EMB

Practical Considerations for Reserve Variability

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- Introduction

Introduction

Why should we care about reserve variability?

- Traditional deterministic methods give no insight into what could happen beyond the mean estimate
- Regulators and management are starting to ask for more information than the point estimate
  - Actuarial Opinion Summary has encouraged actuaries to determine reasonable range of reserves
  - Management is starting to ask questions about likely ranges of results and reserve distributions
- Enhancement in computing power has made simulation based approaches to evaluating distributions feasible

Introduction

- Potential Uses for reserve variability methods:
  - Determination of range for Actuarial Opinion Statement
  - Enterprise Risk Management
  - Asset / Liability Matching
  - Commutations
  - Merger and acquisition
  - Determine reasonability of loss emergence (Actual vs. Expected)
  - Management reporting and change in ultimate loss

Introduction: Agenda

- Introduction
- Overview of Models
- Brief Bootstrap Illustration
- Data Considerations
- Scaling of Results
- Impacts of Reinsurance
- Aggregation of Results

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- **Overview of Models**
- Brief Bootstrap Illustration
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Models for Reserve Variability

What are we trying accomplish?

- Construct a model which describes loss reserve process
- Obtain an estimate of variability or obtain full predictive distribution
  - Prediction error is a function of:
    - Parameter Error – Uncertainty around parameter estimates
    - Process Error – Uncertainty of the underlying reserve process
    - Model Error – Uncertainty surrounding which is the correct model
    - Simulation Error – Minimized by modern computing power
  - Prediction error depends on the quality of your model – if the model is inappropriate, your estimate of reserve variability will be unreliable

High Level Overview

Two basic ways to estimate the variance of a given process:

1. Analytic Approach
   - Mathematically define the underlying process
   - From the process, derive the formula for prediction error
   - Formulas can become intractable as you tweak the model
   - Mack's approach estimates first two moments of reserve distribution
2. Simulated Approach
   - Prediction error can be calculated directly full distribution of outcomes simulated from a model
   - Bootstrapping and Bayesian methods fall in this category

A Few More Thoughts

Before constructing or using a model for reserve variability, consider:

- Basic assumptions underlying the model
- Does your data reasonably fulfill the assumptions?
- Check the fit of the model (examine residuals, statistics, etc.)
- Validate results – Does it pass common sense reasonability tests?
- Try different models and run sensitivity tests
- Have reasonable expectations – models cannot solve all problems

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- **Brief Bootstrap Illustration**
- Data Considerations
- Scaling of Results
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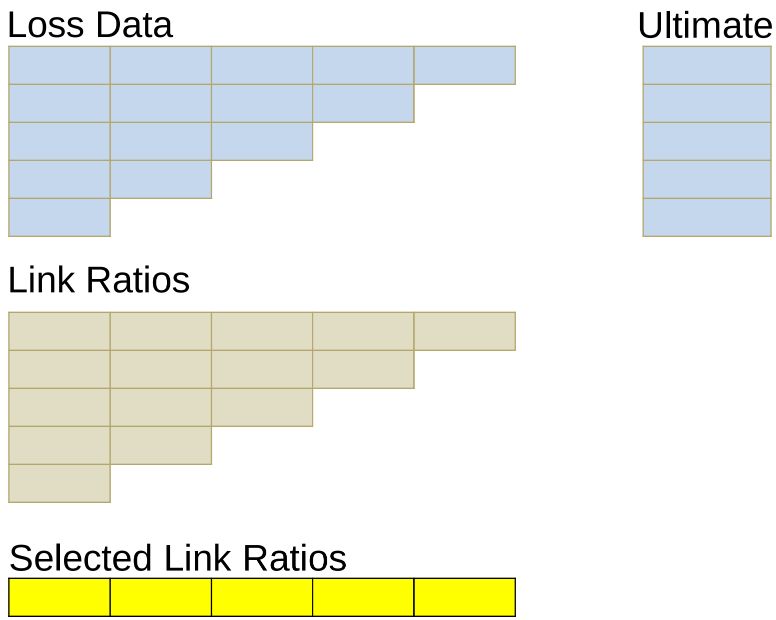
Bootstrap Reserve Models

- Bootstrapping is a sampling technique which allows modeler to incorporate estimation error (i.e. parameter variance)
- Often applied to the chain ladder reserve method with an assumed underlying stochastic model. Common models include:
  - Over-dispersed Poisson (ODP)
  - Mack's Model
  - Others
- Process error is handled through simulation of future incremental emergence of loss
- England and Verrall (2002) "*Stochastic Claims Reserving in General Insurance* " gives excellent overview of stochastic reserve models and has extensive bibliography

Generic Bootstrap Illustration

- Step 1: Construct a chain ladder method for loss triangle



- All period weighted average factors are commonly used
- Data can be excluded and curves can be fit to tail if necessary
- Remember, if chain ladder is inappropriate for the data then bootstrap will be inappropriate

Generic Bootstrap Illustration

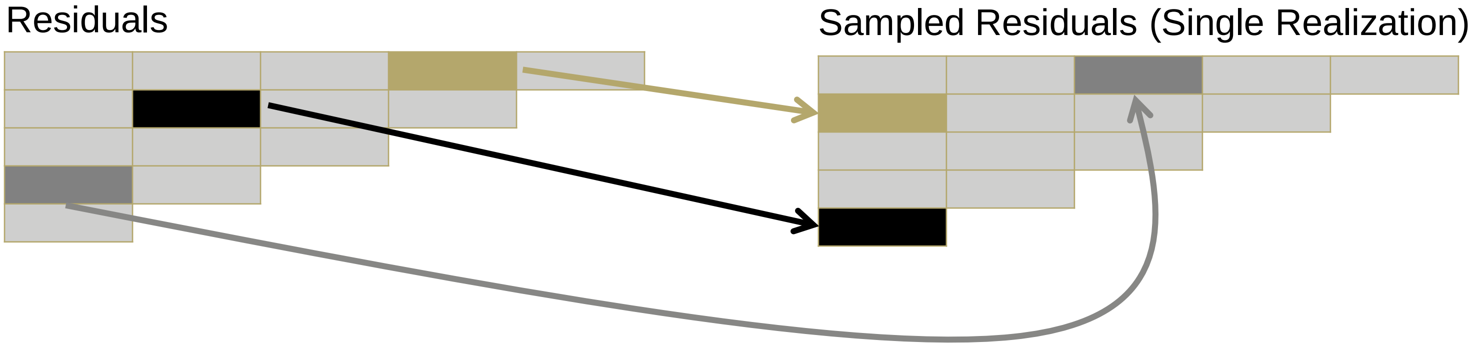
- Step 2: Calculate residuals

Residuals

- Compare actual triangle to backwards predicted results
- Residuals will need to be further adjusted to make them independent and identically distributed. Adjustments vary by underlying model selected (Mack, ODP, etc.)
- Check residuals – are they reasonable?

Generic Bootstrap Illustration

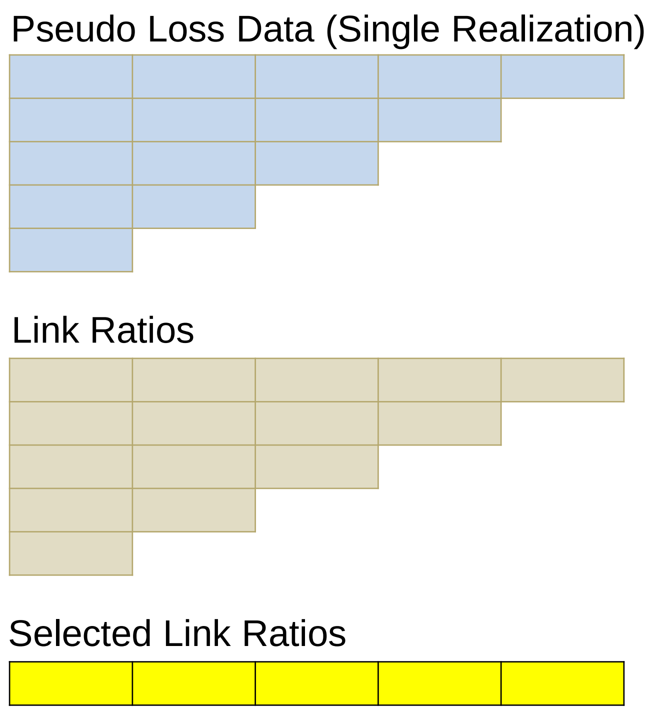
- Step 3: Bootstrap (sample) residuals with replacement to construct a new triangle of residuals



- Bootstrap requires data be iid and thus we use adjusted residuals
- If triangle is small, the universe of residuals to sample from will be small, though this can be dealt with if problematic

