Stochastic GBM Methods for Modeling Market Prices (McNichols and Rizzo)

The method evolved from various actuarial consulting projects, including;

- Risk Finance Strategic Planning for Corporations
- Equity Investment Asset Risk Module for Dynamic Risk Modeling (and/or Economic Capital Models) for Captives
- Foreign Exchange Portfolio Risk Hedge(s) Structured as Insurance for placement in there Captives.

Louis Bachelier (1840-1926)

The Father of Modern Financial Mathematics

Theorie de la speculation (1900)

Calcul des probabilites (1912)

Le Jeu, la Chance et le Hasard (1914)

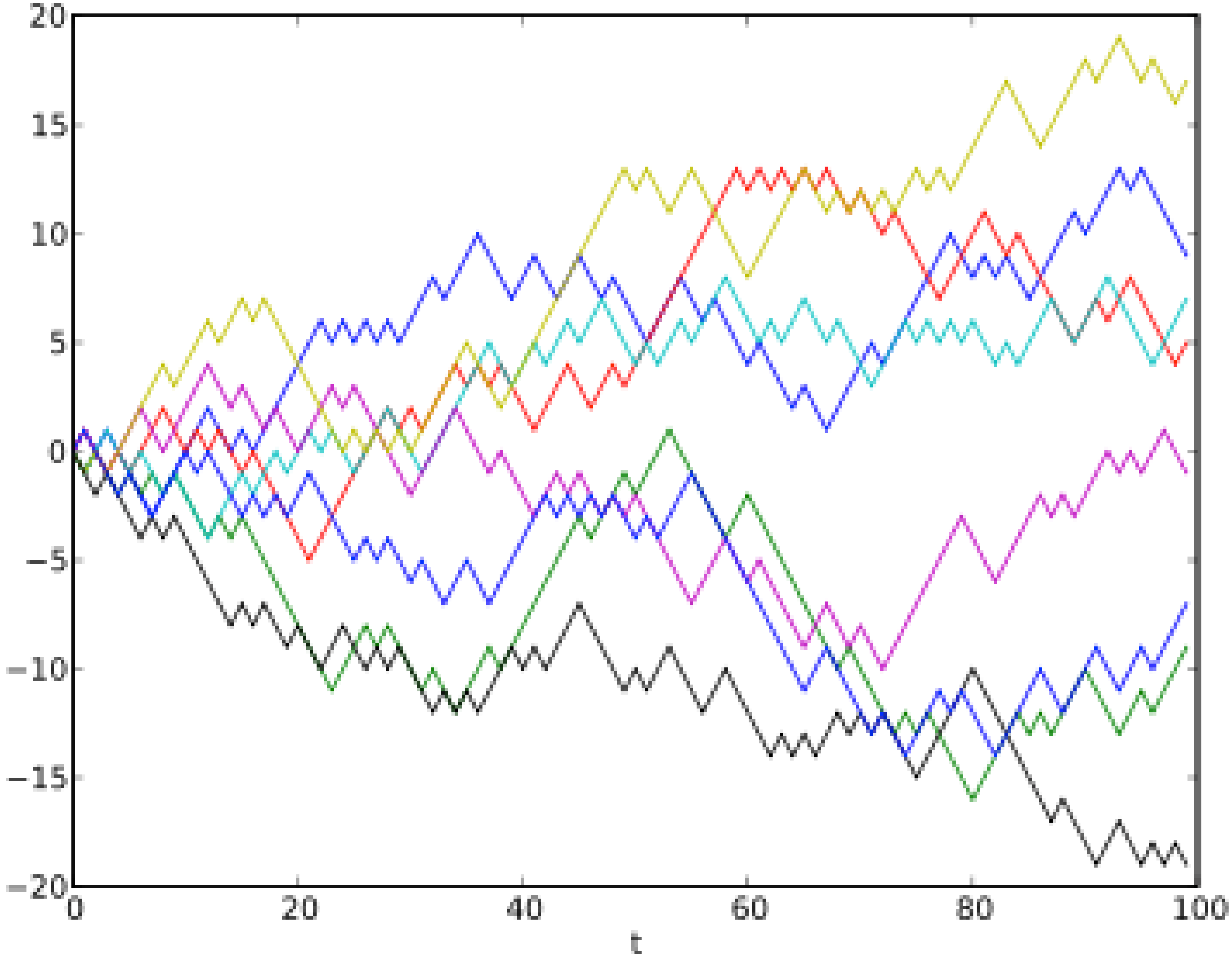
(e.g., Games, Chance and Randomness)

Research in random walks from the discrete case to the continuous case.

