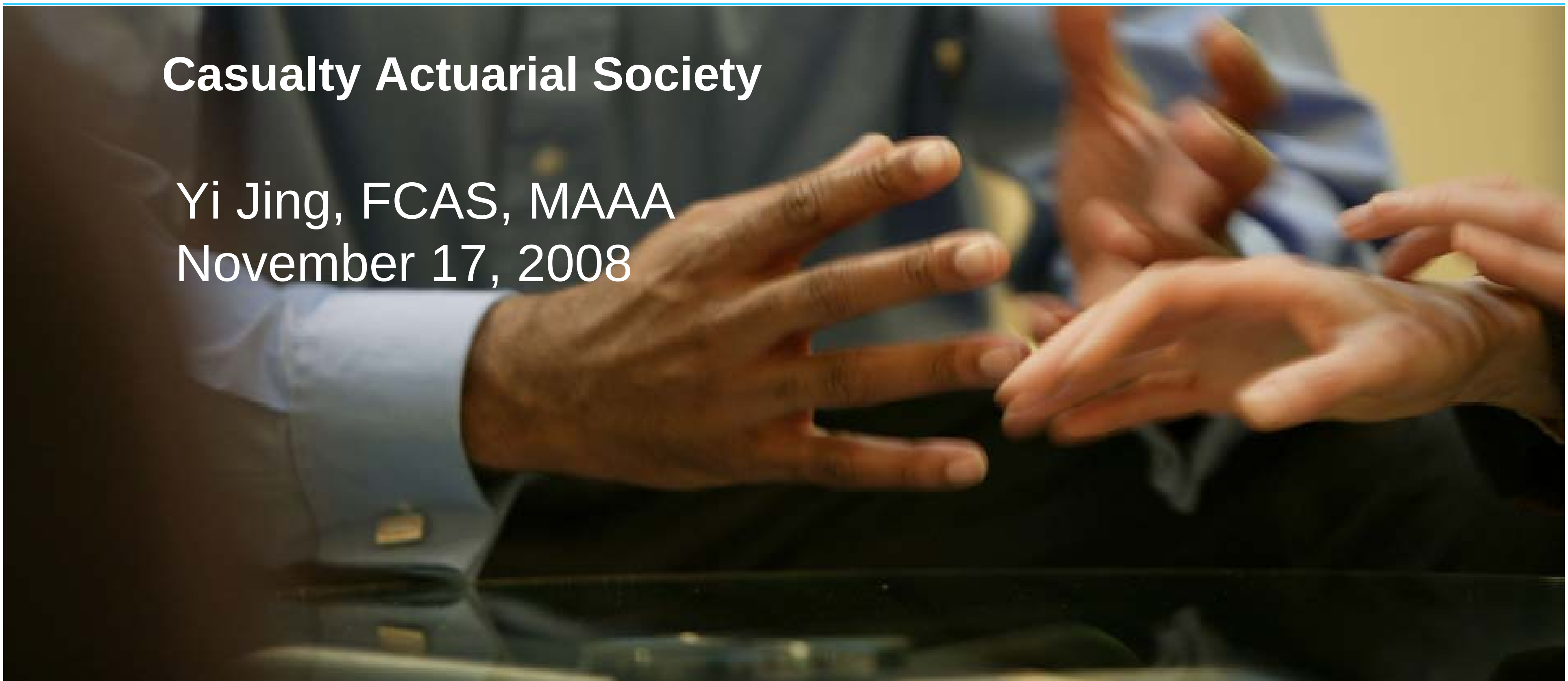
Risk Horizon and the Measurement of General Insurance Risk



