Illiquidity Risk Premium

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Background

- Thanks to the CAS Committee on Theory of Risk for sponsoring this project on "illiquidity risk premiums" -- completed in April 2012.
- Three researchers:
  - Professor Shaun Wang (Georgia State U)
  - Mr. Phillip Heckman
  - Professor Dilip Madan (U. of Maryland)
- Produced a theoretical paper: "A theory of risk for two price market equilibria"

Concept of Liquidity

- Liquidity is a necessity: Like Fish needs water, firms (markets) need financial liquidity
- Too much liquidity, like a flood, can cause asset price bubble and runaway inflation
- Too little liquidity, like draught, can force business shutdowns
- How to measure illiquidity?

An Insurer’s Illiquidity Concern

- Insurer is concerned about the cash flow squeeze:
  - Catastrophic risk exposures
  - Negative reserve developments
  - Changing market shares with fixed operating expense
- Insurer is concerned about the threat of rating downgrade (loss of clients, loss of confidence)

3 Levels of Illiquidity

1) System-wide illiquidity (e.g. 2008 financial crisis)
2) A firm's own funding illiquidity (LTCM)
3) Illiquidity risk for individual assets and liabilities (e.g., insurance contracts)

- 3 levels of illiquidity may interact with each other

Measure of illiquidity for traded assets

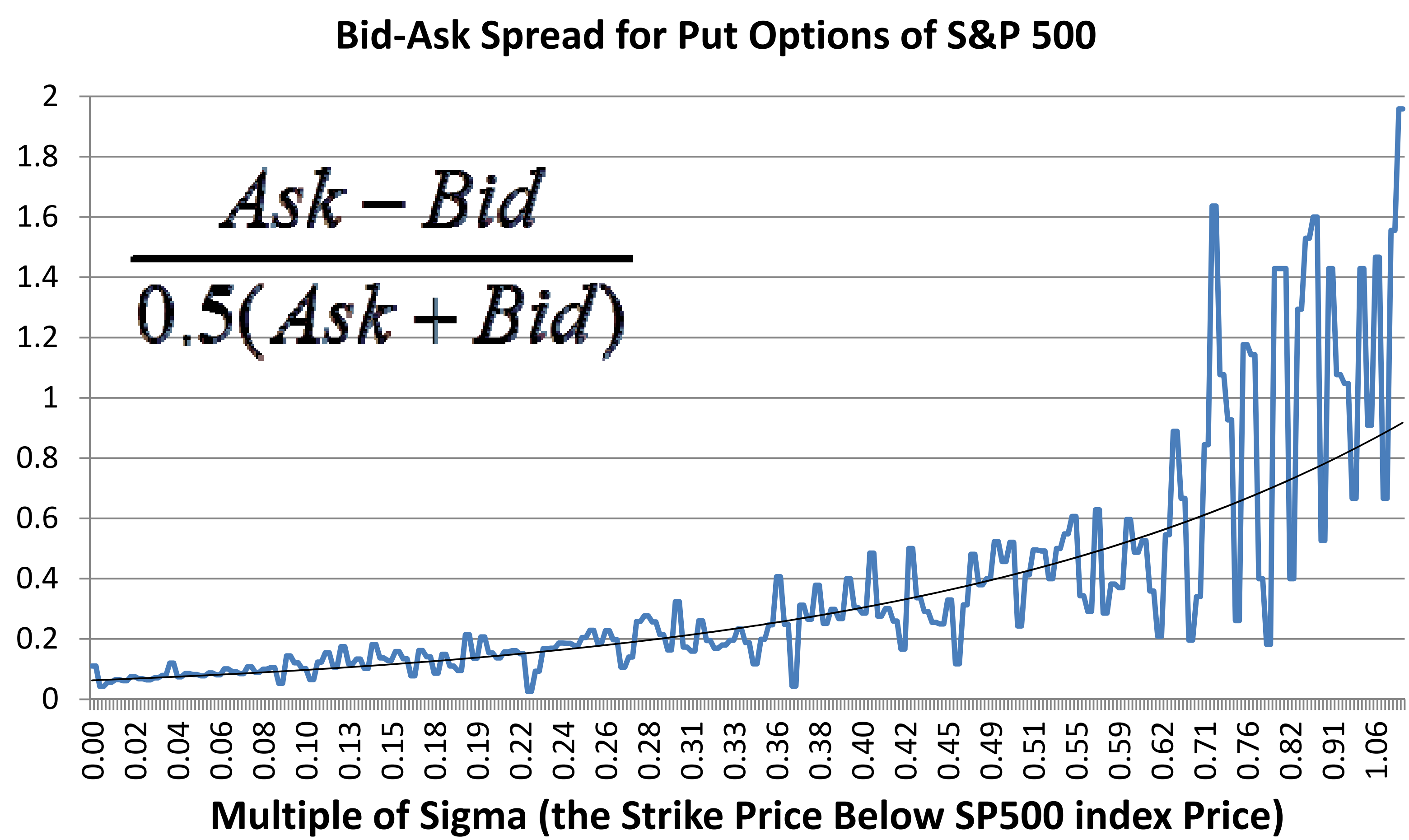
- Bid-Ask Spread (simultaneous)