Demonstrated that if a random walk on the y-axis is represented as a graph in time with the "drunkard" making ***n*** steps in time ***t***, each step of length +/- ***d***, in total length **D**, the resulting path was such that the tangent of the path angle {*i.e.,* **D** divided by **t**} became increasingly large {in the ratio ***n***^(1/2)}, as n increased.

BUT MORE IMPORTANTLY that the resulting distribution of where the drunkard might be located became increasingly regular.

Random Walk – Discrete Sample Trials



Random Walks (Markov Processes)

The sum of the upward/downward steps gives the height of the drunkard at time t above/below the origin.

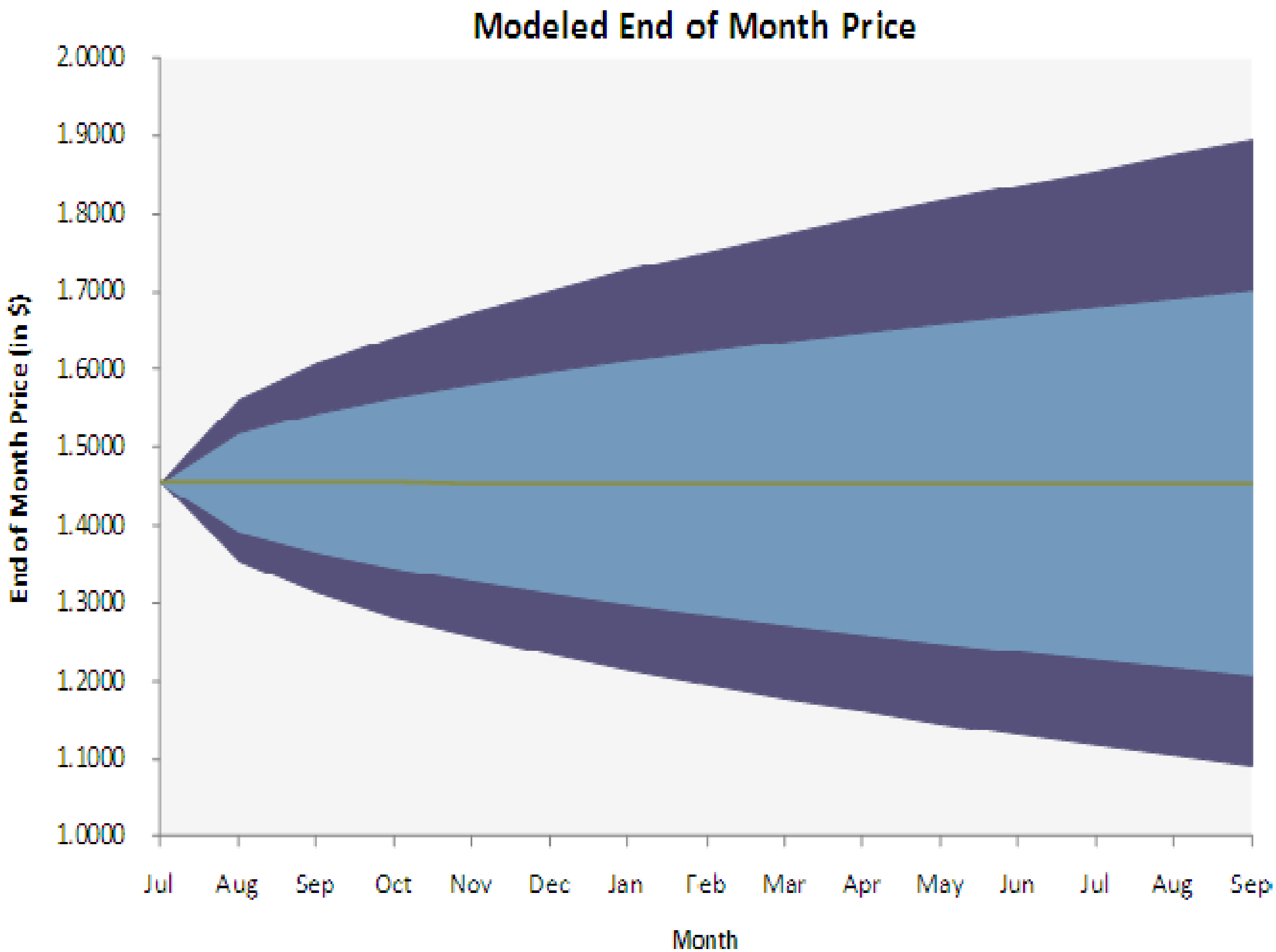
The sum of the squares of the steps taken is equal to t.

