

CAPITAL ALLOCATION BY PERCENTILE LAYER

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OUTLINE

- Introduction
- Scope
- Sample data set
- Capital Allocation by Percentile Layer methodology
- Results of application to sample data set
- Other applications
- Concluding remarks

INTRODUCTION

- Why allocate capital?
 - measure risk adjusted profitability
 - set target “risk loads” aka “margins”
 - other
- We will focus on setting target “risk loads” aka “margins”

SCOPE

- “Risk Load” / “margin” should be disaggregated into components
 - margin for
 - cost of capital arising from measured variability
 - biases and unmeasured variability in estimated parameters
 - “winner’s curse”
 - data quality
 - future changes [legislative, judicial, societal, etc]
- We discuss allocating capital for setting target margins only for cost of capital from measured variability
 - winner’s curse, etc, are out of scope
- Cost of capital itself might need to be disaggregated into “risk cost” and “frictional cost”
 - allocation method for “risk cost” ought to be different from allocation method for “frictional cost”?
 - we will gloss over this point today; leave for ‘future topics of debate’

SCOPE

- Only analyze capital allocation for
 - underwriting risk
 - short tail

- Topics deferred to Q&A are
 - assets
 - long tail lines of business
 - actual held capital different than required capital
 - other

SAMPLE DATA SET

- Available on CAS meeting website
- 3 LOBs, lognormal, mean 1m, CVs of 20%, 50%, 100%