Casualty Actuarial Society

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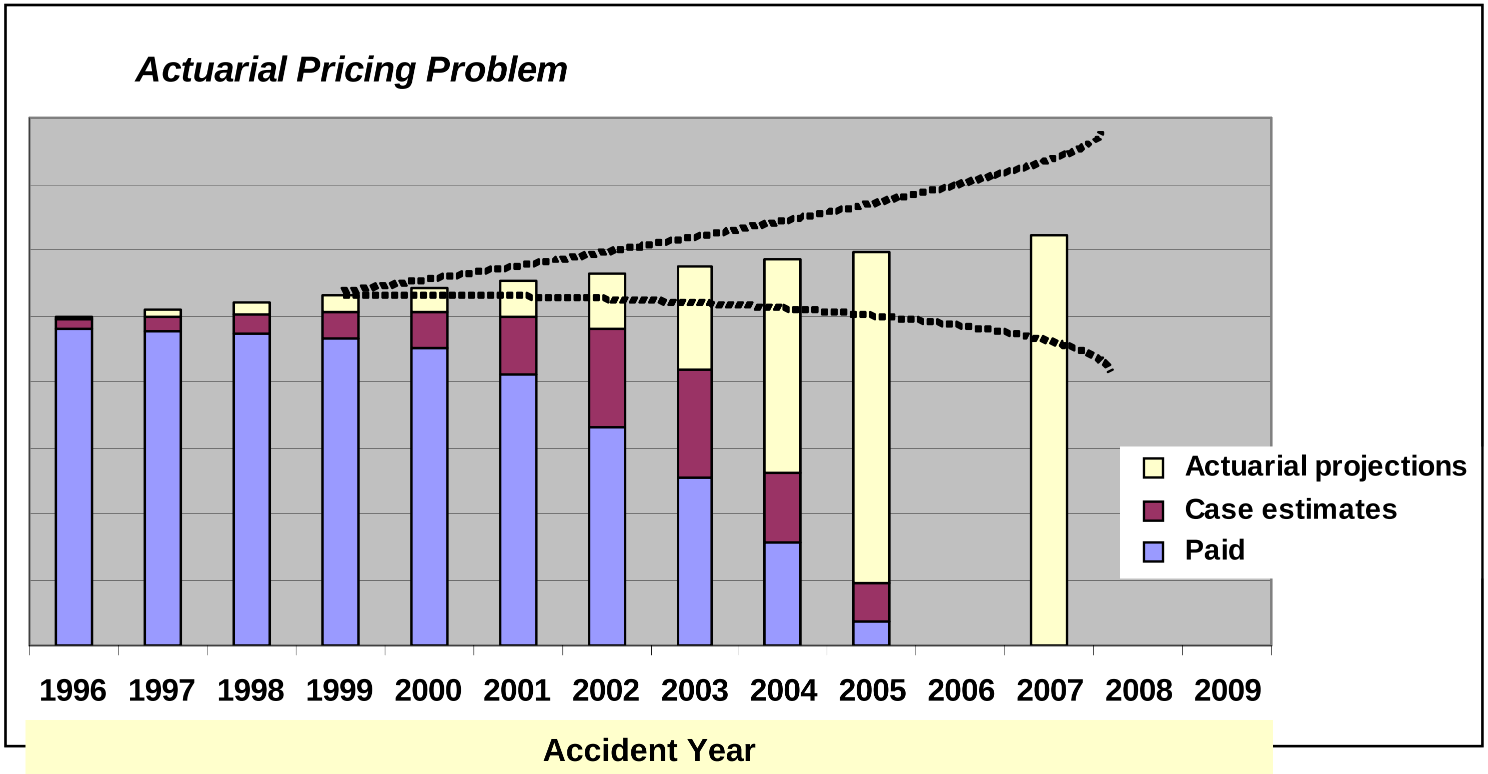
Today's Agenda

- Definition of risk horizon
- Methods of assessing insurance risk over each risk horizon
- Risk horizon in the context of economic capital (EC)
- Pros and Cons of two alternative risk horizons

- This presentation is based on the paper "Risk Horizon and the Measurement of General Insurance Risk", authored by Yi Jing, Stephen Lowe, and François Morin, that has been submitted for publication in ASTIN Bulletin.

Definition

General insurance risk stems from the uncertainty of estimated future claim payments



Each risk horizon asks a different question

- **In the context of a run-off risk horizon:**

  What is the potential adverse variation in the ultimate cost of the claim liabilities (net of any offsetting premium or tax benefit) from the current actuarial central estimate?

- **In the context of a one-year risk horizon:**

  What is the potential adverse change in the actuarial central estimate of ultimate claim costs (net of any offsetting premium or tax benefit) that could occur with the benefit of one additional year of actual claim emergence and other relevant information?

  - One-year risk horizon introduces additional complexity, requires us to go beyond estimating uncertainty of estimates; to measure risk over a one-year horizon, we must also model how the uncertainty resolves over time and in particular how much of the uncertainty resolves during the upcoming year.