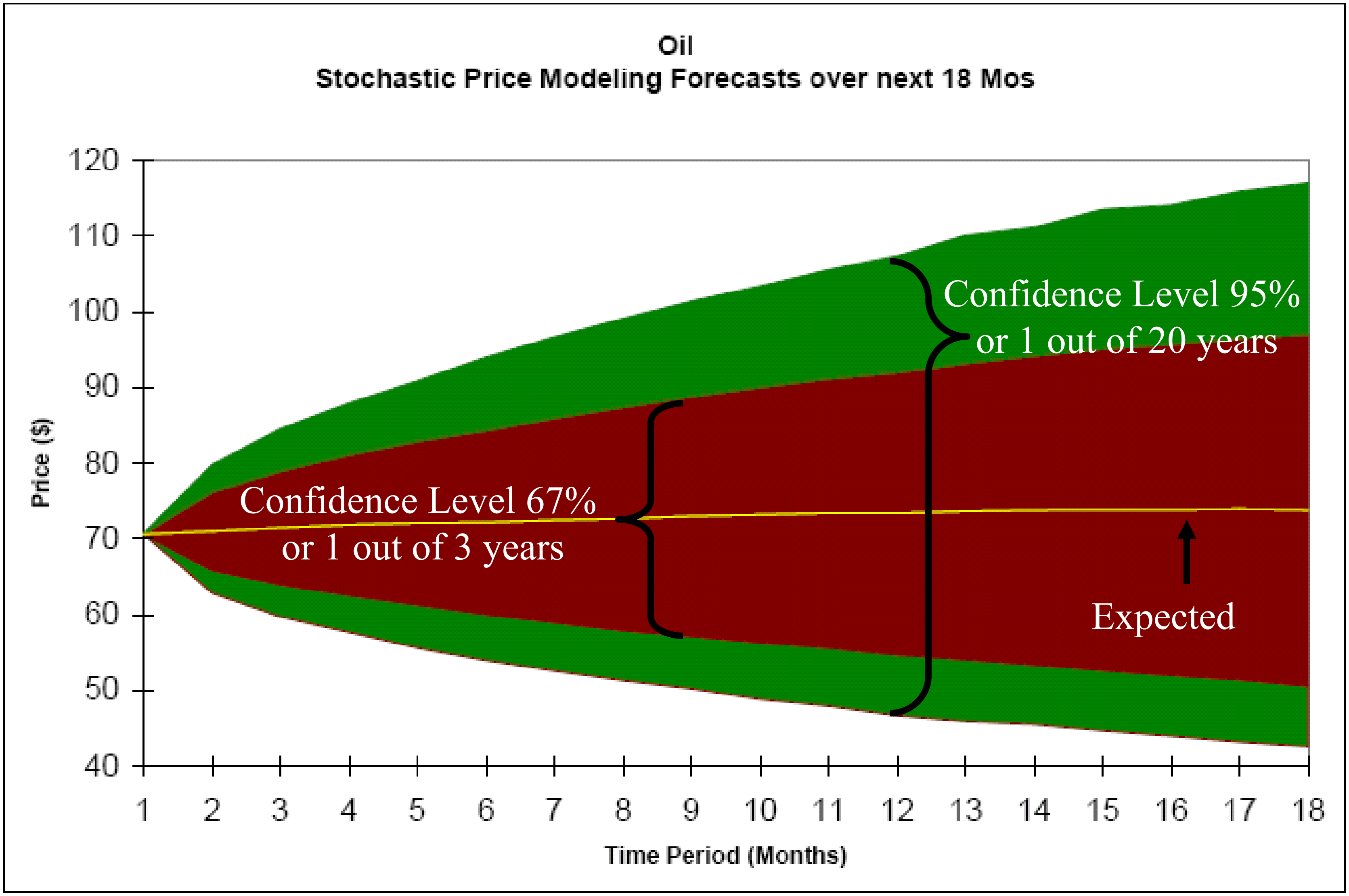
The sum of the cubes of the upward/downward steps (and higher powers) approaches zero, as n increases.