Generic Bootstrap Illustration

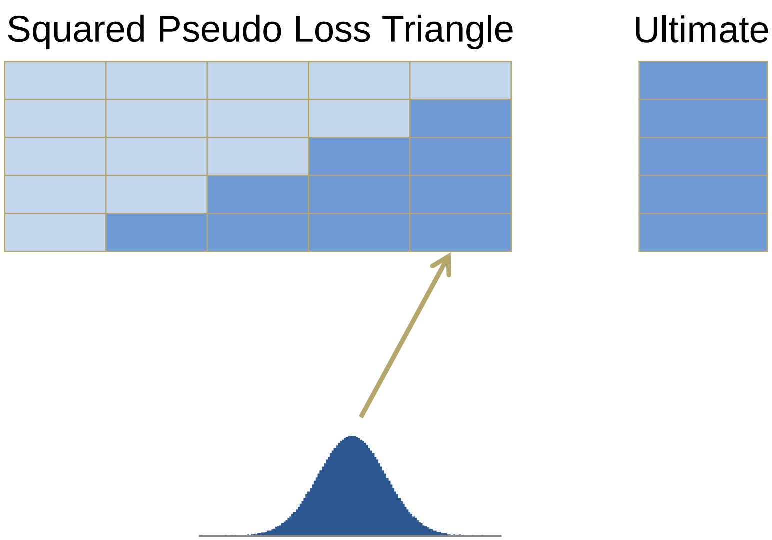
- Step 4: Invert residuals to construct pseudo-triangles based on residuals and apply original chain ladder model to each simulated triangle



- The variability in the pseudo-triangles captures the parameter error of the model

Generic Bootstrap Illustration

- Step 5: Square the triangle with future incremental losses through simulation – Repeat steps 3 – 5 10,000 times



- Select distribution for future incremental losses – Gamma? Lognormal? Normal?
- Mean of incremental loss is defined based off chain ladder method
- Variance is based on underlying model (Mack, ODP)
- End result is a full distribution of ultimate loss taking into account variation in parameter estimates as well as variation in loss process

Bootstrap Overview

Advantages

- Can be implemented in a spreadsheet
- Can handle complications such as data exclusion or curve fitting (although this complicates the math)
- Numerous articles exist which offer formulas and examples for implementation
- Statistical framework allows testing of model fitness

Considerations

- Flexibility of model leaves it open to potential manipulation
- Complexity of model increases risk of improper implementation, especially as chain ladder model becomes more complicated than all year weighted average with no tail

Data Considerations

Paid vs. Incurred

- A Mack or Bootstrap analysis is a process that will produce a predictive distribution around a selected chain ladder method that has been chosen by the modeler to be most representative of the underlying development variability
- Any analysis using either variability model will only reflect the information contained within the underlying model and data triangle used within that model, so
  - Bootstrapping a **paid** triangle will reflect the variability associated with the historical observed payments to estimate the uncertainty around the **ultimate**, the **reserves** and the **future cash flows**
  - Bootstrapping an **incurred** triangle will reflect the variability associated with the setting of historical case reserves to estimate the uncertainty around the **ultimate**, the **IBNR** and the future **emergence of IBNR** (a slightly obscure concept)

Data Considerations

Paid vs. Incurred

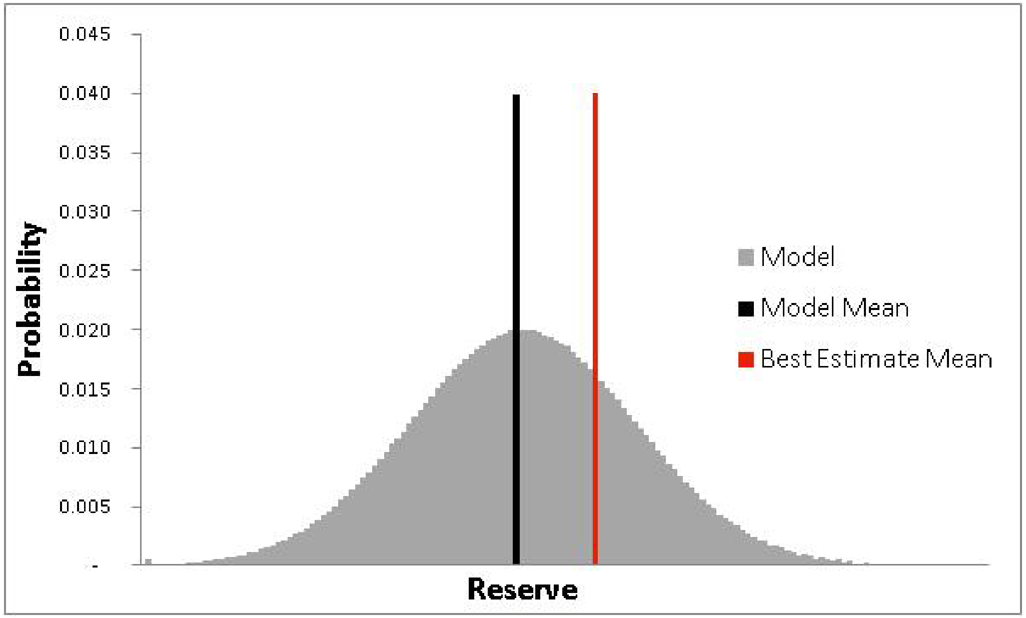
- As a general guideline, if the chief concern is the variability surrounding the final deterministic best estimate **ultimate**, then the model most closely relied-upon to reach that deterministic best-estimate might be the most informative
- Another consideration is the output required of the model
- Volatility model based on paid losses will produce future cash flows which can be useful for:
  - Projection of actual cash flows for financial model
  - Testing impact of inflation on future paid losses
  - Asset / Liability management and probabilistic cash flow matching

Scaling

- Reserve variability models generate a process which will produce a predictive distribution that has been chosen by the modeler to be most representative of the underlying development variability
- However, it is likely that your final estimate is reliant on the results of a variety of methods or sources, therefore it may be the case that:

Final Selected Ultimate ≠ Modeled Mean Ultimate

- In order to align the deterministic best estimate analysis and the variability analysis, the specified mean reserves can be targeted to scale the distribution of reserves accordingly.



Additive Scaling

Simulation	Reserve	Operation	Scaled Reserve
1	15.00		20.00
2	17.50		22.50
3	20.00	**+ 5**	25.00
4	22.50		27.50
5	25.00		30.00
Mean	20.00		25.00
St Dev	**3.95**		**3.95**
CoV	0.20		0.16

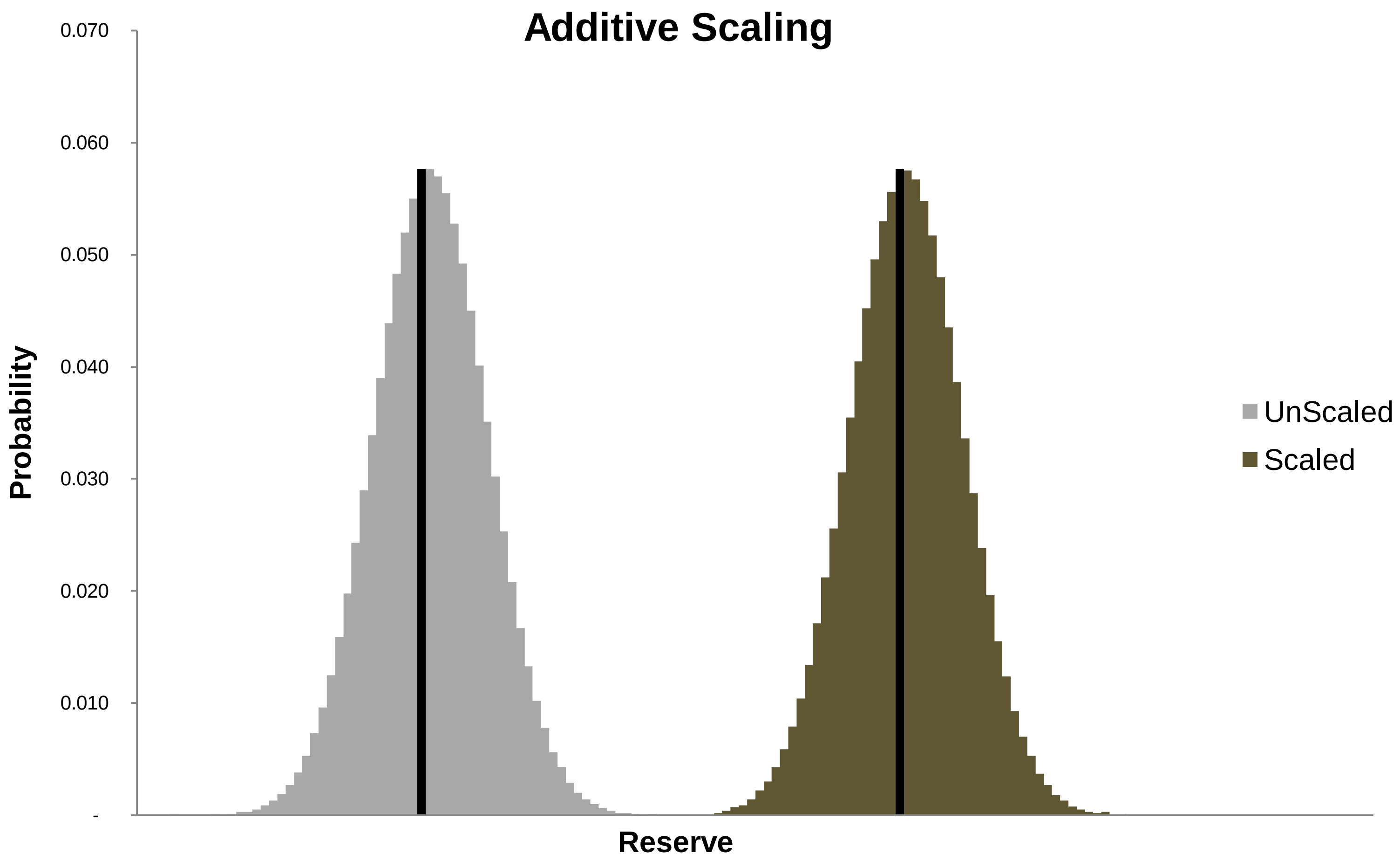
- Additive scaling preserves the standard deviation of the model