$$\frac{Ask - Bid}{0.5(Ask + Bid)}$$

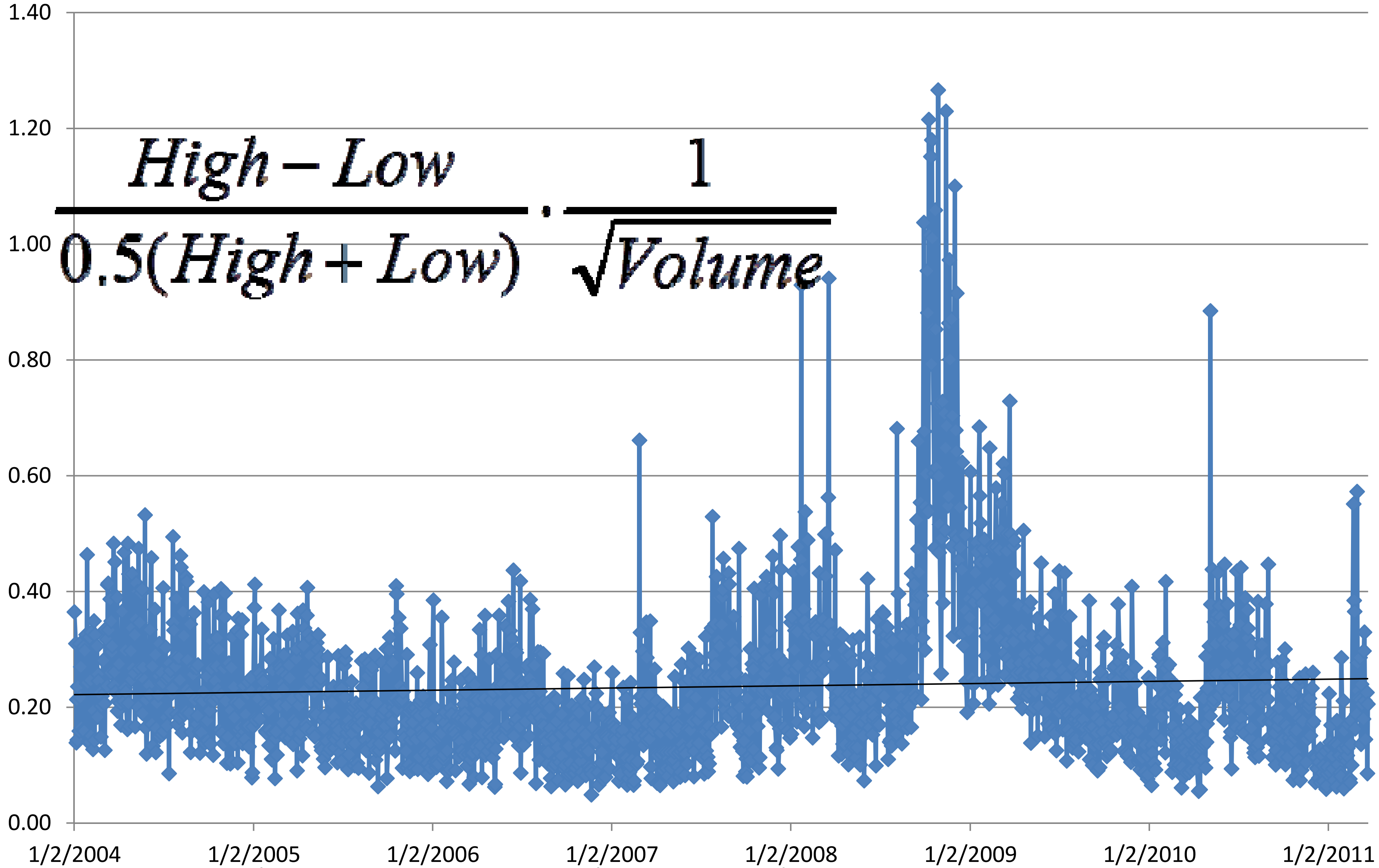
- High-Low Spread (during a time interval), account for trading volume (thin, normal, heavy) and its impacts on price change