- 250 year VaR: \$10.1m

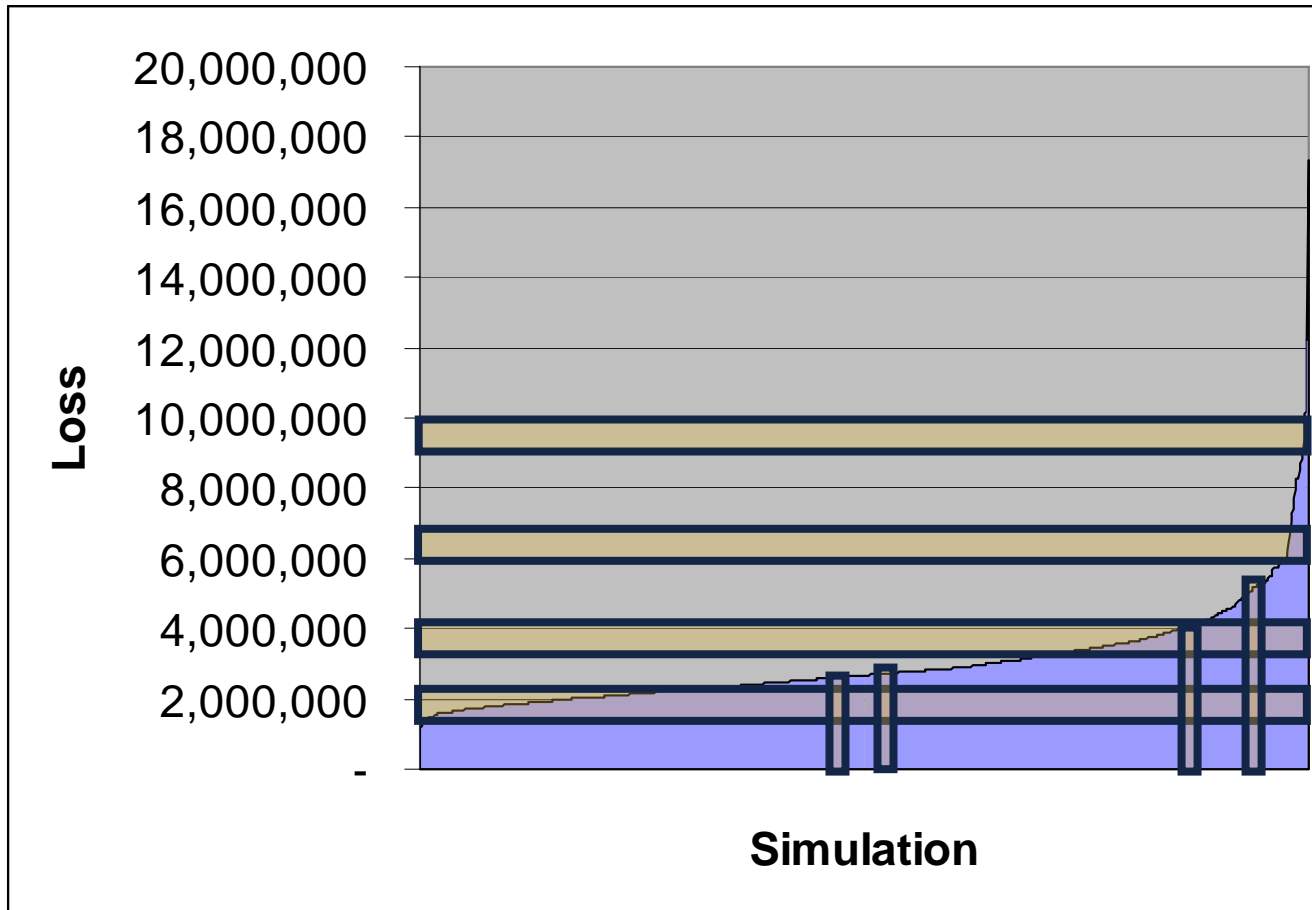
- Largest loss: \$17.3m

These metrics
are based on
the pure Loss
distribution

CAPITAL ALLOCATION BY PERCENTILE LAYER

- Allocate capital costs from total company level to scenario level
 - based on Kreps's "co-measures" and others
 - a scenario is a simulated loss for the entire company
- "Capital Allocation by Percentile layer" assigns capital costs to each scenario across multiple percentile layers of capital
 - initially designed for VaR capital
 - can be tweaked to apply to TVaR
- Allocate capital from scenario level to component level
 - component = LOB, state, underwriter, policy...
- Calculate capital costs, target margin, and target premium
 - if using Loss distribution, then incorporate credit for "contribution to capital" from premium

CAPITAL ALLOCATION BY PERCENTILE LAYER



- Partition total capital into incremental “percentile layers” of capital
- The cost of each layer of capital should only be allocated to scenarios that cause the firm to hold the layer of capital

CAPITAL ALLOCATION BY PERCENTILE LAYER

Allocating from Scenario to LOBs

Simulation	LOB 1	LOB 2	LOB 3	total	LOB 1	LOB 2	LOB 3	total
152	1,195,450	432,090	15,743,032	17,370,572	7%	2%	91%	100%
78	1,344,801	649,606	10,230,559	12,224,966	11%	5%	84%	100%
207	999,982	496,927	9,192,411	10,689,319	9%	5%	86%	100%
211	1,231,477	727,594	8,233,323	10,192,394	12%	7%	81%	100%
890	1,121,540	732,353	8,260,723	10,114,617	11%	7%	82%	100%
469	1,335,332	977,992	7,042,521	9,355,845	14%	10%	75%	100%
821	1,136,602	1,328,601	6,491,581	8,956,784	13%	15%	72%	100%
973	630,563	417,992	7,852,700	8,901,255	7%	5%	88%	100%
323	839,199	579,627	7,434,963	8,853,789	9%	7%	84%	100%
451	688,269	934,708	7,075,494	8,698,471	8%	11%	81%	100%
632	970,182	1,409,193	6,168,811	8,548,186	11%	16%	72%	100%
137	926,653	569,720	6,794,409	8,290,781	11%	7%	82%	100%
208	800,660	982,868	6,504,748	8,288,275	10%	12%	78%	100%
243	804,822	1,395,411	6,045,181	8,245,414	10%	17%	73%	100%
827	1,097,579	1,186,482	5,804,553	8,088,614	14%	15%	72%	100%
445	670,322	1,785,701	5,489,771	7,945,793	8%	22%	69%	100%
696	1,357,838	480,177	5,876,477	7,714,492	18%	6%	76%	100%
292	779,529	572,468	6,044,066	7,396,063	11%	8%	82%	100%
774	904,447	2,671,713	3,731,368	7,307,527	12%	37%	51%	100%
802	1,041,401	3,836,785	2,401,870	7,280,056	14%	53%	33%	100%

