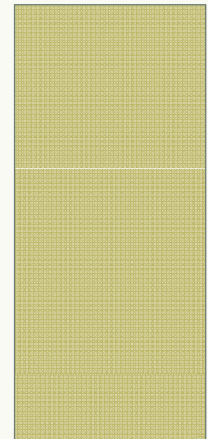


# CHAIN LADDER RESERVING METHODS FOR LIABILITIES WITH PER OCCURRENCE LIMITS

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# BACKGROUND

- Response to CAS Call for Non-Technical Papers 2013
- As a regulator I see many actuarial reports and many techniques. I wrote this paper to improve the quality of actuarial work. Don't remove large losses when LDFs come from net/limited triangles. Schedule P triangles are often limited.

# INTRODUCTION

- Case Inc. Limited Losses: \$3M
- Insured Limit: \$500K
- Losses exceeding 100K: 120K, 450K, 500K
- Applicable Incurred LDF: 1.2
- Accident Year: 20XX

450K and 500K multiplied by the LDF exceed the limit. What should you do?

$$\$3M \times 1.2 = \$3.6M$$

OR

$$(\$3M - \$0.45M - \$0.5M) \times 1.2 + \$0.5M + \$0.5M = \\ \$2.05M \times 1.2 + \$1M = \$3.46M$$

Multiply all by the LDF

Assume that 450K and 500K develop to 500K then stop.

## SOME ASSUMPTIONS/METHODS

- There is some set of “best” LDFs that is applicable to losses that do not reach the self-insured retention or retained limit.
- Used 500K limit/retention throughout the paper
- LDFs are calculated using an all year weighted average.

# DIFFERENT TRIANGLES FROM THE SAME DATA

Limited LDFS

A) Unlimited Triangle

	1	2	3	4	5
2009	415	853	1,258	1,654	2,051
2010	180	370	546	717	-
2011	580	1,192	1,758	-	-
2012	180	370	-	-	-
2013	415	-	-	-	-

	1	2	3	4	5
f(d)	2.06	1.48	1.32	1.24	1.00
F(d)	4.94	2.41	1.63	1.24	1.00

B) Limited Triangle 500K per Occ

	1	2	3	4	5
2009	415	839	1,000	1,158	1,316
2010	180	370	546	717	-
2011	580	1,178	1,500	-	-
2012	180	370	-	-	-
2013	415	-	-	-	-

	1	2	3	4	5
f(d)	2.03	1.28	1.21	1.14	1.00
F(d)	3.58	1.76	1.38	1.14	1.00

C) Small Only - Only Losses <= 450K

	1	2	3	4	5
2009	165	339	500	658	816
2010	180	370	546	717	-
2011	180	370	546	-	-
2012	180	370	-	-	-
2013	415	-	-	-	-

	1	2	3	4	5
f(d)	2.06	1.48	1.32	1.24	1.00
F(d)	4.94	2.41	1.63	1.24	1.00

D) Large Only Limited to 500K per Occ

	1	2	3	4	5
2009	250	500	500	500	500
2010	-	-	-	-	-
2011	400	808	955	-	-
2012	-	-	-	-	-
2013	-	-	-	-	-

	1	2	3	4	5
f(d)	2.01	1.11	1.00	1.00	1.00
F(d)	2.24	1.11	1.00	1.00	1.00

# METHOD-A VS. METHOD-X

Method-A multiplies all losses by the development factors

Method-X tests each loss. If the loss is “large enough” then the ultimate loss is set equal to the limit.

The result from method X is always less than or equal to the result from method A.

## CONCLUSIONS - CONTINUED

- If the LDFs were created using a limited triangle, then it is more accurate to multiply ALL the losses by the LDF.
- If the LDFs were created using a limited triangle, then removing large losses understates the reserve. *This is due to the fact that losses reaching the limits no longer develop over time and hence the LDFs estimated using all losses are smaller than the LDFs estimated using only the losses below the limits.*
- If you can separate large losses from small losses then consider creating two sets of LDFs.

# CONCLUSION

- Case Inc. Limited Losses: \$3M
- Insured Limit: \$500K
- Losses exceeding 100K: 120K, 450K, 500K
- LIMITED LDF: 1.2
- Accident Year: 20XX

450K and 500K multiplied by the LDF exceed the limit. What should you do?

$$\$3M \times 1.2 = \$3.6M$$

Multiply all by the LDF

OR

$$(\$3M - \$450K - \$500K) \times 1.2 + \$500K + \$500K =$$
$$\$2.05M \times 1.2 + \$1M = \$3.46M$$

Assume that 450K and 500K develop to 500K then stop.