$$\frac{High - Low}{0.5(High + Low)} \cdot \frac{1}{\sqrt{Volume}}$$

Bid-Ask Spread Increases for out-of money Options

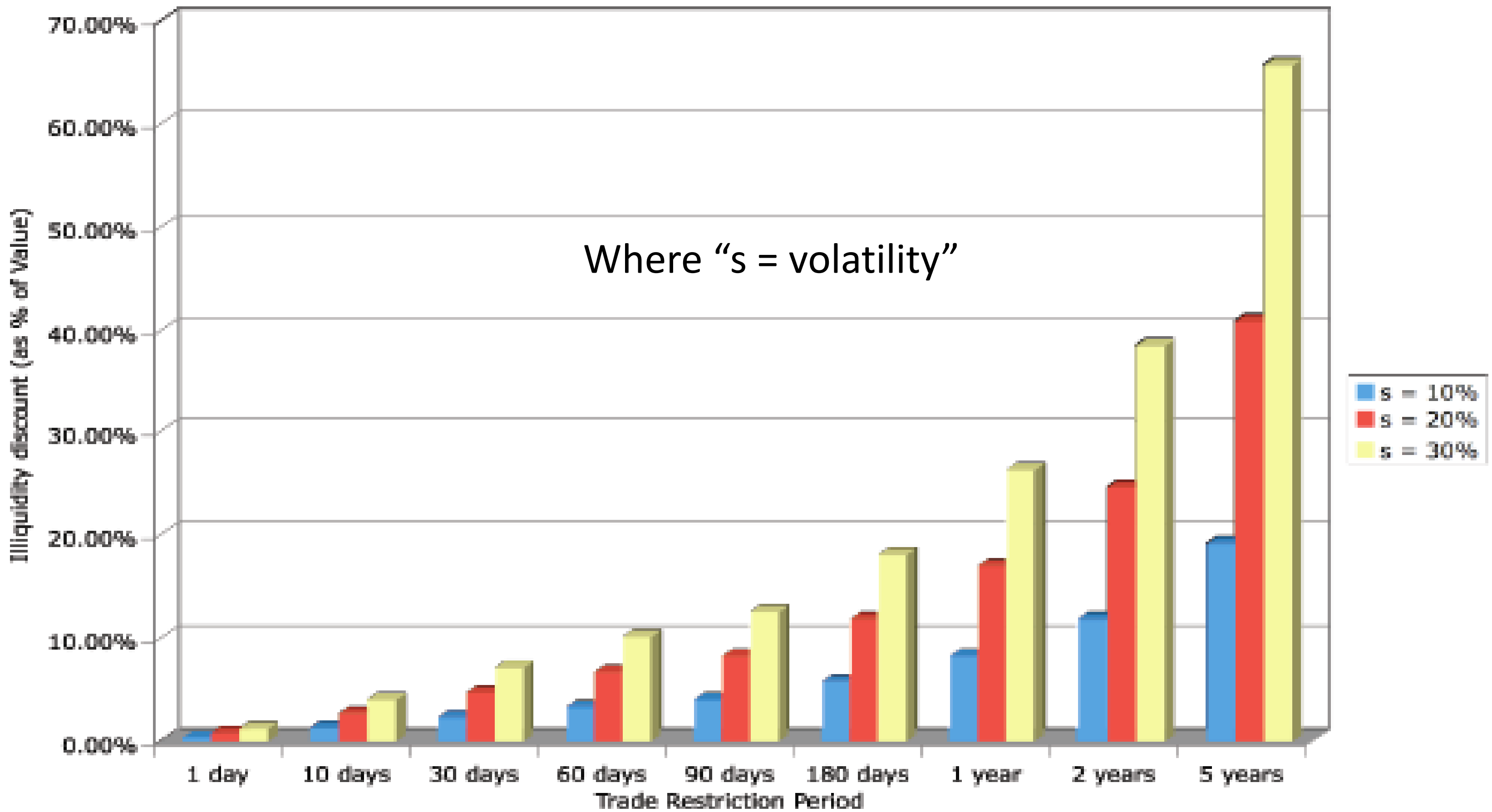


"Y-axis: High-Low Spread per SQRT Volume"
S&P500 Daily Price Data



$$\frac{High - Low}{0.5(High + Low)} \cdot \frac{1}{\sqrt{Volume}}$$

Illiquidity Risk Premium increases with Time Horizon (F. Longstaff, 1995 J. of Finance paper)



