LOSS RESERVING WITH GROWTH CURVES

David R. Clark, FCAS

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LDF Curve-Fitting and Stochastic Reserving

Goals:

1. Describe loss emergence in a mathematical model to assist in estimating needed reserves

2. Calculate the variability around the estimated reserves

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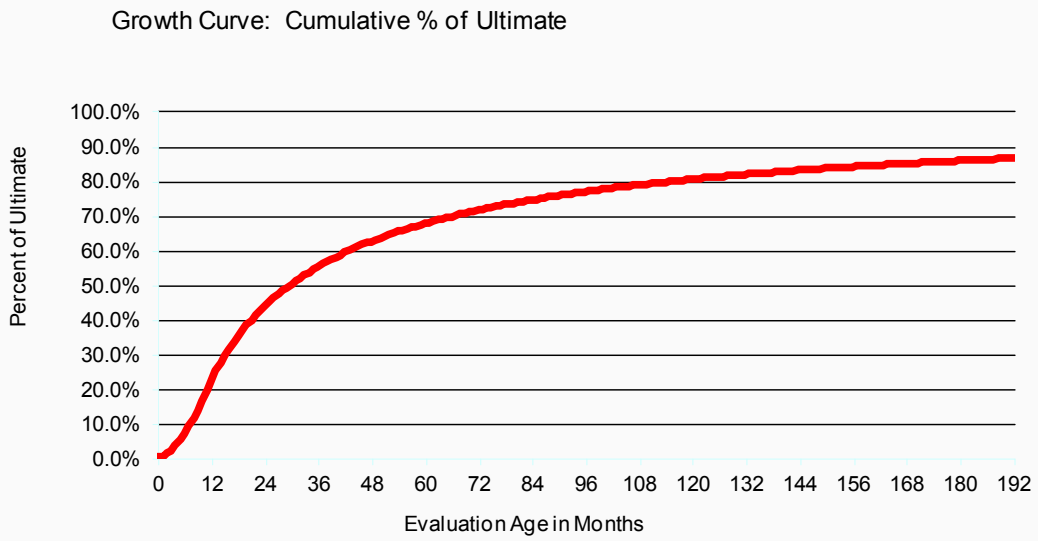

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Growth Curve = Cumulative % of Ultimate

$$G(t) = 1 / LDF_t$$

Inverse Power Curve:

$$G(t|\theta,\omega) = 1 / [1+(\theta/t)^{\omega}]$$

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Why use a continuous curve?

1. Smoothing of Development Pattern
2. Interpolation & Extrapolation (including tail factor)
3. Handle irregular evaluation dates (e.g., latest diagonal less than 12 months from penultimate diagonal)
4. Avoid Over-Parameterization

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Disadvantages of using a continuous curve:

1. Need curve-fitting engine (answers not in “real time”)
2. May not fit well unless the “right” curve form is used

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Basic Model:

1. Convert loss development triangle to an incremental basis
2. For each "cell" of the triangle, we have

$c_{i,t}$ = actual loss for AY *i,* between ages *t* and *t-1*

$\mu_{i,t}$ = expected loss for AY *i,* between ages *t* and *t-1*

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Two Methods for calculating the Expected Incremental Loss:

1. LDF

   Allows each accident year reserve to be estimated independently

2. Cape Cod

   Requires on level premium or other exposure base for each accident year

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LDF Method:

$\mu_{i,t} = Ultimate_i \times [G(t|\theta,\omega) - G(t\text{-}1\ |\theta,\omega)]$

n+2 Parameters:

$Ultimate_i$	expected ultimate loss for accident year *i*
θ	"scale" parameter of G(*t*)
ω	"shape" parameter of G(*t*)

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Cape Cod Method:

$\mu_{i,t} = Premium_i \times ELR \times [G(t|\theta,\omega) - G(t\text{-}1\ |\theta,\omega)]$

3 Parameters:

ELR	expected loss ratio for all years
θ	"scale" parameter of G(*t*)
ω	"shape" parameter of G(*t*)

An on level $Premium_i$ entry for each accident year must be supplied by the user

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Why We Prefer the Cape Cod Method:

- Provides the model with more information (an exposure base in addition to the triangle)
- Requires estimation of fewer parameters - resulting in lower parameter variance
- More stable estimate of immature year(s)

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And now for the Stochastic part…

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Assumptions:

- The expected development in each cell, $\mu_{i,t}$ is treated as the mean of a distribution.
- Each cell has a different mean, but assumed to have the same ratio of Variance/Mean, σ^2.
- The distribution for each cell follows an *Over-dispersed Poisson* with a constant Variance/Mean ratio.
- The model parameters are estimated using Maximum Likelihood Estimation (MLE).

