INTRODUCTION TO RESERVE RANGE THEORY AND PRACTICAL MODEL APPLICATION

CAS ANNUAL MEETING, NOVEMBER 2009

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Agenda

- Motivation
- Terminology
- Popular stochastic techniques
  - Mack
  - Monte carlo simulation
  - Bootstrapping
- Aggregation of liabilities

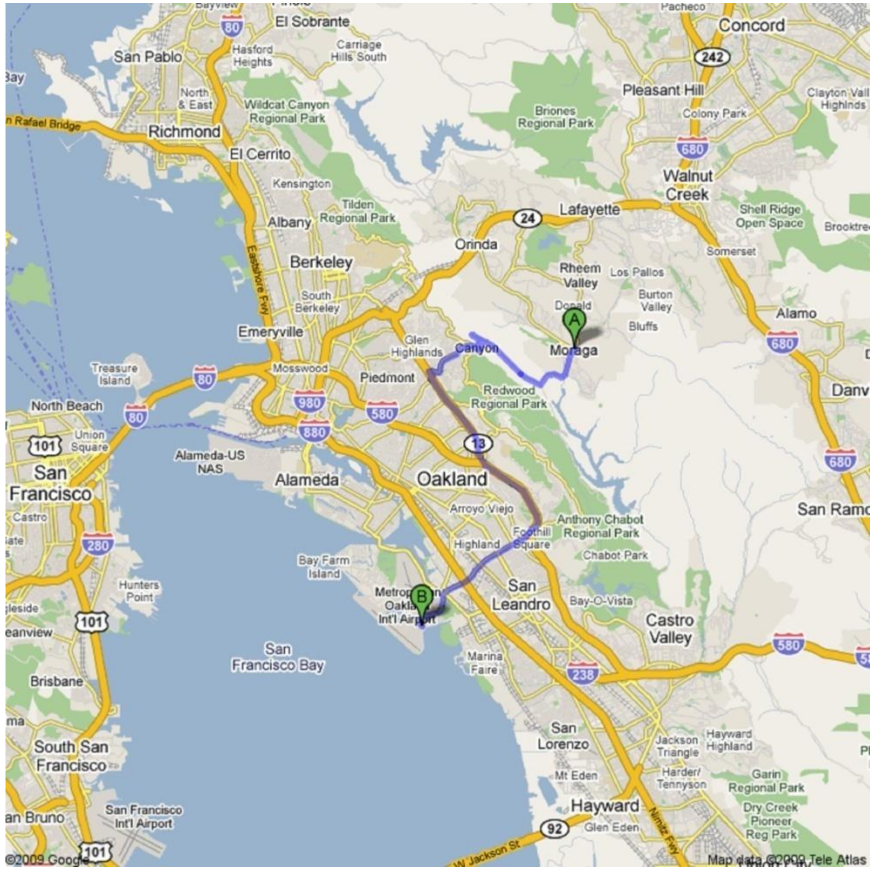
Why measure ranges?

- NAIC
  - "We know carried reserves can't be perfectly omniscient. We'll settle for *reasonable*, with justification."
- Rating Agencies
  - Looking for ways to objectify rating
  - Moving from *reserve adequacy* to *economic capital*
- Fair Value Accounting
  - Value of an asset recognizes uncertainty of future cash flows
  - Concept being applied to liabilities
- Economic Capital
  - Sufficient capital to be 99.5% sure that balance sheet entries will not change over the next year by amounts large enough to ruin the firm
    - Solvency II Solvency Capital Requirement (SCR)
- Transparency
  - If Wall Street understood our company better maybe we'd get a better rating

Practical reasons to provide ranges

- People think we already do. We’re the math geeks after all!
- If actuaries don’t, somebody else will
- Knowing uncertainty of an estimate can improve decisions based on that estimate