Critical Findings

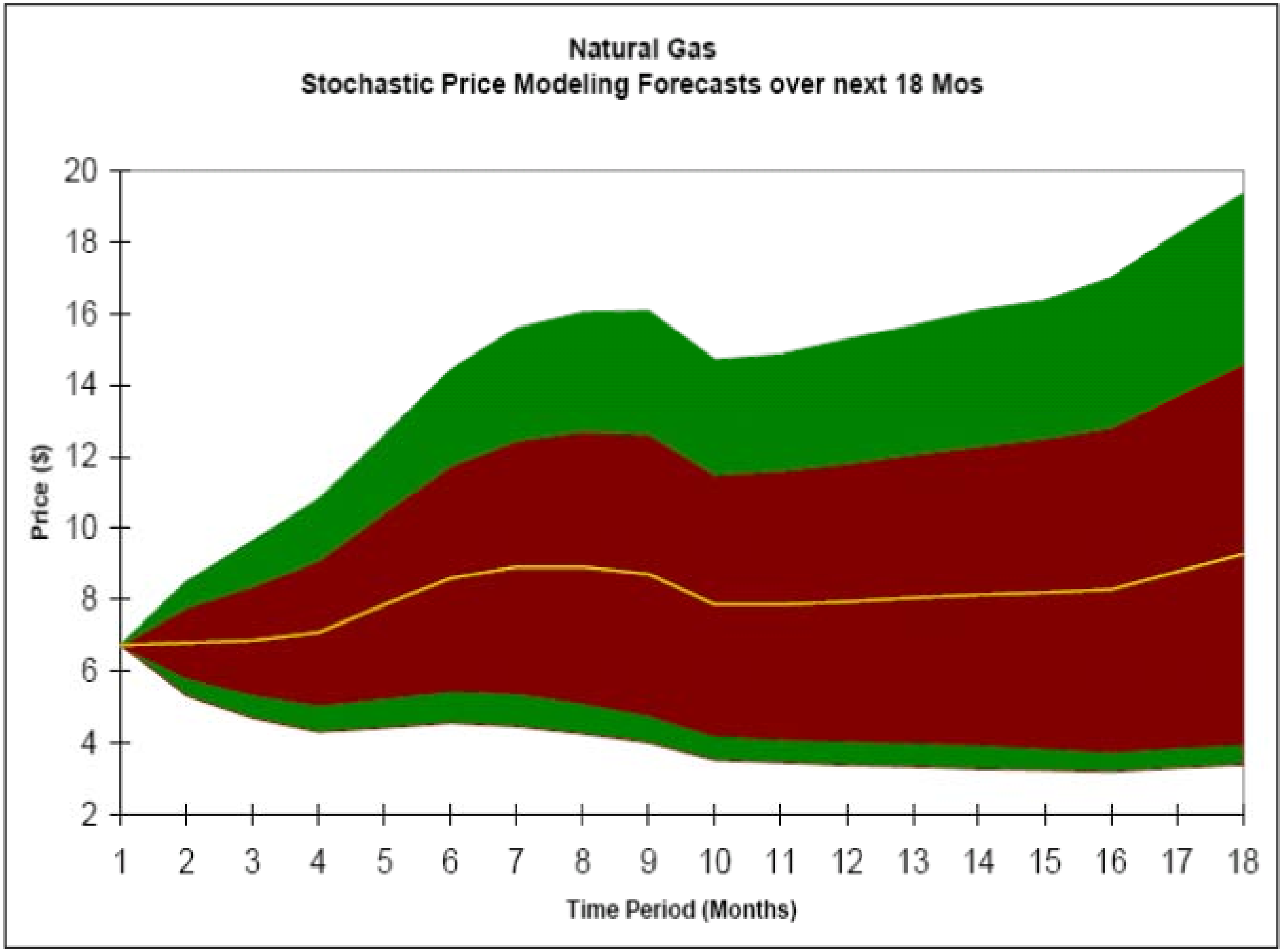
These properties of continuity, non-differentiability, infinite 1^{st} order variation, finite 2^{nd} order variation and zero 3^{rd} (or higher) order variation yields the drunkard's walk and, in the limit, Brownian Motion which in turn led to Ito's Lemma(1950) and subsequently to Black-Sholes (1974).