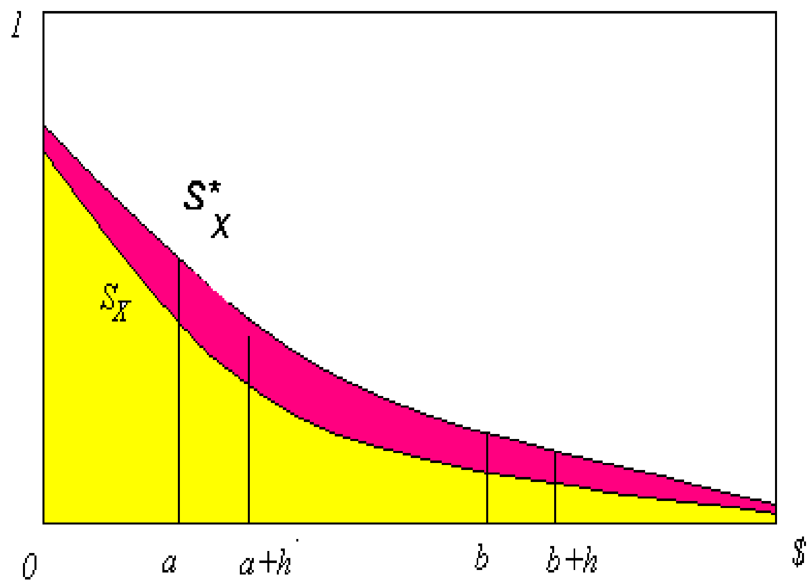
Illiquidity Risk Premium

- Non-Actively Traded Contracts such as property-casualty insurance contracts
  - P-measure: Physical probability measure
  - Q-measure: Risk-adjusted (or price implied) probability measure
  - There is a spread (difference) between the P-measure and the Q-measure

P-measure vs. Q-measure

Mapping between

1. ***Loss Curve***
   - ***physical measure***
   - ***S(x) = 1- F(x)***
2. ***Pricing Curve***
   - ***risk-neutral measure***
   - ***S*(x) = 1- F*(x)***



Wang Transform

- **Map <u>loss curve</u> to a <u>price curve</u>:**

  $F^*(x) = \Phi[\Phi^{-1}(F(x)) - \lambda]$ or

  $F^*(x)$ = normsdist(normsinv($F(x)$) – λ)

  e.g. $\mathbf{0.97 = \Phi[\Phi^{-1}(0.99) - 0.45]}$

- If F_X is normal(μ,σ), F_X^* is normal($\mu+\lambda\sigma$, σ):
  - $E^*[X] = E[X] + \lambda\, \sigma[X]$

- If F_X is lognormal(μ, σ), F_X^* is lognormal($\mu+\lambda\sigma$, σ)

Benchmark Pricing based on Empirical Data: 2-factor Wang Transform

$$F^*(y) = t_5\left(\Phi^{-1}(F(y)) - 0.45\right)$$

- ➢ Φ is standard Normal Distribution,
- ➢ t_5 is Student-t with 5 degrees-of-freedom

- ✓ Using student-t to replace Normal distribution is a way to reflect parameter uncertainty.
- ✓ Compiling evidence from Cat pricing data

Costs of Holding Capital versus Buying Reinsurance

- Assume solvency capital=the 99.5th percentile
- Assume hurdle rate is 10% over risk-free rate
- There is a cost of holding more capital
- Buying reinsurance can reduce the capital requirement, thus the cost of holing the capital
- We need to evaluate the trade-off.