Formal definitions of run-off and one-year risk horizons

- $C_{a,d}$ = actual incremental paid claims on the a^{th} accident year in d^{th} development period

$$\begin{bmatrix} C_{1,0} & C_{1,1} & C_{1,2} & C_{1,3} & C_{1,4} \\ C_{2,0} & C_{2,1} & C_{2,2} & C_{2,3} & \\ C_{3,0} & C_{3,1} & C_{3,2} & & \\ C_{4,0} & C_{4,1} & & & \\ C_{5,0} & & & & \end{bmatrix}$$

- $\hat{C}^{(t)}_{a,d}$ = estimate of the expected incremental paid claims on accident year a in development period d
- $L^{(t)} = \sum_{a,d} (v_{a+d-1-t})(C^{(t)}_{a,d})$ for all $a+d-1>t$
- $\hat{L}^{(t)} = \sum_{a,d} (v_{a+d-1-t})(\hat{C}^{(t)}_{a,d})$ for all $a+d-1>t$
- Under run-off risk horizon, $e_r = \hat{L}^{(t)} - L^{(t)}$
- Under one-year risk horizon, $e_1 = \hat{L}^{(t)} - v_1(\sum_{a,d} C_{a,d} + \hat{L}^{(t+1)})$

$C_{a,d}$ are the $a+d-1=t+1$ diagonal of payments in the next calendar year

Example of calculating run-off and one-year reserve risk - Data

Commercial Auto Liability
Schedule P Part 2

	Net Ultimate Loss & ALAE Reported at Each Maturity									
Accident Year	12	24	36	48	60	72	84	96	108	120
1983	51,067	69,925	70,174	72,533	72,808	72,342	71,901	72,169	72,126	71,852
1984	96,872	97,380	102,019	102,075	101,734	103,337	102,539	103,560	103,453	103,552
1985	115,045	118,884	122,359	121,038	122,034	122,345	122,477	122,973	122,945	123,297
1986	139,963	145,364	144,324	146,321	147,301	149,175	149,751	149,666	150,036	150,326
1987	157,723	164,069	166,012	168,934	171,807	171,388	171,357	172,264	172,308	172,408
1988	172,262	180,112	187,158	188,775	190,569	190,701	190,344	191,752	192,087	194,099
1989	192,154	198,550	196,977	197,877	198,265	198,957	200,108	200,044	201,101	201,380
1990	215,766	218,769	222,694	225,682	226,110	226,247	226,698	226,739	226,744	226,263
1991	208,425	207,219	207,811	211,186	210,054	208,456	207,581	207,273	206,990	207,092
1992	216,172	217,665	216,400	214,510	212,937	214,979	214,145	214,513	214,899	215,141
1993	237,378	237,679	237,253	233,699	231,958	232,944	233,677	233,264	233,786	233,698
1994	249,592	248,849	242,493	243,140	242,907	244,737	244,854	243,805	243,678	244,692
1995	256,434	249,225	238,468	236,970	239,422	241,843	241,497	240,784	241,707	241,793
1996	237,860	227,353	231,394	232,722	238,184	240,732	240,776	238,824	238,056	237,781
1997	220,029	226,968	228,101	230,540	232,679	234,313	233,572	232,942	231,590	232,101
1998	139,047	140,490	147,785	151,858	154,989	156,396	156,903	157,870	158,584	158,446
1999	134,725	144,002	151,640	155,435	156,787	157,243	158,179	159,058	159,334	160,071
2000	139,275	153,675	161,654	168,684	171,068	173,890	175,689	175,508		
2001	140,194	146,159	145,503	147,129	150,249	152,184	151,727			
2002	158,856	160,571	164,819	174,470	177,425	178,957				
2003	153,638	161,267	169,677	180,888	183,939					
2004	145,570	161,958	172,548	177,466						
2005	159,494	182,822	206,850							
2006	174,143	190,583								
2007	181,760									

Example of calculating run-off and one-year reserve risk (cont'd)