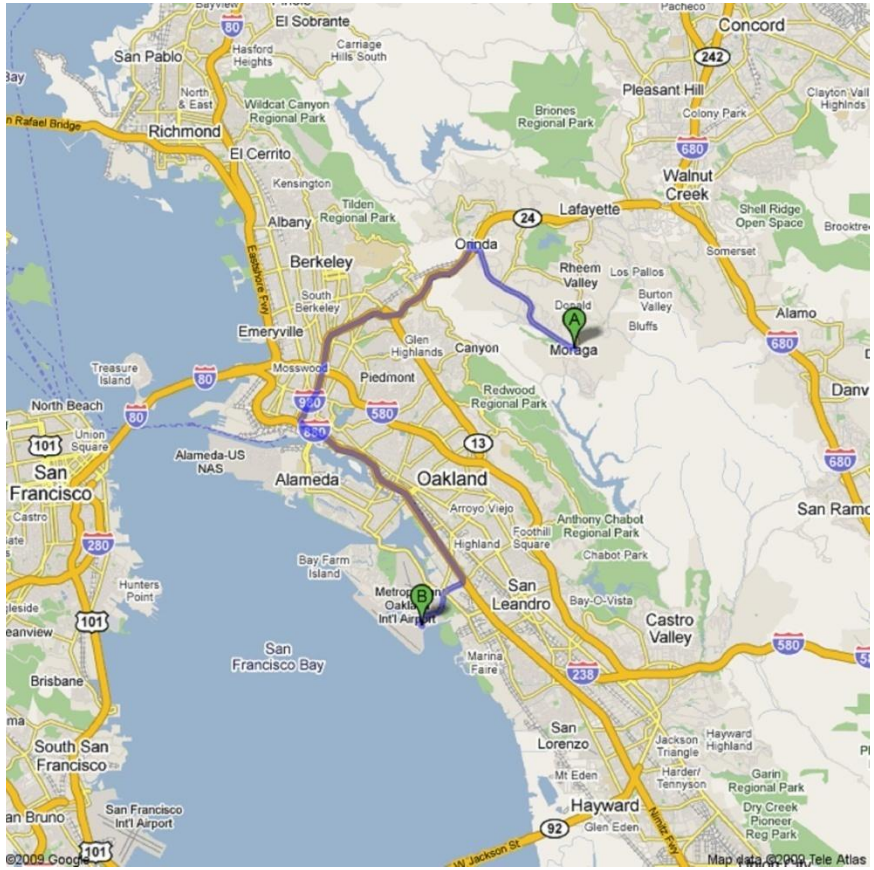
A practical situation where knowing a range can help



Home to airport via back roads

Average = 43 minutes

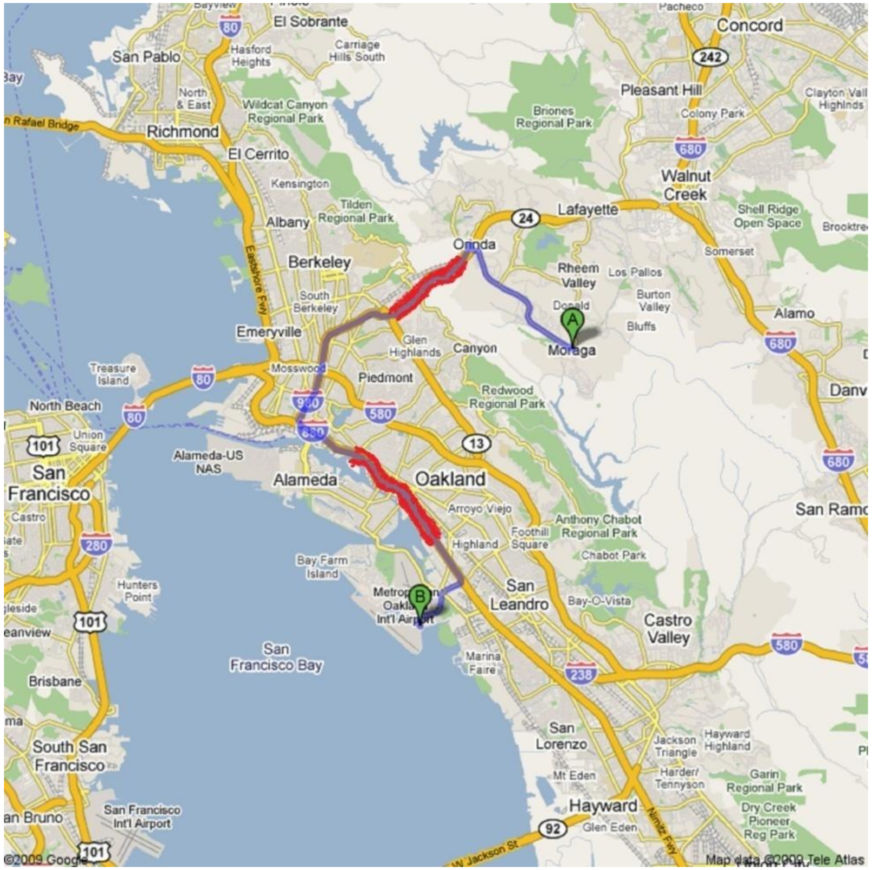
Hmm, if I took the freeway I could get in a power nap



Home to airport via freeway

Average = 32 minutes (per google maps)

Do I risk being late for you-know-who or take sure bet?