Kreps et al:
use actual simulated losses to allocate capital for each individual scenario down to the LOB level; the allocation varies for every single scenario

CAPITAL ALLOCATION BY PERCENTILE LAYER

Application to sample data set

- Required capital amount = \$18m
- Required rate of return on capital from underwriting = 10%
- Required target profit for the overall company = $\$18m * 10\% = \$1.8m$
- Simulated Expected Loss = \$2.99m
- Total Target Premium = $\$2.99m + \$1.8m = \$4.79m$
- What is the total amount of funds available to pay losses?
 - Premium + Capital = $\$4.79m + \$18m = \$22.79m$

CAPITAL ALLOCATION BY PERCENTILE LAYER

Capital	18,000,000
Required Rate of Return	10%
Required profit	1,800,000
Expected Loss	2,990,581
Calculated Premium	4,790,581
Total funds for losses (= capital + premium)	22,790,581

- Why do we require the firm to hold this amount of funds? What is the “required capital rule” that generates this requirement?
 - multiple of VaR? TVaR?
 - based on Loss distribution or Profit distribution?
 - does calculated required capital provide “offset credit” for contributed premium?
 - example: S&P formula for Cat perils provides offset credit for available premium funds
- In our sample data set, the answer is unclear
- Strongly recommend that one should clarify this question when allocating capital costs

CAPITAL ALLOCATION BY PERCENTILE LAYER

- Should you allocate capital costs based on the Loss distribution or the Profit distribution?
 - both are reasonable
 - I prefer allocating capital costs based on the Loss distribution, then afterwards crediting for premium contributions
 - rationale:
 - firm needs sufficient funds to pay severe losses
 - funds derive from premium and capital
 - a reasonable required capital rule → Loss Distribution risk measure (such as VaR or TVaR) minus the funds contributed by premium
- In our example, what is the “risk measure” or “required capital rule” that generates the required \$22.8m of funds?
 - inherently unclear; should be clarified
 - \$22.8m could be $\{\text{VaR}(250 \text{ year}) * 2.25\}$ or $\{\text{TVaR}(10 \text{ year}) * 3.781\}$ or other
 - “percentile layer” procedure depends upon the type of risk metric
 - TVaR is fundamentally different type of criterion than VaR, so allocation must be different

CAPITAL ALLOCATION BY PERCENTILE LAYER

- Let
 - EL = expected loss;
 - P = premium (net of expenses)
 - r = % required rate of return [cost of capital] from underwriting
- Some important formulas
 - $P = EL + \text{cost of capital } \$$
 - $P = EL + \text{cost of capital } \% * (\text{net allocated capital})$
 - $P = EL + \text{cost of capital } \% * (\text{gross allocated capital} - \text{contributed capital})$
 - $P = EL + r * (\text{gross allocated capital} - P)$
 - $P = EL + \{r/(1+r)*(\text{gross allocated capital} - EL)\}$

Once you've allocated capital based on the pure Loss distribution, use this formula to calculate target premium

Caveat: formula might need to change if / when "required capital rule" is different

RESULTS

22.8m gross capital [18m required capital net of premium] is a stipulated assumption; here we interpret the required 22.8m as a multiple of 250 year VaR

Total Required Funds = VaR(250 Year) * 2.25; allocation via Capital Allocation by Percentile Layer