Funnel Plot of USDvGBP





Oil
Stochastic Price Modeling Forecasts over next 18 Mos
Price ($)
120
110
100
90
80
70
60
50
40
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18
Time Period (Months)
Confidence Level 95%
or 1 out of 20 years
Confidence Level 67%
or 1 out of 3 years
Expected


Natural Gas
Stochastic Price Modeling Forecasts over next 18 Mos
Price ($)
20
18
16
14
12
10
8
6
4
2
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
Time Period (Months)

Modeling Markov Processes

- Wiener Process:

$$\Delta Z \Rightarrow N(0, \Delta t)$$

- The Generalized Wiener Function:

$$\Delta x = a\Delta t + b\Delta z$$

- Ito's Lemma:

$$\Delta x = a(x,t)\Delta t + b(x,t)\Delta z$$

Geometric Brownian Motion (GBM)

$$\Delta S = \mu S \Delta t + \sigma S \Delta z$$

Used for Option Markets Pricing and Foreign Exchange Rates.

μ = expected price appreciation or differential.

S => often assumed to follow a lognormal.

GBM Theory into Practice

$$\ln(S_T) = \ln(S_t) + [\mu - \sigma^2/2]\tau + \sigma\sqrt{\tau}\varepsilon_{N(0,1)}$$

$$\Delta S/S \approx (\mu\Delta t, \sigma\Delta t)$$

$$\ln(S_{t+1}) = \ln(S_t) + \ln([\mu\Delta t + \sigma\varepsilon_{N(0,1)}\sqrt{\Delta t}])$$

This is the formula applied in the model to simulate market price risk exposure.

Interest Rate Risk Functions

GBM processes are widely used for stock prices and currencies but are not appropriate for interest yields.

<u>General model of interest rate dynamics</u>

k = Speed of mean reversion ; b = Long term mean

$$\Delta r_t = k(b - r_t)\Delta t + \sigma\, r^{\gamma}_t\, \Delta z_t$$

$\gamma = 0$, generates returns that are Normally distributed.

$\gamma = 1$, generates returns that are Lognormal.

If $\gamma = 0.5$, results in Cox-Ingersol-Ross (CIR) method.

Practical Applications

The recent interest in these models has been from corporate clients that are directly affected by commodity price fluctuations, such as;

- Oil & Gas companies
- Mining companies

Many of these actuarial engagements are for international corporations which also introduces foreign exchange risk exposures.

Applications to Insurance Problems

P&C insurance is also affected - for example, with business interruption coverage, commodity prices are a key factor when evaluating insurance coverage and limits. Pricing volatility can influence:

- Amount of coverage limits actually provided
- Impacts on deductible payout:
  - Day deductible
  - Flat dollar deductible
- Cost of insurance coverage

Sample Output with Unlimited Results

This table shows the modeled losses before the application of any insurance structure.

The "Expected" column applies the forward curve from the commodity options market.

The "Floating" column implements a stochastic GBM price model.

The introduction of a volatility dimension does not impact the mean, but increases the standard deviation and introduces effects on the tail risk and implied capital charges.

$ in millions

Confidence Level	Years per Event	Unlimited Expected	Unlimited Floating
Expected Loss		$ 336.15	$ 338.04
Standard Deviation		$ 348.12	$ 362.43
TVaR @ 95%		$ 1,320.15	$ 1,396.35
TVaR @ 99%		$ 1,752.19	$ 1,936.95
10%		$ 13.97	$ 13.90
20%		$ 47.51	$ 47.47
30%		$ 91.51	$ 89.71
40%		$ 139.15	$ 138.66
50%	2.0	$ 211.99	$ 210.80
60%	2.5	$ 336.81	$ 323.34
70%	3.3	$ 447.43	$ 440.38
80%	5.0	$ 570.03	$ 574.72
90%	10	$ 824.77	$ 820.56
95%	20	$ 1,038.62	$ 1,062.49
99%	100	$ 1,488.38	$ 1,602.06
99.5%	200	$ 1,669.67	$ 1,833.47
99.90%	1000	$ 2,098.51	$ 2,387.59
99.95%	2000	$ 2,280.98	$ 2,597.65
99.99%	10000	$ 2,612.05	$ 3,053.69

Impact of Insurance Structures

To the prior model we now compare two deductible options

– fixed versus monthly

Under a 3 month deductible, the insured will be retaining much more risk than under a flat deductible.