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What the heck is an *Over-dispersed Poisson* distribution?

A discretized version of the aggregate loss amount, with the same shape as a standard Poisson - commonly used in Generalized Linear Models (GLM).

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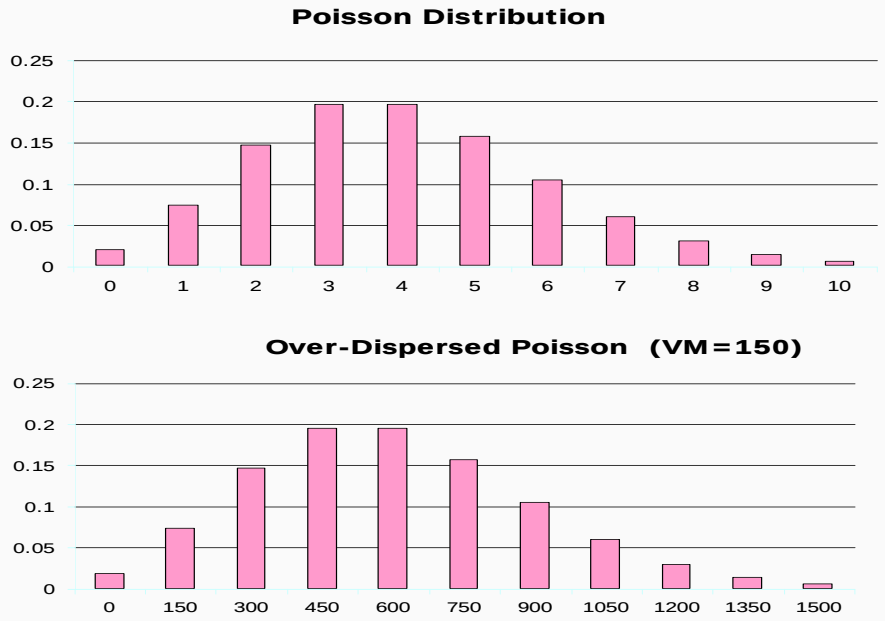

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Advantages of the *Over-dispersed Poisson* distribution:

1. Sum of ODP is also ODP
2. Can always match mean & variance
3. Reflects mass point at zero
4. Very convenient mathematics

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Maximizing the Likelihood means solving for $\boldsymbol{\theta}$ and $\boldsymbol{\omega}$ that maximize the expression:

$$\sum_{i,t} c_{i,t} \cdot \ln(\hat{\mu}_{i,t}) - \hat{\mu}_{i,t}$$

Note this is actually a “quasi-likelihood” – not including the dispersion parameter σ.

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The Variance/Mean Ratio, σ^2, is estimated by:

$$\sigma^2 \approx \frac{1}{n-p} \cdot \sum_{i,t} \frac{\left(c_{i,t} - \hat{\mu}_{i,t}\right)^2}{\hat{\mu}_{i,t}}$$

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Why use Maximum Likelihood Estimation (MLE)?

1. Familiar methods (LDF and Cape Cod) are exact MLE results

2. MLE provides estimate of the uncertainty in the parameters ("delta method" in Loss Models)

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Comments & Limitations

- Assumption that incremental losses are independent draws from identical distributions (the old "iid")

- Sources of Variance:
  1. Process
  2. Parameter or estimation error
  3. Model or specification error
  4. "State of the World" risk

These we can do!

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We begin with sample triangle from Taylor – Ashe:

Accident Year	12	24	36	48	60	72	84	96	108	120
1997	357,848	1,124,788	1,735,330	2,182,708	2,745,596	3,319,994	3,466,336	3,606,286	3,833,515	3,901,463
1998	352,118	1,236,139	2,170,033	3,353,322	3,799,067	4,120,063	4,647,867	4,914,039	5,339,085	
1999	290,507	1,292,306	2,218,525	3,235,179	3,985,995	4,132,918	4,628,910	4,909,315		
2000	310,608	1,418,858	2,195,047	3,757,447	4,029,929	4,381,982	4,588,268			
2001	443,160	1,136,350	2,128,333	2,897,821	3,402,672	3,873,311				
2002	396,132	1,333,217	2,180,715	2,985,752	3,691,712					
2003	440,832	1,288,463	2,419,861	3,483,130						
2004	359,480	1,421,128	2,864,498							
2005	376,686	1,363,294								
2006	344,014									