Example One: Optimal Reinsurance

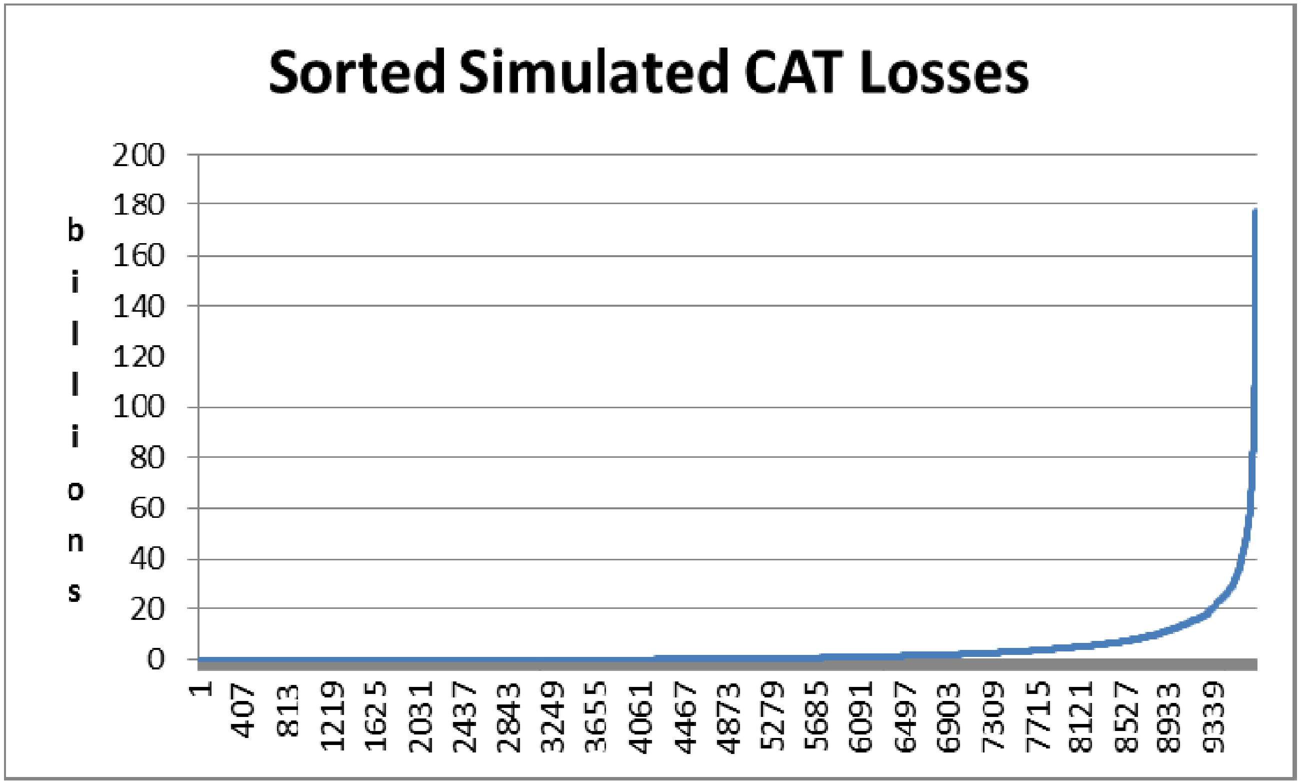
- Simulated Florida Hurricane Losses

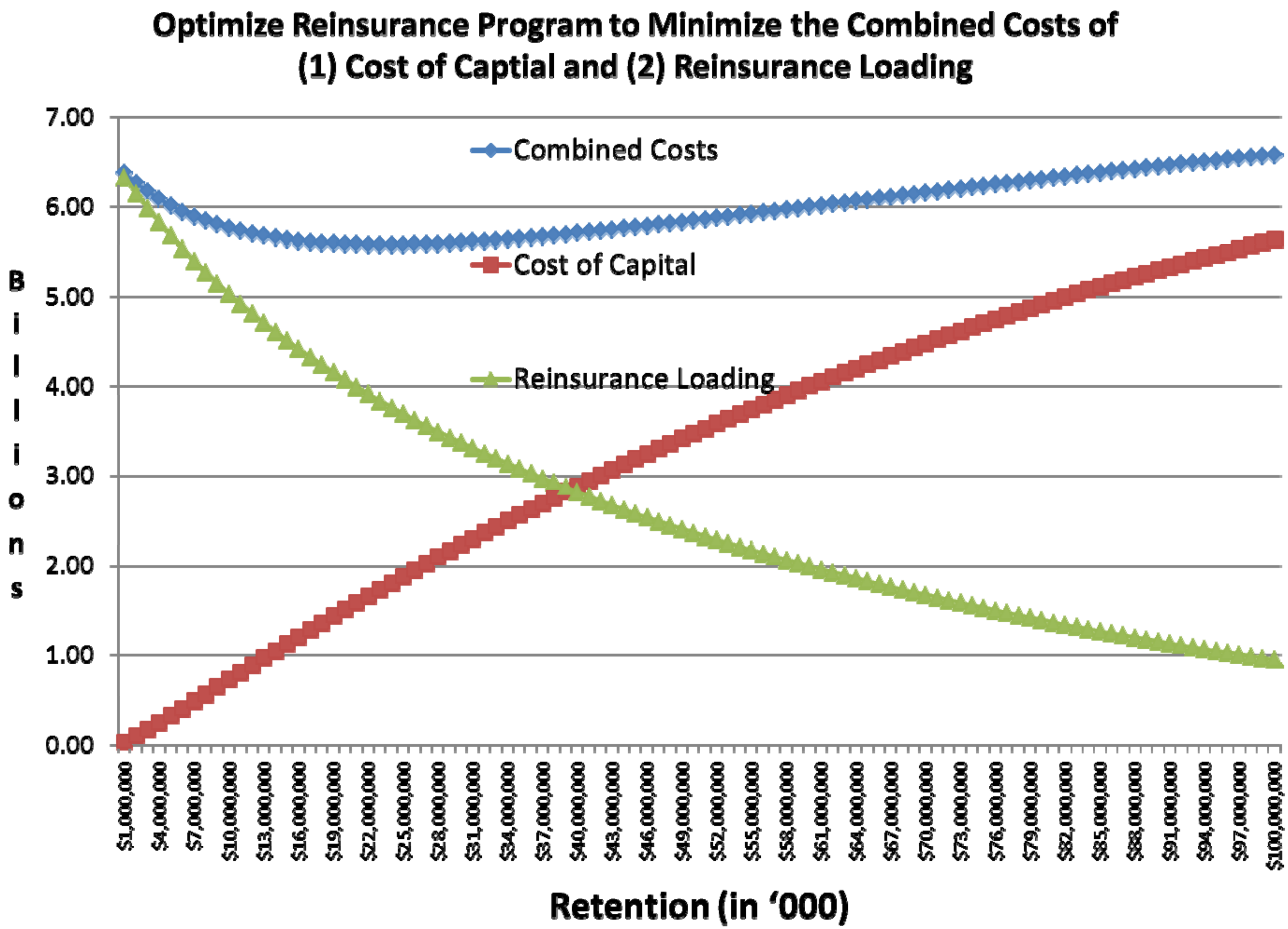
  Summary statistics (in billions)

