Parameterizing Payout Lag Time Distributions

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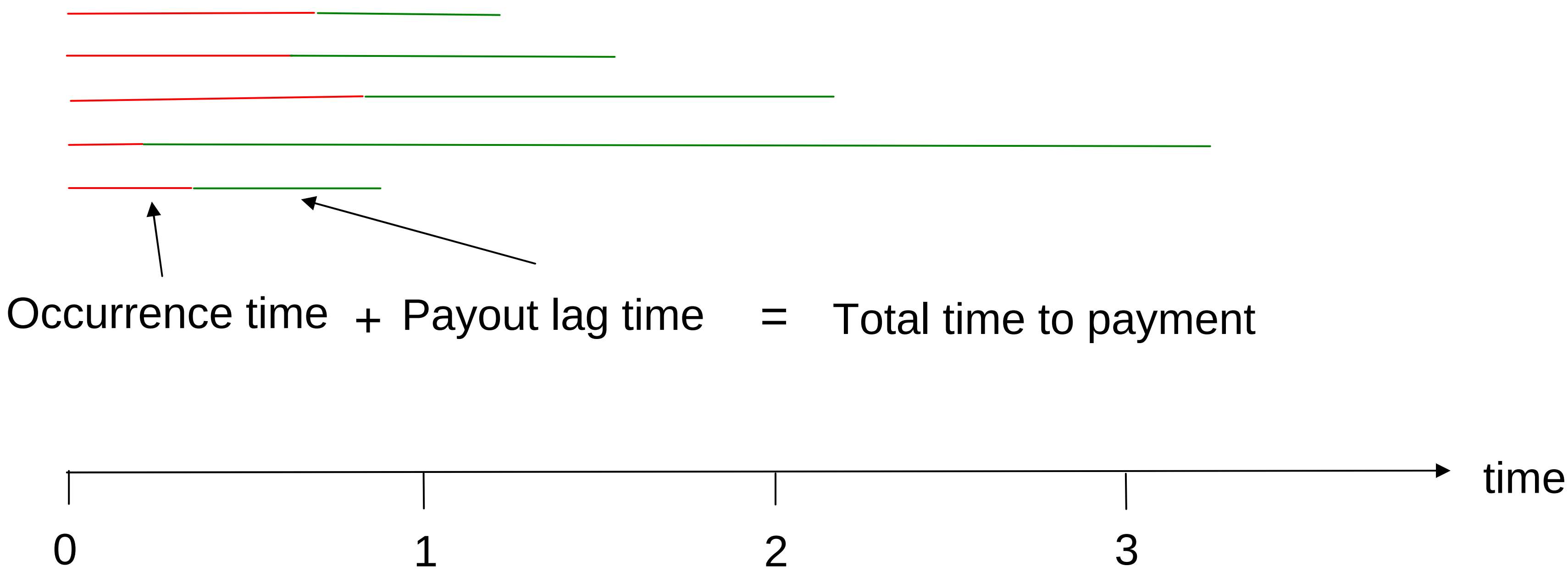
Fundamental Notion

- During an accident year, there is a random time to claim occurrence which is usually taken as a uniform distribution.
- There is a random time from occurrence to to a single payout, given by a distribution we wish to parameterize.
- The observed data – accident year payout time – is the sum of these two random variables.

Accident Year (AY) Data



Motivation

Starting with the distribution of payout time from occurrence will allow

- Consistent treatment of partial accident periods, both in using such data and in constructing its development factors.
- Consistent smoothing of accident period development factors for noisy data.
- Simulation in a timeline approach.

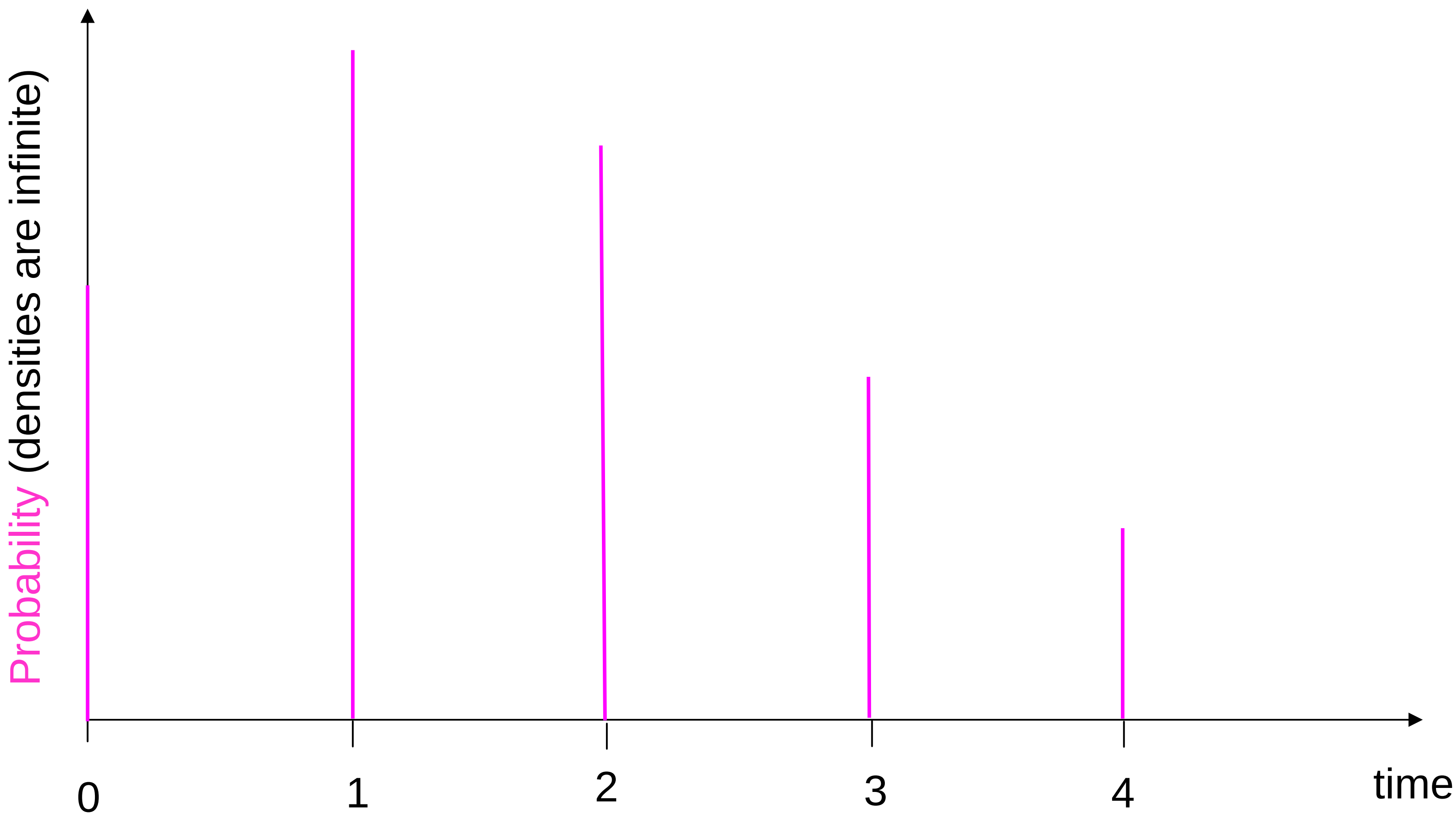
AY Data as It Is

- We have paid (ideally, count) data by accident year.
- We get development factors and payout fractions.
  - More completely, we get development factors *and* uncertainties.
  - We also make up a tail.
- We want to choose a payout distribution by fitting to the AY probability of events per year.
- We will use a variance-weighted least squares fit (with some modification).

An Obvious Solution

- Put probability masses at 0,1,2,…
- If x% of AY paid is in lag year n, put x% probability at that point.
- Since the occurrence is always in year 0, this makes the right AY payment pattern.
- Problem: this implies that there is NO claims payment except at anniversary dates.

Payout Lag Time Density



We Expect That

- the density would have no zeros.
- the density would be smooth (no jumps).
- there would be some probability of (almost) immediate payment.
- the density would decrease to zero gradually at large times.
- the density would be unimodal, except for special cases.

So, What Now?

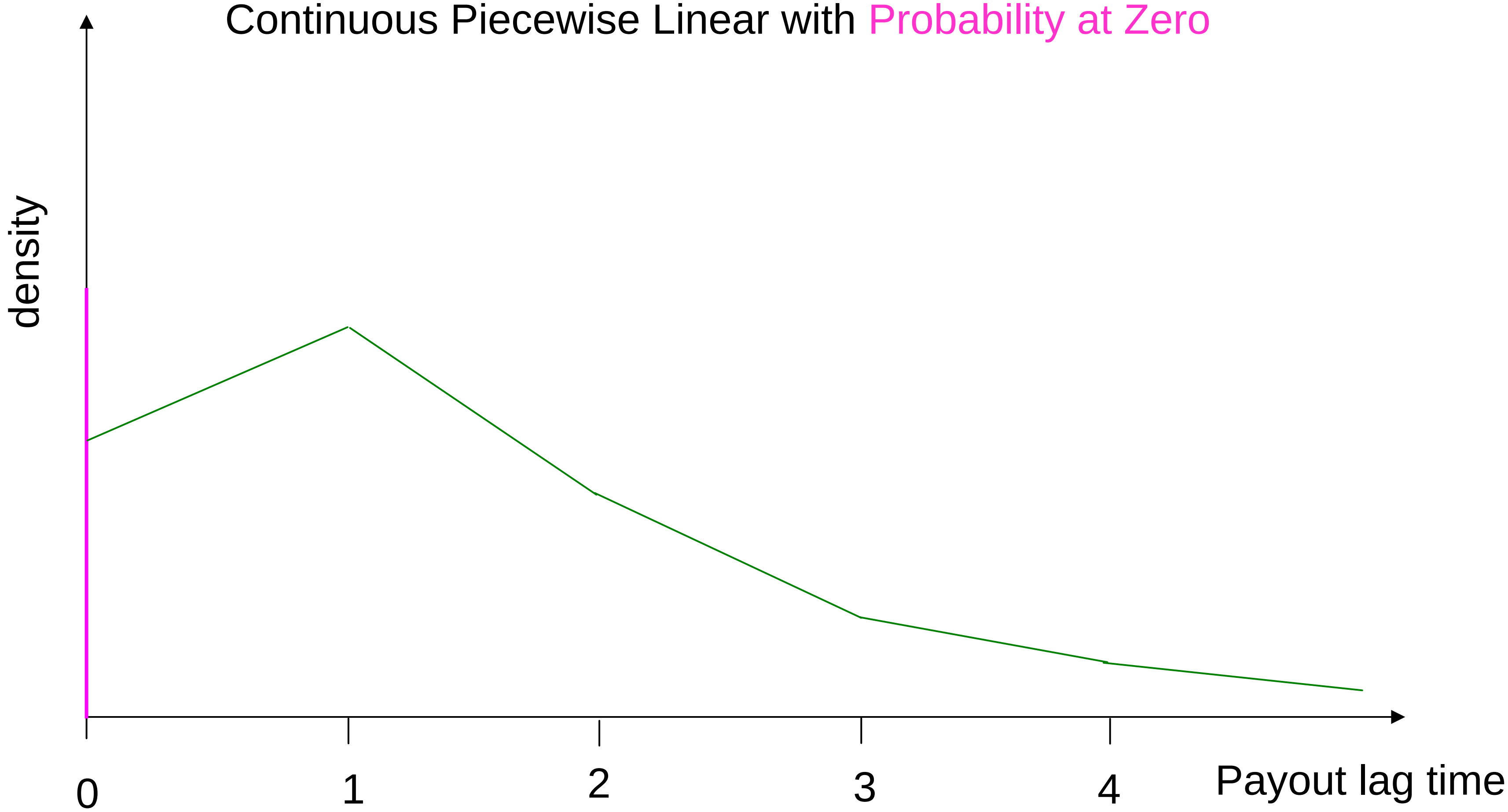
- We can use our favorite parameterized distribution – e.g. gamma with a probability mass at zero.
- Or, to mock up a more general case, use a continuous piecewise linear distribution with a probability mass at zero.
- In any case, we calculate the payment fractions in the calendar periods.

How to calculate?

- The paper shows the probabilities of payment in terms of the Cumulative Distribution Function of the payout time lag for any distribution.
- We will concentrate on AY by year, but it also does AY by quarter and policy year by year.
- It specializes to the case of a continuous piecewise linear density with probability at zero.