Multiplicative Scaling

Simulation	Reserve	Operation	Scaled Reserve
1	15.00		18.75
2	17.50		21.88
3	20.00	**x 1.25**	25.00
4	22.50		28.13
5	25.00		31.25
Mean	20.00		25.00
St Dev	3.95		4.94
CoV	**0.20**		**0.20**

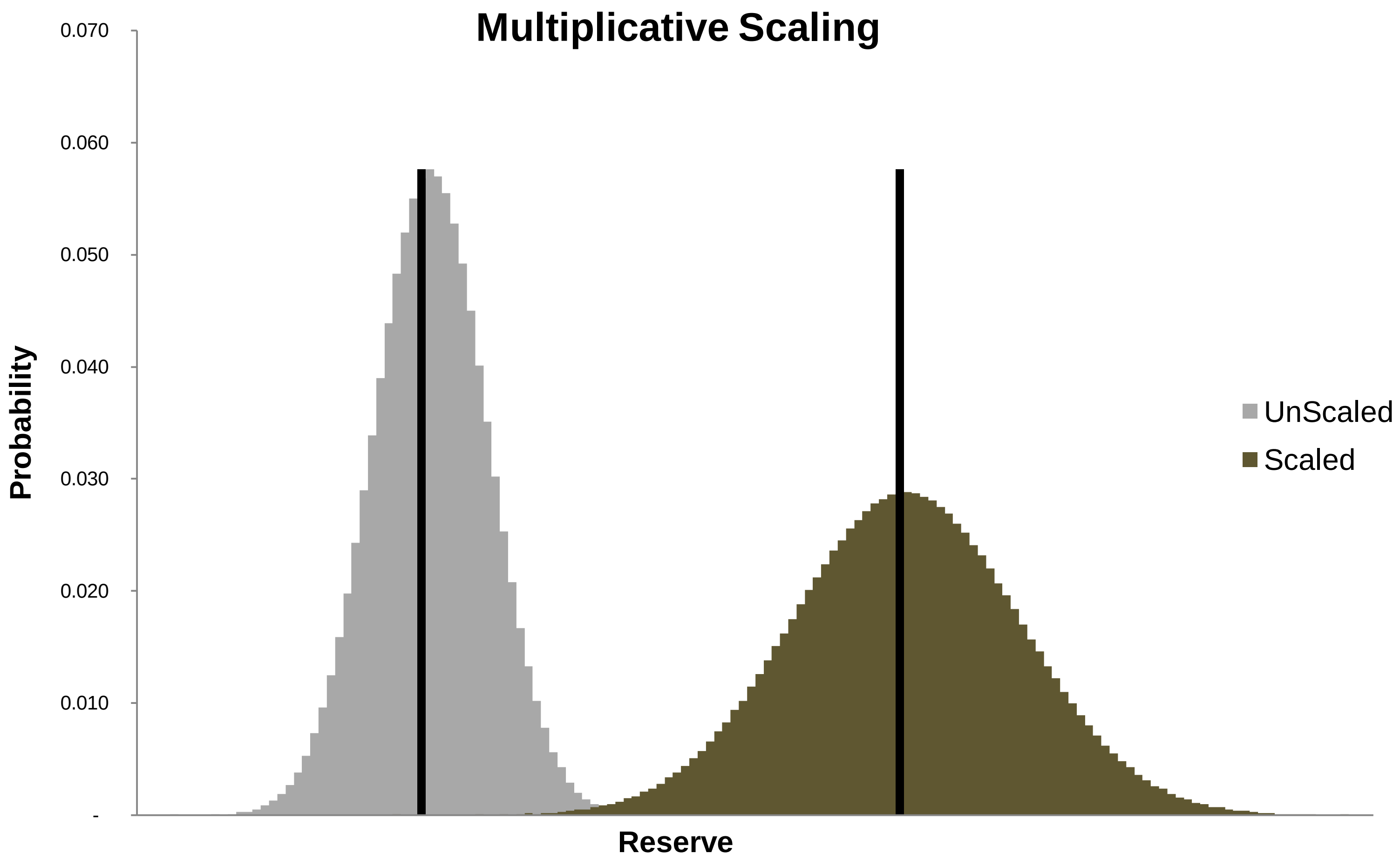
- Multiplicative scaling preserves the coefficient of variation of the model

Additive Scaling



Note that this is an extreme scaling example

Multiplicative Scaling



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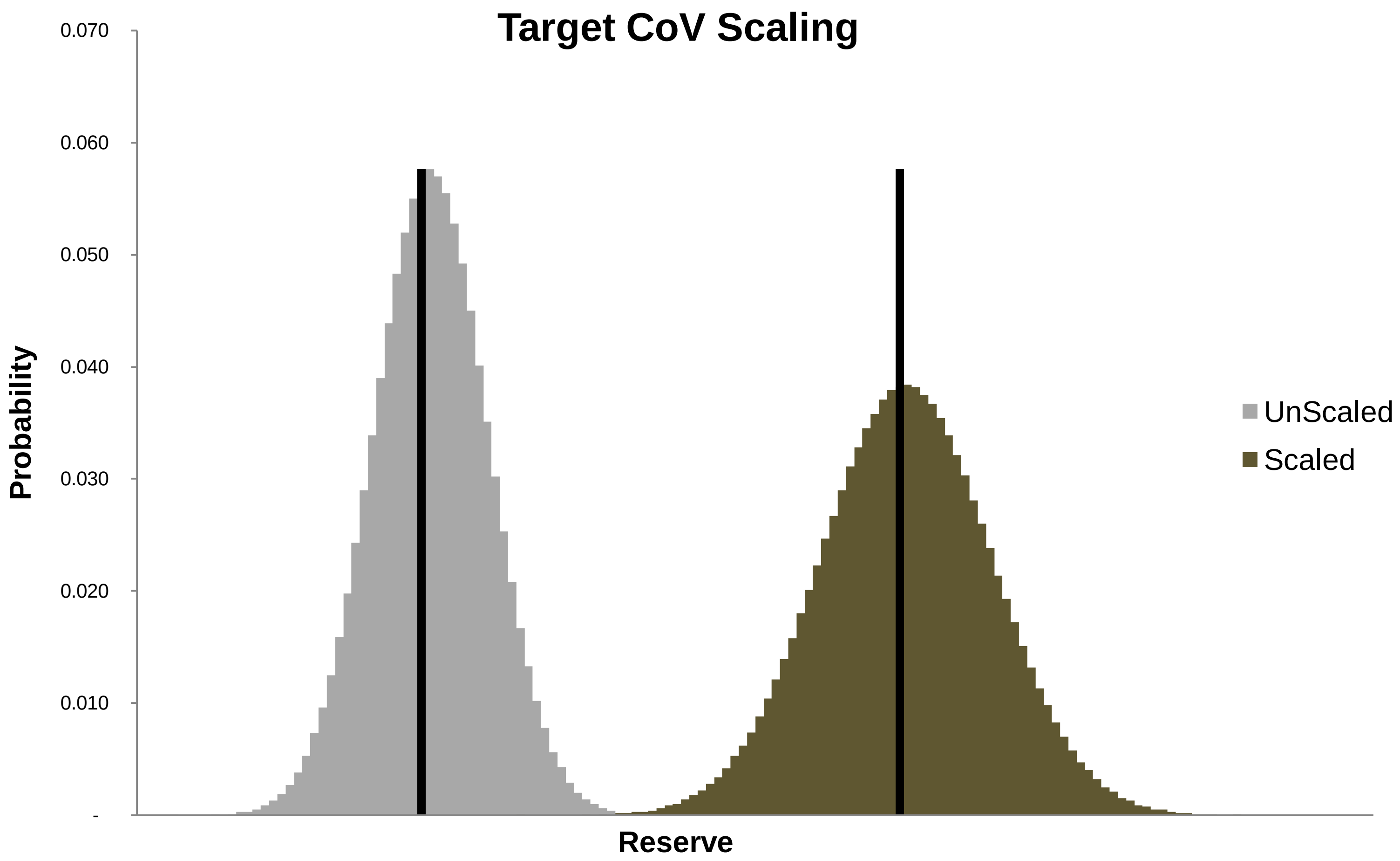
Target CoV Scaling