mean	3.64
Stdev	9.35
Max	177.03

- Question: what is the optimal retention?

Simulated Florida Hurricane Loss Curve





Optimize Reinsurance Program to Minimize the Combined Costs of
(1) Cost of Captial and (2) Reinsurance Loading
Combined Costs
Cost of Capital
Reinsurance Loading
Billions
7.00
6.00
5.00
4.00
3.00
2.00
1.00
0.00
$1,000,000
$4,000,000
$7,000,000
$10,000,000
$13,000,000
$16,000,000
$19,000,000
$22,000,000
$25,000,000
$28,000,000
$31,000,000
$34,000,000
$37,000,000
$40,000,000
$43,000,000
$46,000,000
$49,000,000
$52,000,000
$55,000,000
$58,000,000
$61,000,000
$64,000,000
$67,000,000
$70,000,000
$73,000,000
$76,000,000
$79,000,000
$82,000,000
$85,000,000
$88,000,000
$91,000,000
$94,000,000
$97,000,000
$100,000,000
Retention (in '000)

Calculated Costs for the case that retention = $20 billion

Costs	Actuarial Exp. Loss	Reins Loading	Cost of Holding Capital
Retained	2.78	2.41	**1.51**
Ceded	0.76	**4.07**	5.65

✓For the retained loss, the cost of capital is $1.51 billion, which is lower than reinsurance loading of $2.41 billion.

✓For the ceded loss, the cost of capital is $5.65 billion, which is higher than the reinsurance loading of $4.07 billion.

Optimal Retention changes with pricing & capital requirements

- Everything else equal, if we lower the capital requirement from 99.5^{th} to 99^{th} percentile, the optimal retention will increase from $24 billion to $33 billion
- Everything else equal, if we lower Wang transform lambda from 0.45 to 0.3, the optimal retention will decrease from $24 billion to $10 billion.

Example: Reinsurer Credit Risk

- Ln(X) has a normal distribution mu=4 and sigma=0.5
- Regular Deductible = 50
- Pricing is based on applying Wang transform with lambda=0.6

- Assume that the reinsurer has a 2% chance of default on paying claims (zero recovery rate).

Correct way of reflecting reinsurance credit risk

1-step approach: Apply Wang transform to the ceded loss distribution reflecting reinsurer credit risks

Implied Premium Discount
= 1.36% (less than the 2% default probability).

This is counter intuitive.

2-steps Approach

1) Transform ceded loss distribution w/o considering credit risk

2) Transform the Bernoulli reinsurer credit risk

Implied Premium Discount
= 7.3% (higher than the 2% default probability).