	LOB 1	LOB 2	LOB 3	Total
1 Expected Loss	1,009,960	992,037	988,584	2,990,581
2 Gross Allocated Capital	4,686,143	4,897,645	13,206,794	22,790,581
3 Allocated Margin	334,198	355,055	1,110,746	1,800,000
4 Allocated Margin % of Total Margin	18.6%	19.7%	61.7%	100.0%
5 Calculated Premium	1,344,159	1,347,093	2,099,330	4,790,581
6 Calculated Premium % of Total Premium	28.1%	28.1%	43.8%	100.0%
7 Net Allocated Capital	3,341,984	3,550,552	11,107,464	18,000,000
8 Margin % of Net Allocated Capital	10.0%	10.0%	10.0%	10.0%
9 Target LR % [no expenses]	75.1%	73.6%	47.1%	62.4%
10 Target Profit Margin % [no expenses]	24.9%	26.4%	52.9%	37.6%
11 Margin % of Expected Loss	33.1%	35.8%	112.4%	60.2%

Notes

- 1 stipulated simulated losses
- 2 via Capital Allocation by Percentile Layer
- 3 $r/(1+r) * (\text{allocated gross capital} - \text{EL})$
- 4 row 3 / row 3 total
- 5 row 1 + row 3
- 6 row 5 / row 5 total
- 7 row 2 minus row 5
- 8 row 3 / row 7
- 9 row 1 / row 5
- 10 row 3 / row 5
- 11 row 3 / row 1

Catastrophe prone LOB 3 gets larger Target Profit Margin...but LOBs 1 and 2 still receive “reasonable” targets; contrast to other popular methods

RESULTS

22.8m gross capital [18m required capital net of premium] is a stipulated assumption; here we interpret the required 22.8m as a multiple of 10 year TVaR

Total Required Funds = TVaR(90%) * 3.78; allocation via Capital Allocation by Percentile Layer

	LOB 1	LOB 2	LOB 3	Total
1 Expected Loss	1,009,960	992,037	988,584	2,990,581
2 Gross Allocated Capital	6,087,630	6,490,290	10,212,661	22,790,581
3 Allocated Margin	461,606	499,841	838,552	1,800,000
4 Allocated Margin % of Total Margin	25.6%	27.8%	46.6%	100.0%
5 Calculated Premium	1,471,566	1,491,879	1,827,136	4,790,581
6 Calculated Premium % of Total Premium	30.7%	31.1%	38.1%	100.0%
7 Net Allocated Capital	4,616,063	4,998,412	8,385,525	18,000,000
8 Margin % of Net Allocated Capital	10.0%	10.0%	10.0%	10.0%
9 Target LR % [no expenses]	68.6%	66.5%	54.1%	62.4%
10 Target Profit Margin % [no expenses]	31.4%	33.5%	45.9%	37.6%
11 Margin % of Expected Loss	45.7%	50.4%	84.8%	60.2%

Notes

- 1 stipulated simulated losses
- 2 via Capital Allocation by Percentile Layer
- 3 $r/(1+r) * (\text{allocated gross capital} - \text{EL})$
- 4 row 3 / row 3 total
- 5 row 1 + row 3
- 6 row 5 / row 5 total
- 7 row 2 minus row 5
- 8 row 3 / row 7
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- 10 row 3 / row 5
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- Using a lower critical percentile (10 year vs 250 year) redistributes the required margin from LOB 3 to LOB 1 and LOB 2
- but LOB 3 still receives a significantly larger margin than LOB 1 and LOB 2

RESULTS

22.8m gross capital [18m required capital net of premium] is a stipulated assumption; here we interpret the required 18m as a multiple of {250 Year VaR – Premium}

Total Required Capital = {VaR(250 yr) - Premium} * 3.38; allocation via Capital Allocation by Percentile Layer