Home to airport via freeway

"With traffic add 20-30 minutes"

"Risk comes from not knowing what you're doing."

- Warren Buffet

Top 5 List for Not Giving a Range

Attendance at session required to see list!

☺

SSAP 55 vs. GAAP: Who gave accountants all that say anyway?

- SSAP 55 Effective 2001
  - Management's estimate
    - Management shall record its "best estimate"
  - Ranges of estimates
    - Management may consider a range of reserve estimates
    - The range shall not include the set of all possible outcomes but only those outcomes that are considered reasonable
    - When no estimate within the range ***is better than*** any other, the ***midpoint*** of the range is to be accrued
    - When the high end of the range cannot be quantified, management's best estimate shall be recorded
- GAAP
  - When a range of estimates exists and no estimate ***is better than*** any other, the company shall accrue the ***lowest*** estimate in the range

Actuarial standards regarding "ranges" originally couched in terms of *actuarial methods*

ASOP 36 (2000): Statements of Actuarial Opinion Regarding Property/Casualty Loss and Loss Adjustment Expense Reserves

- Company's stated reserve amount should be within the actuary's *range of reasonable reserve estimates*
- A range of reasonable estimates is a range of estimates that could be produced by appropriate ***actuarial methods*** or alternative sets of assumptions that the actuary judges to be reasonable
- The reasonable range need not be disclosed

ASOP "range" wording is evolving: becoming broader, more mathematical

ASOP 43 (2007): Property/Casualty Unpaid Claim Estimates

- One *should consider uncertainty* associated with one's estimate
- Sources of uncertainty may include ***model risk, parameter risk, and process risk***
- If a range is specified, its basis should be disclosed, e.g.,
  - Based on individual estimates, each of which is a reasonable estimate on a stand-alone basis
  - A confidence interval produced by a ***model or models***
  - A confidence interval reflecting certain risks, such as ***process risk and parameter risk***

Excel-erate Your Mack Method

- What motivates the model behind the Mack methodology?
- How can the calculations be done in a spreadsheet?
- References
  - Mack, “Distribution Free …,” *Astin* 1993, http://www.casact.org/library/astin/vol23no2/213.pdf
  - Murphy, “Unbiased LDFs,” *PCAS* 1994, http://www.casact.org/pubs/proceed/proceed94/94154.pdf
  - Bardis, Majidi, Murphy, “Flexible Factor Chain Ladder Model,” summer *eForum* 2009, http://www.casact.org/pubs/forum/09sumforum/01_Murphy.pdf
  - Barnett, Zehnwirth, “Best Estimates for Reserves,” PCAS 2000, http://www.casact.org/pubs/proceed/proceed00/00245.pdf

Does historical variability have anything to say about future variability in a chain ladder application?

ABC Insurance Company

Chain Ladder Loss Projection

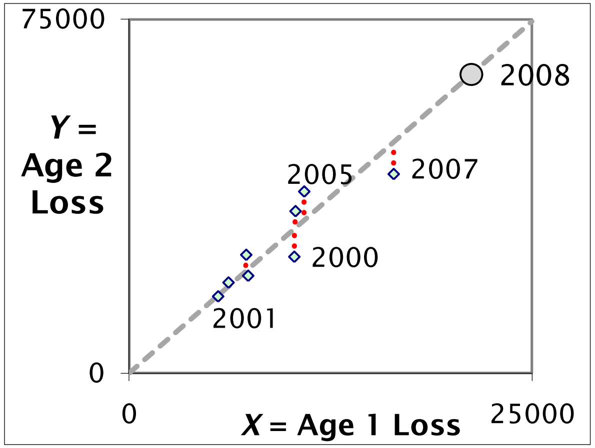
AY \ Age	1	2	3	4	5	6	7	8	9 = Ult
2000	10,238	24,654	38,025	46,550	52,842	58,722	65,227	67,604	69,559
2001	5,508	16,235	25,586	32,863	38,111	42,315	45,171	47,666	49,045
2002	7,374	20,620	34,220	43,438	50,898	55,475	58,367	60,943	62,706
2003	6,153	19,182	31,005	40,424	46,949	50,942	54,931	57,354	59,014
2004	7,253	25,066	40,134	51,063	58,376	64,144	69,166	72,218	74,307
2005	10,855	38,520	62,348	82,710	95,382	104,806	113,011	117,998	121,411
2006	10,313	34,341	51,110	65,632	75,688	83,166	89,677	93,634	96,343
2007	16,411	42,228	66,770	85,743	98,879	108,649	117,155	122,324	125,863
2008	21,234	63,281	100,059	128,491	148,177	162,818	175,564	183,311	188,614