If the insurance market is pushing for this change, then the client may argue for a corresponding credit to adopt this coverage change.

$ in millions

		Fixed Option 1	Fixed Option 2	Floating Option 1	Floating Option 2
	Retention	$ 65.00	3 Months	$ 65.00	3 Months
	Limit	$ 500.00	$ 500.00	$ 500.00	$ 500.00
	Expected Loss	$ 125.32	$ 133.35	$ 129.31	$ 137.33
	Standard Deviation	$ 196.77	$ 205.36	$ 215.60	$ 223.87
	TVaR @ 95%	$ 820.15	$ 830.42	$ 896.35	$ 906.79
	TVaR @ 99%	$ 1,252.19	$ 1,261.70	$ 1,436.95	$ 1,447.46
Confidence Level	Years per Event				
10%		$ 13.97	$ 13.11	$ 13.90	$ 13.03
20%		$ 17.57	$ 17.50	$ 17.74	$ 17.55
30%		$ 30.00	$ 30.00	$ 30.00	$ 30.00
40%		$ 45.00	$ 38.98	$ 45.00	$ 38.96
50%	2.0	$ 65.00	$ 51.74	$ 65.00	$ 52.02
60%	2.5	$ 82.15	$ 73.23	$ 81.96	$ 73.42
70%	3.3	$ 100.50	$ 109.02	$ 101.64	$ 110.36
80%	5.0	$ 147.28	$ 191.56	$ 147.32	$ 191.42
90%	10	$ 325.03	$ 372.46	$ 320.81	$ 374.42
95%	20	$ 538.62	$ 548.72	$ 562.49	$ 579.18
99%	100	$ 988.38	$ 1,002.95	$ 1,102.06	$ 1,109.77
99.5%	200	$ 1,169.67	$ 1,180.95	$ 1,333.47	$ 1,340.63
99.90%	1000	$ 1,598.51	$ 1,599.01	$ 1,887.59	$ 1,889.77
99.95%	2000	$ 1,780.98	$ 1,780.98	$ 2,097.65	$ 2,111.53
99.99%	10000	$ 2,112.05	$ 2,112.05	$ 2,553.69	$ 2,623.49

Mining Company – Example

The minerals they sold were historically within a fairly stable and low volatility market price regime.

The market underwent a sizable transformation with prices jump shifting to a higher and suddenly more volatile range.

Project Analysis Goals

- Demonstrate to the client what the “new normal” market would have done to the prior profit/loss history.
- Demonstrate what the impact from new insurance structures would have on their loss retention strategy.
- Enable the client to proactively request significant structural changes to their insurance program rather than passively reviewing what the insurance market offers.

Energy Company – Example 1

High commodity prices for natural gas and oil was going to result in a “touch renewal”.

Project Analysis Goals

- Show what the array of future commodity prices would mean in terms of their potential losses.
- Show what the commodity price ranges would do to their ability to absorb losses.
- Enable the client to determine the likely consequences of self-insuring layers of their own risk programs and to efficiently price that risk.

Energy Company – Example 2

Incorporate the potential impacts from interest and commodity price fluctuation into the strategic planning process

