Fall 2009 SWAF meeting

Collateralized Debt Obligations

Subprime Meltdown

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Financial Engineering 101

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Investment Bankers Gone Wild

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Where did the trillion dollars go?

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The Mathematics of the Subprime Meltdown

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Zero-coupon Bonds

These are very simple securities that pay a fixed amount of money after a fixed amount of time.

For example: a $100 one-year, zero-coupon bond pays exactly $100 in exactly one year.

If the payment is certain, we call the bond risk-free; if it is uncertain, we call the bond risky.

Zero-coupon Bonds

Our example: a $100 one-year, zero-coupon bond pays exactly $100 in exactly one year.

Investors might pay $97 for such a bond today if it is risk-free. (US government bonds are usually considered to be risk-free.)

Zero-coupon Bonds

Our example: a $100 one-year, zero-coupon bond pays exactly $100 in exactly one year.

Suppose that there is a 90% chance that the bond pays $100 and a 10% chance that it pays nothing.

Investors might pay $85 for such a bond today.

Our examples

Risk-free

$100 in one year (certain)

Cost today: $97

Yield: 3.1%

=100/97 - 1

Risky

$100 with 90% probability

$0 with 10% probability

Expected payoff: $90

Cost today: $85

Expected yield: 5.9%

= 90/85 - 1

Risk-load

The difference between 3.1% and 5.9% is called the risk-load.

It is the extra amount of compensation that investors require in order to bear the risk of losing their bet (and getting 0 back).

The marketplace sets the risk-load.

Mortgages

Potential homeowners seldom have enough cash to purchase their homes outright. Instead they go to a bank and borrow money promising to pay the bank back with interest over time.

Typically mortgages have monthly payments.

Mortgages can be fixed-rate or variable-rate (adjustable-rate).

Mortgages (cont.)

Perhaps the simplest mortgage is the "standard" 30-year fixed-rate mortgage.

The monthly payment is always the same. First, the interest on the outstanding balance is paid, then the remainder is applied to lower the outstanding balance. The payment is such that exactly 360 payments are needed to reduce the outstanding balance to zero.

Mortgages (cont.)

Example: 6% fixed-rate, 30-year mortgage for $100,000 principal:

Payment amount: $599.55

First payment: Interest=$500.00

First payment: Principal=$99.50

181st payment: Interest=$355.25

181st payment: Principal=$244.30

Mortgages are complex

Way too complex to include in our talk

Instead we will use simple zero-coupon bonds

These will illustrate the important points without the unnecessary complications that more realistic mortgages would introduce.

Mutual funds

Investors pool their money
- Say, 10 investors contribute \$500 each.

They purchase a pool of assets
- \$5,000 worth, each owns 10% of the whole

Each investor has a pro-rata share of the result

Ignoring transaction costs, each investor could do this on their own if they wanted to

New Idea: Senior/Subordinated

We have two classes of investors:
- Senior investors: get paid back first
- Subordinated investors: get paid second

In the real world there are often more than 2.

The final tranche (last to get paid back) is called the equity tranche.

Our example

One-year, zero-coupon bonds are available for a price of 85. They have a 90% chance of maturing at 100 and a 10% chance of being worthless.

We will purchase 100 of these. To do that we need $8,500.

Where to get $8,500?

We will sell some bonds to investors and put in the additional money ourselves.

What is the probability that 22+ of the 100 bonds default? Each has a 10% chance.

If they are independent, the probability is less than 0.01%

Where do we get $8,500?

We know that investors will pay 97 for a risk-free one-year bond. Suppose that they will pay 96 for a bond with a 0.01% chance of default.

We can sell 78 of these bonds at 96 each and raise 7,488. We need to supply $1,012 to get the $8,500.

What do we get for our $1,012?

We will have the right to all of the proceeds from the 100 bonds after we pay off the senior debt. We will owe them $7,800, because we sold 78 bonds.

The expected value of the portfolio is $9,000.

So our expected return is $1,200 on an investment of $1,012.

That is an expected return of 18.6%.

What do we make on our $1,012?

Our <u>expected</u> return is 18.6%.

Our actual return will be very different.

If 12 or more of the bonds default we will actually lose money. Why? Because we have to pay the senior lenders $7,800 and if exactly 12 bonds default we will have only $8,800. In that case we would lose $12.

(Remember that we put in $1,012, so if we only get $1,000 back ....)

Some probabilities

Assuming independence …

Probability of 12 or more losses is 29.7%

This is the probability that we lose money.

