Measuring the Robustness of Different Claims Reserving Methods

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Motivation

- Efforts to find a "best" estimate of the outstanding claims liability
- In general, different forecasting models give different estimates

  → *How to compare them? Which one is better?*

Motivation

- Complexity of the underlying claims generating process
- Complexity of the process of claims handling from the time they are notified to their finalization

  → *Variability in the amount paid in any particular calendar year for claims from a given accident year*

Problem

To study the impact of (small) perturbations in each entry of the runoff triangle on the forecast of the outstanding claims liability, given a particular forecasting model.

Robustness

Measuring one aspect of the robustness of a model by looking at *how sensitive* it is relative to the entries of a runoff triangle.

→ *How sensitive are the forecast values to (small) perturbations in the data?*

A measurement of the sensitivity of a statistic

The rate of change of a statistic to a small change in a particular observation

$$\frac{\partial T}{\partial X_i}$$

Leverage and Influence

Studies on *Leverage and Influence* in Regression or Linear Models, Non-liner Regression, Two-Way Table, etc

→ Example: The statistic analyzed is the fitted value

Sensitivity Analysis

"The study of how the variation in the output of a model can be apportioned, qualitatively or quantitatively, to different sources of variation, and how a given model depends upon the information fed into it".

Saltelli, A., et al. (Editors). 2000. *Sensitivity Analysis*, John Wiley & Sons, page 3

Measurement of Sensitivity

$$\text{Leverage} \equiv \frac{\Delta\text{estimate O/S}}{\Delta\text{entry}}$$

The Importance of Leverage

- Gain insights on the forecasting methodology used:
  → *Very or Moderately or Not Sensitive?*
- Gain insights on the data:
  → *Absolute and Relative importance*
- Gain insights on the uncertainty of the estimate of the outstanding claims liability
  → *Example: if the leverage is high then the estimate is uncertain*

Leverage

- High leverage (positive or negative) is not desirable:
  → *the forecasting methodology used is very sensitive to small perturbations*
  → *significant difference in the estimates of the unperturbed and the perturbed data (there is an uncertainty in the estimate)*

Leverage

- Zero (close to zero) leverage is not desirable
  → *the estimate of the outstanding claims liability is not affected by the perturbations*
- Moderate leverage values are desirable
  → *gain insights on the behaviour of the estimate of the outstanding claims liability to small perturbations in the data*

Mack's Data ($'000)

	0	1	2	3	4	5	6	7	8	9
0	5012	3257	2638	898	1734	2642	1828	599	54	172
1	106	4179	1111	5270	3116	1817	-103	673	535	
2	3410	5582	4881	2268	2594	3479	649	603		
3	5655	5900	4211	5500	2159	2658	984			
4	1092	8473	6271	6333	3786	225				
5	1513	4932	5257	1233	2917					
6	557	3463	6926	1368						
7	1351	5596	6165							
8	3133	2262								
9	2063									

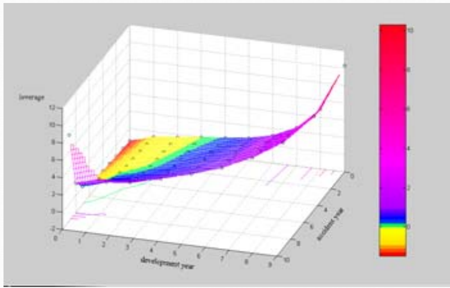
Chain Ladder

Chain Ladder Estimate of the Outstanding Claims Liability of Mack's Data: 52 135

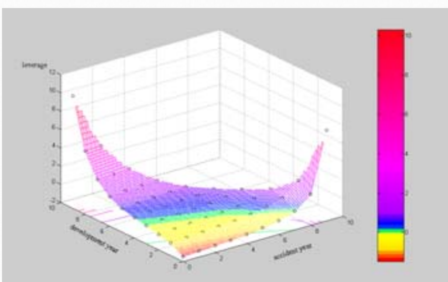
Chain Ladder Leverage

	0	1	2	3	4	5	6	7	8	9
0	-1.48	-0.637	-0.344	-0.005	0.253	0.571	1.226	2.453	4.922	10.316
1	-1.375	-0.532	-0.24	0.099	0.357	0.675	1.331	2.557	5.026	
2	-1.273	-0.43	-0.138	0.201	0.459	0.777	1.433	2.659		
3	-1.152	-0.309	-0.016	0.323	0.581	0.899	1.554			
4	-1.045	-0.202	0.091	0.43	0.688	1.006				
5	-0.817	0.026	0.318	0.658	0.915					
6	-0.488	0.355	0.647	0.986						
7	0.05	0.893	1.185							
8	1.412	2.255								
9	7.92									