Run-off Errors - Expressed as Percent of Current Estimate

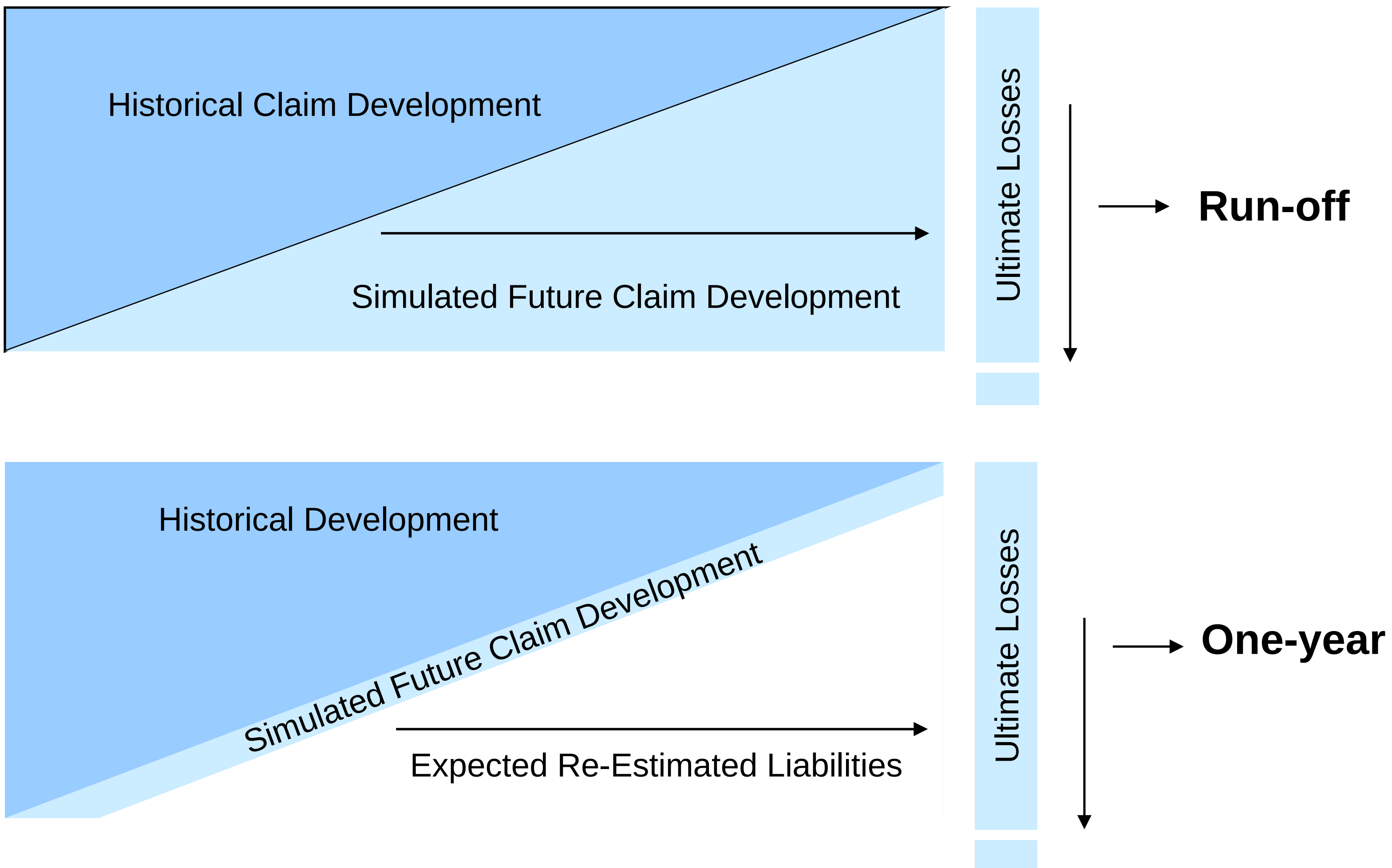
Accident Year	12	24	36	48	60	72	84	96	108	120
1983	40.70%	2.76%	2.39%	-0.94%	-1.31%	-0.68%	-0.07%	-0.44%	-0.38%	
1984	6.90%	6.34%	1.50%	1.45%	1.79%	0.21%	0.99%	-0.01%	0.10%	
1985	7.17%	3.71%	0.77%	1.87%	1.03%	0.78%	0.67%	0.26%	0.29%	
1986	7.40%	3.41%	4.16%	2.74%	2.05%	0.77%	0.38%	0.44%	0.19%	
1987	9.31%	5.08%	3.85%	2.06%	0.35%	0.60%	0.61%	0.08%	0.06%	
1988	12.68%	7.77%	3.71%	2.82%	1.85%	1.78%	1.97%	1.22%	1.05%	
1989	4.80%	1.43%	2.24%	1.77%	1.57%	1.22%	0.64%	0.67%	0.14%	
1990	4.86%	3.43%	1.60%	0.26%	0.07%	0.01%	-0.19%	-0.21%	-0.21%	
1991	-0.64%	-0.06%	-0.35%	-1.94%	-1.41%	-0.65%	-0.24%	-0.09%	0.05%	
1992	-0.48%	-1.16%	-0.58%	0.29%	1.04%	0.08%	0.47%	0.29%	0.11%	
1993	-1.55%	-1.67%	-1.50%	0.00%	0.75%	0.32%	0.01%	0.19%	-0.04%	
1994	-1.96%	-1.67%	0.91%	0.64%	0.73%	-0.02%	-0.07%	0.36%	0.42%	
1995	-5.71%	-2.98%	1.39%	2.04%	0.99%	-0.02%	0.12%	0.42%	0.04%	
1996	-0.03%	4.59%	2.76%	2.17%	-0.17%	-1.23%	-1.24%	-0.44%	-0.12%	
1997	5.49%	2.26%	1.75%	0.68%	-0.25%	-0.94%	-0.63%	-0.36%	0.22%	
1998	13.95%	12.78%	7.21%	4.34%	2.23%	1.31%	0.98%	0.36%	-0.09%	
1999	18.81%	11.16%	5.56%	2.98%	2.09%	1.80%	1.20%	0.64%	0.46%	
Mean	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	
Std Dev	13.04%	5.61%	3.17%	2.10%	1.39%	0.96%	0.83%	0.48%	0.35%	
Security %tile	99.60%	99.60%	99.60%	99.60%	99.60%	99.60%	99.60%	99.60%	99.60%	
Needed Asset Ratio	139.9%	115.8%	108.7%	105.7%	103.7%	102.6%	102.2%	101.3%	100.9%	