Probability of 10 or fewer losses is 58.3%

This is the probability that we actually achieve at least our expected return.

Bigger Projects

We would like to expand the program.

New program is to be 10 times bigger.

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All dollar amounts would be multiplied by 10.

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There is a better way!

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We purchase 1,000 different bonds each for 85 instead of 100 different bonds each for 850.

Our cost is the same either way: $85,000.

Remember our Senior lenders?

They will pay $96 for a zero-coupon bond that pays them $100 with probability 99.99%

Initially, we could sell them 78 bonds because the probability that we have more than 22 defaults out of 100 was less than 99.99%.

Increasing the dollar amounts by a factor of ten did not increase how many they would buy.

Watch what happens when we increase the number of bonds

The probability of 137 or more bonds out of 1,000 defaulting is less than 0.01%

That means that we can sell the senior lenders 864 bonds for $96 each, raising $82,944

We only need $85,000, so we have to contribute $2,056. Our project is 10 times larger, but our contribution is barely twice as much!

Watch what happens when we increase the number of bonds

Our expected return is enhanced also.

We expect to receive $90,000. We will need to pay the senior lenders $86,400, leaving us with 3,600.

E(return) = 3,600/2,056 – 1 = 75.1% (!)

There’s more!

Suppose we increase the number of bonds to 2,000. Now we can sell 1,750 bonds to the senior lenders and we need to contribute:

2000(85)-1750(96) = $2,000.

Notice that this is less than the $2,056 we had to come up with to do a 1000 bond deal.

I-Banker's Nirvana

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6000(85)-5314(96) = -$144.

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We collect $144 more from the senior lenders than we need to buy the collateral ….

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We collect $144 more from the senior lenders than we need to buy the collateral ….

This is better than no money down ….

We get change back!

And at the end of the year we expect to receive $540,000 and pay out $531,400

So we expect to pocket an additional $8,600!

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Mortgages are not independent

- Macro-economic factors such as interest rates
- Regional factors such as property values
- Local factors such as large employer layoffs
- Deal specific problems such as servicers

Correlation

- One way to measure how pairs of random variables co-vary
- Correlation is a number between -1 and 1
- A correlation value near the extremes (-1 and 1) means that the two variables are nearly co-monotonic.

Monte Carlo Simulation

- Generates many possible future states of the world, say 1 million states
- If all the possible states are equally likely, it is easy to compute probabilities from this randomly generated sample.
- For example, if in our 1 million states we lose money in 50,000 of them, then we would conclude that the probability of losing money is about 50,000/1,000,000 = 5%

Gaussian Copula

- Correlation is implemented in Monte Carlo simulations in different ways. One common way is to use a Gaussian copula
- Gaussian copula is obtained from a multi-normal random variable.
- Multi-normal random variables are completely defined by specifying the means and the covariance matrix

Generating multi-normal RVs

- A valid covariance matrix is symmetric and positive semi-definite
- Has a LL^T decomposition
- Decomposition can be obtained through Cholesky decomposition

Alternative Method

- Regime shifting
- Suppose that our 10% probability of loss is really ….
  - A 9% probability of loss 90% of the time and
  - A 12% probability of loss 10% of the time
- Such a distribution is called a <u>mixture</u>.
- The 90%/10% random variable is called the mixing variable.
- We can impose correlation by using a common mixing variable.

What if it's 12% instead of 10%?

- With 100 bonds the senior lenders were willing to take the last 78 losses ($Pr<0.01\%$)
- At 12% their probability of loss is about 0.15%

What if it's 12% instead of 10%?

- With 100 bonds the senior lenders were willing to take the last 78 losses (Pr<0.01%)
- At 12% their probability of loss is about 0.15%

- With 1,000 bonds the senior lenders were willing to take the last 864 losses (Pr<0.01%)
- At 12% their probability of loss is about 5.62%

What if it’s 12% instead of 10%?

- With 2,000 bonds the senior lenders were willing to take the last 1,750 losses (Pr<0.01%)
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- With 2,000 bonds the senior lenders were willing to take the last 1,750 losses (Pr<0.01%)
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- With 6,000 bonds the senior lenders were willing to take the last 5,314 losses (Pr<0.01%)
- At 12% their probability of loss is about ….

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- With 2,000 bonds the senior lenders were willing to take the last 1,750 losses (Pr<0.01%)
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- With 6,000 bonds the senior lenders were willing to take the last 5,314 losses (Pr<0.01%)
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90.9% !!