# METHOD USED IN PAPER

- Simulate many incurred losses and associated triangles where the losses have per occurrence limits.
- Apply the limited LDFs to the incurred losses both with and without large losses
- See which one is more accurate



# UNLIMITED - TO- LIMITED

## Unlimited Occurrences

Year	d=1	d=2	d=3	d=4	d=5	d=6	d=7	d=8
2006	250	514	758	996	1,236	1,483	1,742	1,995
2006	150	308	455	598	741	890	1,045	1,197
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
				Several Rows				
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
Total	580	1,194	1,753	2,314	2,865	3,441	4,047	4,632

## Limited Occurrences





Year	d=1	d=2	d=3	d=4	d=5	d=6	d=7	d=8
2006	250	500	500	500	500	500	500	500
2006	150	308	455	500	500	500	500	500
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
				Several Rows				
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
2006	15	31	45	60	74	89	105	120
Total	580	1,180	1,495	1,720	1,888	2,068	2,260	2,440

# EXAMPLE LIMITED TRIANGLE & LIMITED LDFS

	1	2	3	4	5	6	7	8	9	10
2004	180	370	516	...	890	1,068	1,251	1,436	1,616	1,791
2005	180	370	516	...	890	1,068	1,251	1,436	1,616	.
2006	180	370	516	...	890	1,068	1,251	1,436	.	.
2007	180	370	516	...	890	1,068	1,251	.	.	.
2008	180	830	1,000	1,158	1,316	1,479	.	.	.	.
2009	180	370	516	...	890	.	.	.	.	.
2010	225	462	692	897	.	.	.	.	.	.
2011	180	808	955	.	.	.	.	.	.	.
2012	180	370	.	.	.	.	.	.	.	.
2013	285	.	.	.	.	.	.	.	.	.

	1	2	3	4	5	6	7	8	9	10
F(4)	1,016	1,936	2,720	3,380	3,920	4,344	4,656	4,956	5,244	5,520
F(5)	1,196	2,236	3,066	3,776	4,360	4,824	5,176	5,524	5,868	6,200

# AN ITERATION OF METHOD A

Accident Year	Age - d	Incurred \$ (000)	F(d)	Method A Estimate \$ (000)	True Ultimate \$ (000)	Method A IBNR \$ (000)	True IBNR \$ (000)	Error \$ (000)	Error as a % of True IBNR
		(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
		modeled	modeled	=(a)*(b)		=(c)-(a)	=(d)-(a)	=(f)-(e)	=(g)-(f)
2004	10	1,794	1.00	1,794	1,794	-	-	-	-
2005	9	1,616	1.11	1,794	1,794	178	178	-	0%
2006	8	2,436	1.23	3,042	2,794	606	357	248.7	
2007	7	1,254	1.39	1,746	1,794	491	539	(47.7)	-9%
2008	6	1,479	1.59	2,358	2,144	871	666	203.6	
2009	5	890	1.83	1,624	1,794	735	904	(169.2)	-19%
2010	4	897	2.13	1,928	2,242	1,031	1,345	(314.2)	-23%
2011	3	955	2.66	2,541	1,995	1,586	1,040	546.0	
2012	2	370	3.33	1,305	1,794	933	1,434	(488.7)	-34%
2013	1	285	7.19	2,050	1,845	1,765	1,560	204.4	
<b>Total</b>		<b>11,975</b>		<b>20,173</b>	<b>19,988</b>	<b>8,198</b>	<b>8,013</b>	<b>185.1</b>	<b>2%</b>
<b>2004-2011</b>		<b>11,320</b>		<b>16,818</b>	<b>16,349</b>	<b>5,498</b>	<b>5,019</b>	<b>469.3</b>	<b>9%</b>

# AN ITERATION OF METHOD X

Accident Year	Age - d	F(d)	Incurred \$'(000)	Large Losses	Inc X Known Large Losses	Method X Estimate \$'(000)	True Ultimate \$'(000)	Method X IBNR \$'(000)	True IBNR \$'(000)	Error \$'(000)	Error as a % of True IBNR
		(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)
		modeled	modeled	modeled	$= (b) - (c)$	$= (a) * (d) - (c)$		$= (g) - (b)$	$= (f) - (b)$	$= (i) - (h)$	$= (j) - (i)$
2004	10	1.00	1,774	-	1,774	1,774	1,774	-	-	-	0%
2005	9	1.11	1,515	-	1,515	1,724	1,724	173	173	-	0%
2006	8	1.25	1,434	1,000	1,434	2,724	2,724	357	357	(0)	0%
2007	7	1.32	1,254	-	1,254	1,724	1,724	491	332	(43)	-13%
2008	6	1.52	1,472	500	72	2,033	2,144	577	636	(32)	-5%
2009	5	1.59	590	-	590	1,724	1,724	735	304	(169)	-13%
2010	4	2.15	327	-	327	1,223	2,242	1,031	1,345	(314)	-23%
2011	3	2.52	355	500	455	1,710	1,393	755	1,040	(253)	-24%
2012	2	3.53	370	-	370	1,305	1,724	336	1,424	(452)	-32%
2013	1	7.12	355	-	355	2,030	1,545	1,755	1,530	204	13%
<b>Total</b>			11,275	2,000		18,799	12,988	6,824	8,013	(1,189)	-15%
2004-2011			11,320	2,000		15,444	11,349	4,124	5,029	(905)	-18%