Example of calculating run-off and one-year reserve risk (cont'd)

One-year Errors - Expressed as Percent of Current Estimate of Ultimate

Accident Year	12	24	36	48	60	72	84	96	108	120
1983	36.93%	0.36%	3.36%	0.38%	-0.64%	-0.61%	0.37%	-0.06%	-0.38%	
1984	0.52%	4.76%	0.05%	-0.33%	1.58%	-0.77%	1.00%	-0.10%	0.10%	
1985	3.34%	2.92%	-1.08%	0.82%	0.25%	0.11%	0.40%	-0.02%	0.29%	
1986	3.86%	-0.72%	1.38%	0.67%	1.27%	0.39%	-0.06%	0.25%	0.19%	
1987	4.02%	1.18%	1.76%	1.70%	-0.24%	-0.02%	0.53%	0.03%	0.06%	
1988	4.56%	3.91%	0.86%	0.95%	0.07%	-0.19%	0.74%	0.17%	1.05%	
1989	3.33%	-0.79%	0.46%	0.20%	0.35%	0.58%	-0.03%	0.53%	0.14%	
1990	1.39%	1.79%	1.34%	0.19%	0.06%	0.20%	0.02%	0.00%	-0.21%	
1991	-0.58%	0.29%	1.62%	-0.54%	-0.76%	-0.42%	-0.15%	-0.14%	0.05%	
1992	0.69%	-0.58%	-0.87%	-0.73%	0.96%	-0.39%	0.17%	0.18%	0.11%	
1993	0.13%	-0.18%	-1.50%	-0.74%	0.43%	0.31%	-0.18%	0.22%	-0.04%	
1994	-0.30%	-2.55%	0.27%	-0.10%	0.75%	0.05%	-0.43%	-0.05%	0.42%	
1995	-2.81%	-4.32%	-0.63%	1.03%	1.01%	-0.14%	-0.30%	0.38%	0.04%	
1996	-4.42%	1.78%	0.57%	2.35%	1.07%	0.02%	-0.81%	-0.32%	-0.12%	
1997	3.15%	0.50%	1.07%	0.93%	0.70%	-0.32%	-0.27%	-0.58%	0.22%	
1998	1.04%	5.19%	2.76%	2.06%	0.91%	0.32%	0.62%	0.45%	-0.09%	
1999	6.89%	5.30%	2.50%	0.87%	0.29%	0.60%	0.56%	0.17%	0.46%	
2000	10.34%	5.19%	4.35%	1.41%	1.65%	1.03%	-0.10%			
2001	4.25%	-0.45%	1.12%	2.12%	1.29%	-0.30%				
2002	1.08%	2.65%	5.86%	1.69%	0.86%					
2003	4.97%	5.21%	6.61%	1.69%						
2004	11.26%	6.54%	2.85%							
2005	14.63%	13.14%								
2006	9.44%									
Mean	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	
Std Dev	9.57%	4.33%	2.65%	1.25%	0.91%	0.45%	0.48%	0.29%	0.35%	
Security %tile	99.60%	99.60%	99.60%	99.60%	99.60%	99.60%	99.60%	99.60%	99.60%	
Needed Asset Ratio	128.2%	112.1%	107.3%	103.4%	102.4%	101.2%	101.3%	100.8%	100.9%	