Payout Time Lag Density

Continuous Piecewise Linear with Probability at Zero



Solving (1)

- There are enough parameters to actually solve the equations for the density heights.
- But – with some real data these can come out negative
  - Could be noise in data.
  - Could be another model is needed, such as multiple payments.
- So, we enforce positive densities.

Solving (2)

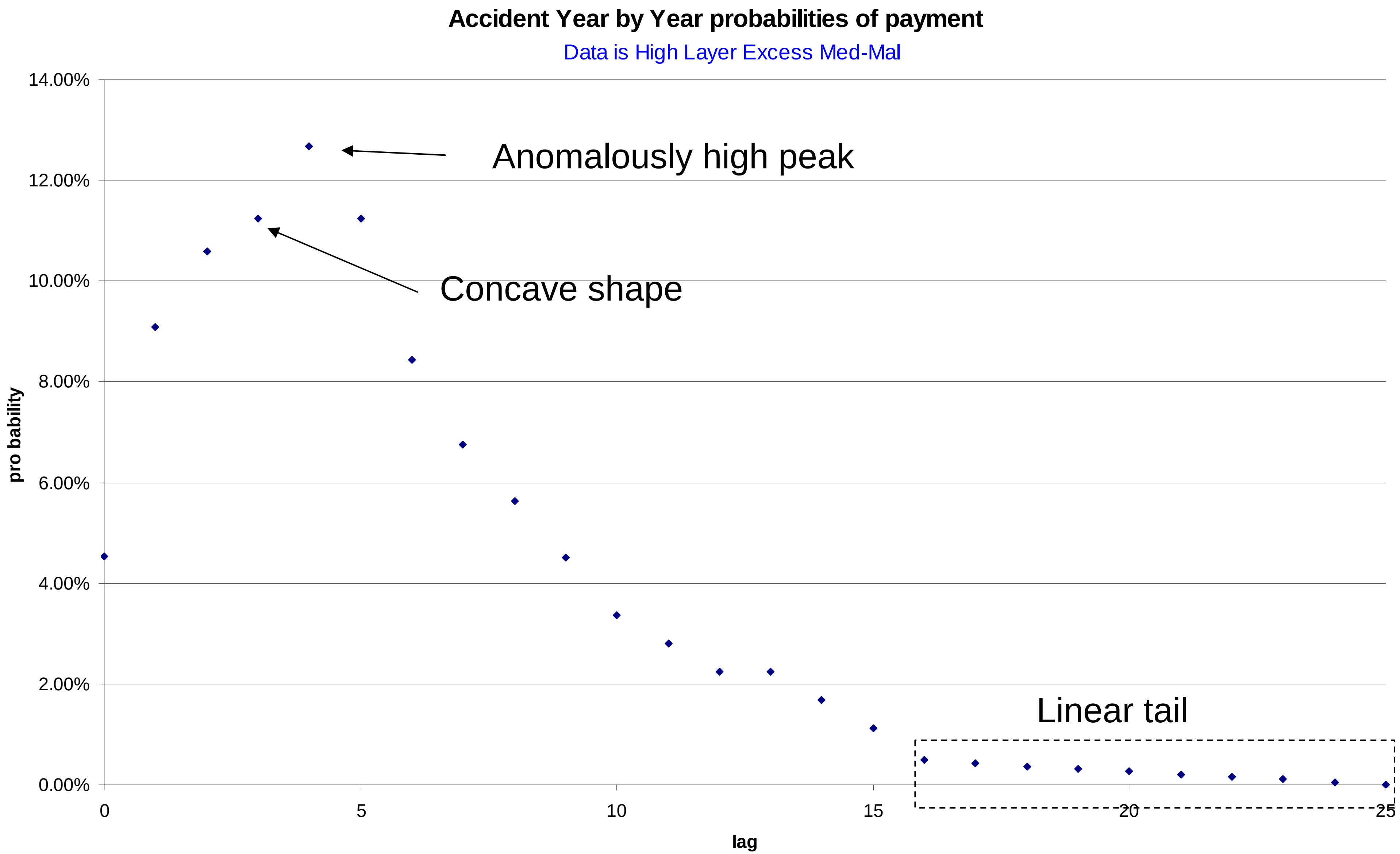
- Enforcing positive densities can mean not fitting exactly. This is a good thing.
  - Some data are outliers.
  - We are then consistent, and can use the density for arbitrary time intervals.
- Even with positive densities, either distribution may not be entirely believable because of its shape.

Solving (3)

- If the density has many modes – too much like a saw – we may want to enforce a smoothing requirement.
- A useful criterion is the sum of the square of the change of slope at each transition.
- The more this is weighted in with the probability fit, the more important smoothing is compared to best data fit.
- THIS IS A PURE JUDGMENT CALL.

Example Payout Probability data

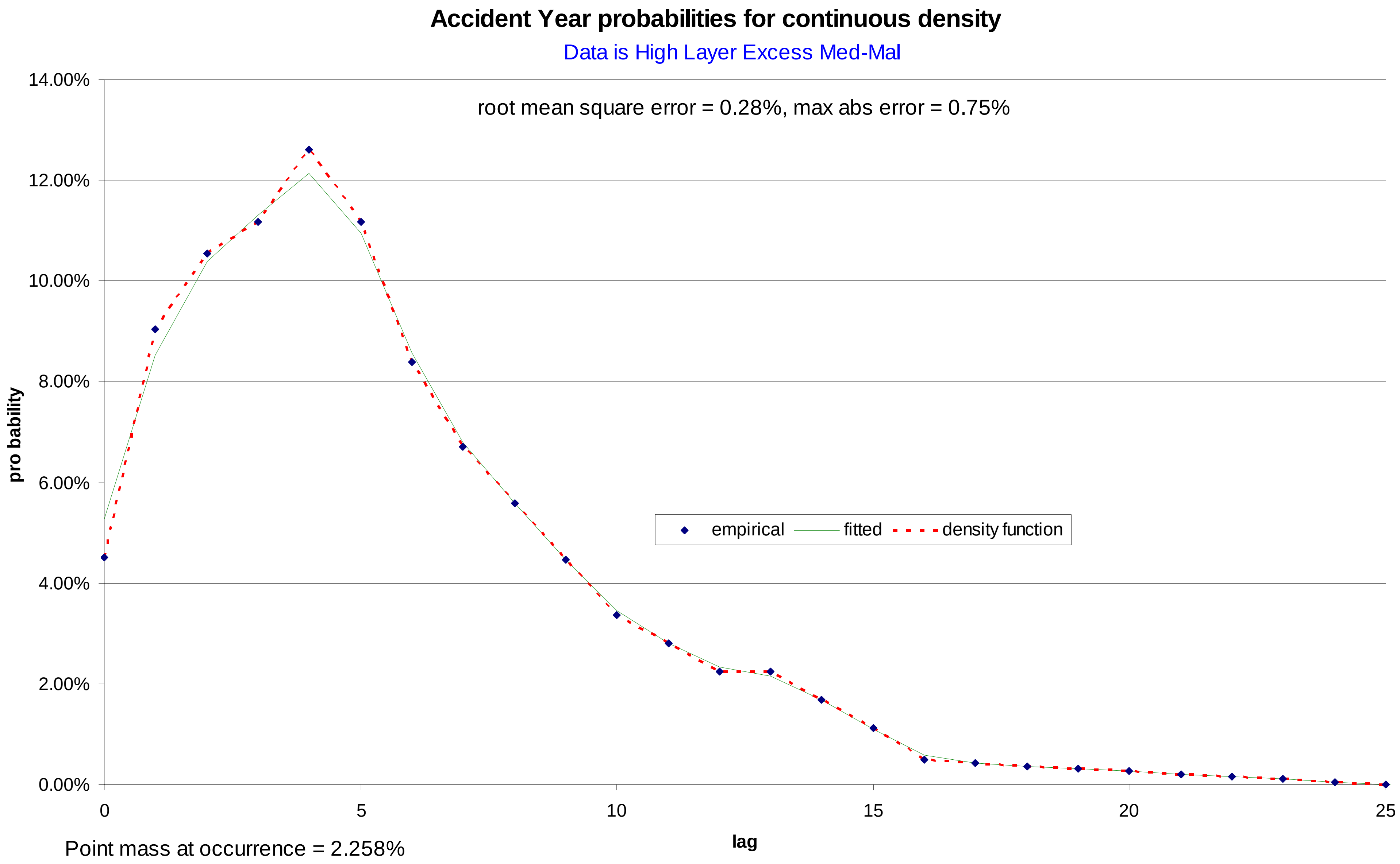
- High Excess Med-Mal.
- No uncertainties from the original data.
- Made up tail.



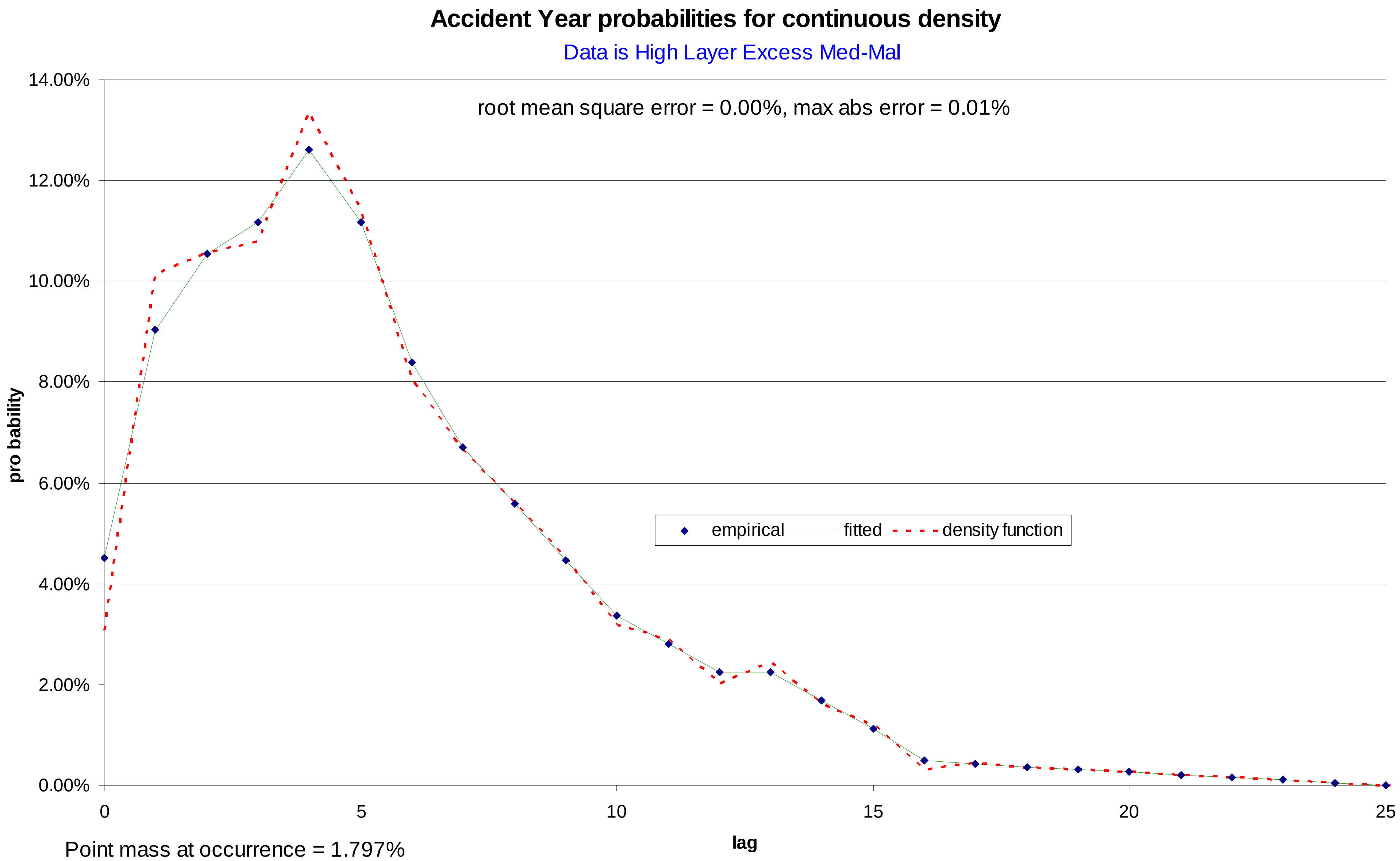
Time axis is the lag in years to payment from the accident year