Chain Ladder Leverage
(1 unit increase)



Chain Ladder Leverage
(1 unit increase)



Chain Ladder Leverage

1. *What happens if claim payments are delayed?*

   For a particular accident year:

   Pay early → a "decrease" in outstanding claims liability estimate

   Pay later → an "increase" in outstanding claims liability estimate

Chain Ladder Leverage

2. *What happens when there are very few observations to forecast?*

   Large leverage in the last accident year and at the tail

Hertig's Model

$$l_{ij} \sim N\left(\mu_j, \sigma_j^2\right) \quad , \quad i = 0, 1, \cdots, n-2$$
$$j = 1, 2, \cdots, n-i-1$$

Hertig's Model

$$\hat{E}\left[U_i \middle| c_{i,n-i-1}\right] = c_{i,n-i-1} e^{\hat{g}_i} e^{0.5 v_i^2}$$
$$\hat{g}_i = E\left[g_i\right] = \mu_{n-i} + \mu_{n-i+1} + \cdots + \mu_{n-1}$$
$$Var\left[g_i\right] = v_i^2 = \sigma_{i,n-i}^2 + \sigma_{i,n-i+1}^2 + \cdots + \sigma_{i,n-1}^2$$

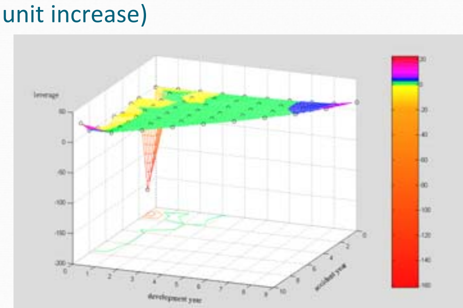
Hertig's Model

Hertig's Model Estimate of the Outstanding Claims Liability of Mack's Data: 86 889

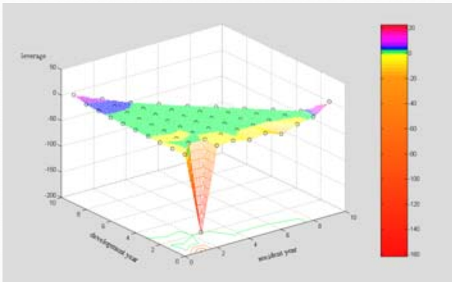
Hertig's Model Leverage (1 unit increase)

	0	1	2	3	4	5	6	7	8	9
0	-1.292	-1.311	-0.513	-0.11	0.48	1.201	2.116	3.237	5.489	12.161
1	-161.585	1.03	-1.596	0.762	0.877	1.323	2.073	3.707	6.455	
2	-1.352	-0.629	-0.034	0.257	0.643	1.142	1.677	2.678		
3	-0.659	-0.469	0.025	0.47	0.626	0.996	1.528			
4	-7.935	0.318	0.254	0.626	0.804	0.996				
5	-3.322	0.037	0.671	0.842	1.454					
6	-13.908	0.367	1.51	1.405						
7	-3.344	1.177	1.664							
8	2.265	2.309								
9	22.815									

Hertig's Model Leverage (1 unit increase)



Hertig's Model Leverage
(1 unit increase)



Hertig's Model Leverage

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Hertig's Model Leverage

- *What happens when there are very few observations to forecast?*

  Large leverage in the last accident year and at the tails
- Extremely large leverage in entry (1,0) →unusual observation

CONCLUSION

The (triangle of) Leverage:

1. Show some characteristics/properties of the forecasting model used

   → same leverage pattern across different runoff triangles

   Chain Ladder and Hertig's Model:

   The Negative-Zero-Positive Zones

CONCLUSION

Chain Ladder:

- High leverage in the last accident year and at the tails
- Smooth leverage

Hertig's Model:

- High leverage in the last accident year and at the tails
- More variability in leverage

CONCLUSION

2. Show some characteristics of the data

   → Hertig's Leverage reflected the unusual observation in the data whereas that of the Chain Ladder did not.