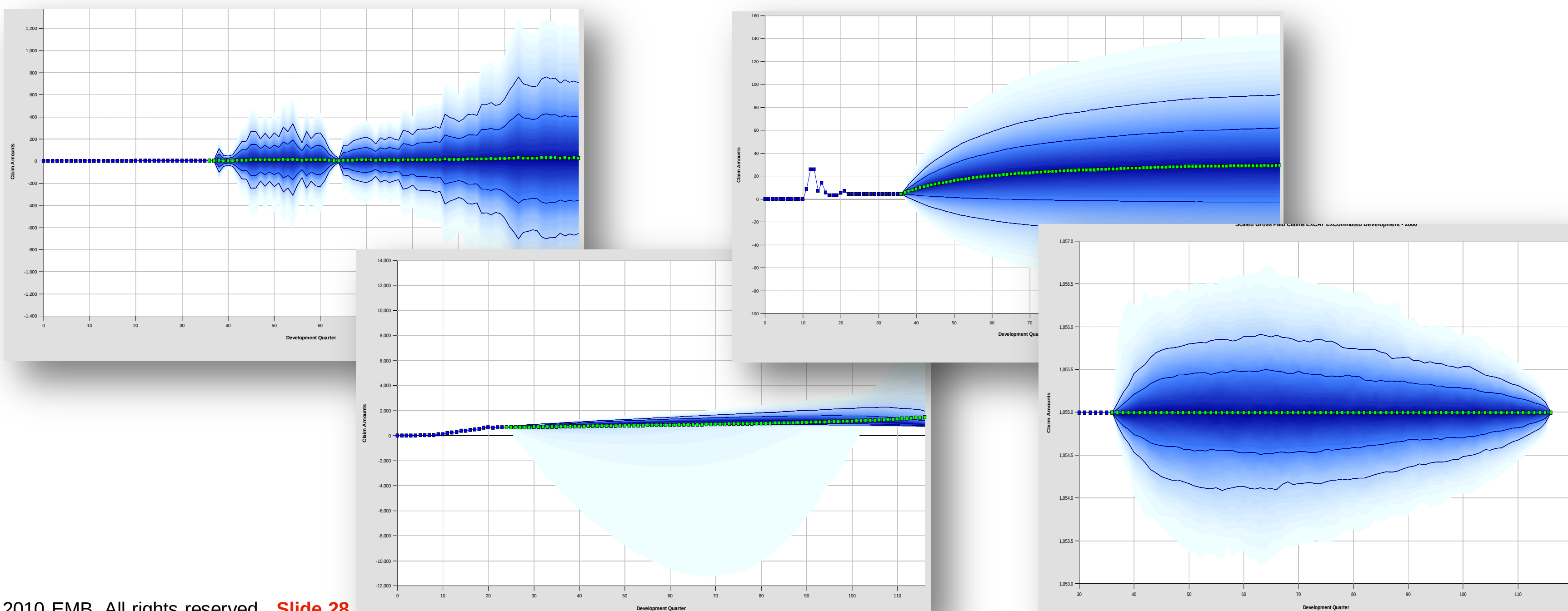


Note that this is an extreme scaling example

Scaling

Interpreting the Results

- When selecting a scaling method the size and maturity of the reserves should be borne in mind and the effect on *both* the CoV and the absolute prediction error. Special precaution should be taken when either the degree of scaling is large or volumes are low.
- If the scaling is having a significant impact on either the CoV or the absolute prediction error, then the validity of that model being used should be re-assessed for reasonability
- When using bootstrapping, also be sure to check that the cash flows are not being adversely distorted by inappropriate scaling, especially where volumes are small or cash flows volatile:

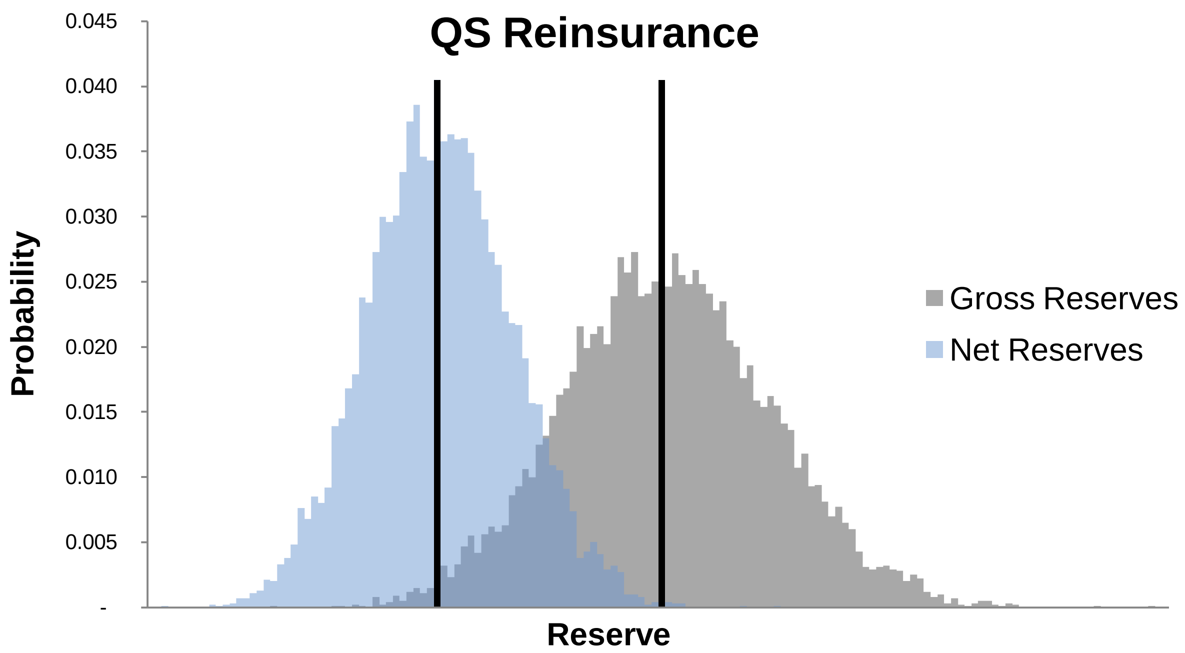


Reinsurance Modeling

- Modeling reinsurance will pose similar challenges in stochastic framework relative to traditional deterministic framework
- Some variability models are more suited to handle specific reinsurance scenarios and thus reinsurance should be considered at outset of modeling project
- Common Reinsurance Contracts
  - Quota share
  - Aggregate excess of loss
  - Individual excess of loss

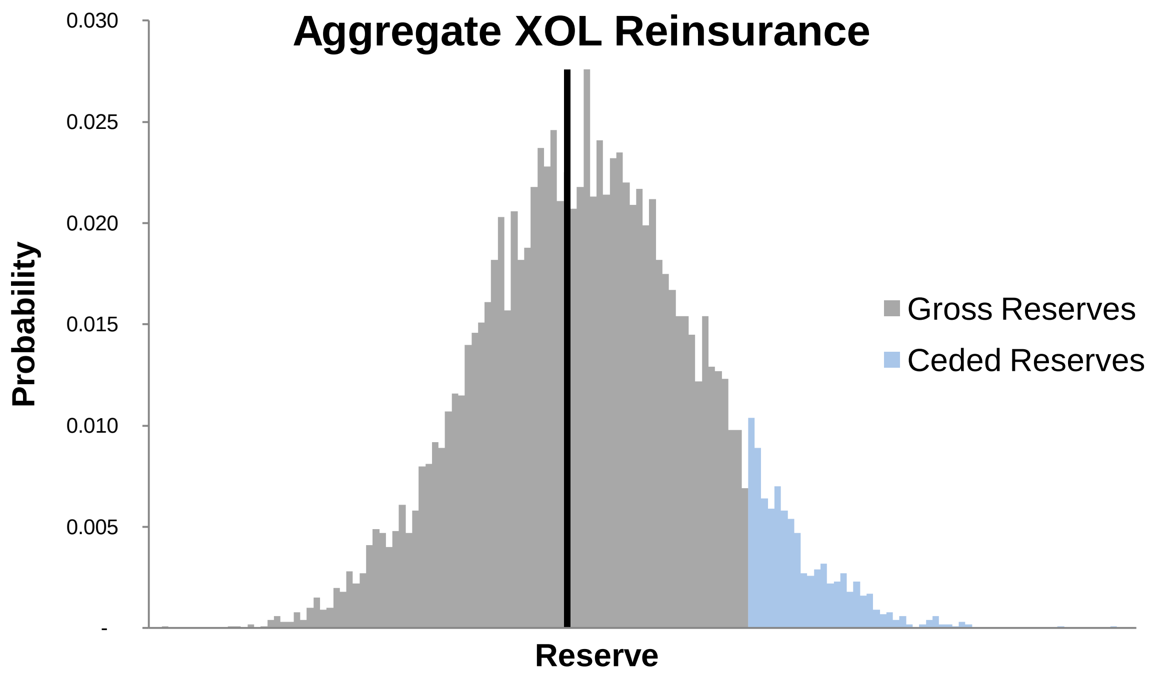
Reinsurance Modeling – Quota Share

- Modeling of quota share reinsurance is straightforward as long as model generates results for the period of interest
- Rather than applying QS formula to just best estimate, apply to every simulation
- QS impact on reserve distribution is identical to multiplicative scaling



Reinsurance Modeling – Aggregate XOL

- Modeling of aggregate XOL reinsurance is straightforward as long as model generates results for the period of interest
- Again, apply XOL contract terms to every simulation to determine net reserve distribution
- Upper part of gross reserve distribution is truncated



Reinsurance Modeling – Individual XOL

- Individual XOL reinsurance causes difficulties in traditional reserve methods
- There are several commonly applied techniques:
  - Apply development method to net triangle
  - Use experience or industry to determine expected ratio of net to gross losses
  - Use series of limited triangles to determine ultimate by origin period
  - Use a method which explicitly allows for application of reinsurance structure
- How do each of these options fare in a stochastic model?