Spreadsheet Examples

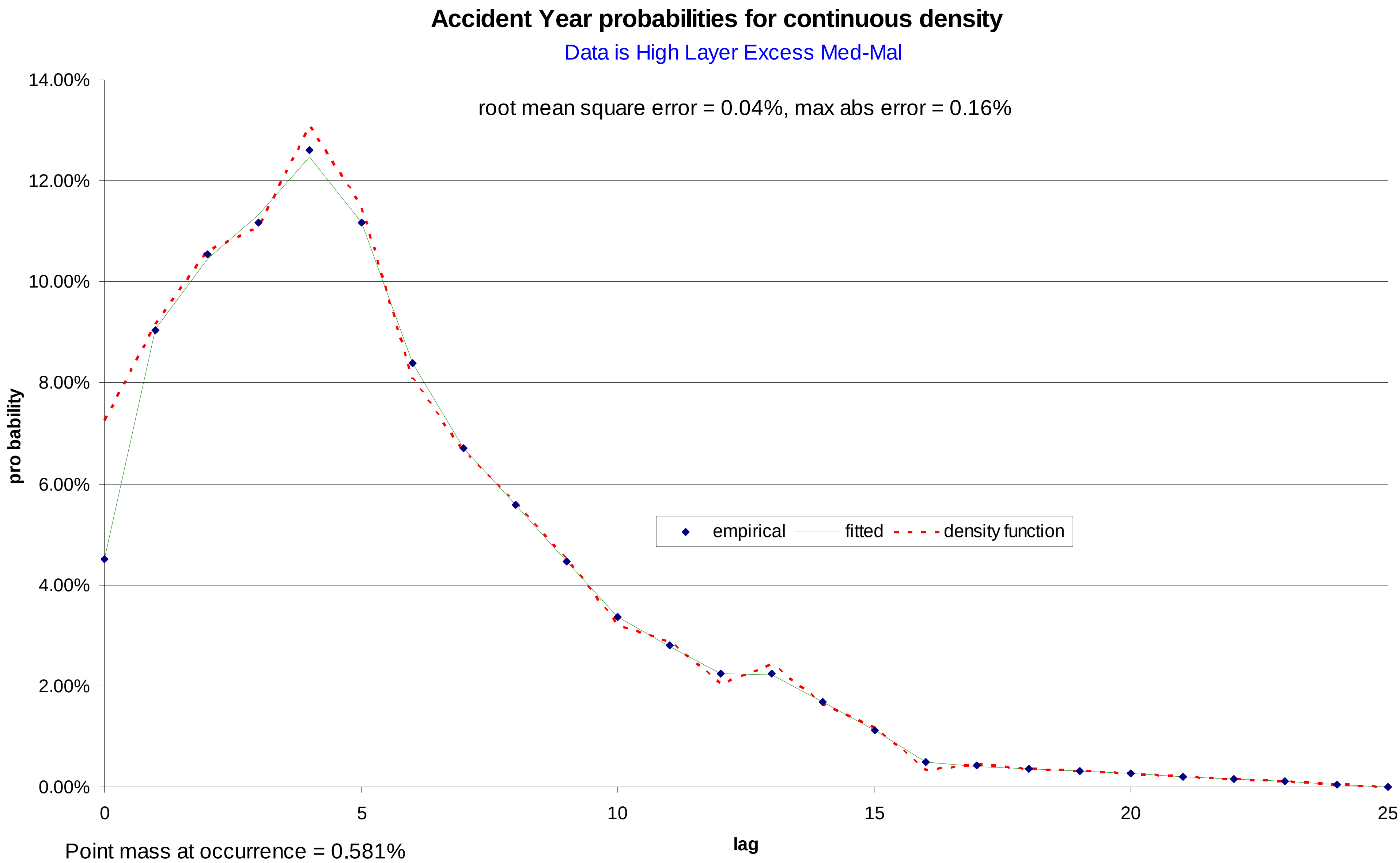
- On the appropriate spreadsheet, we guess at the start values to begin. This is an approximate solution, but often not too bad.
- Then we run Solver to get best fit
- Then we add weight to smoothing for best compromise of fit and distribution



