



# Optimal Growth for Property and Casualty Insurance Companies

Luyang Fu, Ph.D., FCAS

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# Agenda

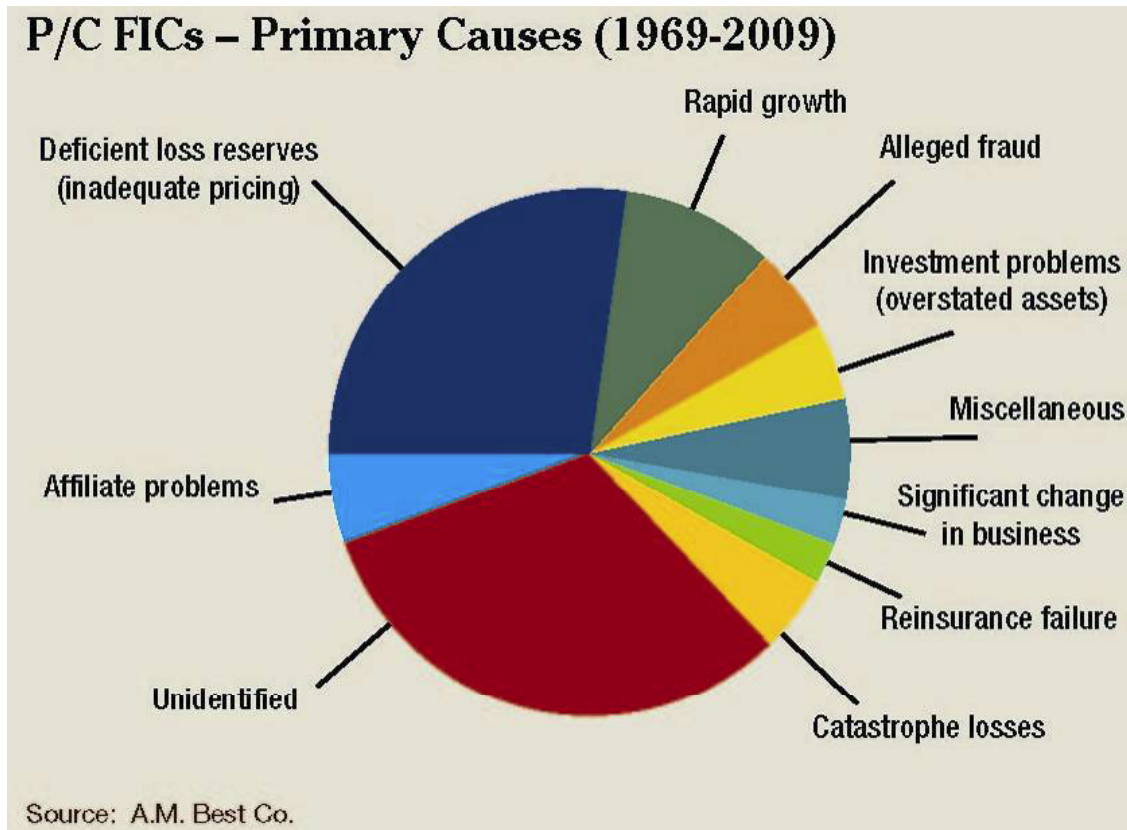
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- Introduction
- Equilibrium New Business Percentage
- Growth Impact Curve
- Growth Limit Curve
- Constrained Maximum Growth
- Optimal Growth
- Case Study

# Introduction

Rapid Growth is one of top causes of financial impairment



# Introduction

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## Conflicts between Growth and Profitability

- Faster growth may reduce profitability
  - Lower price
  - Loose underwriting
  - Attract more NB, NB has higher loss and expense ratios

# Introduction

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## Conflicts between Growth and Profitability

- Aghion and Stein (2008): constraints on management time and other resources
- Harrington, Danzon, and Epstein (2008): insurance companies often sacrifice profit margins by cutting price excessively in the soft market to maintain sales volume
- Ma (2009): profitability will be eroded significantly when a high growth target is achieved by lowering underwriting standards

# Introduction

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## Aging Phenomenon

- D'Arcy and Doherty (1989; 1990): loss ratio improves with policy age
- Cohen(2005): Evidence from personal auto
- Wu and Lin (2009)
  - 8 lines of business, 25 books, \$29 billion premium
  - New business has loss ratios 7% (GL) to 18% (BOP) higher than renewal business
  - New business has retentions 3% (personal auto) to 19% (personal home) lower than renewal business

# Introduction

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## D'Arcy and Gorvett (2004) Optimal Growth Paper

