummy values for a, b and c
3. $linear\ predictor$ = ln(a) + ln(b) + ln(c)
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Finding a, b, c

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6. Do for all incremental losses

Finding a, b, c

Steps:

1. y = 123
2. Fill in dummy values for a, b and c
3. $linear\ predictor$ = ln(a) + ln(b) + ln(c)
4. $E(y)$ = exp(linear predictor) = a * b * c
5. $log(p(y)) = -E(y) + ylog(E(y)) - log(y!)$

$$= -(a*b*c) + 123 * log(a*b*c) - \cancel{log\ (123!)}$$

6. Do for all incremental losses
7. Sum and solve for a, b, c that maximizes the sum

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Finding a, b, c

$E(y)$

a	a * b * c	a * b * b * c * c	a * b * b * b * c * c * c
a * c	a * b * c * c	a * b * b * c * c * c	
a * c * c	a * b * c * c * c		
a * c * c * c			

Reserving with a GLM

$E(y)$

a	a * b * c	a * b * b * c ^ 2	a * b * b * b * c ^ 3
a * c	a * b * c ^ 2	a * b * b * c ^ 3	a * b * b * b * c ^ 4
a * c ^ 2	a * b * c ^ 3	a * b * b * c ^ 4	a * b * b * b * c ^ 5
a * c ^ 3	a * b * c ^ 4	a * b * b * c ^ 5	a * b * b * b * c ^ 6

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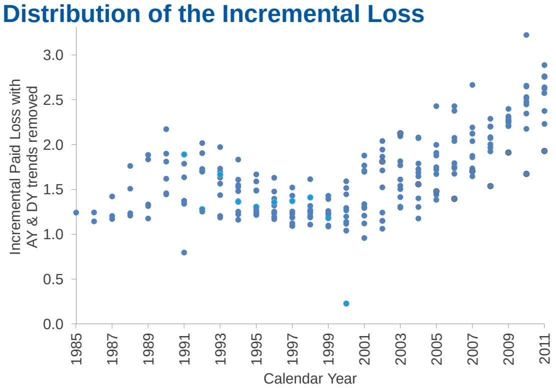
Distribution of the Incremental Loss



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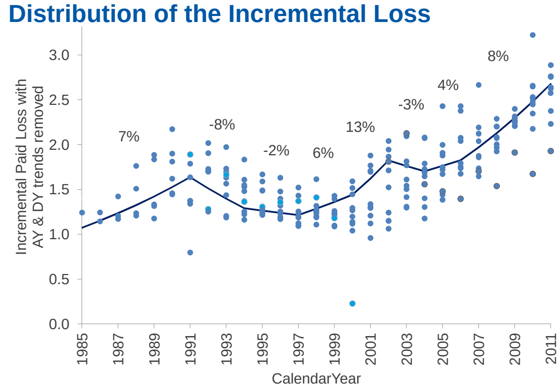

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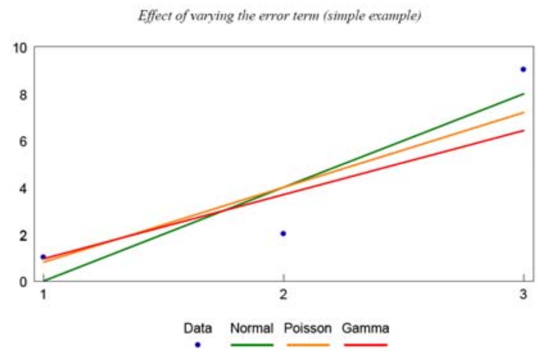

Questions

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