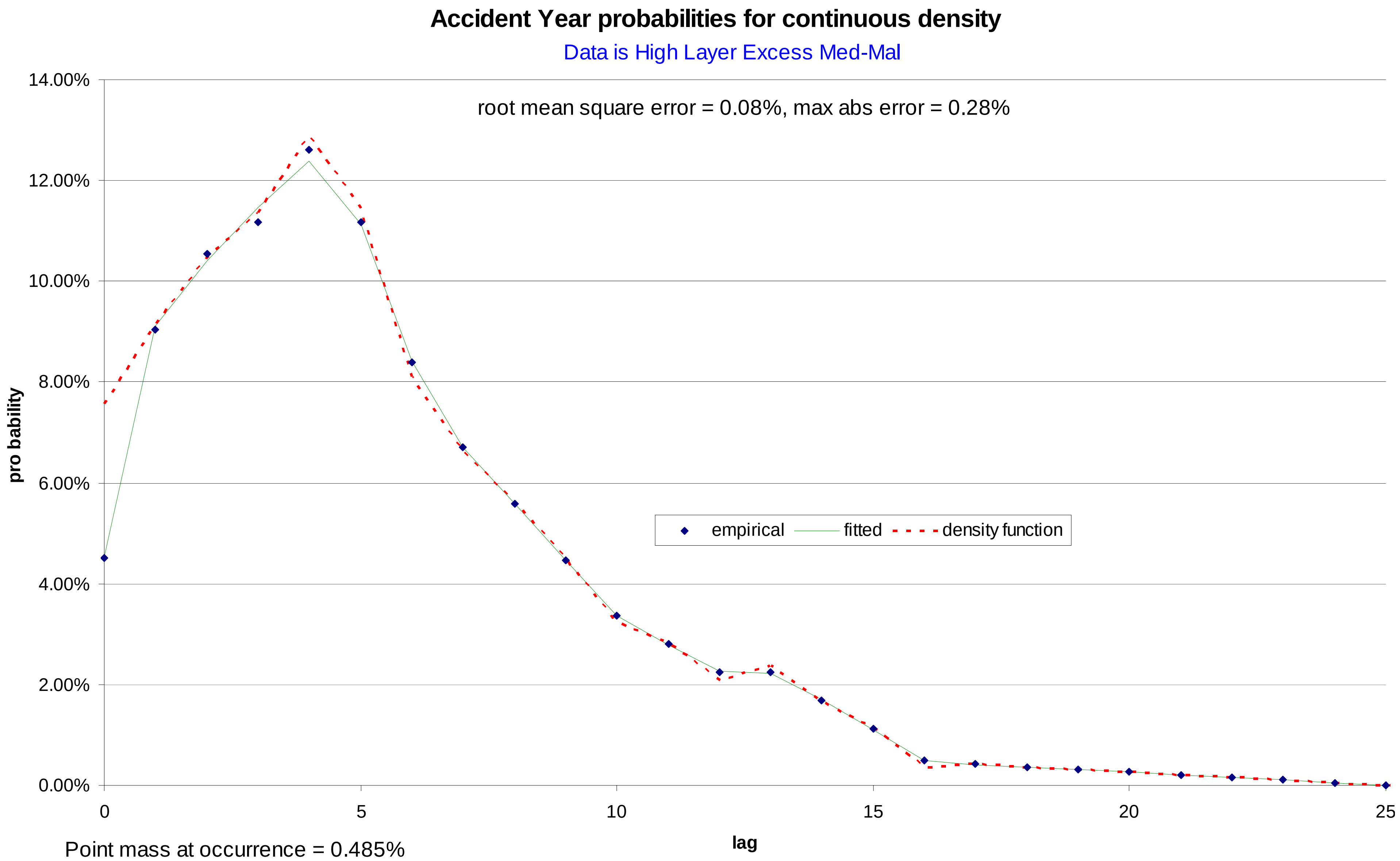
Start values, no smoothing



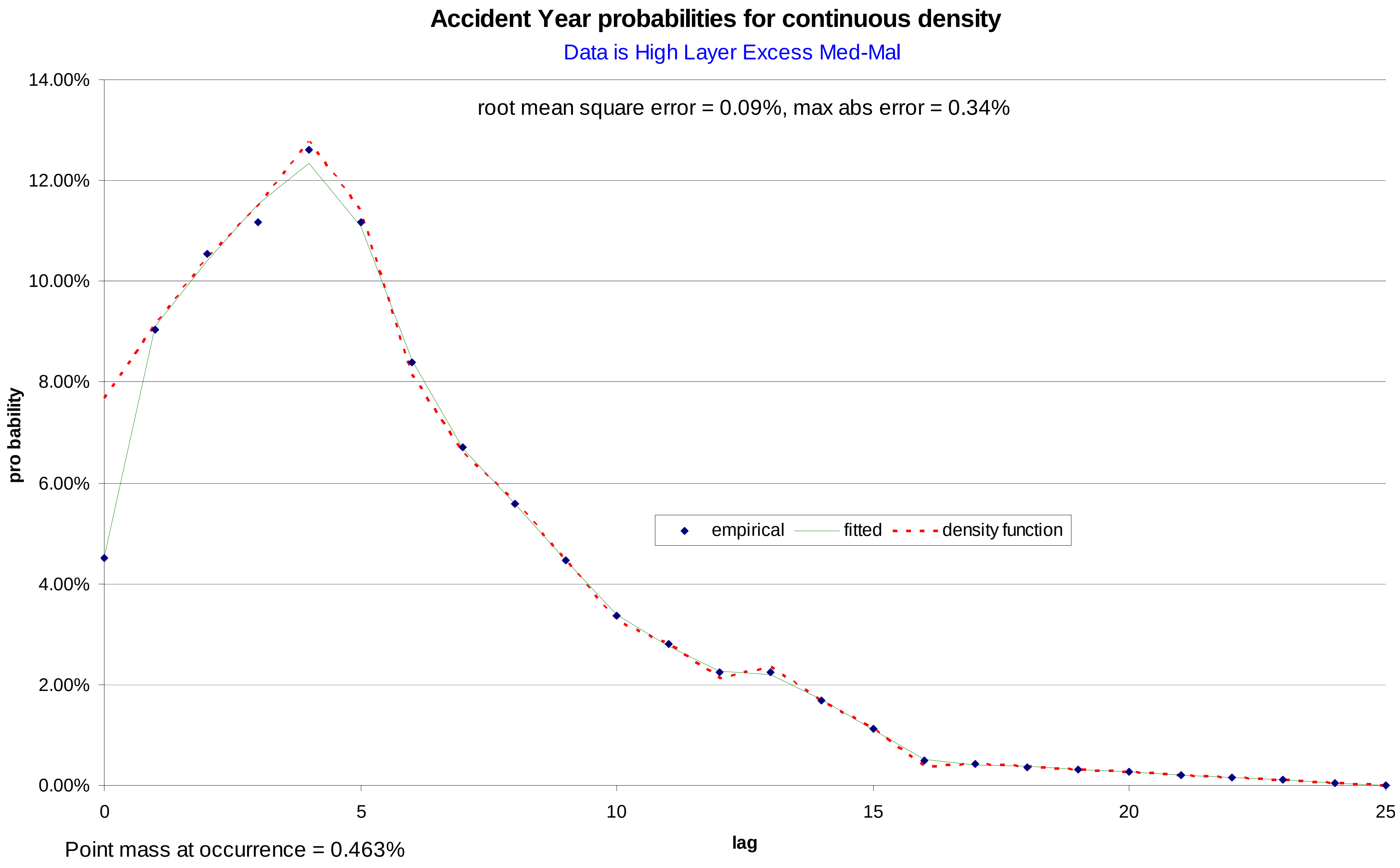
Best fit with no smoothing



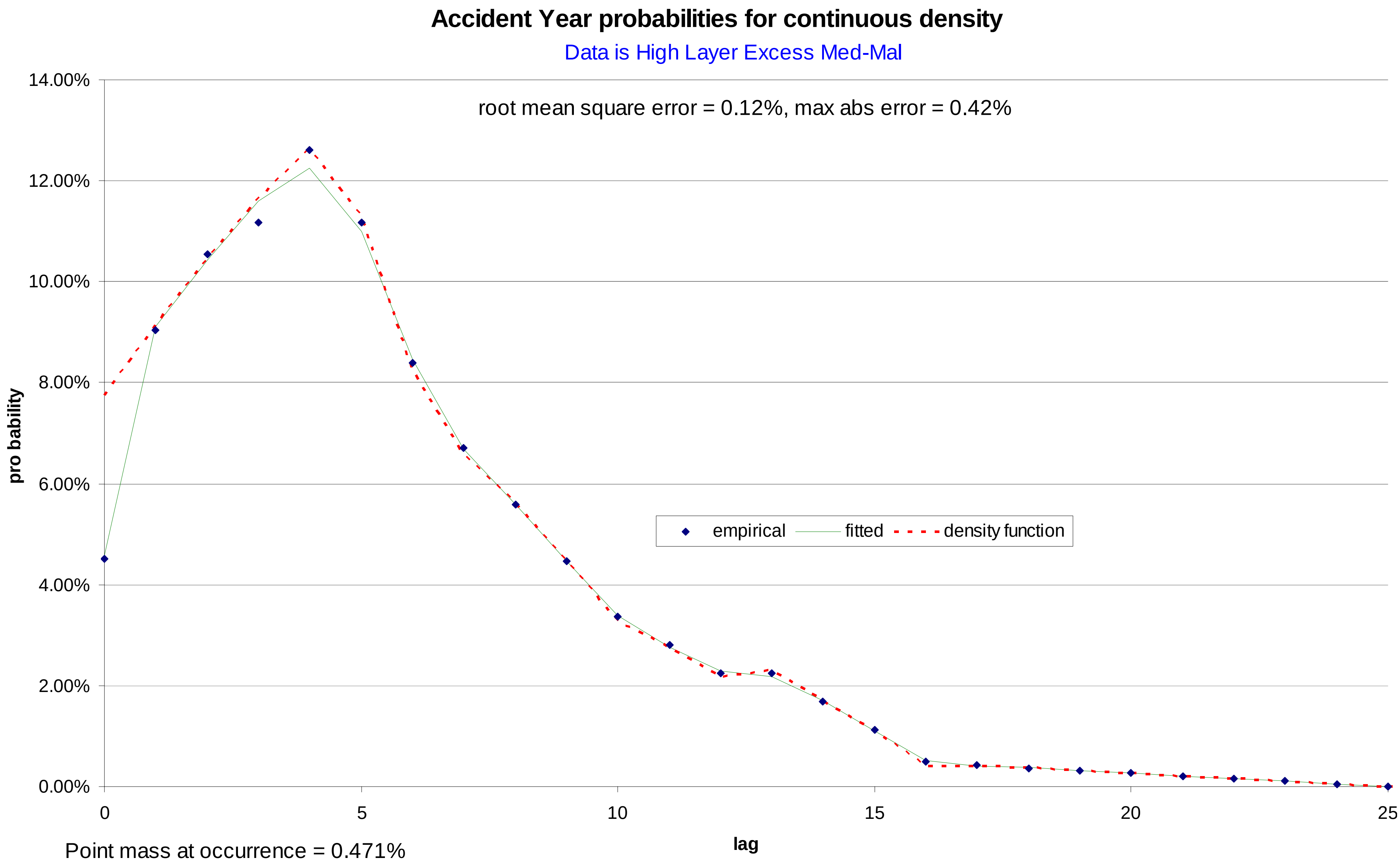
Smoothing 0.001



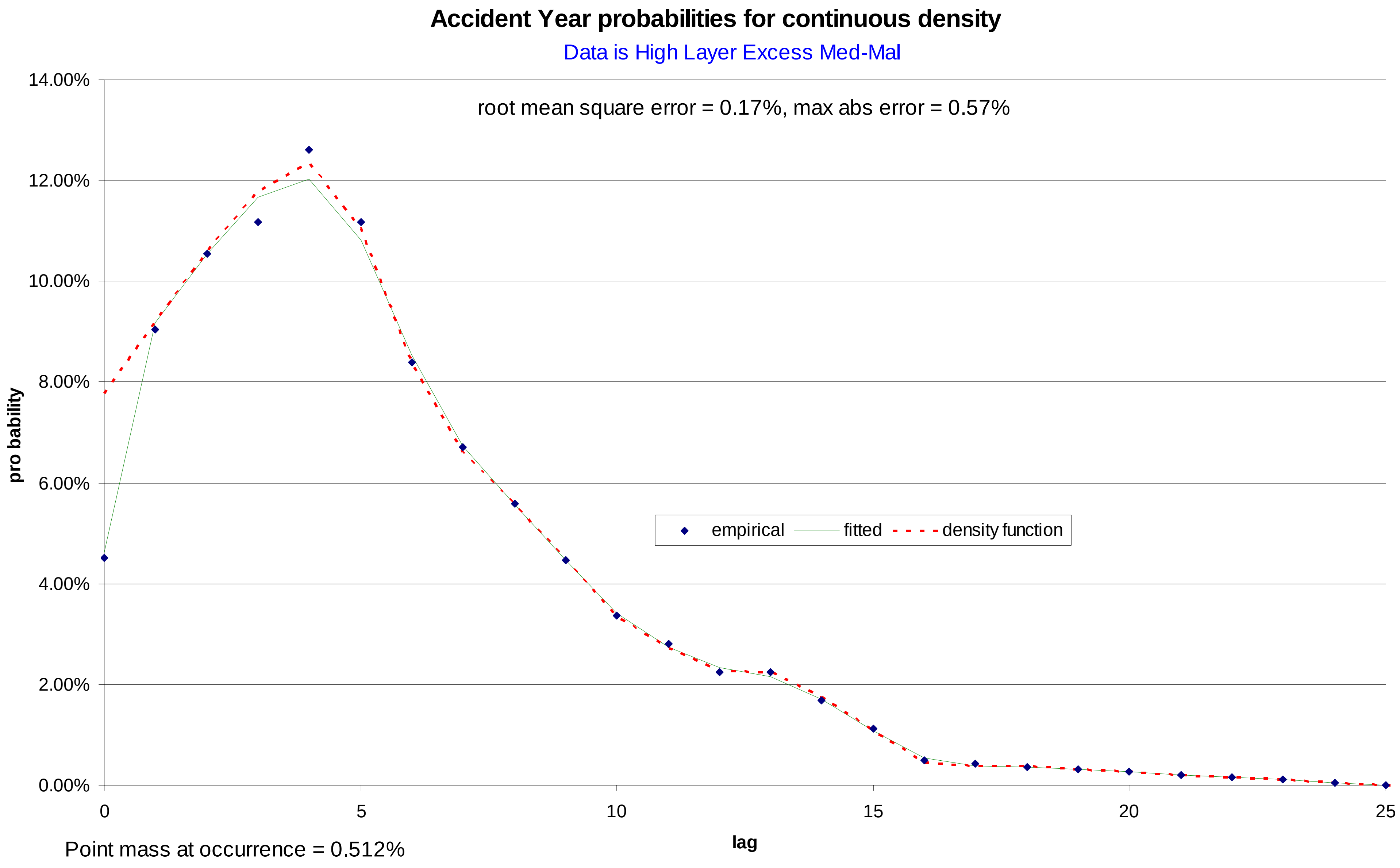
Smoothing 0.003



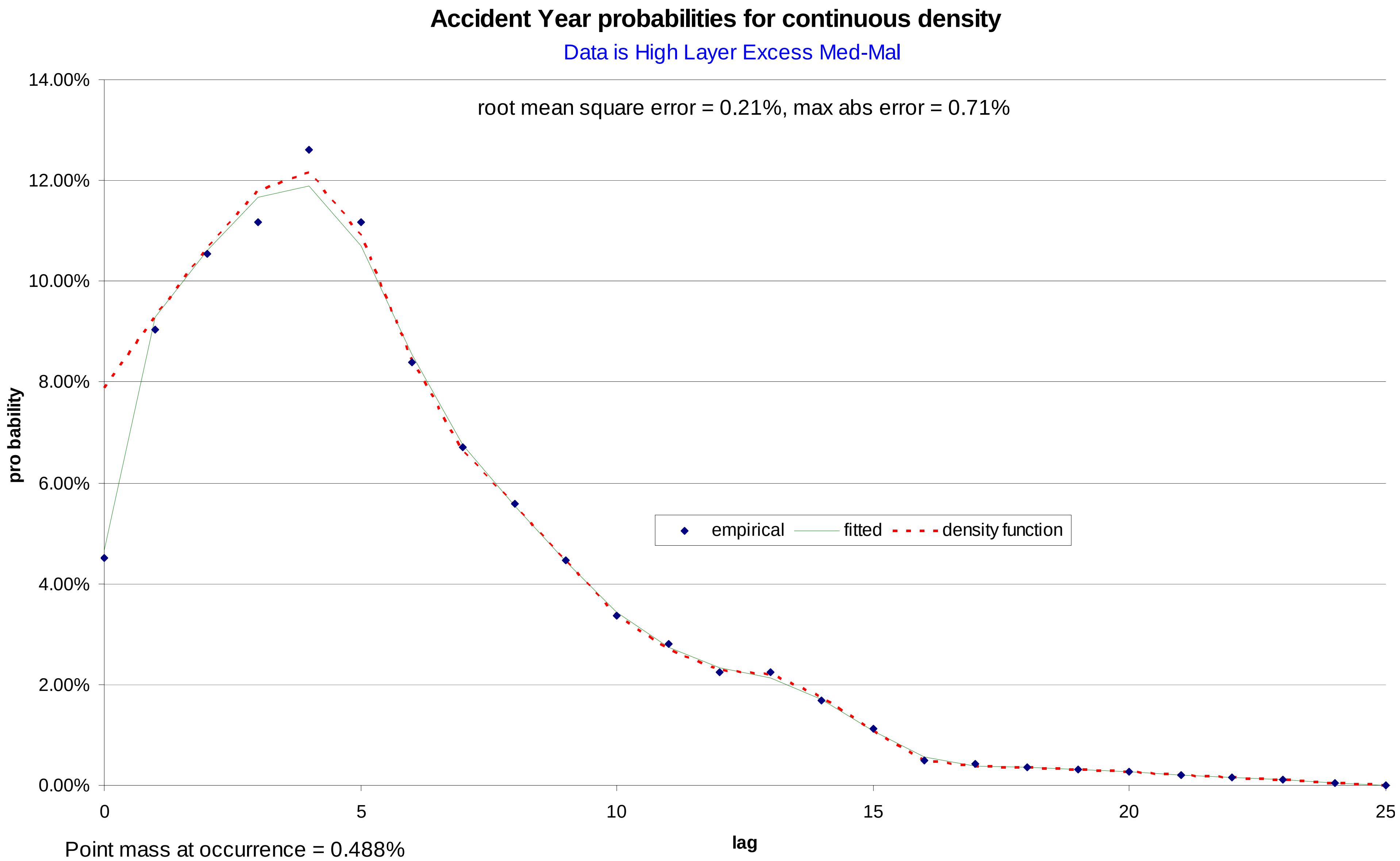
Smoothing 0.005



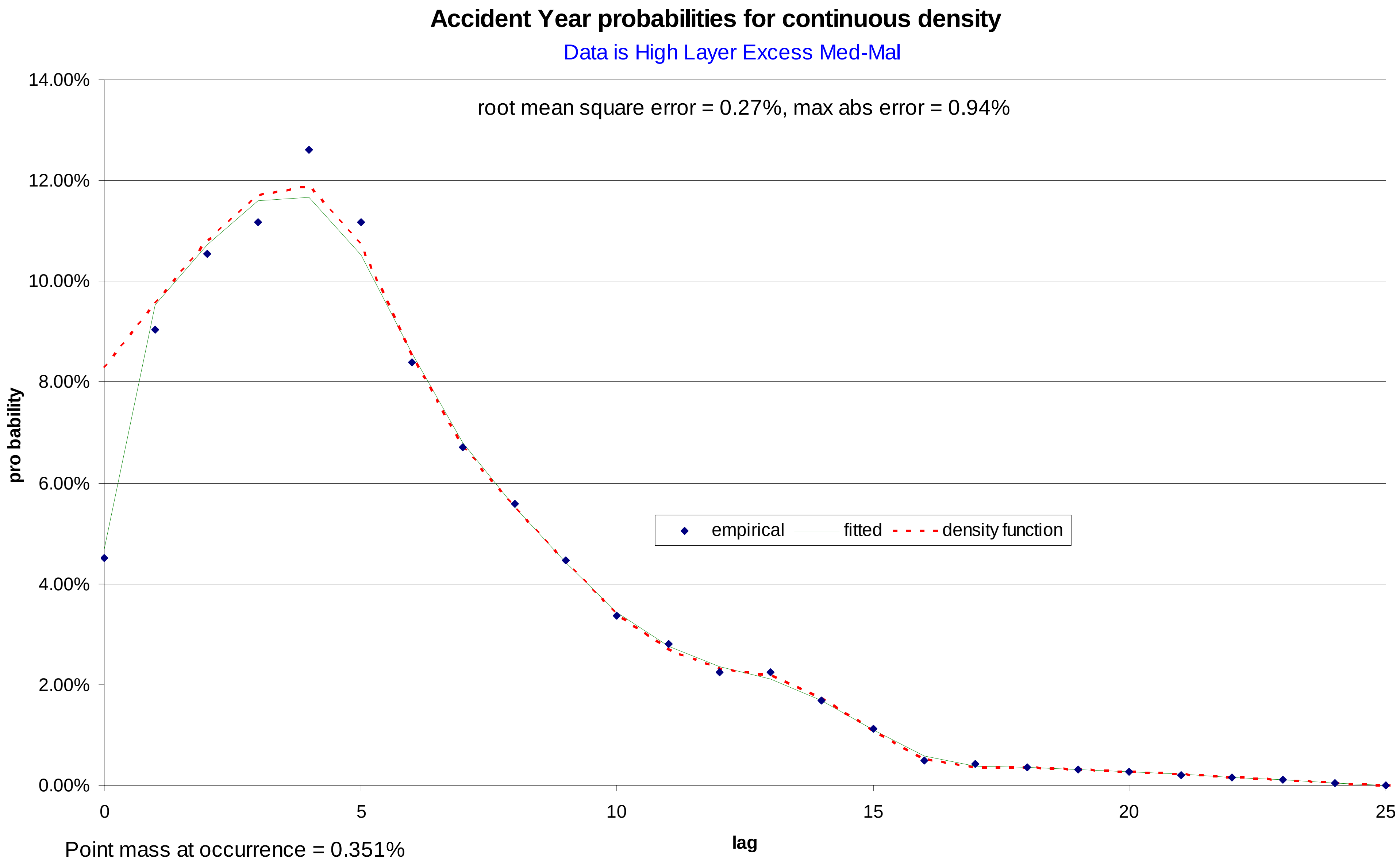
Smoothing 0.010