- A milestone: first study
- Three-factor econometrics model
  - Market value =  $a + b * \text{surplus} + c * \text{NWP} + d * \text{combined ratio}$
  - 15 companies:  $b = 2.13$ ,  $c = 1.57$ ,  $d = -23,878,168$
  - 14 companies (Excluding AIG):  $b = 1.85$ ,  $c = 0.28$ ,  $d = -2,076,192$
- Run DFA simulations
- Does optimal growth rate exist?
  - Using 14-company parameters: optimal growth = 0%
  - Using 15-company parameters: optimal growth = 10%

# Introduction

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## Practical concerns of applying D'Arcy and Gorvett (2004)

- Data availability: Mutual, reciprocal, subsidiary, and privately-held companies do not have observed market values
- Parameter Risks: Volatile results by including AIG or not
- Complicated DFA simulations: not easy to understand and apply.



# Introduction

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## Improvements from Fu (2012):

- Data availability: traditional actuarial database.
- Parameter Risks: no regression which is subject to volatility of equity market.
- Deterministic: easy to understand and apply.
- Study the conditions for the existence of optimal positive growth.

## Disadvantages of Fu (2012): no stochastic insights

- Cannot be analyzed in the classical mean-variance framework of modern financial economics.
- No risk frontier.

# Equilibrium New Business Percentage

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Required NB percentage to achieve 15% overall growth when 10% of the current book of business consists of NB

| Year (t) | RB Exposure<br>(1) | NB Exposure<br>(2) | RB %<br>(3)=(1)/(5) | NB %<br>(4)=(2)/(5) | Exposure<br>(5)=(1)+(2) |
|----------|--------------------|--------------------|---------------------|---------------------|-------------------------|
| 1        | 0.900              | 0.100              | 90.0%               | 10.0%               | 1.000                   |
| 2        | 0.890              | 0.260              | 77.4%               | 22.6%               | 1.150                   |
| 3        | 1.009              | 0.314              | 76.3%               | 23.7%               | 1.323                   |
| 4        | 1.159              | 0.362              | 76.2%               | 23.8%               | 1.521                   |
| 5        | 1.333              | 0.416              | 76.2%               | 23.8%               | 1.749                   |

Assume RB retention ratio 90% and NB retention 80%

$0.89 = 0.9 \cdot 90\% + 0.1 \cdot 80\%$ ;  $0.26 = 1.15 - 0.89$

$1.009 = 0.89 \cdot 90\% + 0.26 \cdot 80\%$ ;  $0.314 = 1.15^2 - 1.009$

# Equilibrium New Business Percentage

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Required NB percentage to achieve 15% overall growth when 25% of the current book of business consists of NB

| Year (t) | RB Exposure | NB Exposure | RB %  | NB %  | Exposure |
|----------|-------------|-------------|-------|-------|----------|
| 1        | 0.750       | 0.250       | 75.0% | 25.0% | 1.000    |
| 2        | 0.875       | 0.275       | 76.1% | 23.9% | 1.150    |
| 3        | 1.008       | 0.315       | 76.2% | 23.8% | 1.323    |
| 4        | 1.159       | 0.362       | 76.2% | 23.8% | 1.521    |
| 5        | 1.333       | 0.416       | 76.2% | 23.8% | 1.749    |

# Equilibrium New Business Percentage

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- Percentage of NB exposure converges at 23.8%.
- Notation: Q - exposure; G - growth rate; A - NB percentage
  - Exposure = prior (1+growth):  $Q_t = Q_{t-1}(1 + G_t)$
  - NB= total \* NB percentage:  $Q_{n,t} = Q_t A_t = Q_{t-1}(1 + G_t)A_t$
  - RB = prior NB Renewal+ prior RB renewal:  $Q_{r,t} = Q_{t-1}A_{t-1}R_{n,t} + Q_{t-1}(1 - A_{t-1})R_{r,t}$
  - Total = NB+RB:  $Q_{t-1}(1 + G_t) = Q_{t-1}(1 + G_t)A_t + Q_{t-1}A_{t-1}R_{n,t} + Q_{t-1}(1 - A_{t-1})R_{r,t}$
- Solving for ENBP:

$$A_t = \frac{1 + G_t - R_{r,t}}{1 + G_t + R_{n,t} - R_{r,t}} = 1 - \frac{R_{n,t}}{1 + G_t + R_{n,t} - R_{r,t}}$$

