OLIVER WYMAN

Financial Services

16 November 2009

Capital Allocation

CAS Annual Meeting

GARY G. VENTER

Senior Advisor to Oliver Wyman

gary.venter@gmail.com

Some Basics of Capital Allocation

- How to depends on why
- Will assume purpose is to measure risk-adjusted return
  - Purpose of that is:
    - Decide on which business units to grow or shrink
    - Set target profit levels – i.e., pricing risk
- Capital will be allocated in proportion to a risk measure
  - Starts with Capital = K * Risk Measure
    - E.g., Capital = 3 * $TVaR_{98}$ or Capital = 7 * $TVaR_{60}$ or Capital = 6 * std
    - Used to be K = 1, but not necessary

Proposed Answer

- Use distortion measures
  - Basically risk measure is mean under transformed probabilities
- Capital is multiple of risk measure
- Allocation by such a risk measure and setting target returns equal across business units ends up pricing by mean of transformed probabilities
- That has some good properties as a pricing rule
- Pricing by other allocations is a back-door into risk pricing that often ignores basic pricing principles
  - Like pricing for all the risk, not just the tail

Pricing Theory 1 – CAPM

- Price for risk is proportional to covariance of risk with market
- Many problems with that including:
  - If risk is not normally distributed, basic investor utility calls for using higher moments as well
  - Empirical studies (Fama-French especially) find that other factors influence prices
  - FF factors may be proxies for risk measures
    - Two higher co-moments do as well as FF
    - Using even more co-moments eliminates FF factor effect
  - Jump risk probably influences prices in addition to moments
    - Jumps make market incomplete and are systematic and not hedgeable
- But even super-CAPM taking all that into account would be wrong for insurance because …

In Insurance, Specific Risk Matters

- Opacity of accounting makes security analysts and investors sensitive to earnings fluctuations
  - Impact on market price can be much greater than capital loss
- Cost of raising new capital can be high especially when it is needed – distressed firms – so losses from specific risk can be more expensive than market capital costs
- Policyholders tend to be not diversified in their insurance purchases so are adverse to all risk of the insurer
  - Recent market turmoil illustrates that weakening financial position of insurer will force it to lower prices
- Empirical studies support these ideas

Pricing Theory 2 – Arbitrage-Free Pricing

- Arbitrage = positive expected return with no risk of loss
- May exist for a short time but is quickly competed away
- Some so-called arbitrage actually has hidden risks
- In incomplete market same principle applies
  - In complete market no-arbitrage uniquely determines prices
  - In incomplete market no-arbitrage restricts but does not determine prices
- Either way, basic rule to be arbitrage-free is prices have to be means under equivalent transformed probabilities
- Equivalent = no positive probabilities transformed to zero or vice-versa
- Also transforms have to be on event probabilities, not on outcomes of deals – all deal prices have to use same probabilities for the underlying events

Arbitrage-Free Pricing in Insurance

- Main implication is that prices have to be additive
- In effect, policyholders get the benefit of diversification
  - Insurers still get fair price for risk they take on
  - If pricing does not give diversification benefit to policyholders, insurer will eventually lose that business to competitors
  - Advantage of diversification is to improve competitive position
- Suppose not, e.g., suppose insurer can charge pre-diversification prices, pool risk, and cede 100% to a reinsurer at a lower price
  - That would be an arbitrage profit
  - Not likely in competitive insurance market
- Pricing then through equivalent probability transforms
  - Transform probabilities of firm simulations and lines inherit
  - Transform does not change losses, just probabilities of losses
  - Unfortunately still have to decide among transforms
  - Gives target price based on firm risk, to judge market prices

Some Transforms That Have Worked in Market

- Esscher transform:
  - Parameter w, with $c = G^{-1}(1 - 1/w)$
  - $g^*(y) = Kg(y)\exp(y/c)$, $K = 1/E[\exp(Y/c)]$
  - Doesn't always exist but will if there is a maximum loss
- Wang transform, original normal
  - $G^*(y) = \Phi[\Phi^{-1}(G(y)) + \lambda]$
    - Probability transform from shifting normal percentile by λ
    - λ may be positive or negative depending on losses vs. profit, etc.
    - In simulation get g^* by differencing G^*
- Wang transform with T distribution q dof
  - $G^*(y) = T_q[\Phi^{-1}(G(y)) + \lambda]$
  - Puts more weight in tail
  - Unfortunately in both tails
- Empirical studies on bonds, cat bonds, cat reinsurance give support to Esscher and Wang T_5

Example Company

Simulated Company – 1000 Simulations

	LOB 1	LOB 2	LOB 3	TOTAL
Log N Mu	13.796	13.691	13.316	
Log N Sigma (~CV)	20.0%	50.0%	100.0%	
Actual CV	20.2%	53.3%	131.1%	
Skew	61.4%	175.0%	618.5%	
Expected Loss	1,000,000	1,000,000	1,000,000	3,000,000
Simulated	1,009,960	992,037	988,584	2,990,581
Correlation with Total	17%	34%	91%	100%