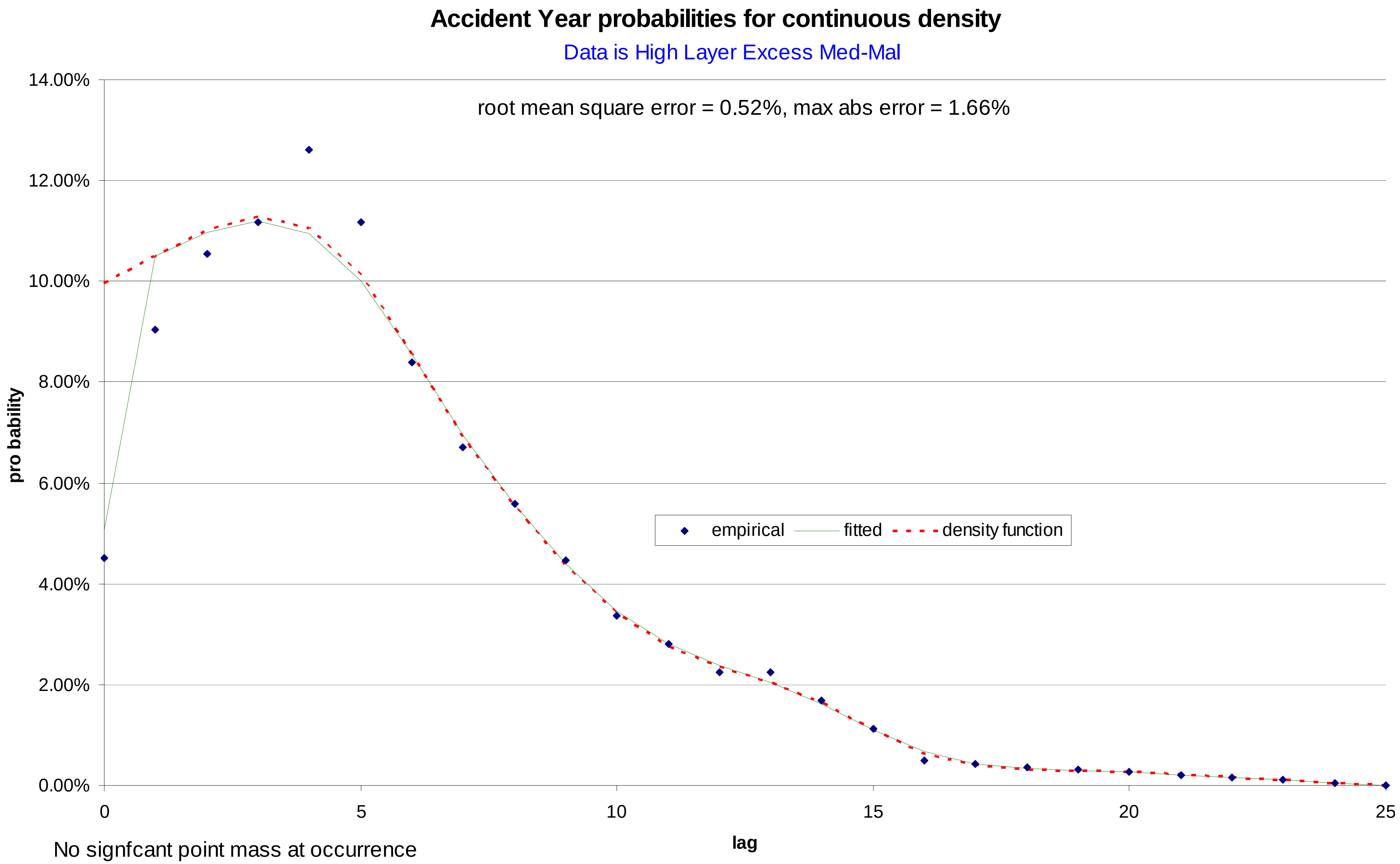
Smoothing 0.030



Smoothing 0.050

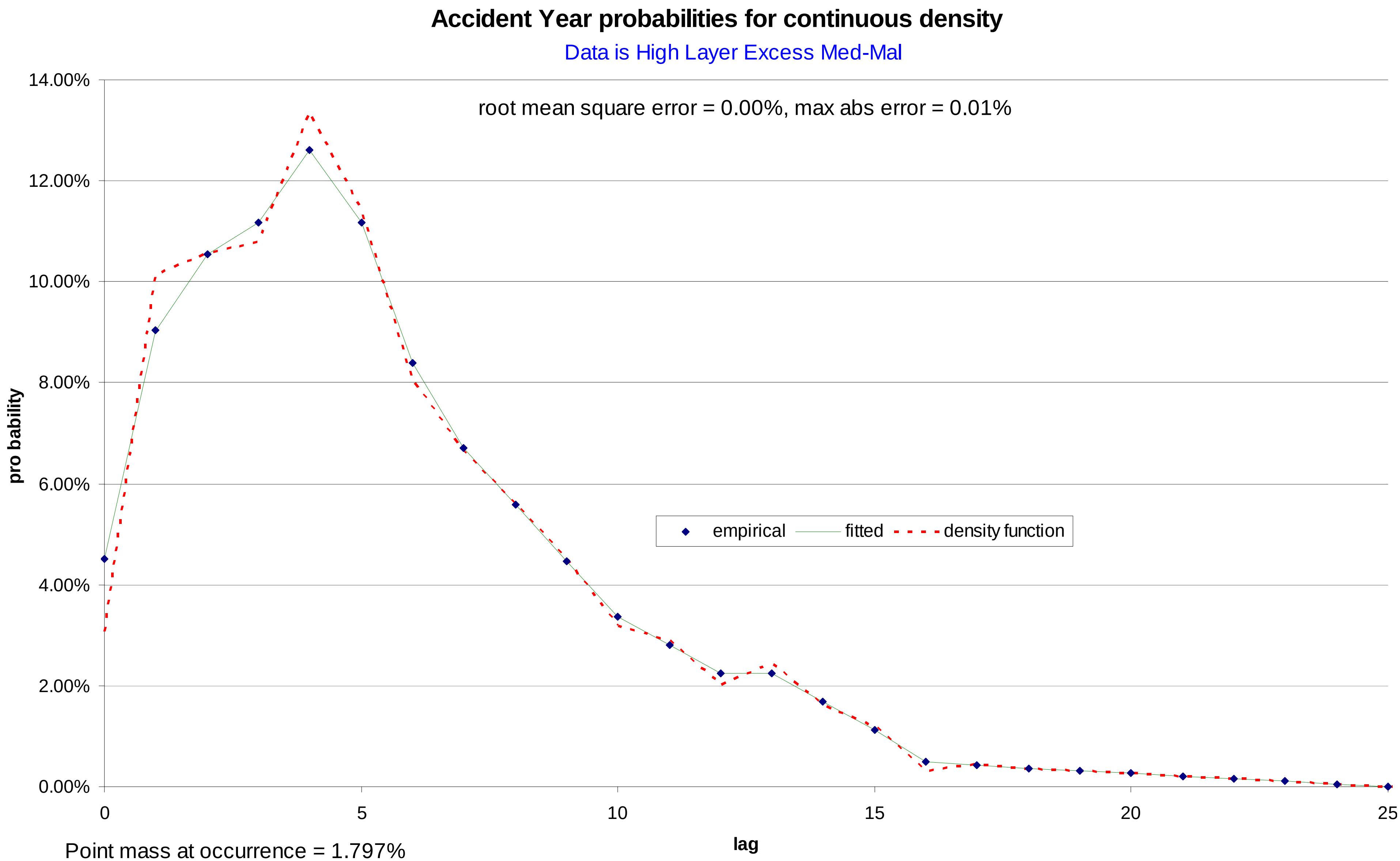


Smoothing 0.100



No signfcant point mass at occurrence

Smoothing 0.500

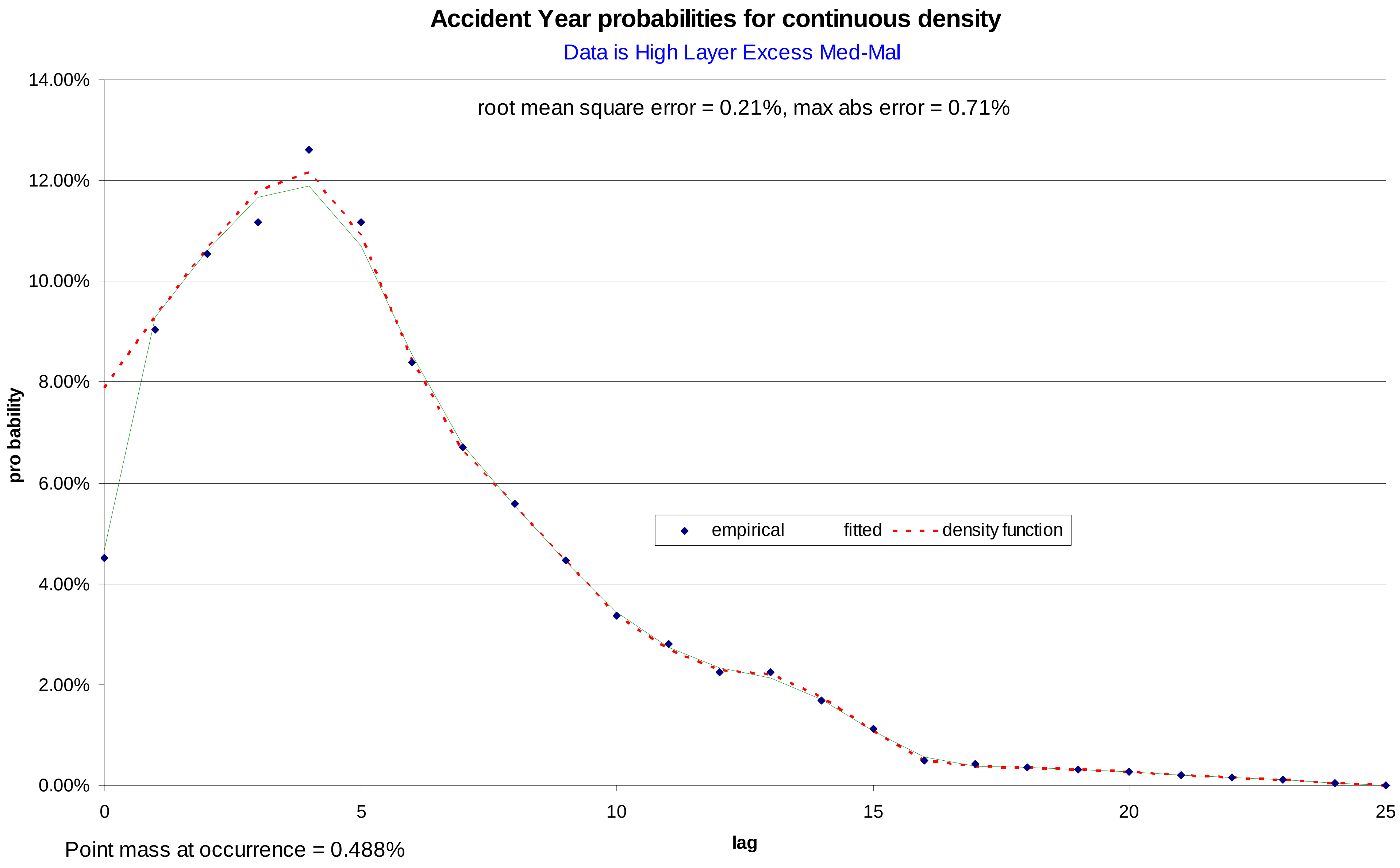


Point mass at occurrence = 1.797%

Best fit with no smoothing

Considerations

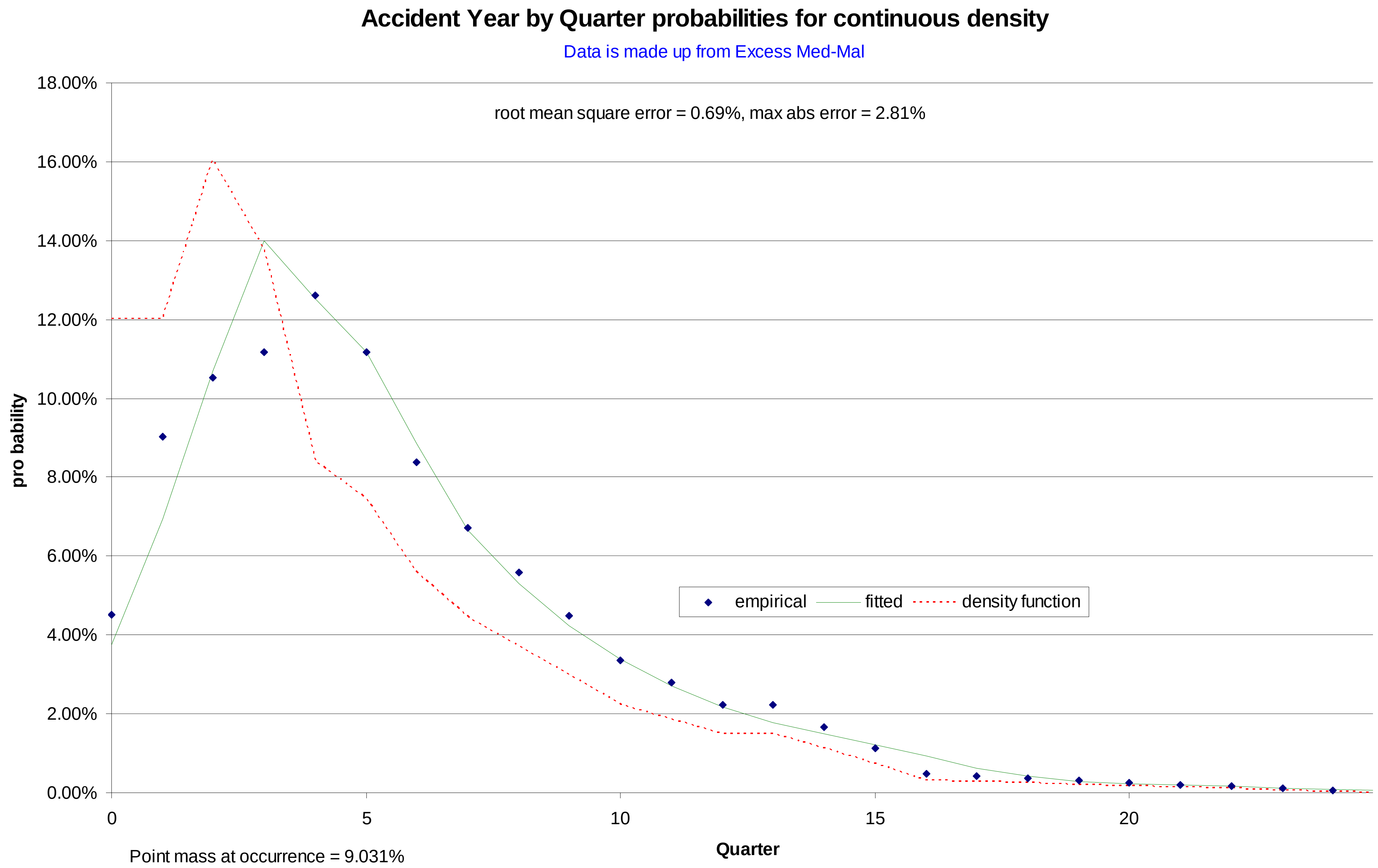