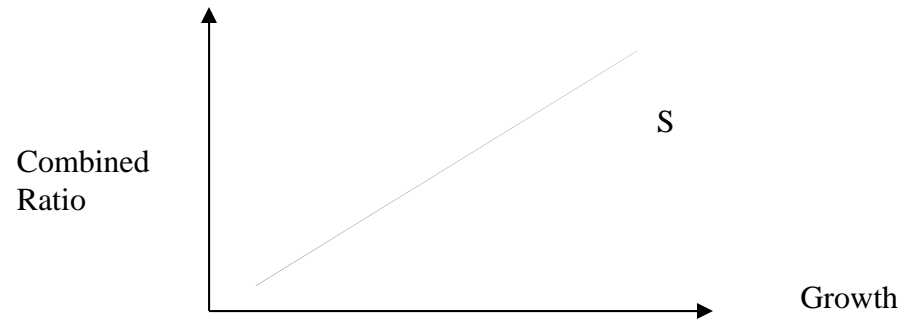
# Growth Impact Curve

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Growth Impact Curve shows the underwriting fact:

- Combined ratio is an increasing function of growth
- Growth reduces underwriting profit margin

$$C_t = A_t C_{n,t} + (1 - A_t) C_{r,t} = \frac{(1 + G_t - R_{r,t}) C_{n,t} + R_{n,t} C_{r,t}}{1 + G_t + R_{n,t} - R_{r,t}}$$



# Growth Limit Curve

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Surplus capacity can constrain the growth of an insurance company

- Premium-to-surplus ratio
  - Regulator;
  - Rating agency
  - Internal ERM
- To avoid over-leverage, the profit growth after tax and dividend has to keep up the pace with sales growth

# Growth Limit Curve

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- Surplus constraints on the growth: evidence from academia
  - Davis 1979;
  - Hagstrom 1981;
  - Gron 1994;
  - Winter 1994;
  - Cummins and Danzon 1997;
  - Wang et al. (2011)

# Growth Limit Curve

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- I is investment, lambda is found-generating coefficient, S is surplus; Investment Asset is:

$$I_t = \lambda * WP_t + S_t$$

- t is tax and Y is investment yield; retained profit after tax and dividend is:

$$\pi_t = [EP_t * (1 - C_t) * (1 - t_u) + I_t Y_t * (1 - t_I)] * (1 - D_t)$$

- To maintain target premium-to-surplus ratio K:

$$\frac{WP_{t+1}}{S_{t+1}} = \frac{WP_t * (1 + G_{t+1})}{S_t + \pi_t} \leq K_t$$



# Growth Limit Curve

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To maintain a target premium-to-surplus ratio  $K$

- Combined Ratio needs to be below a threshold

$$C_t \leq 1 - \frac{WP_t * (1 + G_{t+1}) - K_t * S_t - K_t * I_t * Y_t * (1 - t_l) * (1 - D_t)}{EP_t * (1 - t_u) * (1 - D_t) * K_t}$$

- Or, the growth has to be below a threshold under certain profit level

$$G_{t+1} \leq \frac{K_t * (S_t + \pi_t)}{WP_t} - 1$$

# Growth Limit Curve

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Growth limit curve shows the capital constraint

- Faster growth requires lower combined ratio to generate extra capital to support such growth
- Do not cross the line: if the combined ratio is over the curve, premium growth > surplus growth, the leverage ratio will increase and penetrate the “target”.



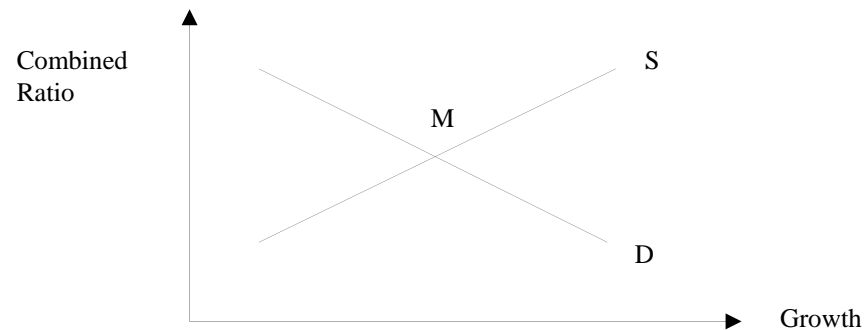
$$C_t \leq 1 - \frac{WP_t * (1 + G_{t+1}) - K_t * S_t - K_t * I_t * Y_t * (1 - t_l) * (1 - D_t)}{EP_t * (1 - t_u) * (1 - D_t) * K_t}$$

# Constrained Maximum Growth

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Balance two conflicting goals:

- Growth Impact curve – faster growth drives up combined ratio from the perspective of underwriting performance
- Growth Limit curve – faster growth requires lower combined ratio from the perspective of capital management
- Max growth rate under the capital constraint: the intersection M between two curves.