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Tabular Format of Development Data

Year	Starting	Ending	**Increment**	Diagonal	Latest
1997	0	12	**357,848**	120	3,901,463
1997	12	24	**766,940**	120	3,901,463
1997	24	36	**610,542**	120	3,901,463
1997	36	48	**447,378**	120	3,901,463
1997	48	60	**562,888**	120	3,901,463
1997	60	72	**574,398**	120	3,901,463
1997	72	84	**146,342**	120	3,901,463
1997	84	96	**139,950**	120	3,901,463
1997	96	108	**227,229**	120	3,901,463
1997	108	120	**67,948**	120	3,901,463
1998	0	12	**352,118**	108	5,339,085
1998	12	24	**884,021**	108	5,339,085
1998	24	36	**933,894**	108	5,339,085
1998	36	48	**1,183,289**	108	5,339,085
1998	48	60	**445,745**	108	5,339,085
1998	60	72	**320,996**	108	5,339,085
1998	72	84	**527,804**	108	5,339,085
1998	84	96	**266,172**	108	5,339,085
1998	96	108	**425,046**	108	5,339,085
1999	0	12	**290,507**	96	4,909,315
1999	12	24	**1,001,799**	96	4,909,315
1999	24	36	**926,219**	96	4,909,315
1999	36	48	**1,016,654**	96	4,909,315
1999	48	60	**750,816**	96	4,909,315
1999	60	72	**146,923**	96	4,909,315
1999	72	84	**495,992**	96	4,909,315
1999	84	96	**280,405**	96	4,909,315
2000	0	12	**310,608**	84	4,588,268

Calculated Fields

Starting	Ending	Diagonal	**Fitted**	Likelihood
0.00%	4.74%	77.24%	**239,295**	4,192,814
4.74%	19.38%	77.24%	**739,686**	9,624,727
19.38%	33.34%	77.24%	**705,171**	7,516,507
33.34%	44.77%	77.24%	**576,987**	5,357,739
44.77%	53.75%	77.24%	**453,829**	6,878,055
53.75%	60.78%	77.24%	**355,106**	6,985,799
60.78%	66.32%	77.24%	**279,911**	1,555,544
66.32%	70.75%	77.24%	**223,278**	1,500,370
70.75%	74.32%	77.24%	**180,455**	2,569,751
74.32%	77.24%	77.24%	**147,745**	661,057
0.00%	4.74%	74.32%	**340,360**	4,144,834
4.74%	19.38%	74.32%	**1,052,089**	11,206,001
19.38%	33.34%	74.32%	**1,002,997**	11,902,020
33.34%	44.77%	74.32%	**820,675**	15,293,216
44.77%	53.75%	74.32%	**645,502**	5,317,578
53.75%	60.78%	74.32%	**505,083**	3,710,390
60.78%	66.32%	74.32%	**398,130**	6,407,657
66.32%	70.75%	74.32%	**317,579**	3,054,416
70.75%	74.32%	74.32%	**256,669**	5,037,510
0.00%	4.74%	70.75%	**328,768**	3,361,574
4.74%	19.38%	70.75%	**1,016,256**	12,840,263
19.38%	33.34%	70.75%	**968,836**	11,798,028
33.34%	44.77%	70.75%	**792,724**	13,016,722
44.77%	53.75%	70.75%	**623,517**	9,394,719
53.75%	60.78%	70.75%	**487,880**	1,436,491
60.78%	66.32%	70.75%	**384,571**	5,993,828
66.32%	70.75%	70.75%	**306,762**	3,235,826
0.00%	4.74%	66.32%	**327,747**	3,616,974