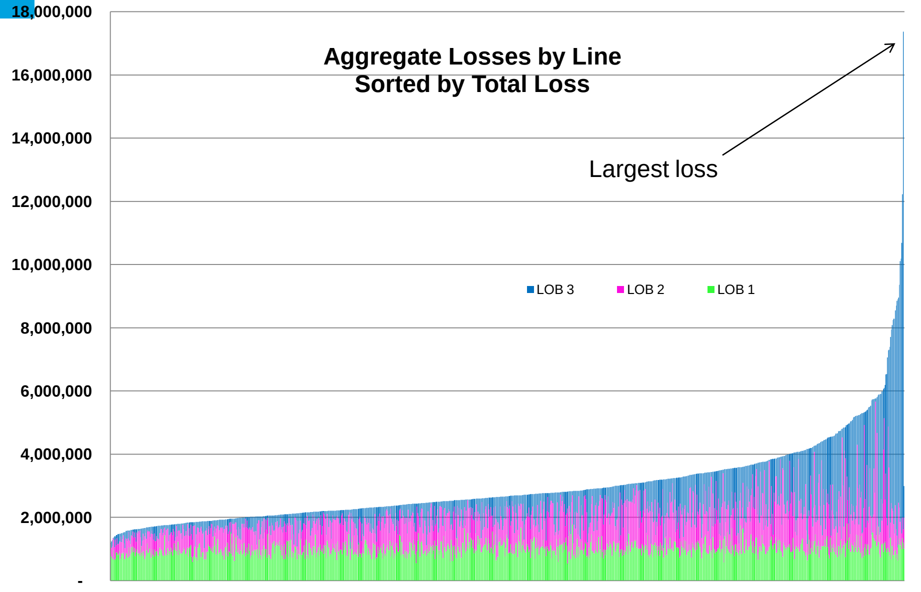
Capital = 18,000,000 ≈ 2.5 $TVaR_{95}$ ≈ 3 $TVaR_{90}$

Target underwriting profit = 10% of capital = 1,800,000

After tax this is maybe 6.5% and investment income taxed down to be in insurance company, so maybe reasonable

Agg mean	**2,990,581**
Stdev	**1,358,875**
CV	**45.4%**
Skew	**318.9%**
Skew if lognormal	**145.7%**

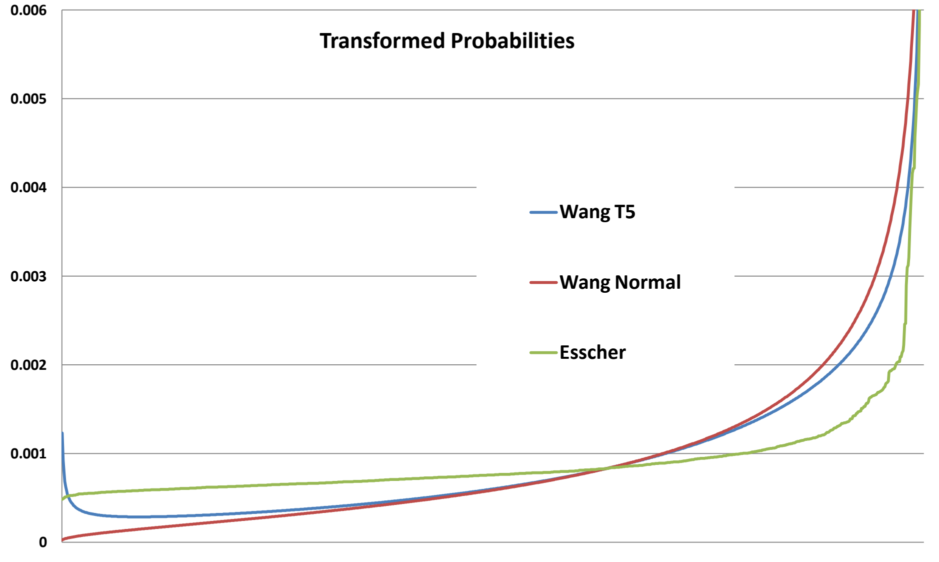
Line Contributions to Total Distribution



Applying Transforms

- Each has a free parameter
- Find parameter value that makes transformed mean = mean + target profit
- Original probability of each scenario = 0.001 = 1/1000
- Esscher probability:
  - w = 3.57, c = 3,281,072, K = 1/E[exp(Y/c)]= 1/2.968
  - g*(y) = Kexp(y/c)/1000
- Normal Wang, set scenario probability = 1/1001 so cumulative probability < 1
  - λ = -1.03005, $G^*(y) = \Phi[\Phi^{-1}(G(y)) + \lambda]$
  - G*(1000/1001) = 0.9803, g*(largest scenario) = 1 – 0.9803 = 0.0197
  - G*(999/1001) = 0.9677, g*(2nd largest) = 0.9803 – 0.9677 = 0.0126

Resulting Probabilities



Allocation and Load

	Total	LOB 1	LOB 2	LOB 3	
Capital	18,000,000	278,954	307,078	17,413,968	***Esscher***
	18,000,000	304,445	1,299,012	16,396,543	***Wang T5***
	18,000,000	390,663	1,892,154	15,717,183	***Wang Normal***
Expected profit	1,800,000	27,895	30,708	1,741,397	***Esscher***
	1,800,000	30,444	129,901	1,639,654	***Wang T5***
	1,800,000	39,066	189,215	1,571,718	***Wang Normal***
Price	4,790,581	1,037,856	1,022,745	2,729,980	***Esscher***
	4,790,581	1,040,405	1,121,939	2,628,238	***Wang T5***
	4,790,581	1,049,026	1,181,253	2,560,302	***Wang Normal***
Load	60.2%	2.8%	3.1%	176.2%	***Esscher***
	60.2%	3.0%	13.1%	165.9%	***Wang T5***
	60.2%	3.9%	19.1%	159.0%	***Wang Normal***