	LOB 1	LOB 2	LOB 3	Total
1 Expected Loss	1,009,960	992,037	988,584	2,990,581
2 Gross Allocated Capital	3,983,231	4,276,006	14,531,345	22,790,581
3 Allocated Margin	270,297	298,543	1,231,160	1,800,000
4 Allocated Margin % of Total Margin	15.0%	16.6%	68.4%	100.0%
5 Calculated Premium	1,280,257	1,290,580	2,219,744	4,790,581
6 Calculated Premium % of Total Premium	26.7%	26.9%	46.3%	100.0%
7 Net Allocated Capital	2,702,974	2,985,426	12,311,601	18,000,000
8 Margin % of Net Allocated Capital	10.0%	10.0%	10.0%	10.0%
9 Target LR % [no expenses]	78.9%	76.9%	44.5%	62.4%
10 Target Profit Margin % [no expenses]	21.1%	23.1%	55.5%	37.6%
11 Margin % of Expected Loss	26.8%	30.1%	124.5%	60.2%

Notes

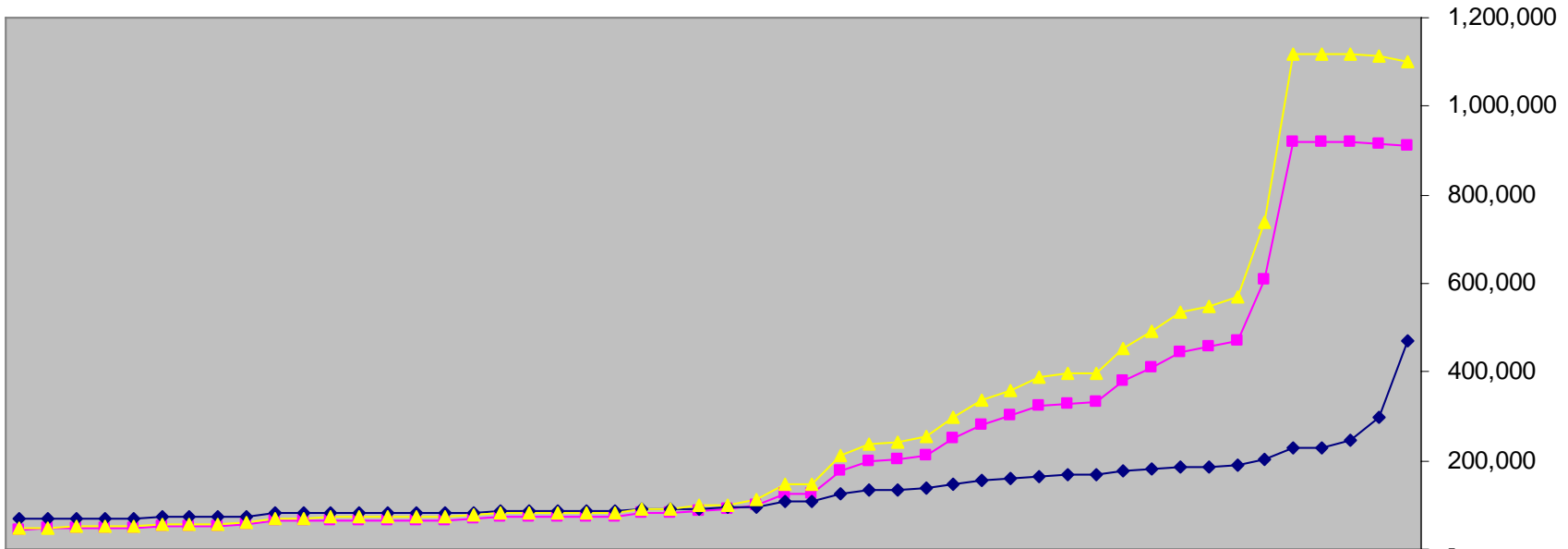
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- 4 row 3 / row 3 total
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- 7 row 2 minus row 5
- 8 row 3 / row 7
- 9 row 1 / row 5
- 10 row 3 / row 5
- 11 row 3 / row 1

- here we interpret required capital rule as {Multiple * (250 Year VaR Loss – Premium)};
- shifts allocation of margin more towards LOB C

RESULTS

Capital Allocation by Percentile Layer: Allocations to Largest 50 Simulated Scenarios

- Capital Rule = {VaR 250 Year * Loss Multiplier} - Premium
- Capital Rule = {TVaR 10 Year * Loss Multiplier} - Premium
- Capital Rule = {VaR 250 Year - Premium} * Capital Multiplier



RESULTS

What if gross capital was based just on 250 Year VaR - Premium?