- Because we do not have the actual uncertainties, it is hard to say how significant the differences are.
- We look at the fitted curve and the payout density and try to find the best judgmental compromise. Here, my choice is probably smoothing of 0.030 or 0.050.



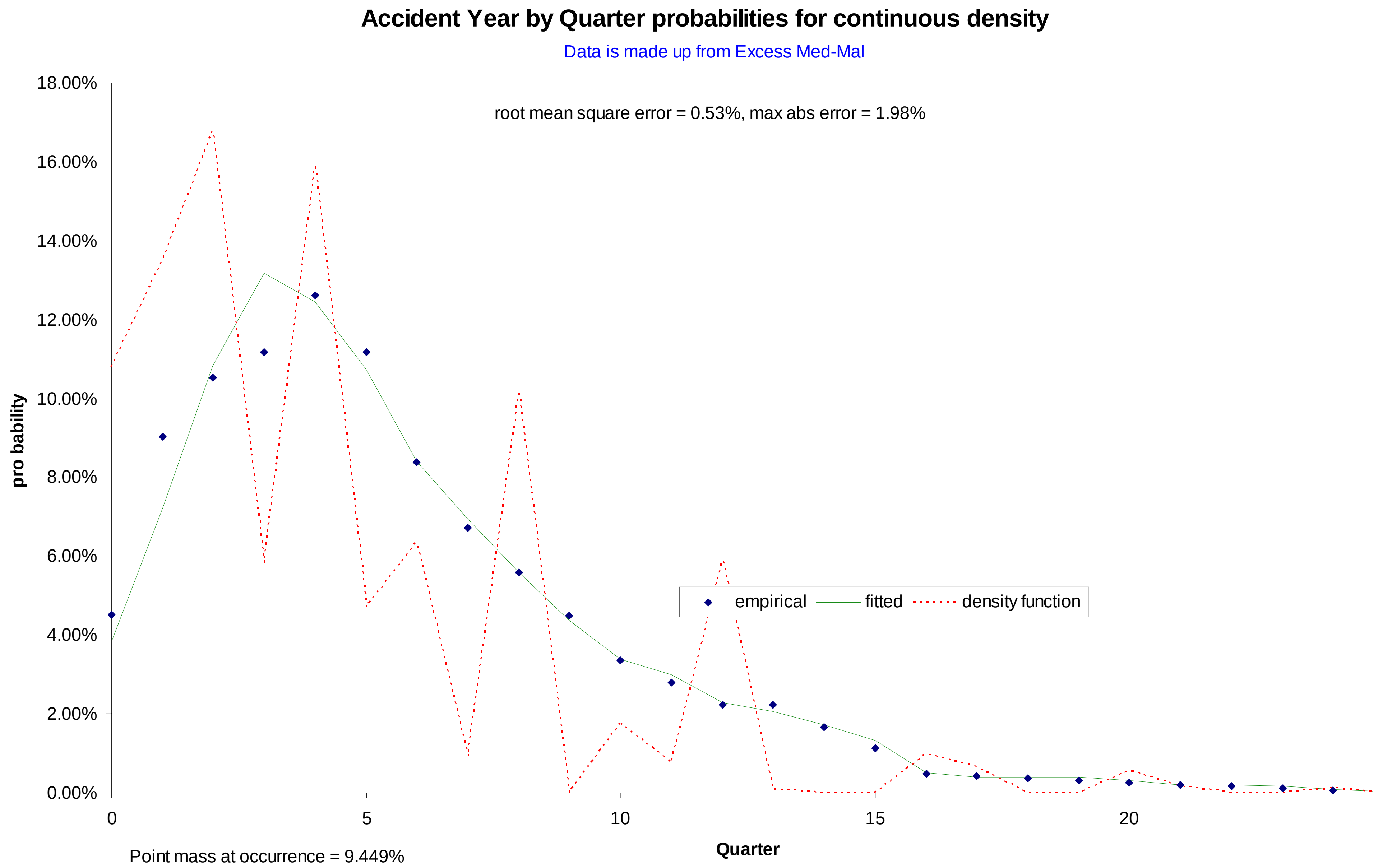
Smoothing 0.050

A More Extreme Case

We use the same data but pretend that it is AY by quarter data. Since it isn't, we expect a mess.