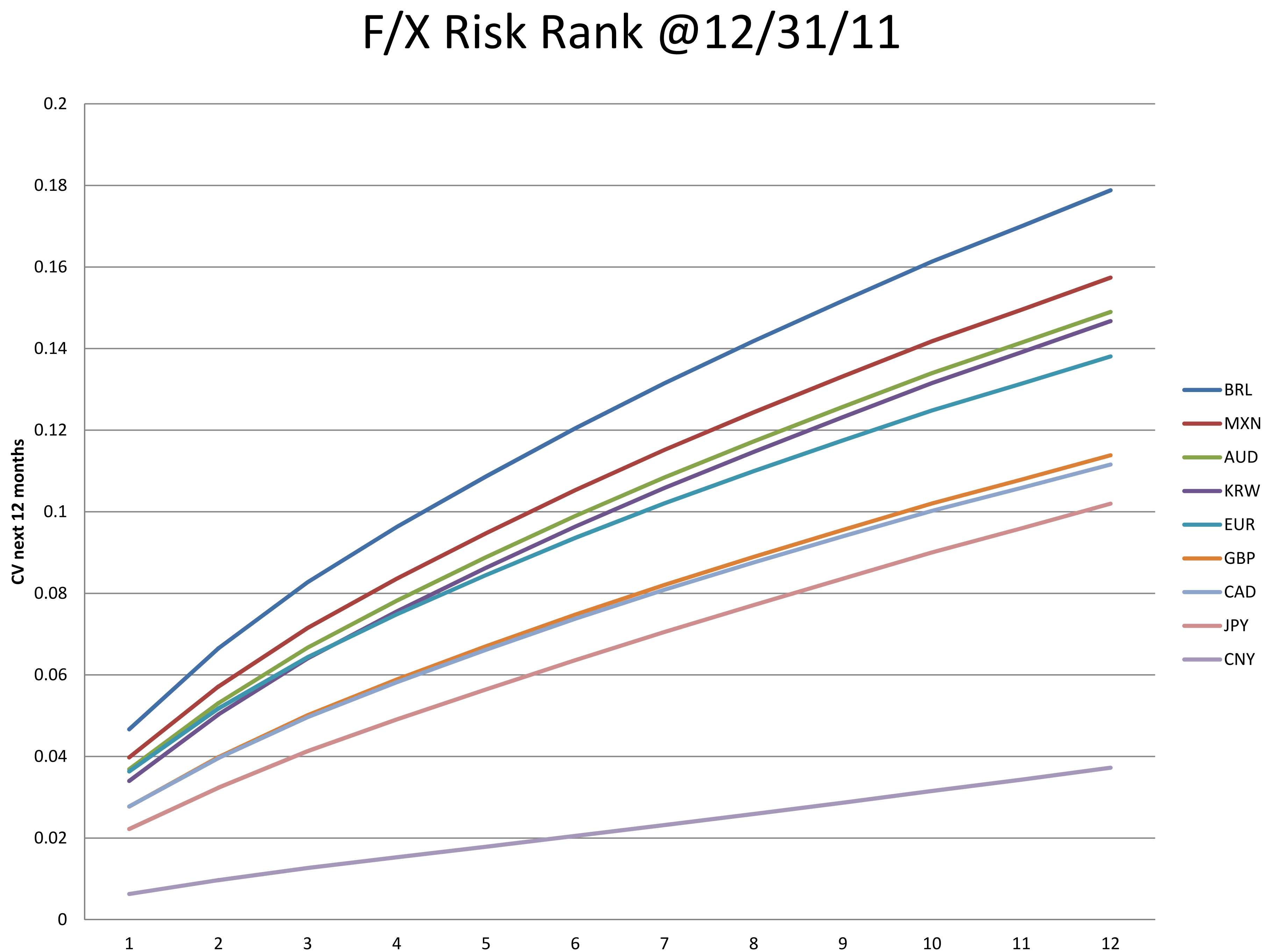
- They used Oracle software which Crystal Ball can plug into the system
- Project Goals
  - Help them understand the formulas and build templates to seed their strategic planning system
  - Coordinate with non-PC actuaries to incorporate their results into the planning software as well