All Yr Wtd	2.980	1.581	1.284	1.153	1.099	1.078	1.044	1.029
Simple Avg	3.022	1.586	1.280	1.154	1.099	1.077	1.046	1.029

- Chain ladder estimate of ultimate loss calculated by squaring the triangle rather than by vector multiplication of diagonal and LDFs
- Variance of chain ladder estimate will also be calculated by squaring
- Start by looking at first future diagonal

Visualization of age 1-2 development suggests the model $Y = bX + \sqrt{X}\sigma z$

AY \ Age	1	2
2000	10,238	24,654
2001	5,508	16,235
2002	7,374	20,620
2003	6,153	19,182
2004	7,253	25,066
2005	10,855	38,520
2006	10,313	34,341
2007	16,411	42,228
2008	21,234	63,281
All Yr Wtd	2.980	



- First term bX expresses expected value of linear relationship
  - Intercept in more general $Y{=}a{+}bX$ does not appear necessary
- Second term $\sqrt{X}\sigma z$ expresses random deviations from expected
  - Form of z unspecified ("Distribution Free") but should be symmetric
  - Heteroscedasticity: higher value of X → higher variability of Y
    - Because of square root, optimal value of b that minimizes the sum of squared residuals ("least squares") is 2.980
- Estimates of b and σ can be calculated by Excel's LINEST function

Remove heteroscedasticity inside LINEST with array version of SQRT

	A	B	C	D	E	F	G
1	**AY \ Age**	**1**	**2**	ata			
2	2000	10,238	24,654	2.408			
3	2001	5,508	16,235	2.948			
4	2002	7,374	20,620	2.796			
5	2003	6,153	19,182	3.118			
6	2004	7,253	25,066	3.456			
7	2005	10,855	38,520	3.549			
8	2006	10,313	34,341	3.330			
9	2007	16,411	42,228	2.573			
10	sum / wtd avg	74,105	220,845	2.980			
11							
12	b	2.980	0	a			
13	se(b)	0.157	#N/A	se(a)			
14	R	98.1%	42.8	s			
15	F	358.5	7	df			
16	ssreg	658159	12851	ssresid			
17							
18	risk	notation	AY 2008	Formula			
19		X	**21,234**				
20		Y	**63,281**	bX	=B12*C19		
21	parameter	$\Delta(Y)$	3,342.10	$X \cdot$se(b)	=C19*B13		
22	process	$\Gamma(Y)$	6,243.48	sqrt(X)$\cdot s$	=sqrt(C19)*C14		
23	total	se(Y)	7,081.71	sqrt($\Delta^2+\Gamma^2$)	=sqrt(B21^2+B22^2)		

LINEST output

AY2008 12-mo value

$$Y = bX + \sqrt{X}\sigma z$$

becomes

$$Y/\sqrt{X} = b\sqrt{X} + \sigma z$$

- Estimates for b, σ will be same in both models

{=LINEST(C2:C9/SQRT(B2:B9),SQRT(B2:B9), FALSE,TRUE)}

"Don't give me an intercept, but give me all the yummy statistics!"

- Δ: Parameter risk = variability in estimate of expected value
- Γ: Process risk = variability due to all other factors not explained by X

Second development period: chained formulas for errors more complicated than for expected values