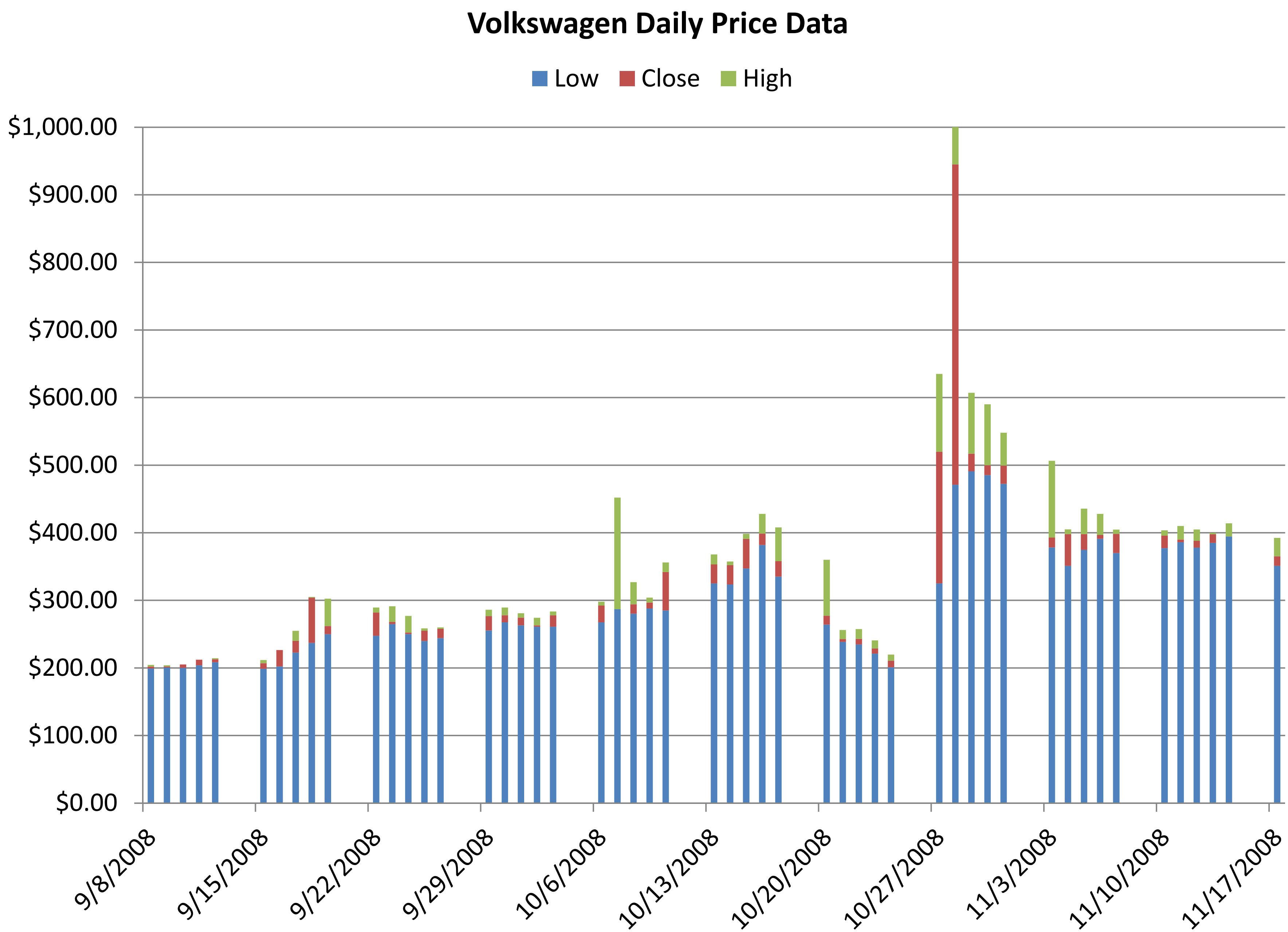
This is the correct way!

Volkswagen Story: Background

- Volkswagen was underperformer in mid-2000
- Market is Generally Short on VW Stock, Hedge funds in particular
- In 2005, Porsche buys 20% of VW matched by Lower Saxony in order to prevent foreign takeover
- In 2007, Porsche ups ownership to 30% but denies any interest in taking over VW
- In 2008, Porsche buys over 42% of cash-settled stock options on VW shares…no disclosure requirements for derivative ownership

Nonlinear Effect of Illiquidity on Price: Volkswagen Story

- October 24, 2008 – VW share price is 200 Euros, over 12% of VW stock is sold short
- October 28, 2008 – Porsche announces it controls 74.1% of VW shares. Lower Saxony holds 20%. 5.9% of shares are available on the market
- Infinite Short Squeeze – situation where the short market struggles to cover their positions in an unavailable market(illiquid)
- October 28, 2008 – VW share price is 1000 Euros
- Hedge Fund Short Sellers lose approximately 10-12 billion Euros
- Porsche makes about 7-8 billion Euros