The two risk horizons require different simulation models



Stochastic methods of calculating run-off and one-year insurance risk

- Mack-Murphy
  - Run-off: Mack paper (1993) and Murphy paper (1994)
  - One-year:
    - Define the **change in the estimate** as $\Delta\hat{C}_{iJ}^{(1)} \equiv \hat{C}_{iJ}^{(1)} - \hat{C}_{iJ}^{(0)}$
    - Define the **change in the noise** as $\Delta\varepsilon_{iJ}^{(1)} \equiv \varepsilon_{iJ}^{(1)} - \varepsilon_{iJ}^{(0)}$
    - Define the calendar year mean square error (CYMSE) as the sum of two components: the variance arising from the change in noise plus the variance arising from the change in the estimate $CYMSE_{ij}^{(1)} = Var(\Delta\varepsilon_{ij}^{(1)}) + Var(\Delta\hat{C}_{ij}^{(1)})$
    - The paper *Uncertainty of the Claims Development Result in the Chain Ladder Method* (2007) by Wuthrich, Merz, and Lysenko analyzes the MSE of this change based on Mack's approach, but with a slightly stronger variance assumption

Stochastic methods of calculating run-off and one-year insurance risk – cont'd

- Bootstrap
  - A simulation approach
    - — generates empirical probability distributions of complex functions
    - — utilizes the sampling-with-replacement technique on the residuals of the historical data
    - — each simulated sampling scenario produces a new "realization" of triangular data that has the same statistical characteristics as the actual data
    - — one-year risk is measured by simultaneously simulating the prospective diagonal and reapplying the method
- Practical/Structural (a forthcoming paper by Gault, Llaguna and Lowe)
  - A Monte-Carlo simulation approach
    - — takes out the impact of inflation from the historical loss development triangle before simulating the RTR factors
    - — adjusts the future loss payment by accident year by calendar year with the simulated inflation simulated from an economic scenario generator
    - — uses ARMA model to estimate the new accident year's loss ratio
    - — one-year risk is measured by reapplying the method to the simulated new diagonal

One-year vs. Run-off reserve risk for commercial auto liability

Method	One-year				Run-off		
	Mean	CV	99%	99th Ratio to Mean	CV	99%	99th Ratio to Mean
Mack Paid	464	3.7%	506	108.9%	3.8%	506	109.1%
Mack Reported	302	5.6%	344	113.8%	5.8%	345	114.2%
Bootstrap Paid	354	5.7%	403	113.8%	6.9%	415	117.2%
Bootstrap Reported	347	4.5%	388	111.8%	4.7%	387	111.5%
P/S Paid	345	4.2%	379	109.8%	5.2%	388	112.3%
P/S Reported	356	4.6%	395	110.8%	5.4%	402	112.8%

Each risk horizon asks a different question in the context of economic capital

- **In the context of a run-off risk horizon:**

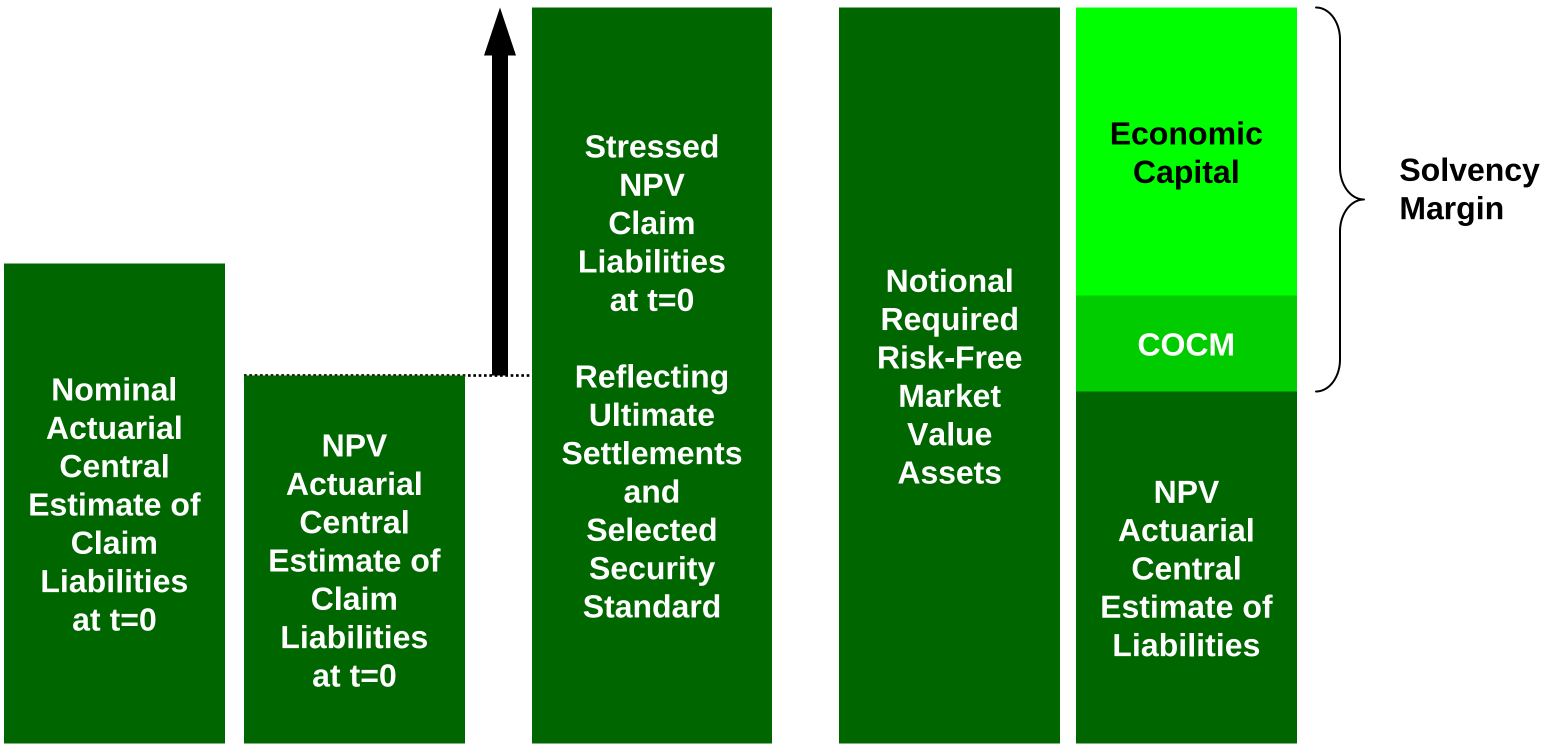
  What is the potential economic capital that could be consumed by adverse development in the net present value cost of the claim liabilities (net of offsetting premium or tax benefits) as they run off and are ultimately paid?