	A	B	C	D
1	**AY \ Age**	**1**	**2**	**3**
2	2000	10,238	24,654	38,025
3	2001	5,508	16,235	25,586
4	2002	7,374	20,620	34,220
5	2003	6,153	19,182	31,005
6	2004	7,253	25,066	40,134
7	2005	10,855	38,520	62,348
8	2006	10,313	34,341	51,110
9	2007	16,411	42,228	66,770
10	2008	21,234	63,281	**100,059**
11				
12	b_2	1.581	0	a
13	se(b_2)	0.023	#N/A	se(a)
14	R	100%	9.6	s_2
15	F	4888.3	6	df
16	ssreg	446571	548	ssresid
17	risk	notation	AY 2008	Formula
18		Y_1	63,281	
19		Y_2	**100,059**	$b_2 Y_1$
20	parameter	$\Delta(Y_2)$	5,475.36	sqrt(Y_1^2*se(b_2)^2 + b_2^2*$\Delta(Y_1)^2$ + se(b_2)^2*$\Delta(Y_1)^2$)
21	process	$\Gamma(Y_2)$	10,324.69	sqrt(Y_1*s_2^2 + b_2^2*$\Gamma(Y_1)^2$)
22	total	se(Y_2)	11,686.69	sqrt(Δ^2+Γ^2)

$$Y_{2008,2} = b_2 Y_{2008,1} + error$$

- Errors are compounded when beginning value Y_1 is estimated
- Use LINEST to find b, s for second development period

```
{=LINEST(D2:D8/SQRT(C2:C8),SQRT(C2:C8),
FALSE,TRUE)}
```

- Error formulas
  - For 2007: same as before
  - For 2008: more formidable

- Formulas relatively easy to copy cell to cell

Error formulas for *AY sum* of unpaid loss are similar – refer to papers

Same as for AY 2008 alone

Uses "Y_1" = 42,228 + 63,281 and Δ, Γ from age 2 sum

ABC Insurance Company
Chain Ladder Loss Projection

AY \ Age	1	2	3	4	5	6	7	8	9 = Ult
2000	10,238	24,654	38,025	46,550	52,842	58,722	65,227	67,604	69,559
2001	5,508	16,235	25,586	32,863	38,111	42,315	45,171	47,666	49,045
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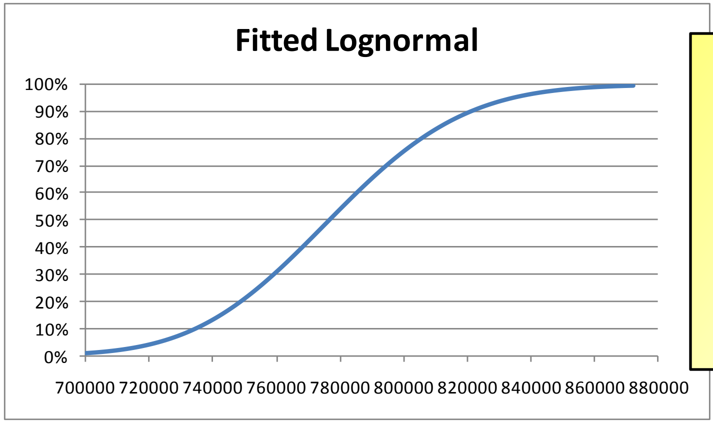
2008	21,234	63,281	100,059	128,491	148,177	162,818	175,564	183,311	188,614
Sum of unpaid loss		63,281	166,829	279,866	418,125	523,583	619,504	707,782	777,301 pt. est.
			= (42228+63281)*1.581						33,566 total risk

Can fit any 2-parameter distribution to first 2 moments of estimated unpaid loss

- Lognormal parameters (method of moments):
  μ=13.6, σ=.04
- Use fitted distribution for risk inferences



Simple average link ratios are the optimal solution of a model with a different variance assumption

	A	B	C	D	E	F	G
1	**AY \ Age**	**1**	**2**	ata			
2	2000	10,238	24,654	2.408			
3	2001	5,508	16,235	2.948			
4	2002	7,374	20,620	2.796			
5	2003	6,153	19,182	3.118			
6	2004	7,253	25,066	3.456			
7	2005	10,855	38,520	3.549			
8	2006	10,313	34,341	3.330			
9	2007	16,411	42,228	2.573			
10				3.022	simple average		
11							
12	*b*	3.022	0	a			
13	se(*b*)	0.147	#N/A	se(a)			
14	R	98.4%	0.4	*s*			
15	F	424.7	7	df			
16	ssreg	73.066	1.204	ssresid			
17							
18	risk	notation	AY 2008	Formula			
19		X	**21,234**				
20		Y_1	**64,172**	bX			
21	parameter	$\Delta(Y_1)$	3,113.93	$X \cdot se(b)$			
22	process	$\Gamma(Y_1)$	60.44	$sqrt(X) \cdot s$			
23	total	$se(Y_1)$	3,114.52	$sqrt(\Delta^2+\Gamma^2)$			

$$Y_1 = b_1 X + X \sigma_1 z$$

- Divide both sides by X to get OLS model with constant variance
- {=LINEST(C2:C9/B2:B9,B2:B9/B2:B9,FALSE,TRUE)}