# 10,000 SIMULATIONS

Error as a Percentage of IBNR								Error as % of Ult
		10th Percentil	25th Percentil	Mean Error	75th Percentil	90th Percentil	Std Dev	Mean
All Years	A	-19%	-10%	3.4%	15%	28%	19%	1.35%
2004-2011	A	-20%	-11%	1.9%	14%	25%	17%	0.54%
All Years	X	-34%	-27%	-20%	-12%	-5%	11%	-7.90%
2004-2011	X	-32%	-25%	-18%	-9%	-3%	11%	-5.54%

# @RISK - GRAPHS OUTPUT

Variable	Mean	St. Dev.	Min	Max	Q1	Q3
"Numbers" large areas	14	2	10	18	12	16
Amount of areas +50000000 on the area data	10000	1000	8000	12000	9000	11000
Area 100000000	0.5	0.1	0.3	0.7	0.4	0.6
Area 100000000	0.5	0.1	0.3	0.7	0.4	0.6
Area 100000000	1.0	0.2	0.6	1.4	0.8	1.2
Area 100000000	1.2	0.2	0.8	1.6	1.0	1.4
Area 100000000	1.4	0.2	1.0	1.8	1.2	1.6
Area 100000000	1.4	0.2	1.0	1.8	1.2	1.6
Area 100000000	2.0	0.3	1.4	2.6	1.8	2.2
Area 100000000	3.5	0.5	2.5	4.5	3.0	4.0
Area 100000000	3.0	0.5	2.0	4.0	2.5	3.5
Area 100000000	1.0	0.2	0.6	1.4	0.8	1.2
Area 100000000	0.8	0.1	0.6	1.0	0.7	0.9



# SENSITIVITY ANALYSIS – 5,000 SIMULATIONS

**Table 1- All Years Combined – Sensitivity of Mean Error to LDF and Percentage of Large Losses**

Ratio of Large Losses to Total Losses - Ultimate Limited Basis	Method	Highest LDF		High LDF		Moderate LDF	
		Mean Error as % of IBNR	Mean Error as % of Ultimate	Mean Error as % of IBNR	Mean Error as % of Ultimate	Mean Error as % of IBNR	Mean Error as % of Ultimate
		<b>15%</b>	A	7%	3%	4%	1%
	X	-33%	-12%	-23%	-6%	-16%	-3%
<b>10%</b>	A	4%	1%	2%	1%	1%	0%
	X	-25%	-10%	-16%	-5%	-12%	-2%
<b>5%</b>	A	1%	1%	1%	0%	0%	0%
	X	-14%	-6%	-9%	-3%	-6%	-1%

# QUESTIONS

- [Kadams@azinsurance.gov](mailto:Kadams@azinsurance.gov)
- 602.364.3249

# OTHER SLIDES - VARIOUS LDFS

Highest

d	1	2	3	4	5	6	7	8	9	10
f(d)	2.055	1.475	1.315	1.240	1.200	1.175	1.145	1.125	1.110	1.000
F(d)	9.964	4.849	3.287	2.500	2.016	1.680	1.430	1.249	1.110	1.000

High

d	1	2	3	4	5	6	7	8	9	10
f(d)	1.541	1.263	1.179	1.138	1.116	1.102	1.085	1.073	1.065	1.000
F(d)	3.973	2.579	2.042	1.733	1.523	1.365	1.239	1.143	1.065	1.000

Moderate

d	1	2	3	4	5	6	7	8	9	10
f(d)	1.296	1.150	1.104	1.081	1.068	1.060	1.050	1.043	1.038	1.000
F(d)	2.289	1.766	1.536	1.391	1.287	1.205	1.137	1.083	1.038	1.000

## OTHER SLIDES - NOTATION

- *The Analysis and Estimation of Loss & ALAE Variability...* by the CAS Working Party on Quantifying Variability in Reserve Estimates.
- $w$ : The accident year
- $d$ : The age of the losses. If the accident year is 2010, then  $d=1$  at 12/31/2010 and  $d=2$  at 12/31/2011
- $f(d)$  : Incremental LDF
- $F(d)$  : Cumulative LDF.
- $f^T(d)$ : true value of  $f(d)$  for unlimited losses.
- $F^T(d)$ : true value of  $F(d)$  for unlimited losses.

## OTHER SLIDES - MORE NOTATION AND METHODS

- $f^T(d)$ : best value of  $f(d)$  for unlimited losses.
- $F^T(d)$ : best value of  $F(d)$  for unlimited losses.
- Losses are in 000's
- The retained limit is always 500K
- LDFs are calculated using an all year weighted average