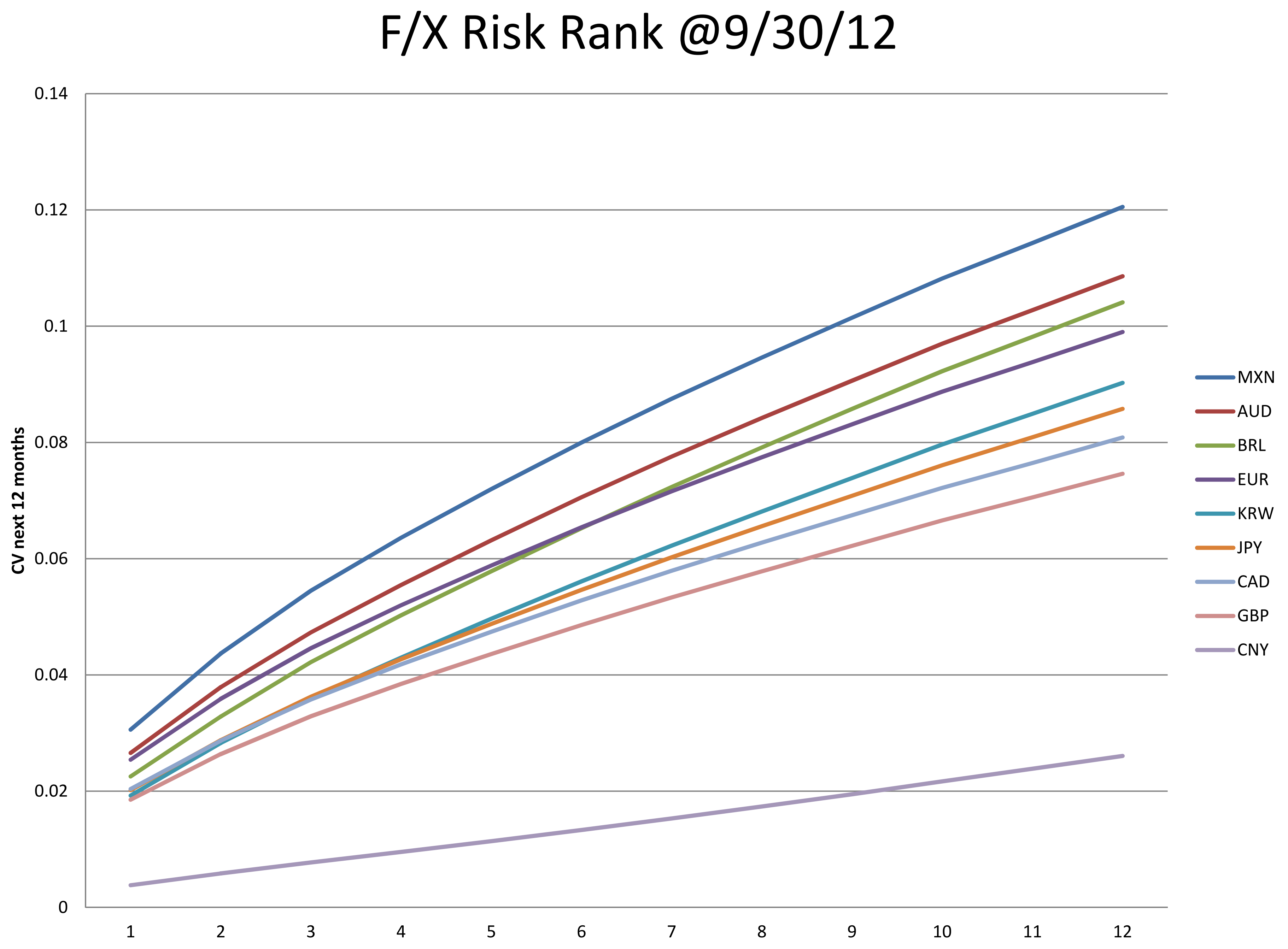
F/X – Example (USD v EUR)

Forward Options	Mean Forward Price (υ)	Options Volatility (σ)
+1 Mos; @01/31/2012	0.7713	12.32%
+2 Mos; 02/28/2012	0.7711	12.55%
+3 Mos.; 03/31/2012	0.7708	12.70%
+4 Mos.; 04/30/2012	0.7704	12.99%
+5 Mos; 5/31/2012….	0.7701	13.07%
+ 12 Mos; ….12/31/2012	0.7669	13.56%

F/X – Example (USD v EUR)

Months Forward	Target Mean	Price BOM	C-I-R random	C-I-R rate	Weiner	Expon	DRV adj.	Price EOM
+1 Mos.	0.7713	0.7715	0.4598	0.0028	0.0162	1.0164	(0.0625%)	0.7713
+2 Mos.	0.7711	0.7835	0.7633	0.0060	0.0278	1.0282	(0.0641%)	0.7711
+3 Mos.	0.7708	0.8051	0.0455	0.0073	0.0016	1.0016	(0.1001%)	0.7708
+4 Mos.	0.7704	0.8056	1.5085	0.0149	0.0575	1.0591	(0.1126%)	0.7704
+5 Mos..	0.7701	0.8523	(0.1487)	0.0149	(0.0051)	0.9949	(0.1437%)	0.7701
..+12 Mos.	0.7669	0.7939	(01067)	0.0109	(0.0040)	0.9960	(0.1653%)	0.7669


F/X Risk Rank @12/31/11
CV next 12 months
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
9
10
11
12
BRL
MXN
AUD
KRW
EUR
GBP
CAD
JPY
CNY



F/X Risk Rank @9/30/12
CV next 12 months
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1
2
3
4
5
6
7
8
9
10
11
12
MXN
AUD
BRL
EUR
KRW
JPY
CAD
GBP
CNY