- **In the context of a one-year risk horizon:**

  What is the potential economic capital that could be consumed by adverse development in the market-consistent value of the claim liabilities (net of offsetting premium or tax benefits) over the course of one year?

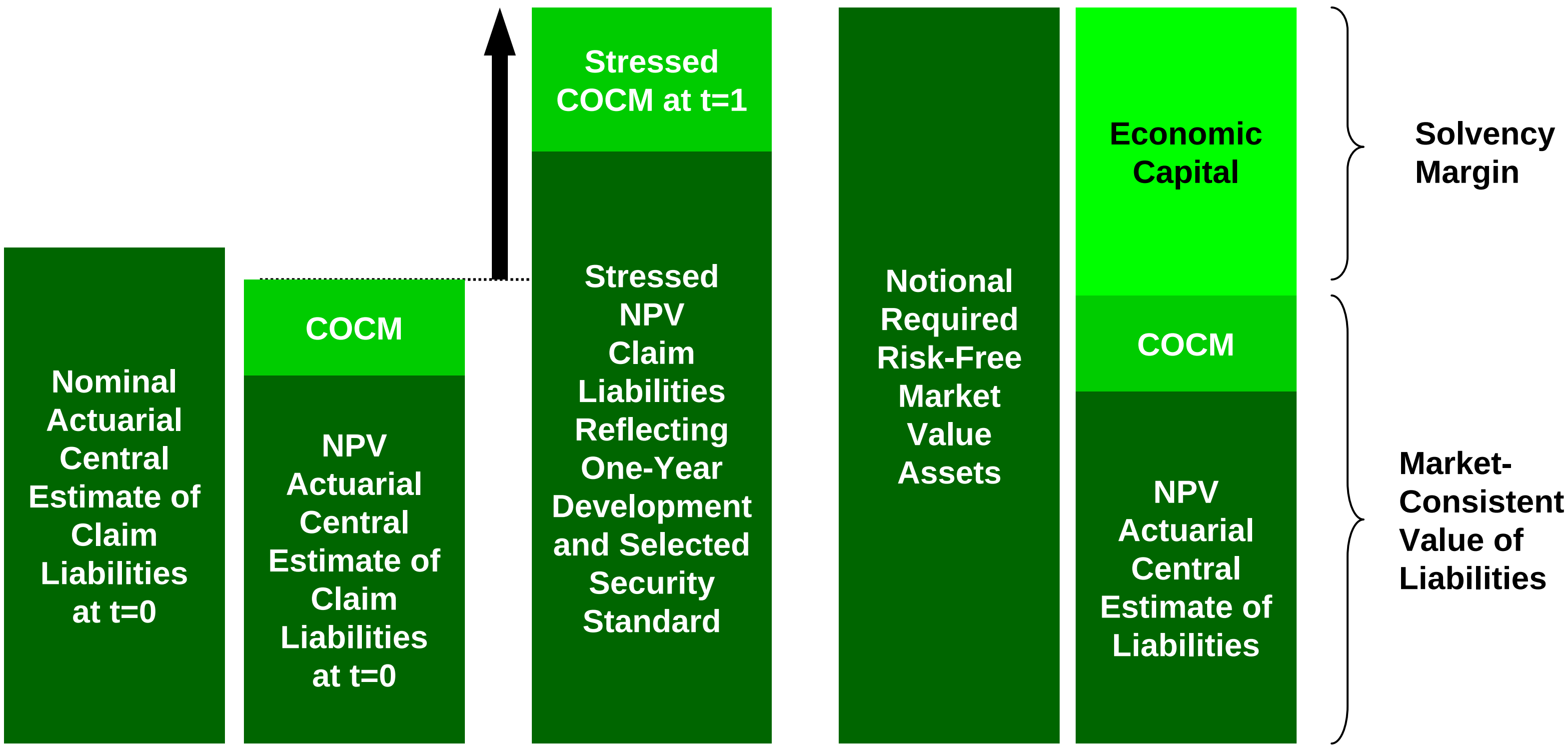
In the run-off view, the claim liabilities are stressed to determine the solvency margin