# Optimal Growth

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## Insurance Company Valuation

- $\phi$  is expected price-to-book ratio;
- $\eta$  is the expected price-to-sales ratio
- $w$  is weight given to surplus-indicated company value

$$V_{t+n} = W * \phi * S_{t+n} + (1 - W) * \eta * WP_{t+n}$$

## To maximize the company value

$$\underset{G}{Max} \quad W * \phi * S_{t+n} + (1 - W) * \eta * WP_t * (1 + G)^n$$

# Case Study

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## Assumptions

At market price level,

- NB loss ratio is 75%, RB loss ratio is 62%;
- NB retention is 78%, RB retention is 84%;
- NB expense ratio is 37%, RB expense ratio is 32%;

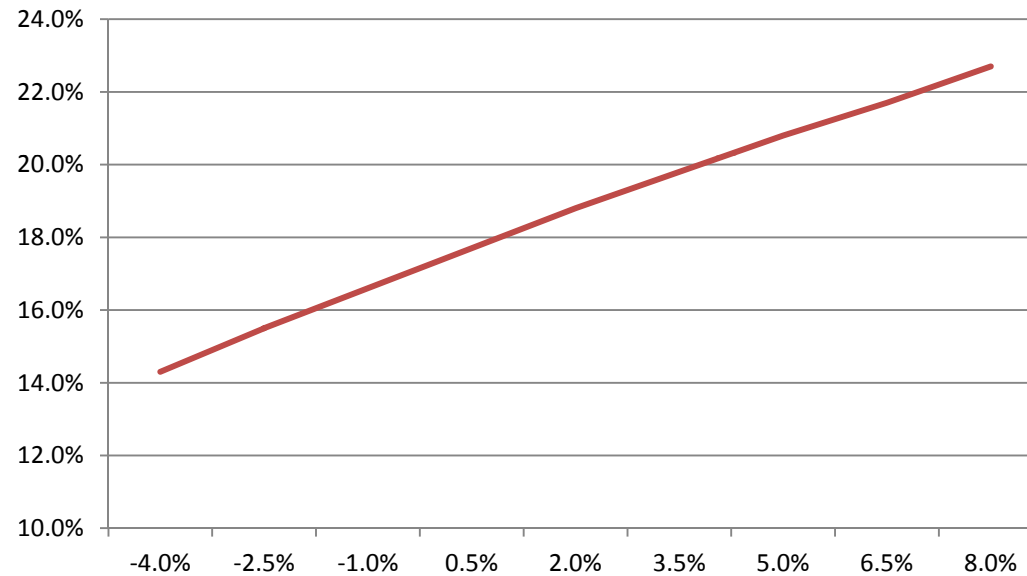
G is the exposure growth rate, R is the retention ratio, dp is the rate difference from market

- $G_t = 2\% - 1.5 * dp_t$  , the lower the price, the faster the growth
- $R_{r,t} = 84\% - 0.2 * dp_t$  , the lower the price, the higher the retention  
 $R_{n,t} = 78\% - 0.3 * dp_t$

# Case Study

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**Equilibrium New Business Percentage**