Total Required Funds = VaR(250 yr); allocation via Capital Allocation by Percentile Layer

	LOB 1	LOB 2	LOB 3	Total
1 Expected Loss	1,009,165	991,712	979,685	2,980,562
2 Gross Allocated Capital	2,079,742	2,173,608	5,861,266	10,114,617
3 Allocated Margin	97,325	107,445	443,780	648,550
4 Allocated Margin % of Total Margin	15.0%	16.6%	68.4%	100.0%
5 Calculated Premium	1,106,491	1,099,158	1,423,465	3,629,113
6 Calculated Premium % of Total Premium	30.5%	30.3%	39.2%	100.0%
7 Net Allocated Capital	973,252	1,074,451	4,437,801	6,485,504
8 Margin % of Net Allocated Capital	10.0%	10.0%	10.0%	10.0%
9 Target LR % [no expenses]	91.2%	90.2%	68.8%	82.1%
10 Target Profit Margin % [no expenses]	8.8%	9.8%	31.2%	17.9%
11 Margin % of Expected Loss	9.6%	10.8%	45.3%	21.8%

Notes

- 1 stipulated simulated losses
- 2 via Capital Allocation by Percentile Layer
- 3 $r/(1+r) * (\text{allocated gross capital} - \text{EL})$
- 4 row 3 / row 3 total
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- 11 row 3 / row 1

- here we interpret required capital rule as
 - required capital = 250 Year VaR Loss – Premium;
- target margins look fairly “realistic”

OTHER APPLICATIONS

- Allocate the 'cost' of 'reinsurance capital'
 - create pricing formulas that directly incorporate the cost of your company's reinsurance program
 - integrated within consistent framework for allocating cost of equity capital and cost of reinsurance capital

CONCLUDING REMARKS

- Mechanics of Capital Allocation by Percentile Layer
 - allocates from “firm level” to “scenario level” to “component level”
 - component can be as granular as you like
 - LOB
 - state
 - individual policy
 - allocates based on asking “which losses cause the firm to hold each dollar of capital”?
 - not a “marginal” method
 - rooted in “equitable cost allocation”
 - see Mango’s paper on Game Theory
 - takes the real world cost of holding capital and assigns it to the LOBs and policies that cause the firm to incur this cost

CONCLUDING REMARKS

- Results of Capital Allocation by Percentile Layer
 - allocates more capital to more severe lines
 - allocates capital to all lines of business that cause the firm to hold capital
 - non-cat lines still receive substantial target pricing margins
 - contrast to other methods that can produce “unrealistically small” target margins for non-cat
 - produces “reasonable” and “realistic” target margins

CORRESPONDENCE

- Workbooks supporting the calculations in this presentation are available from the author
- Send questions to neil.bodoff@willis.com

References

Bodoff, Neil, "Capital Allocation by Percentile Layer", 2007 ERM Symposium,
www.ermsymposium.org/2007/pdf/papers/Bodoff.pdf

Bodoff, Neil, "Capital Allocation by Percentile Layer", 2009 *Variance*,
<http://www.variancejournal.org/issues/03-01/13.pdf>

Kreps, Rodney, "Riskiness Leverage Models", Proceedings of the Casualty Actuarial Society (PCAS) XCII, 2005, 31-60,
www.casact.org/pubs/proceed/proceed05/05041.pdf

Mango, Donald F., "Capital Consumption: An Alternative Methodology for Pricing Reinsurance", 2003 Winter CAS Forum, 351-378,
www.casact.org/pubs/forum/03wforum/03wf351.pdf

Mango, Donald F., "An Application of Game Theory: Property Catastrophe Risk Load," PCAS 85, 1998, pp. 157-186,
<http://www.casact.org/pubs/forum/97spforum/97spf031.pdf>