Average-x-high-low link ratio is optimal solution of a model with a different variance assumption

	A	B	C	D	E	F	G
1	**AY \ Age**	**1**	**2**	ata			
2	2000	10,238	24,654	2.408			
3	2001	5,508	16,235	2.948			
4	2002	7,374	20,620	2.796			
5	2003	6,153	19,182	3.118			
6	2004	7,253	25,066	3.456			
7	2005	10,855	38,520	3.549			
8	2006	10,313	34,341	3.330			
9	2007	16,411	42,228	2.573			
10				3.037	average x hi-lo		
11						α	
12	*b*	3.037	0	a		2.54891151	
13	se(*b*)	0.139	#N/A	se(a)			
14	R	98.55%	0.0	*s*			
15	F	476.5	7	df			
16	ssreg	0.5146	0.0076	ssresid			
17							
18	risk	notation	AY 2008	Formula			
19		X	**21,234**				
20		Y_1	**64,485**	bX			
21	parameter	$\Delta(Y_1)$	2,954.13	$X \cdot se(b)$			
22	process	$\Gamma(Y_1)$	4.79	sqrt(X)·s			
23	total	se(Y_1)	2,954.14	sqrt($\Delta^2+\Gamma^2$)			

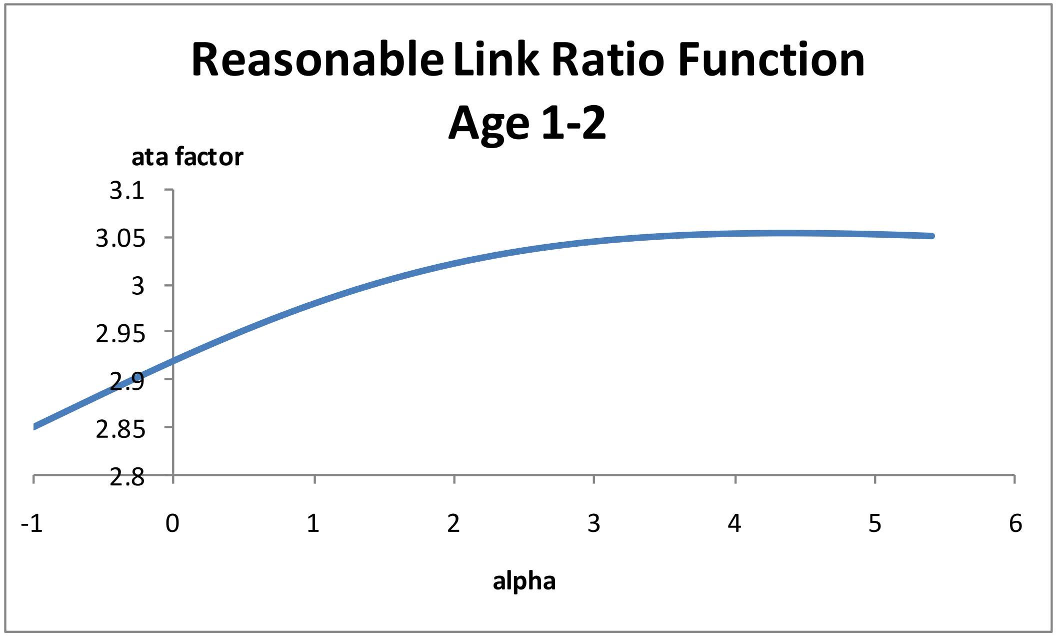
$$Y_1 = b_1 X + X^{\alpha/2}\sigma_1 z$$

α	Type of ata
2	Simple average
1	Weighted average
0	linear regression
other	Actuarial selection

- See eForum paper Bardis, Majidi, Murphy

Many selected link ratios – not necessarily all – can be optimal within this family of α-indexed models

$$Y = bX + X^{\alpha/2}\sigma z$$



- Given triangle data over a development period, *reasonable link ratios* can be viewed as LINEST solutions for some index α
- Use Excel's "What-If" analysis to generate above graph from your own triangle, "Goal Seek" to find α given your selection

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