$$A_t = \frac{1 + G_t - R_{r,t}}{1 + G_t + R_{n,t} - R_{r,t}}$$

# Case Study

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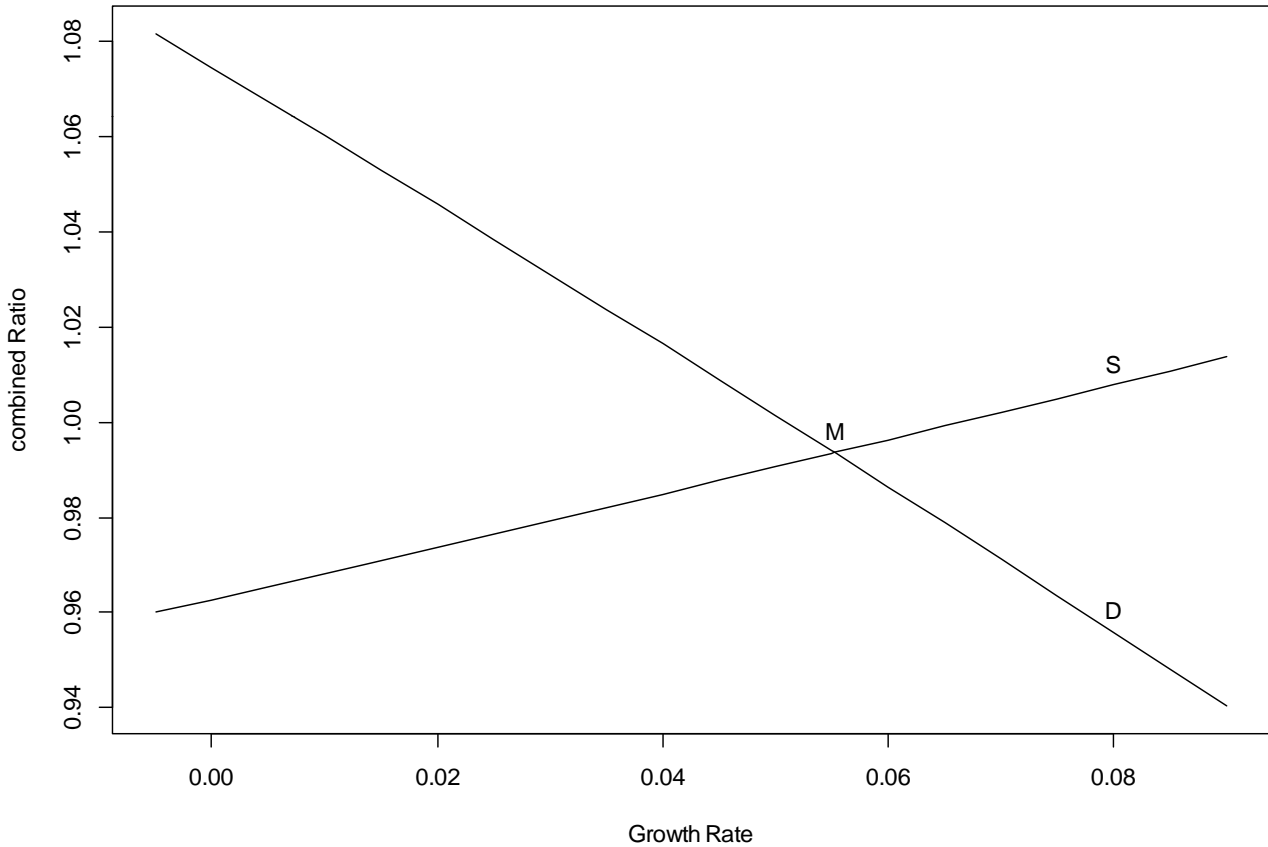
Equilibrium NB percentages, loss and combined ratios by growth

| Exposure Growth | dp  | NB Retention | RB Retention | ENBP  | NB LR | RB LR | NB CR  | RB CR | whole Book CR |
|-----------------|-----|--------------|--------------|-------|-------|-------|--------|-------|---------------|
| 8.0%            | -4% | 79.2%        | 84.8%        | 22.7% | 78.1% | 64.6% | 115.1% | 96.6% | 100.8%        |
| 6.5%            | -3% | 78.9%        | 84.6%        | 21.7% | 77.3% | 63.9% | 114.3% | 95.9% | 99.9%         |
| 5.0%            | -2% | 78.6%        | 84.4%        | 20.8% | 76.5% | 63.3% | 113.5% | 95.3% | 99.1%         |
| 3.5%            | -1% | 78.3%        | 84.2%        | 19.8% | 75.8% | 62.6% | 112.8% | 94.6% | 98.2%         |
| 2.0%            | 0%  | 78.0%        | 84.0%        | 18.8% | 75.0% | 62.0% | 112.0% | 94.0% | 97.4%         |
| 0.5%            | 1%  | 77.7%        | 83.8%        | 17.7% | 74.3% | 61.4% | 111.3% | 93.4% | 96.5%         |
| -1.0%           | 2%  | 77.4%        | 83.6%        | 16.6% | 73.5% | 60.8% | 110.5% | 92.8% | 95.7%         |
| -2.5%           | 3%  | 77.1%        | 83.4%        | 15.5% | 72.8% | 60.2% | 109.8% | 92.2% | 94.9%         |
| -4.0%           | 4%  | 76.8%        | 83.2%        | 14.3% | 72.1% | 59.6% | 109.1% | 91.6% | 94.1%         |

# Case Study

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## Empirical Growth Impact and Limit Curves and Constrained Maximum Growth