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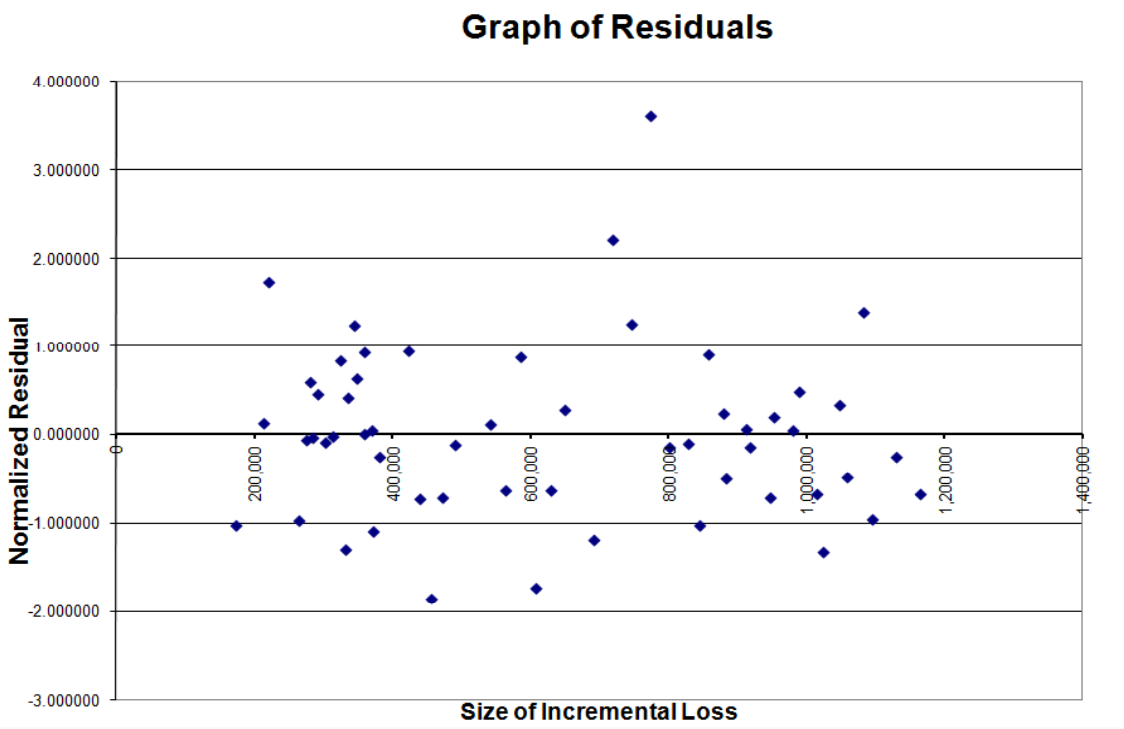

LDF Curve-Fitting and Stochastic Reserving

					From LDF Method	
Accident Year	Age in Months	Actual Losses	LDF	Estimated Ultimate	Process Std Dev	Parameter Std Dev
1997	120	3,901,463	1.2236	4,773,973	238,199	221,376
1998	108	5,339,085	1.2718	6,790,240	307,193	346,324
1999	96	4,909,315	1.3360	6,558,973	327,530	386,159
2000	84	4,588,268	1.4251	6,538,617	356,132	445,399
2001	72	3,873,311	1.5550	6,023,009	373,890	484,507
2002	60	3,691,712	1.7584	6,491,469	426,692	608,202
2003	48	3,483,130	2.1113	7,353,978	501,716	805,536
2004	36	2,864,498	2.8346	8,119,835	584,595	1,076,711
2005	24	1,363,294	4.8765	6,648,066	586,230	1,315,783
2006	12	344,014	19.9502	6,863,141	651,102	2,968,453
All Years Total:		34,358,090		66,161,301	1,438,103	5,373,718

Note: Process Std Dev numbers by year corrected from presentation

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							From Cape Cod	
Accident Year	Age in Months	Premium	LDF	Prem/LDF	Actual Losses	Estimated Ultimate	Process Std Dev	Parameter Std Dev
1997	120	10,000,000	1.2185	8,207,064	3,901,463	5,663,496	250,054	224,953
1998	108	10,400,000	1.2658	8,216,084	5,339,085	5,890,036	275,975	260,609
1999	96	10,800,000	1.3291	8,126,099	4,909,315	6,116,576	305,368	300,400
2000	84	11,200,000	1.4169	7,904,647	4,588,268	6,343,116	339,002	343,886
2001	72	11,600,000	1.5453	7,506,539	3,873,311	6,569,655	377,830	389,688
2002	60	12,000,000	1.7469	6,869,474	3,691,712	6,796,195	422,992	434,769
2003	48	12,400,000	2.0976	5,911,393	3,483,130	7,022,735	475,693	473,606
2004	36	12,800,000	2.8195	4,539,850	2,864,498	7,249,275	536,717	498,388
2005	24	13,200,000	4.8691	2,710,974	1,363,294	7,475,815	604,810	504,489
2006	12	13,600,000	20.1857	673,743	344,014	7,702,355	671,410	511,512
All Years Total:		118,000,000		60,665,868	34,358,090	66,829,253	1,414,028	3,879,758
						LDF Std Dev:	1,438,103	5,373,718