Reinsurance Modeling – Individual XOL

Use a chain ladder method on net of reinsurance triangle

- Works well if retentions are constant over time
- If retentions are changing, lack of fit in chain ladder method will show up as reserve volatility in a bootstrap model

Use experience or industry to determine expected ratio of net to gross losses

- Net to gross ratio is static in deterministic analysis
- When stochastic model is introduced the net to gross ratio should vary by simulation
- Extent of distortion is determined by how volatile results are and where attachment is

Use series of limited triangles to determine ultimate by origin period

- Removes noise associated with changing retentions
- However, individual accident years need to be combined somehow

Use a method which explicitly allows for application of reinsurance structure

- If large losses could be analyzed separately any reinsurance structure could be applied

Reinsurance Modeling – Large Loss Simulation

- Data Requirements
  - Triangle of attritional losses
  - History of individual large losses, their development and status over time
- Method
  - Sample from historical development factors of large losses to develop known large claims to ultimate
  - Use claim count development and historical large losses to simulate IBNR large claims
  - Use bootstrap model to determine variability of attritional claims
- Benefits
  - Individual large claim simulation allows for application of any reinsurance structure including: individual XOL, changing XOL retentions, aggregate deductibles, etc.
- Considerations
  - Large amounts of data are required to calibrate and run this model
  - How can we reconcile the results of the model with expectation?
- See K. Murphy and A. McLennan (2006) "A Method For Projecting Individual Large Claims" for further discussion

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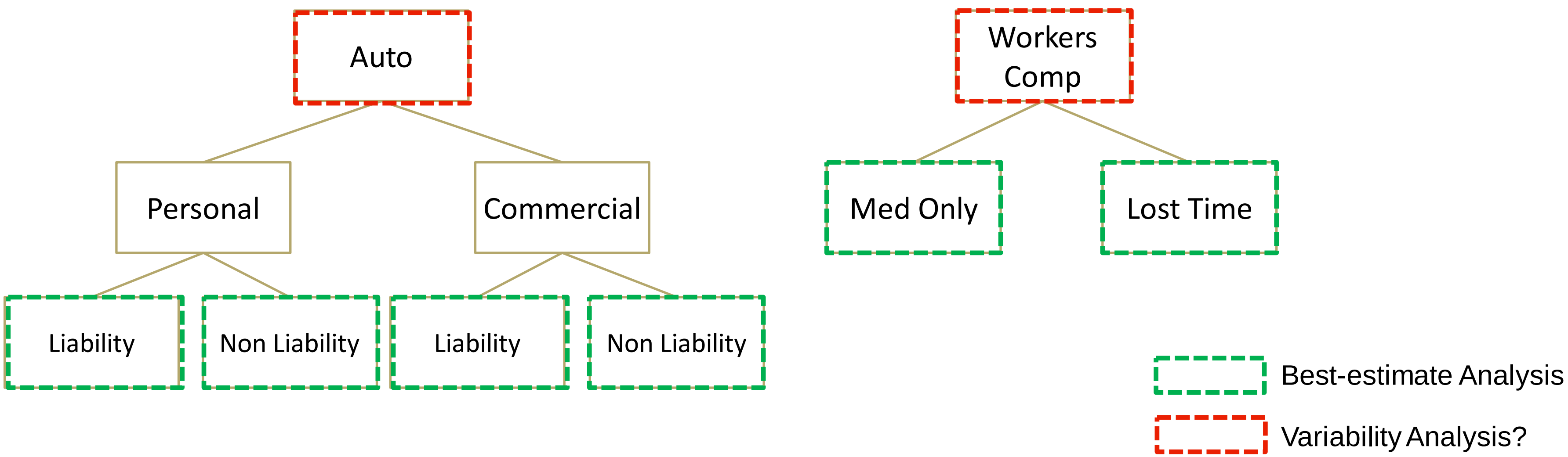
Aggregation
Line of Business

- The business level or reserving classes used for variability analysis may differ from those used to achieve a 'best estimate'
- Though considerations for each of the purposes are similar, the requirement may be different
- Performing variability analyses at a higher level can reduce the need for correlation estimation and remove a degree of the complexity of the model required, without sacrificing the validity of the model to a material degree
- Should bear in mind factors such as class size, degree of homogeneity of the data, susceptibility to common inflationary factors and reporting requirements when deciding upon the level at which to perform analyses



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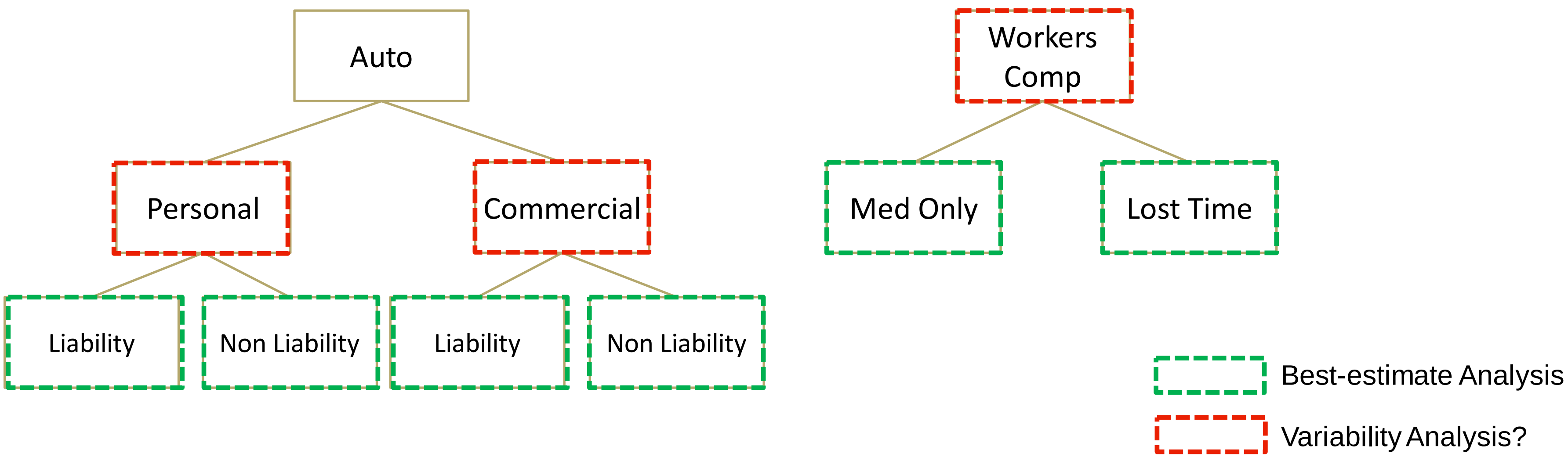


Aggregation
Dependency Assumptions

- Once models are constructed at the appropriate level, how do we estimate total company variability?
- A common approach is to estimate the degree to which the variability is correlated across each of the business segments.
- Things to consider when coming up with a correlation estimate include:
  - To what type of cost influences are the business segments susceptible?
  - Are any of these common across lines?
  - What inflationary pressures are active across the business lines?
  - How could these possibly vary in future?
- It's also important to investigate the degree to which the correlation estimate materially affect the results by stress-testing with various correlation estimates.
- See G. Kirschner, C. Kerley, and B. Isaacs (2008) "Two Approaches to Calculating Correlated Reserve Indications Across Multiple Lines of Business" for an introduction to modeling dependency structure between lines

Aggregation

Example

- We've modeled results from 2 lines of business, now what?

Sim	Line A	Line B	Total
1	10	15	?
2	14	21	?
3	18	12	?
4	16	24	?
5	12	18	?