# Case Study

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Five-year profits, surplus, and leverage ratios  
at constrained maximum growth 5.52%

|   | Beginning Surplus | WP    | EP    | Investment | Inv Profit | UW Profit | Total Profit | Tax rate | After-Tax Profit | Payout % | Year  | End Surplus | Prem/Surplus |
|---|-------------------|-------|-------|------------|------------|-----------|--------------|----------|------------------|----------|-------|-------------|--------------|
| 0 | 0.667             | 1.000 | 0.974 | 1.867      | 0.075      | 0.006     | 0.081        | 35%      | 0.053            | 30%      | 0.037 | 0.703       | 1.500        |
| 1 | 0.703             | 1.055 | 1.028 | 1.970      | 0.079      | 0.007     | 0.085        | 35%      | 0.056            | 30%      | 0.039 | 0.742       | 1.500        |
| 2 | 0.742             | 1.114 | 1.084 | 2.079      | 0.083      | 0.007     | 0.090        | 35%      | 0.059            | 30%      | 0.041 | 0.783       | 1.500        |
| 3 | 0.783             | 1.175 | 1.144 | 2.193      | 0.088      | 0.007     | 0.095        | 35%      | 0.062            | 30%      | 0.043 | 0.827       | 1.500        |
| 4 | 0.827             | 1.240 | 1.207 | 2.315      | 0.093      | 0.008     | 0.100        | 35%      | 0.065            | 30%      | 0.046 | 0.872       | 1.500        |
| 5 | 0.872             | 1.308 | 1.274 | 2.442      | 0.098      | 0.008     | 0.106        | 35%      | 0.069            | 30%      | 0.048 | 0.920       | 1.500        |

Assume 4% investment yield, 35% tax rate, 30% dividend payout ratio, 1.2 fund generating coefficient

# Case Study

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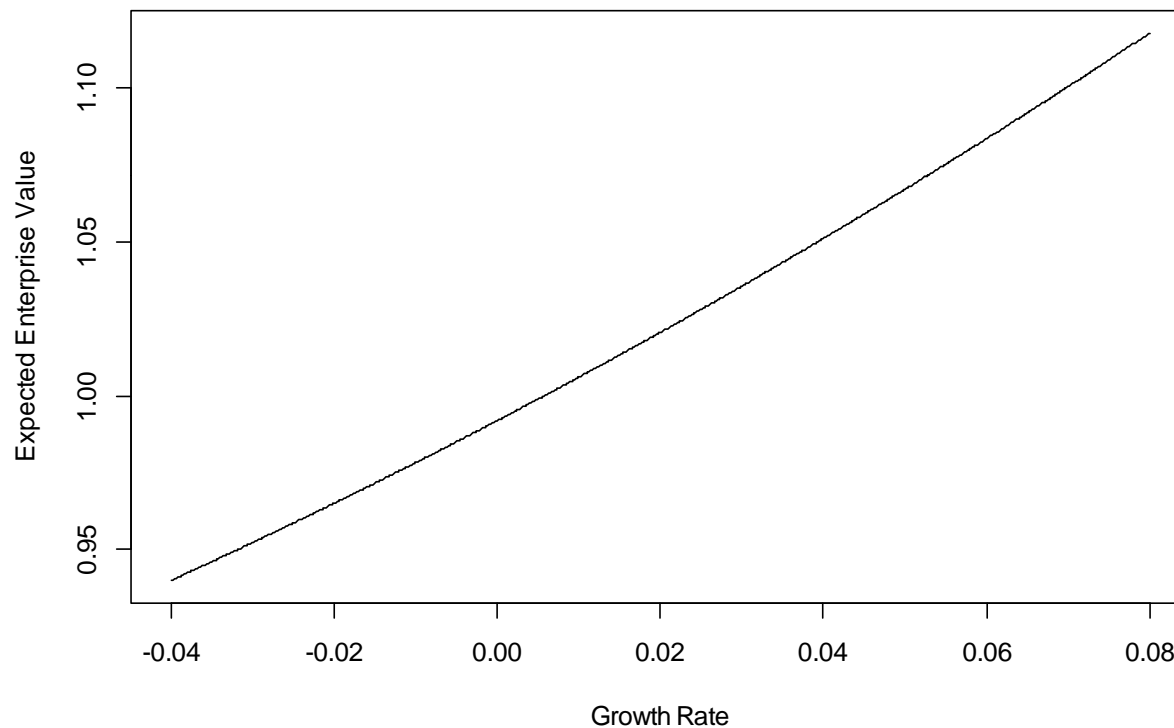
Five-year profits, surplus, and leverage ratios  
at 8% growth