Volkswagen Daily Price Data
Low
Close
High
$1,000.00
$900.00
$800.00
$700.00
$600.00
$500.00
$400.00
$300.00
$200.00
$100.00
$0.00
9/8/2008
9/15/2008
9/22/2008
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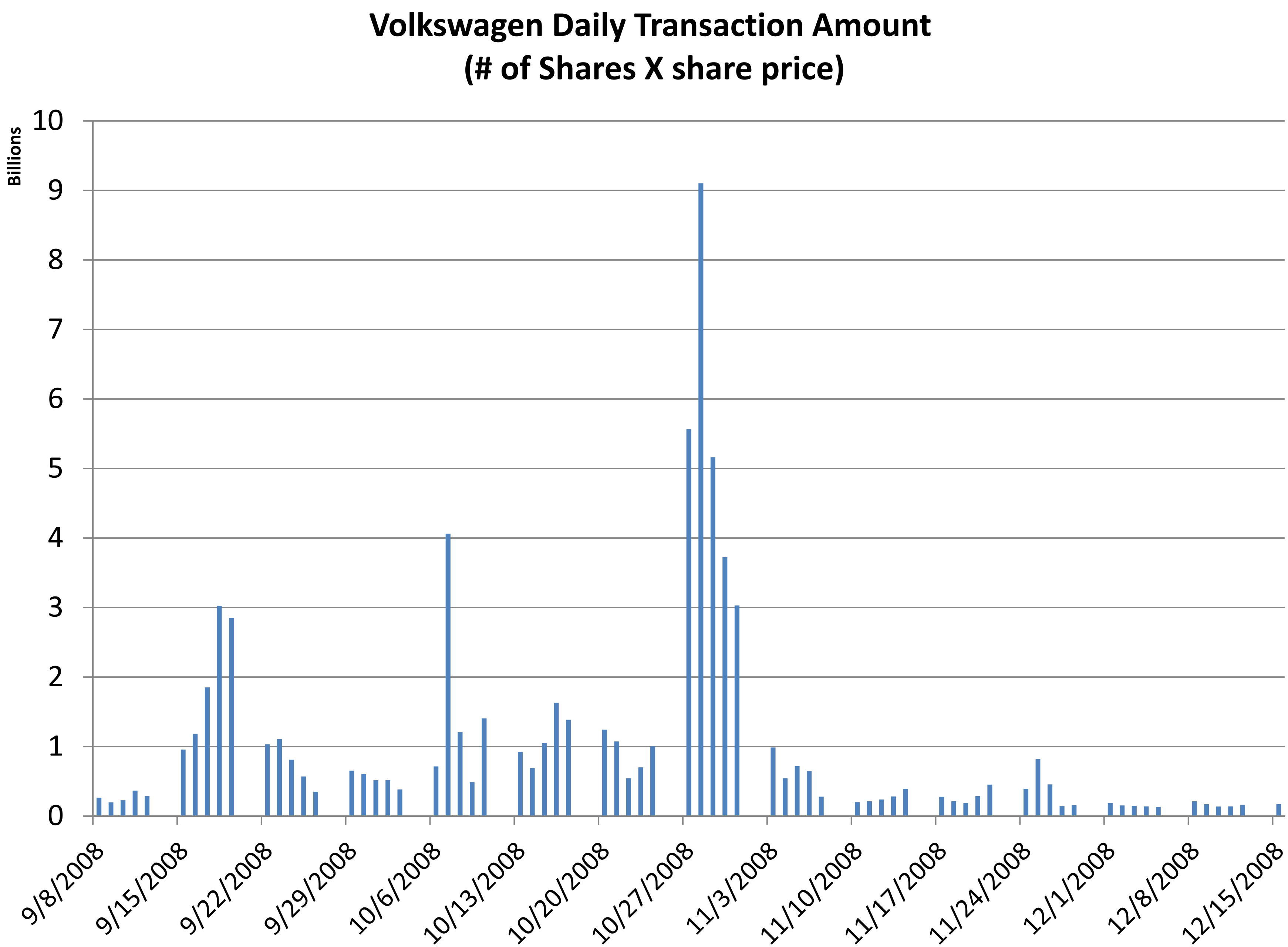
11/10/2008
11/17/2008


Volkswagen Daily Transaction Amount
(# of Shares X share price)
Billions
10
9
8
7
6
5
4
3
2
1
0
9/8/2008
9/15/2008
9/22/2008
9/29/2008
10/6/2008
10/13/2008
10/20/2008
10/27/2008
11/3/2008
11/10/2008
11/17/2008
11/24/2008
12/1/2008
12/8/2008
12/15/2008

Conclusion

- Illiquidity Risk Premium is at the foundation of insurance and reinsurance business
- Wang transform can be used in quantifying illiquidity risk premiums and in selecting optimal reinsurance programs
- Further insights from the Volkswagen example

✓ Size matters: nonlinear effect of illiquidity (demand surge in insurance)

✓ Valuation is a dynamic process.