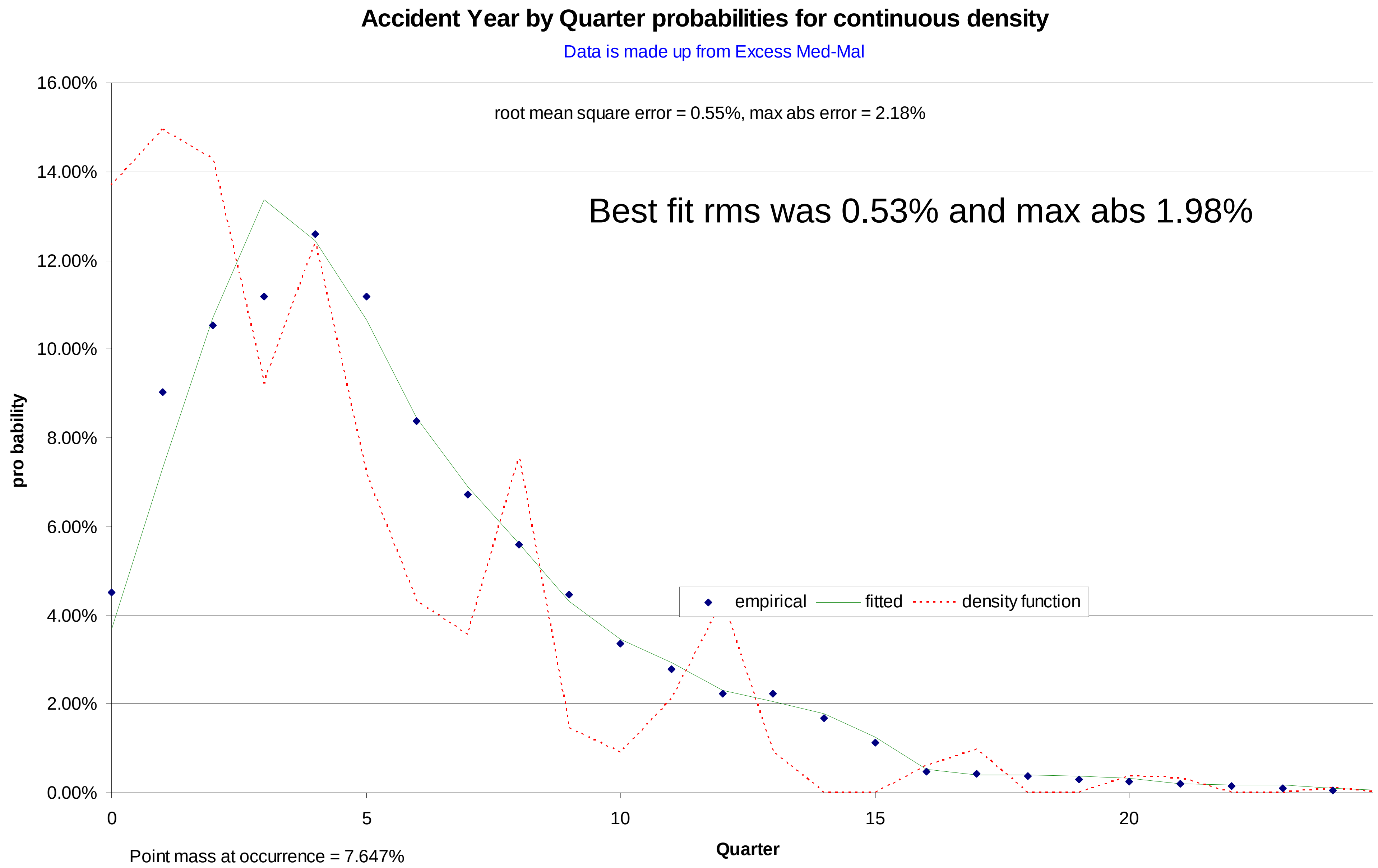
Start values, no smoothing



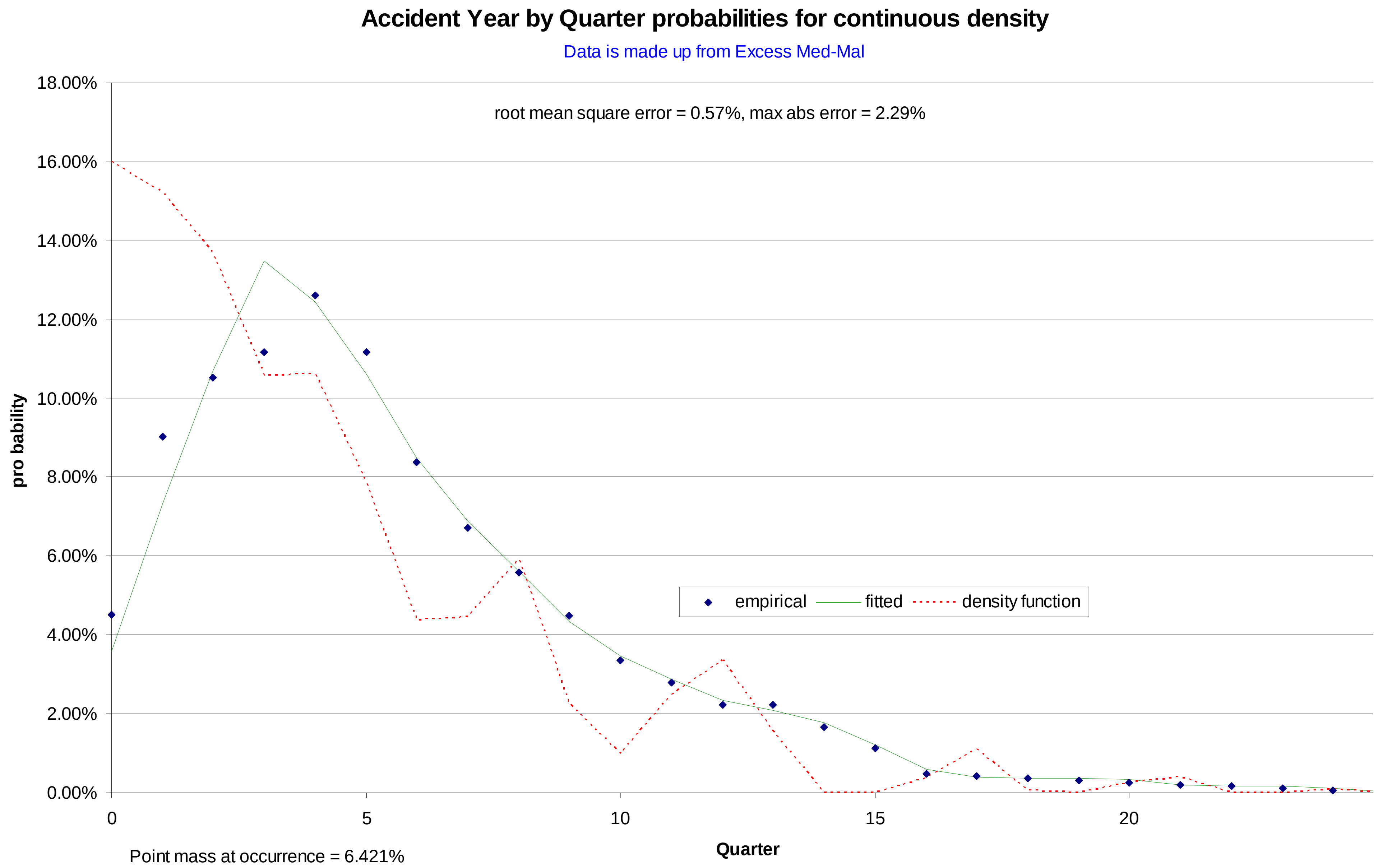
Best fit with no smoothing

Oh My Gosh!

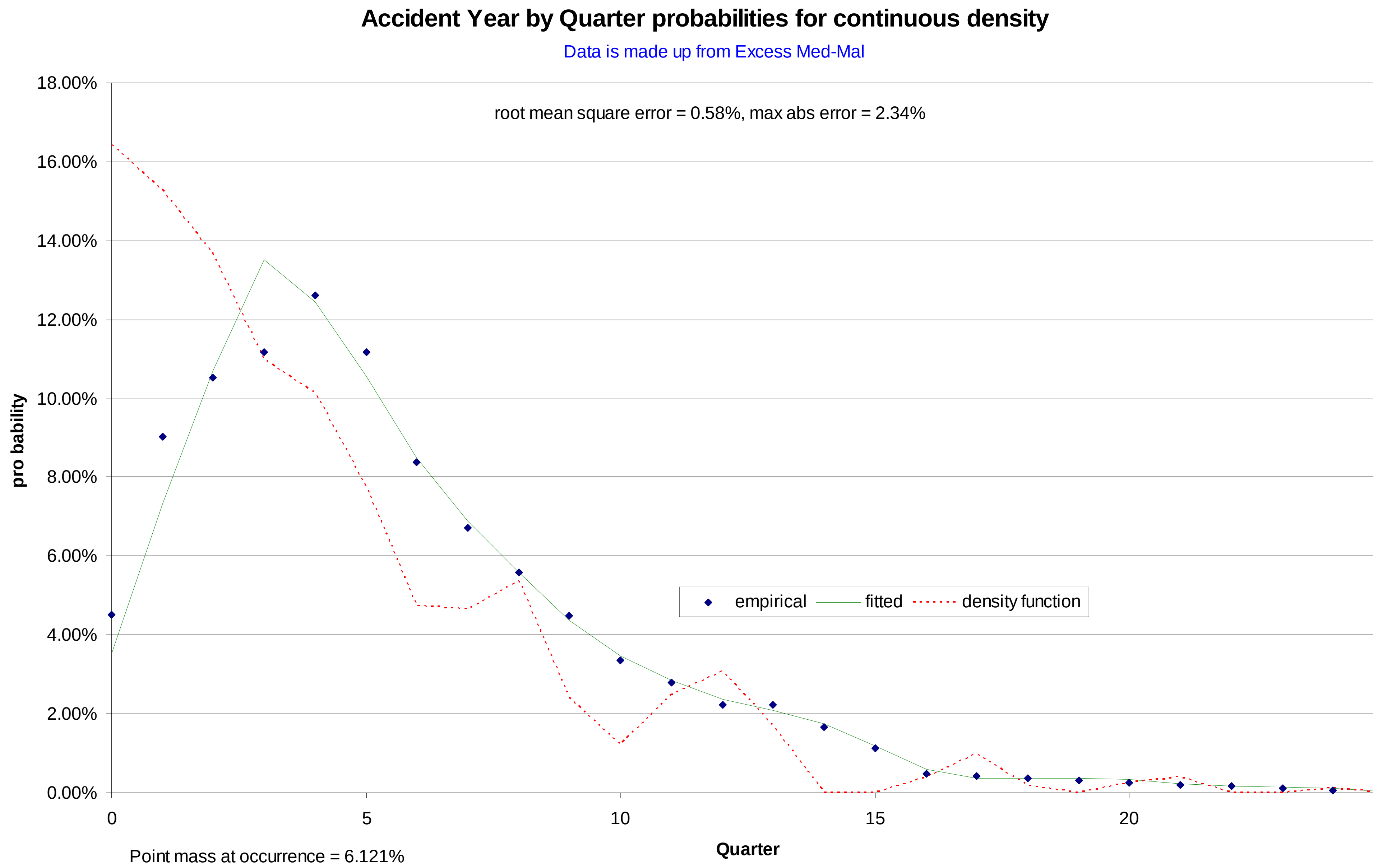
- The fit is not good.
- The payout density is bizarre.
- First question: is this really the data?
- On the assurance that this is good data because we never make mistakes, we try smoothing.