Aggregation
Example

- Simple addition of results is equivalent to assumption of independence

Sim	Line A		Line B	Total
1	10	→	15	25
2	14	→	21	35
3	18	→	12	30
4	16	→	24	40
5	12	→	18	30

Mean	32
St Dev	5.70

Aggregation
Example

- Simulations can be rearranged to emulate any dependency strength

Sim	Line A	Line B	Total
1	10	15	27
2	14	21	37
3	18	12	22
4	16	24	42
5	12	18	32

Mean	32
St Dev	7.91

Aggregation

Example

- Simulations can be rearranged to emulate any dependency strength

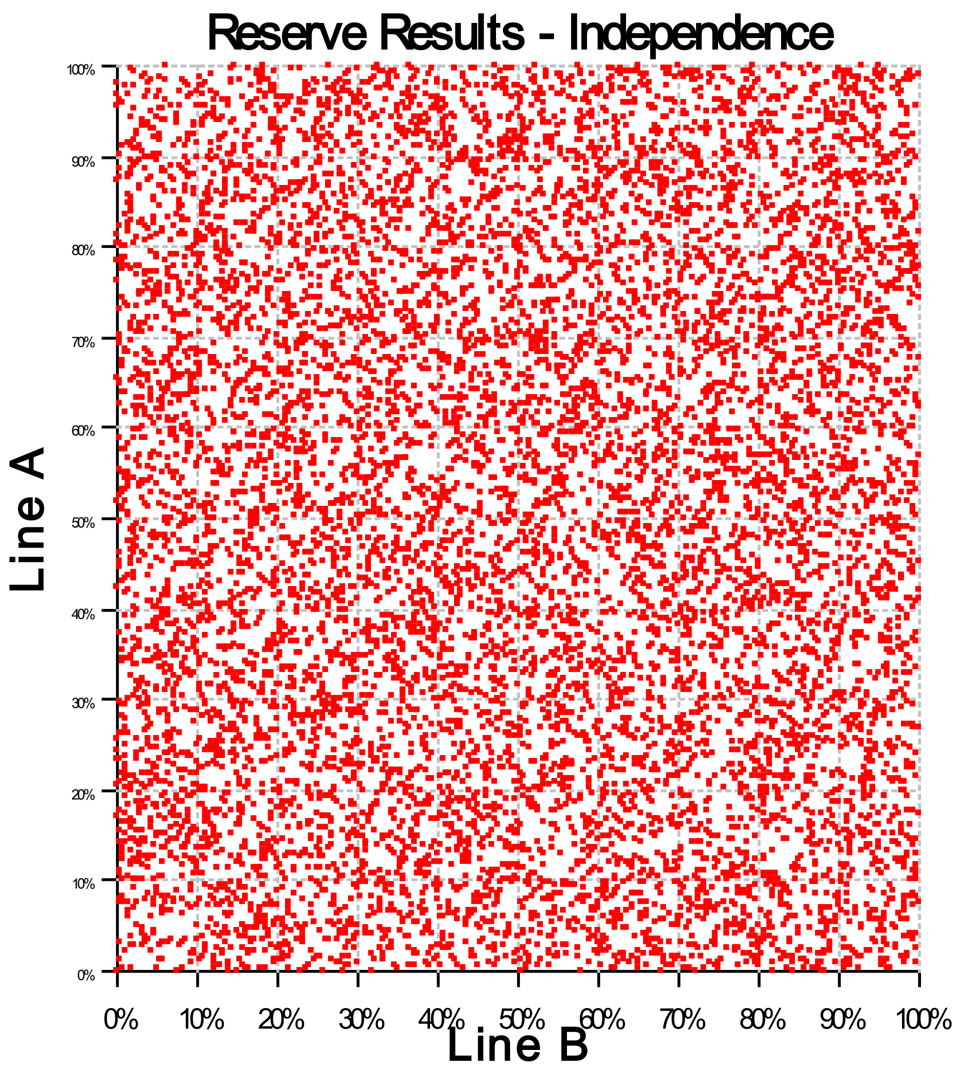
Sim	Line A	Line B	Total
1	10	15	27
2	14	21	37
3	18	12	22
4	16	24	42
5	12	18	32