|   | Beginning Surplus | WP    | EP    | Investment | Inv Profit | UW Profit | Total Profit | Tax rate | After-Tax Profit | Payout % | Year  | End Surplus | Prem/Surplus |
|---|-------------------|-------|-------|------------|------------|-----------|--------------|----------|------------------|----------|-------|-------------|--------------|
| 0 | 0.667             | 1.000 | 0.963 | 1.867      | 0.075      | -0.008    | 0.067        | 35%      | 0.044            | 30%      | 0.031 | 0.697       | 1.500        |
| 1 | 0.697             | 1.080 | 1.040 | 1.993      | 0.080      | -0.008    | 0.072        | 35%      | 0.047            | 30%      | 0.033 | 0.730       | 1.549        |
| 2 | 0.730             | 1.166 | 1.123 | 2.129      | 0.085      | -0.009    | 0.076        | 35%      | 0.050            | 30%      | 0.035 | 0.765       | 1.598        |
| 3 | 0.765             | 1.260 | 1.213 | 2.276      | 0.091      | -0.010    | 0.082        | 35%      | 0.053            | 30%      | 0.037 | 0.802       | 1.648        |
| 4 | 0.802             | 1.360 | 1.310 | 2.434      | 0.097      | -0.010    | 0.087        | 35%      | 0.057            | 30%      | 0.040 | 0.841       | 1.697        |
| 5 | 0.841             | 1.469 | 1.415 | 2.604      | 0.104      | -0.011    | 0.093        | 35%      | 0.061            | 30%      | 0.042 | 0.884       | 1.747        |

# Case Study

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Expected Company Values after 5 years by Growth Rate  
When W=50%



Assume price-to-book ratio =1.2, price-to-sales ratio 0.8:

$$\text{Max}_G \quad 50\% * 1.2 * S_{t+n} + 50\% * 0.8 * WP_t * (1+G)^5$$

# Case Study

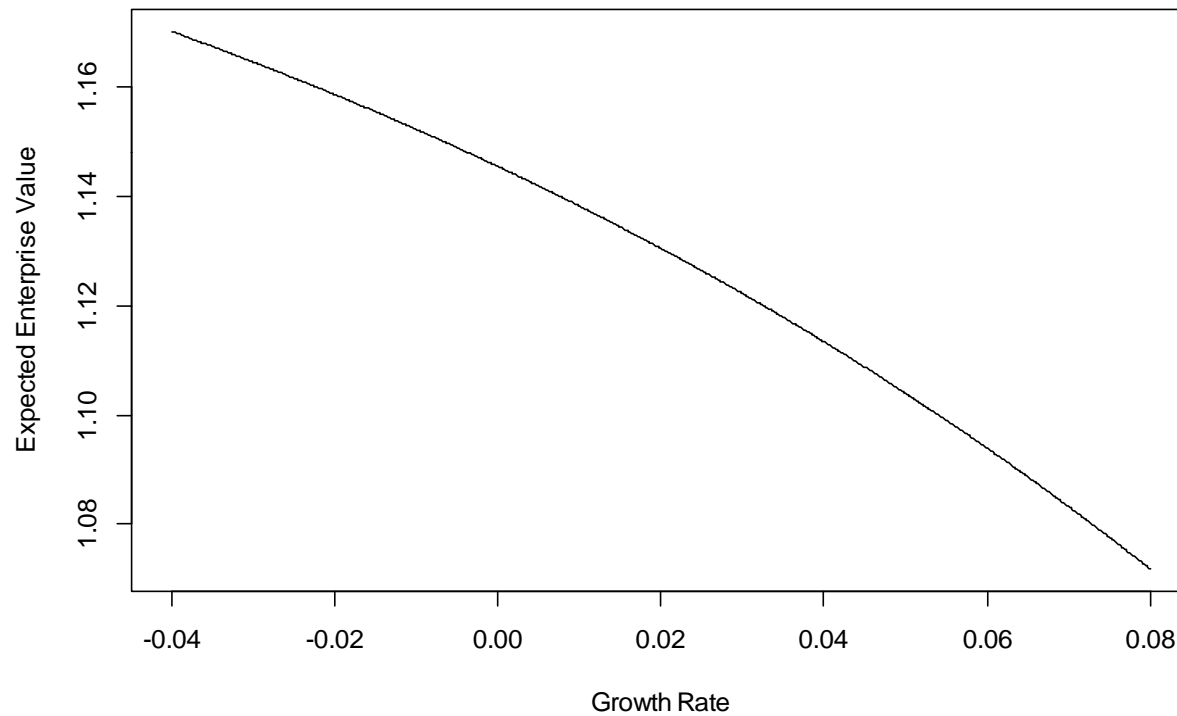
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When the weight on surplus is 50%

- Growth dominates the surplus (it is easier to grow sales volume than to grow underwriting profit).
- The company will grow as fast as possible if no capital constraint
- The optimal growth is the constrained maximum growth under the leverage constraint on capital.