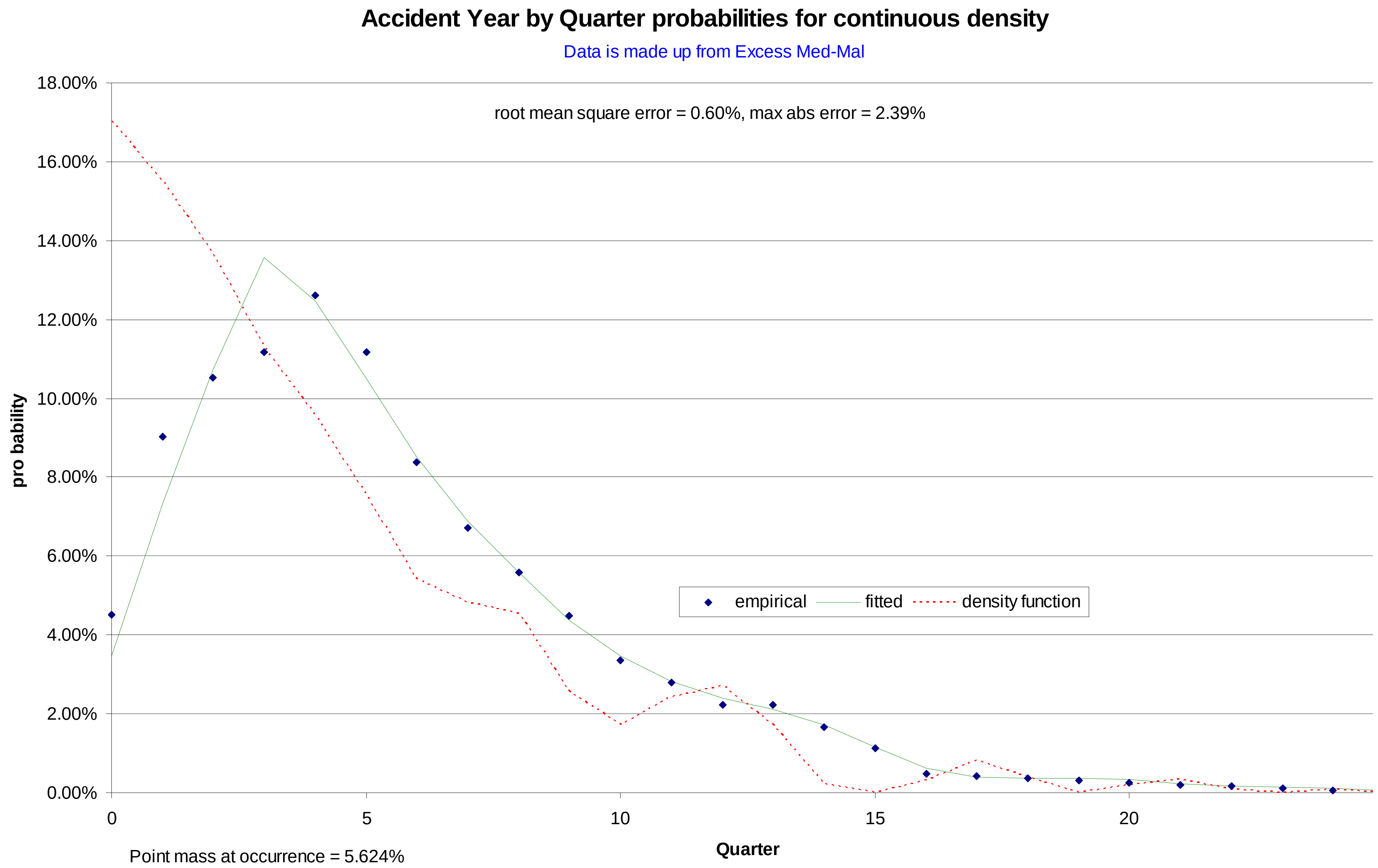
Smoothing 0.0001



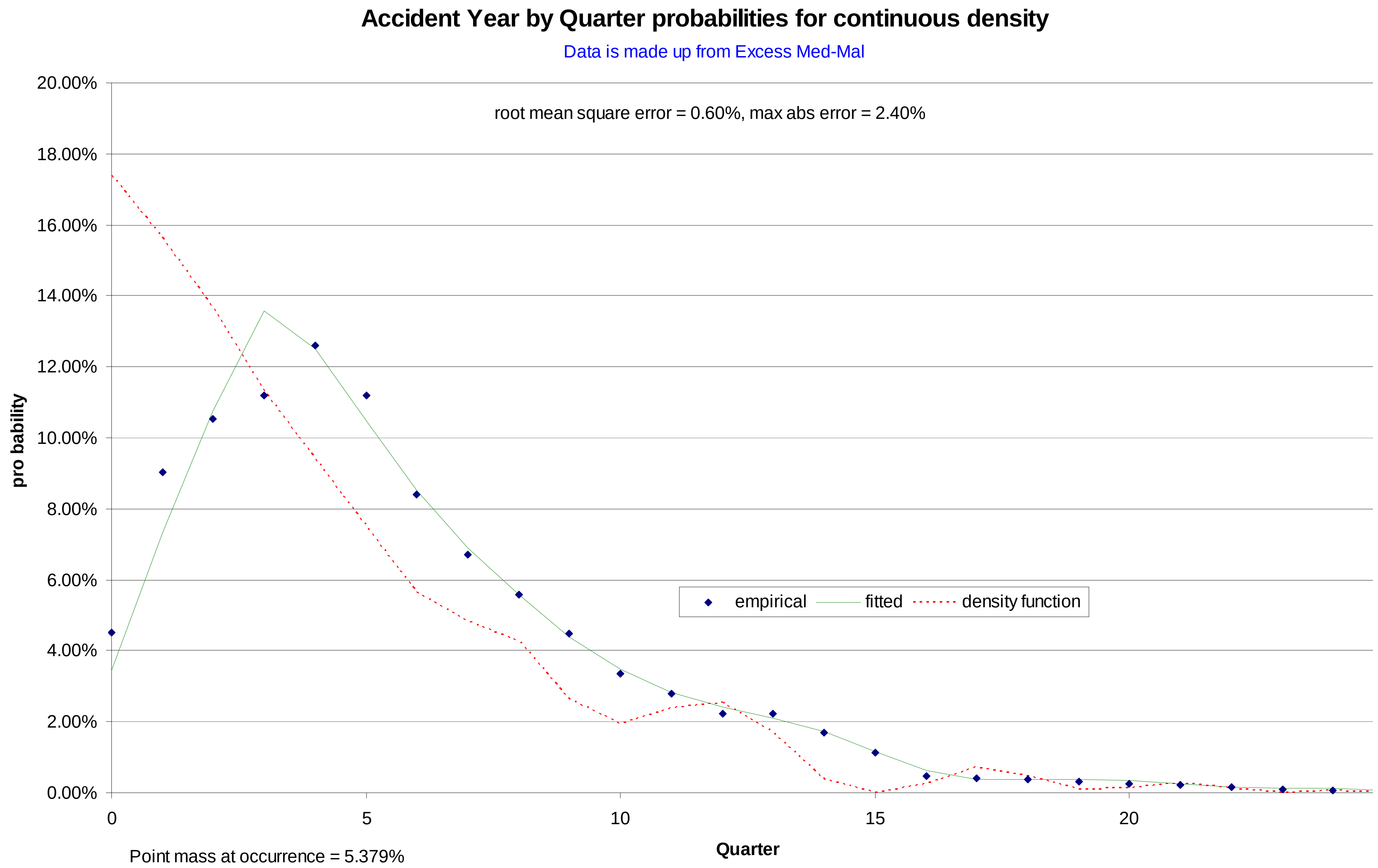
Smoothing 0.0005



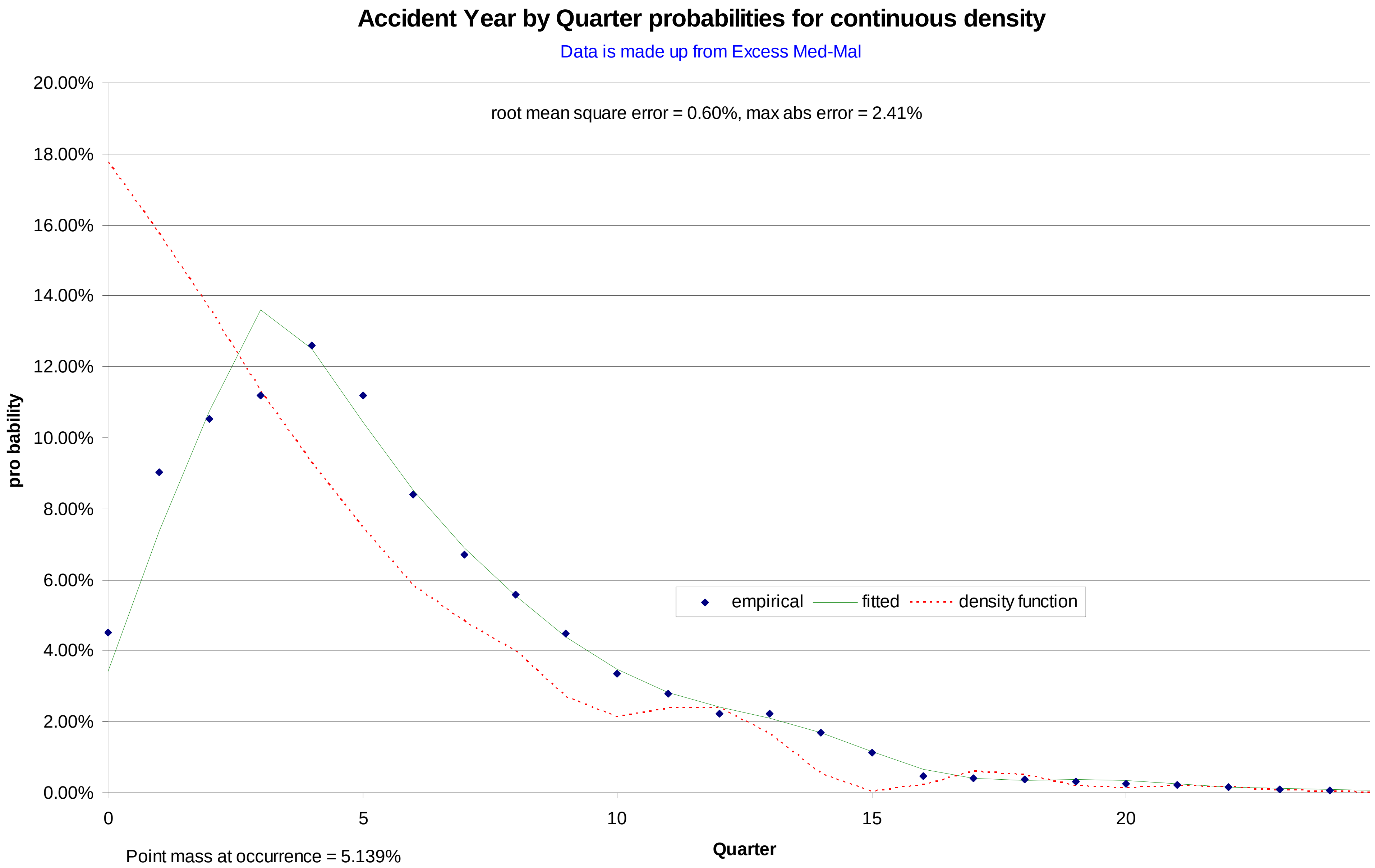
Smoothing 0.001



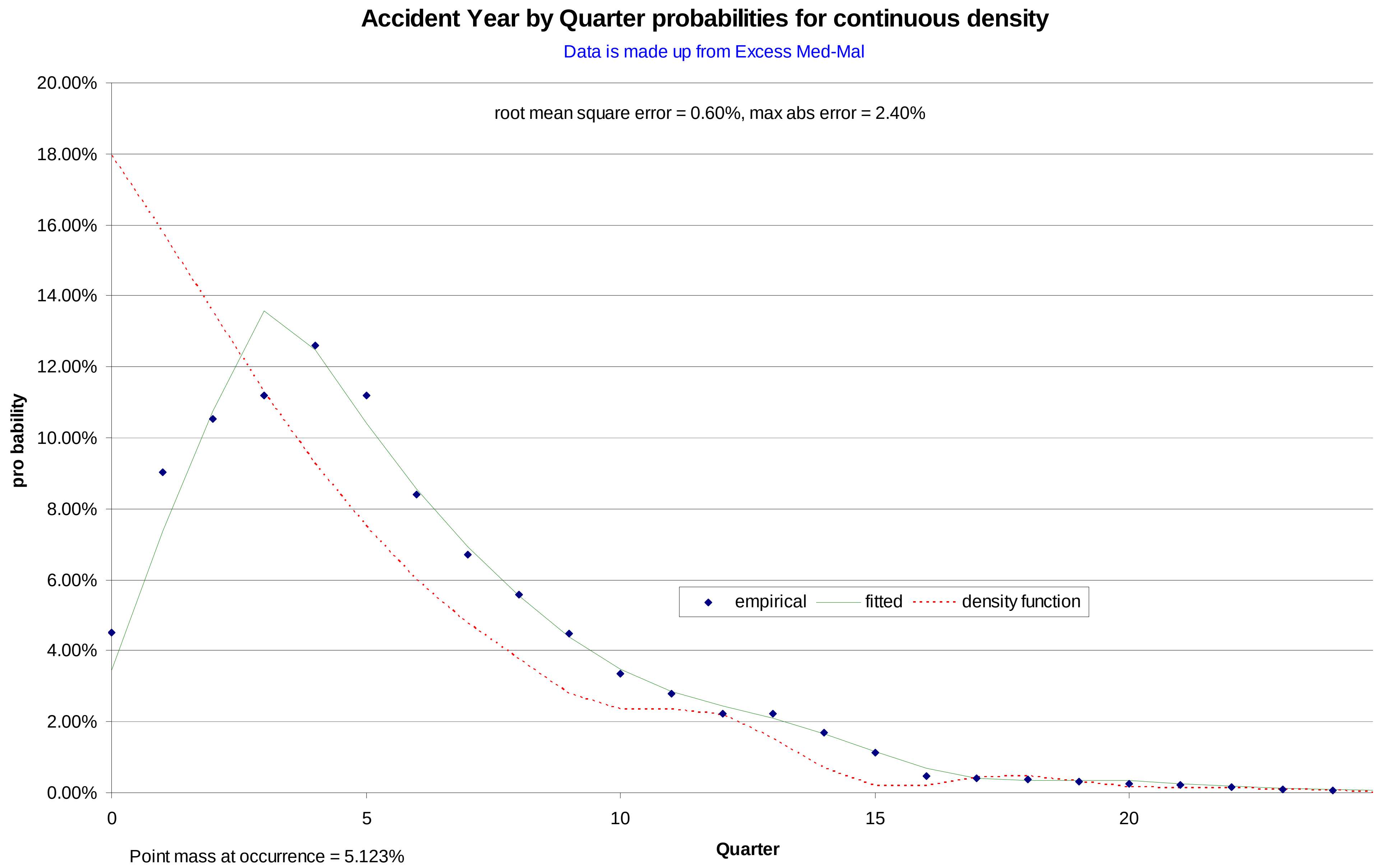
Smoothing 0.003



Smoothing 0.005



Smoothing 0.010



Smoothing 0.030

Huh!

- There is very little curvature left to take out.
- Larger smoothing weights will have little effect.
- The problem is the size of the early quarters relative to the immediately following quarters.
- This also suggests that the data is not real – which it actually isn’t.

Finally

- The spreadsheets also have a simulation capability, so you can see what your fitted distribution will actually do.
- All materials are available from the CAS, and also from me at rkreps8@hotmail.com