- Since there is a presumption is that the liabilities will be held to maturity, all assets in excess of the liabilities are available as a solvency margin



In the one-year view, the market-consistent value of the liabilities are stressed to determine the solvency margin

- Since there is a presumption that the liabilities will be transferred, only the assets in excess of the market-consistent value of the liabilities are available as a solvency margin



Basic principles underlying the cost-of-capital margin

- **The cost-of-capital margin should capture only**
  - **The EC for non-hedgeable risk (EC-NHR)**
    - — Life mortality, morbidity, policyholder behavior
    - — P&C and Life catastrophe risk
    - — P&C non-catastrophe claims risk
    - — P&C and Life reinsurance counterparty risk
    - — Net of diversification across these risks
    - — Including provision for operational risk
  - **The component of the required return that rewards the shareholder for bearing non-hedgeable risk**
    - — Not the same as total shareholder return, as this includes growth in intangible franchise value
    - — Not equivalent to target ROE or WACC
      - – Excludes return on hedgeable risks (market risks, taken up within risk-neutral scenarios)
      - – Excludes return due to distribution and brand value
      - – Does include frictional costs – double taxation, financial distress, and agency

Calculating the cost-of-capital margin

- Basic calculation steps
  1. Project the expected liabilities (from the run-off models)
  2. Determine the EC for non-hedgeable risk (net of diversification)
  3. Ratio the EC-NHR to the current liabilities; use that ratio to project future EC-NHR requirements
  4. Apply the cost-of-capital rate to the projected EC-NHR requirements to obtain projected capital costs
  5. Discount the projected capital costs to present value using the swap curve
- The cost-of-capital rate
  - CRO Forum analysis indicates cost-of-capital rate between 2.5% and 7%, depending on method
  - CRO Forum recommends rate between 2.5% and 4.5%

There are two divergent views on the appropriate risk horizon for economic capital

Horizon	Where Used?	Principal Criticisms	Application
One-Year	■ Solvency II ■ UK ICA ■ Swiss Solvency Test	■ Assumes that risk position can be reduced or additional capital can be raised at the end of the year ■ No secondary market for claim liabilities exists	Companies with: ■ Active capital management ■ Strong earnings momentum ■ Significant franchise value
Run-Off	■ NAIC P&C RBC ■ Rating Agency P&C Capital Adequacy Formulae	■ Requires capital to be held now against possible future consumption later; inefficient ■ Inconsistent with real-world annual balance sheet reviews ■ Makes proper aggregation of risks very difficult	■ Companies that build capital primarily through retained earnings ■ Troubled companies

Why is there a trend towards one-year risk horizon?

Four principal problems with run-off horizon:

1. ***Aggregation of risks with different horizons is an intractable problem***
   — How does one combine one-year equity market risk with workers compensation reserve run-off risk?
   — Putting assets into run-off model dilutes market risk
2. ***In the real world, periodic balance sheets (not cash flows) are the way capital adequacy is monitored***
   — Run-off cash flow testing misses mid-course failures
   — Trapping mid-course failures produces ultra-conservative results
3. ***Projections of future conditions 10-20 years into the future are speculative***
   — One-year projections only require that we articulate possible future states one year from now, not far into the future
4. ***Holding capital now against possible future consumption is inefficient***
   — Efficient markets won't sustain prices at run-off capital levels
   — Firms that have financial flexibility to operate on the one-year model should be allowed that advantage