# Case Study

Expected Company Values after 5 years by Growth Rate  
When W=90%



$$\text{Max}_G \quad 90\% * 1.2 * S_{t+n} + 10\% * 0.8 * WP_t * (1+G)^5$$

# Case Study

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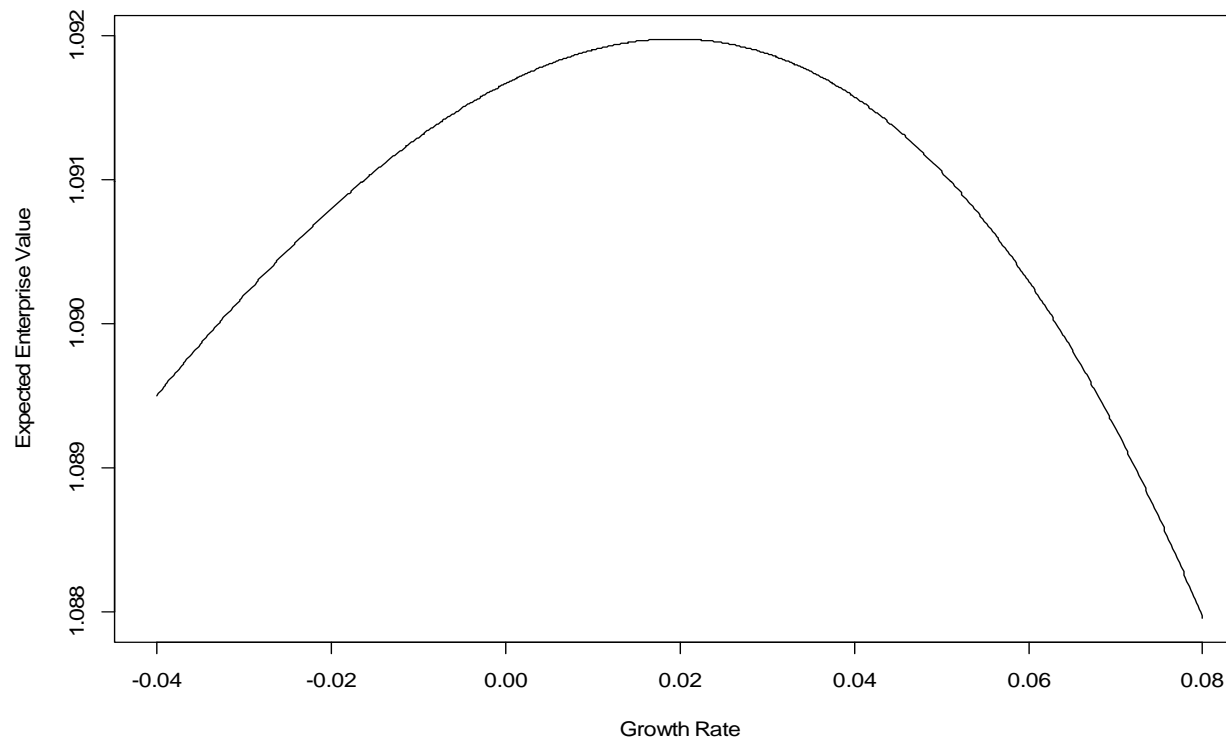


## When the weight on surplus is 90%

- Surplus overweighs the growth.
- The company will not attempt to grow without growth constraints.
  - “If a company is not attempting to grow, its book will gradually ages, so the loss ratio declines. This generates a higher net income in the near future and increase policyholders’ surplus”
- The optimal positive growth does not exist.
- This is equivalent to 14-company case in D’Arcy and Gorvett (2004)

# Case Study

Expected Company Values after 5 years by Growth Rate  
When W=76%



$$\text{Max}_G \quad 76\% * 1.2 * S_{t+n} + 24\% * 0.8 * WP_t * (1 + G)^5$$

# Case Study

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When  $74\% < W < 78\%$ :

- There is a balance between surplus and growth.
- The expected company value is a bell curve of growth.
- The optimal positive growth exists.
- This is equivalent to 15-company case in D'Arcy and Gorvett (2004)