Dependency assumption changes prediction error →

Mean	32
St Dev	7.91

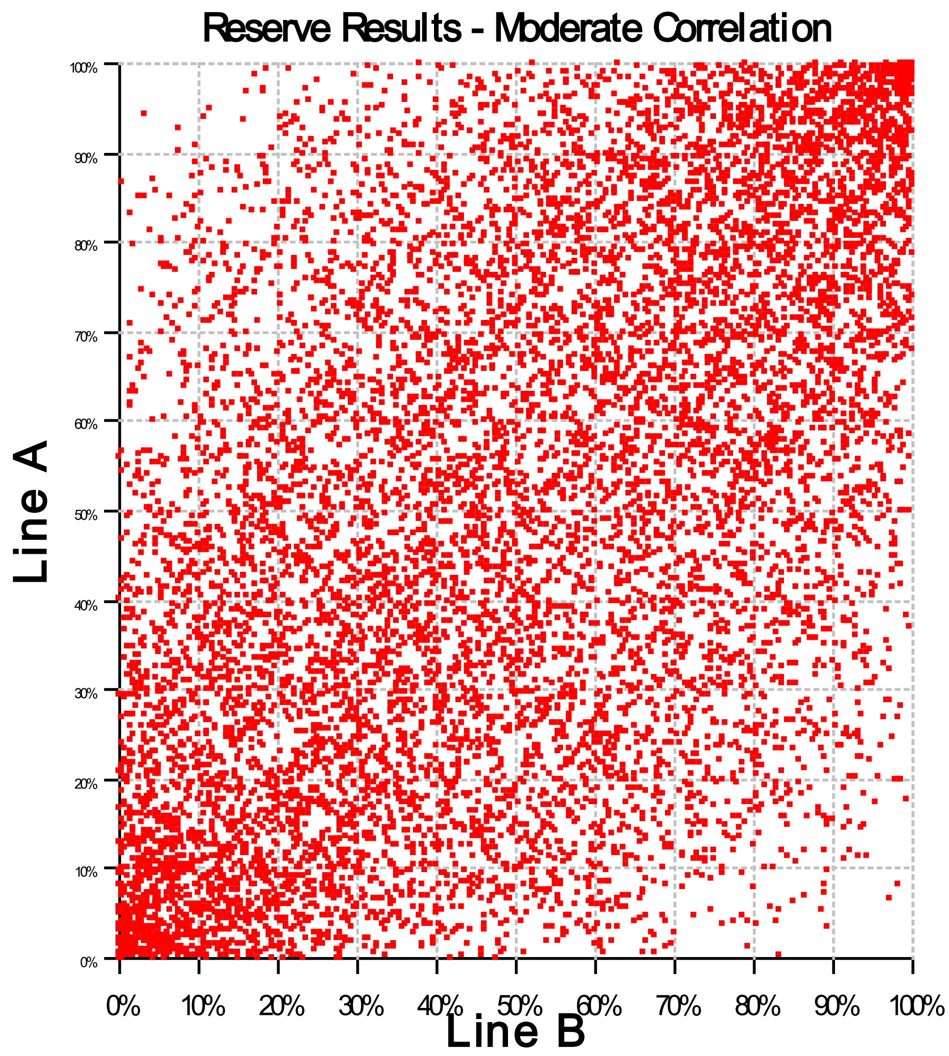
Aggregation

Rank Analysis



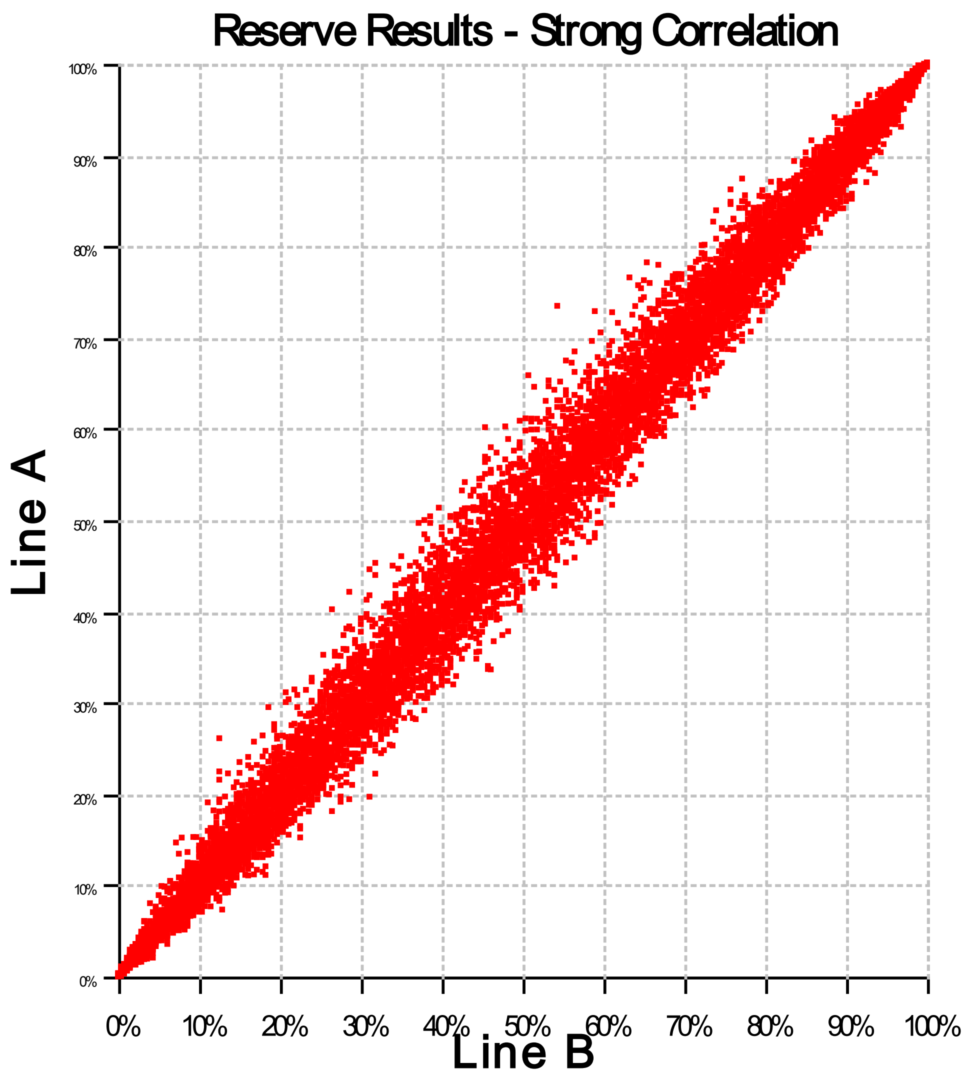
Aggregation

Rank Analysis



Aggregation

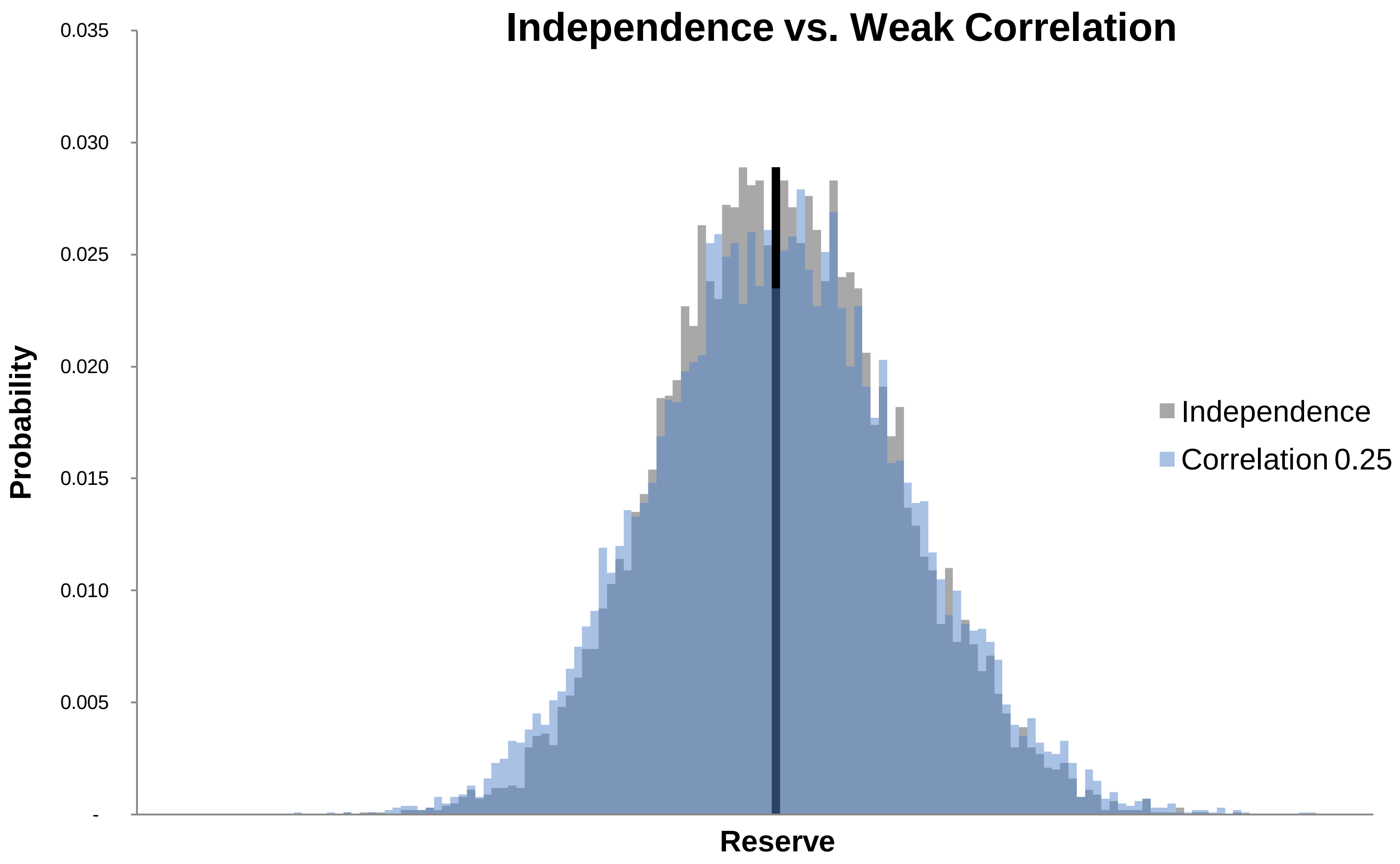
Rank Analysis



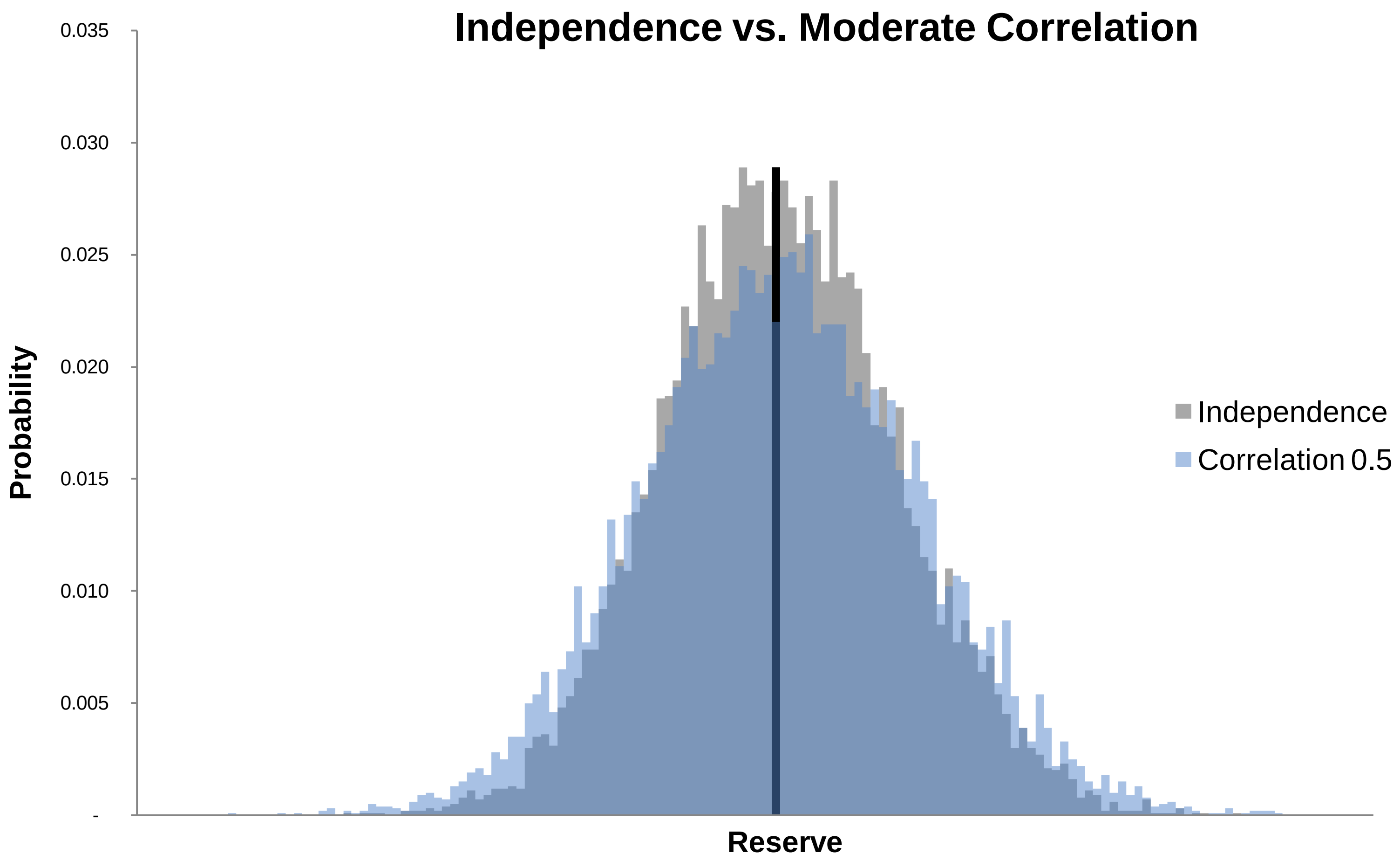
Aggregation

Sensitivity Testing

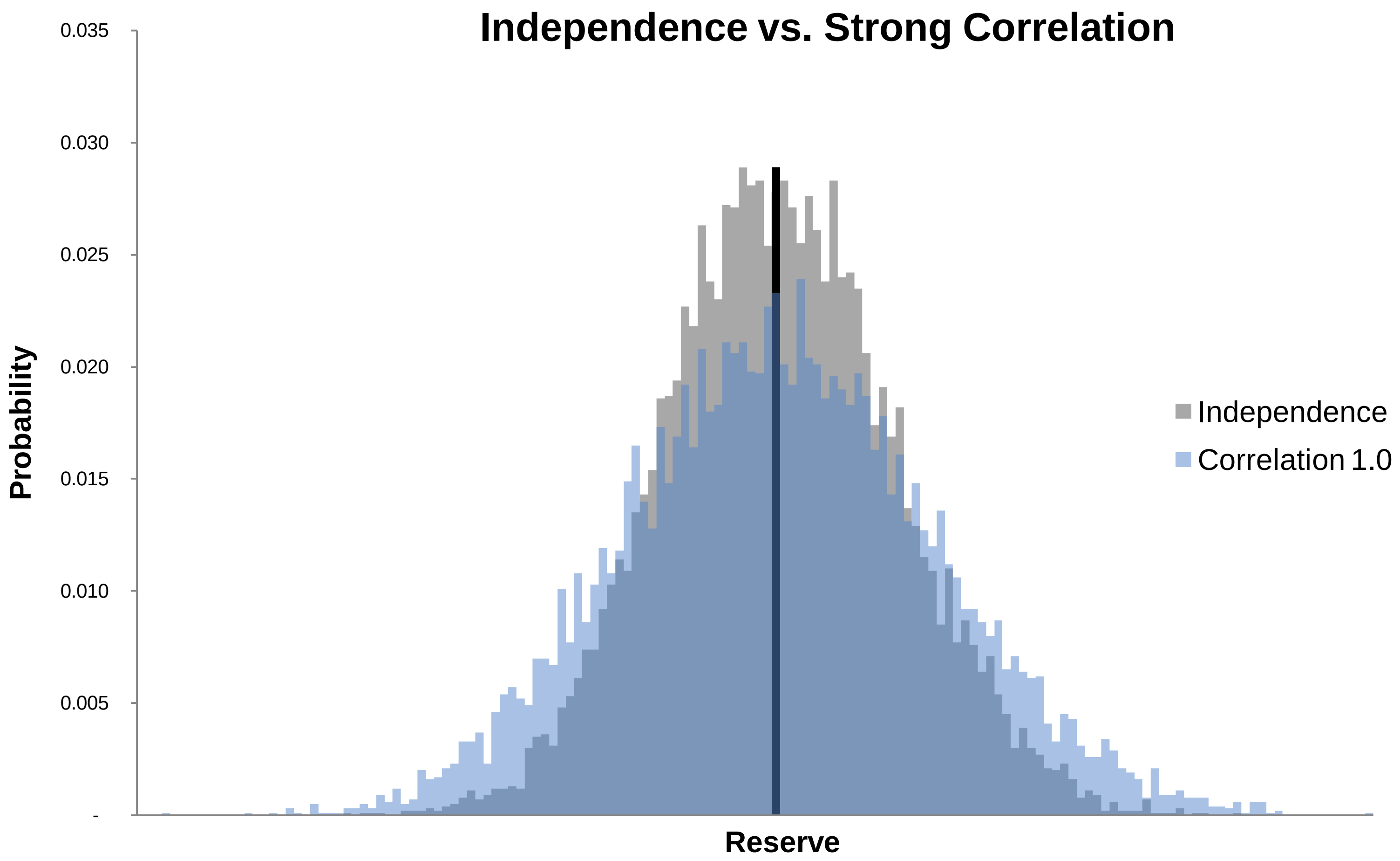
Aggregation
Sensitivity Testing



Aggregation
Sensitivity Testing



Aggregation
Sensitivity Testing



Aggregation
Dependency Analysis

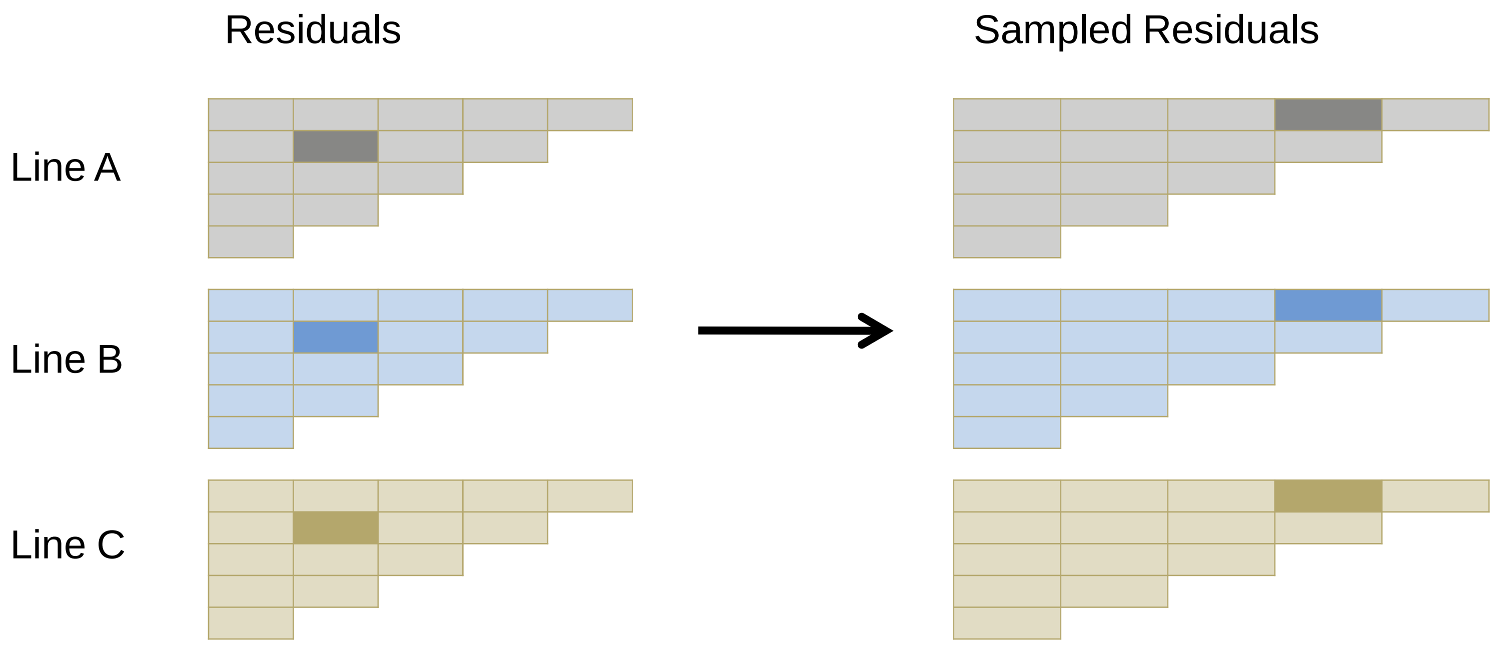
Dependency	Mean	60th %ile	75th %ile	99th %ile
Independence	151,890	155,071	160,548	183,458
Weak Correlation	151,890	155,363	161,376	185,999
Moderate Correlation	151,890	155,559	162,409	188,680
Strong Correlation	151,890	156,111	163,569	194,318

Considerations

- Dependency assumption becomes increasingly important as measurement moves into the tails of the distribution
- As more segments are aggregated, the effect is magnified
- As more volatile segments are aggregated, the effect is magnified
- Copulas allow us to change the shape of the dependency if necessary
- Stress testing and understanding sensitivity of overall reserves to dependency assumption is important
- Can be used to aggregate disparate reserve models, not just bootstrap

Aggregation
Simultaneous Bootstrap

- Simultaneous bootstrap is an alternative to explicitly defining a dependency structure
- When sampling from residuals, take them from the same location on the original triangle of residuals and place them in a consistent location in the new triangle
- This keeps dependency structure from historical triangles intact in bootstrapped pseudo triangles
- Utilizes historical dependency structure for generating forecast parameters



Aggregation
Simultaneous Bootstrap

Key Advantage

- Eliminates the need to define an explicit dependency structure

Considerations

- Method is only as good as the weakest triangle
- Missing data or excluded data from any triangle must be excluded from all triangles
- Since dependency is automatically handled there is less transparency into data structure
- However, one could examine residuals directly and determine implied pairwise correlations
- Beware of company actions that may have affected historical data which may not be true dependency

Final Thoughts

- Reserve variability models are useful tools
- Understand model assumptions and limitations
- Spend time testing and validating models where possible
- Sensitivity test – be aware of the impact of your parameter estimates

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