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A Deeper Understanding of Experience Rating -- Split and Unsplit

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Credibility of Experience – Conceptual Drivers

- Volatility of Actual Loss
  - Process Risk = Noise
  - More Noise ⇒ Less Credibility
- Belief in Initial Estimate
  - Parameter Risk = Uncertainty in Initial Estimate
  - More Parameter Risk ⇒ More Credibility
  - *The Less You Know* …..(in advance)

Pop-quiz 1

- A simple class plan is replaced with one that is more refined. The z for individual risk experience will:
  - A. Increase B. Stay the same C. Decline

- The expected mean loss for a risk is initially $1,000 with standard deviation of $100. The z-weighted estimate is $1,100. The standard deviation of that estimate is:
  - A. Greater than $100 B. =$100 C. Less than $100

Notation

RV	A
Condl Mean	$\mu(\theta)$
Condl Variance	$\sigma^2(\theta)$
UnCondl Mean	μ or E
Process Variance	$\sigma^2 = E[\sigma^2(\theta)]$
Parameter Variance	$\tau^2 = Var(\mu(\theta))$
Total Variance	$\lambda^2 = \sigma^2 + \tau^2$

Credibility Estimate and MSE

$$\mu^*(z) = zA + (1-z)E$$

- *Given arbitrary credibility, z, the mean square error (MSE) is given as:*

$$\varepsilon^2 = E[\left(\mu^*(z) - \mu(\theta)\right)^2]$$

$$= z^2 \cdot \sigma^2 + (1-z)^2 \tau^2$$

Optimal Credibility

- *The z which minimizes mean square error is given as z* where:*

$$z^* = \frac{\tau^2}{\tau^2 + \sigma^2} = \frac{\tau^2}{\lambda^2}$$

What increases optimal credibility?

- *Reducing process risk*
- *Increasing parameter risk*

Error Reduction in Estimate of the Mean

- *It can be proved that Optimal Mean Square Error is given as:*

$$\varepsilon_0^2(NS) = \tau^2\left(1 - z^*\right)$$

- *Initial mean square error is* τ^2
- z^**= the factor by which parameter risk is reduced using optimal weighting*

Example of Error Reduction

- Let $\tau^2 = 100$ and $\sigma^2 = 300$.

It follows that:

$\lambda^2 = 400$ and $z^* = 100/400 = 25\%$

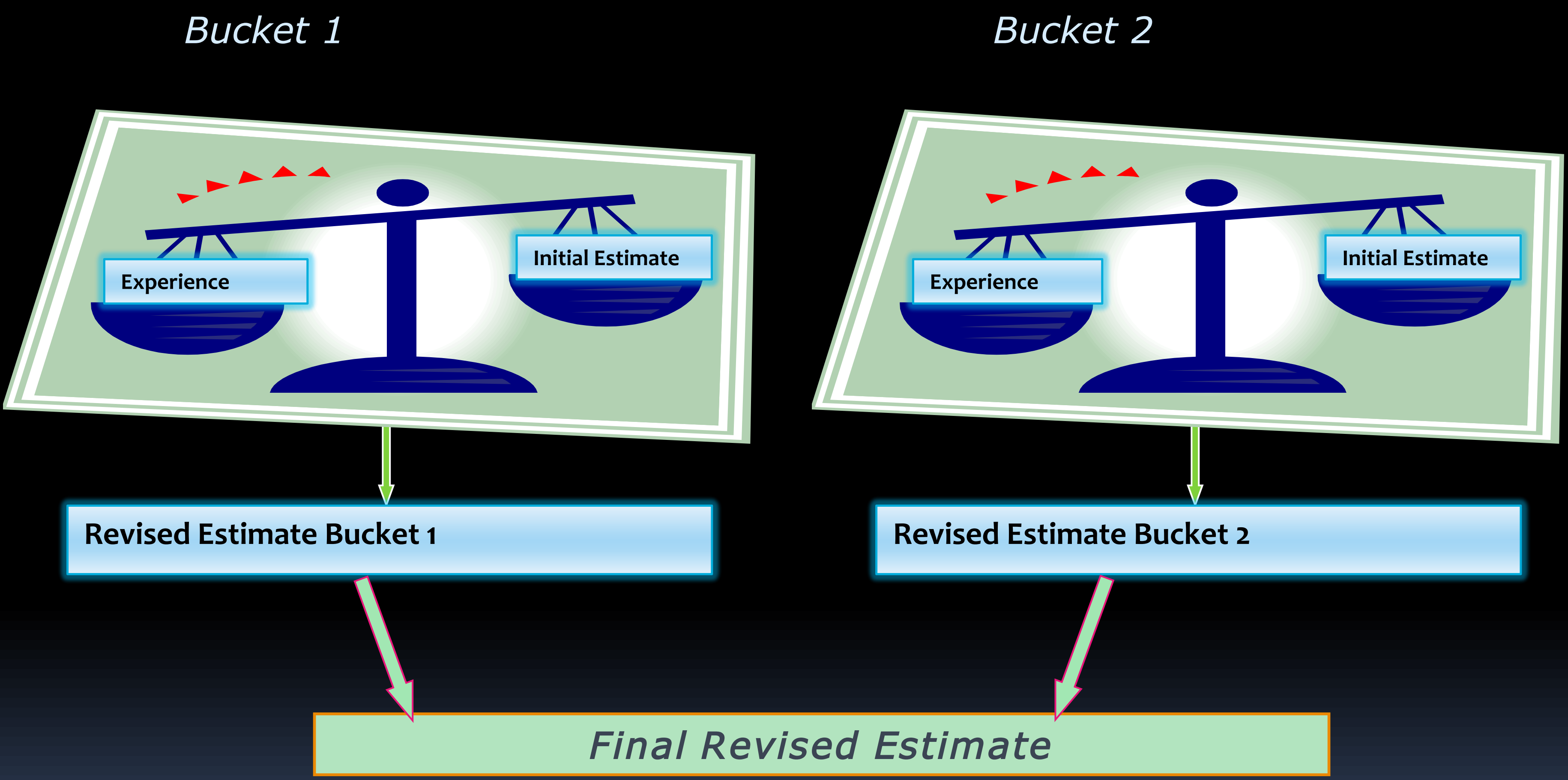
$$\text{Optimal MSE} = \mathrm{E}[(\mu - \mu^*)^2]$$
$$= \tau^2 \cdot (1 - z^*) = 100 \cdot (1 - .25) = 75$$

Split Credibility

Split Credibility – US Workers Comp

- US Workers Comp Experience Rating
- Split-z procedure
- Primary and Excess Split
- Accidents capped at state per accident limit
- Table s of primary and excess Z by risk size
- Tables of expected primary and excess loss rates by class using d ratios

Split Credibility



Split Credibility – Basic Formula

- Final Estimate =Sum of Credibility Weighted Estimates

$$
\begin{aligned}
\mu^* &= \mu_1{}^* + \mu_2{}^* \\
&= \left\{ z_1 A_1 + (1 - z_1) E_1 \right\} \\
&+ \left\{ z_2 A_2 + (1 - z_2) E_2 \right\}
\end{aligned}
$$

Credibility Example-Loss Data

Loss Experience

Claim Number	Total Loss	Primary Loss	Excess Loss
1	1,000	1,000	-
2	1,500	1,500	-
3	2,500	2,500	-
4	4,000	4,000	-
5	15,000	5,000	10,000
6	80,000	5,000	75,000
Total	**104,000**	**19,000**	**85,000**

Based on Split point = 5,000

Credibility Example – Non-Split vs Split

Experience Rating

No Split Plan	**Actual Loss**	**Credibility**	**Expected Loss**	**Z-wtd estimate**
Total	104,000	50%	100,000	**102,000**

Split Plan	**Actual Loss**	**Credibility**	**Expected Loss**	**Z-wtd estimate**
Primary	19,000	70%	30,000	22,300
Excess	85,000	20%	70,000	73,000
Combined Split Estimate				**95,300**

Notation

RV	A_1	A_2	$A = A_1+A_2$
Condl Mean	$\mu_1(\theta)$	$\mu_2(\theta)$	$\mu(\theta)$
Condl Variance	$\sigma_1^2(\theta)$	$\sigma_2^2(\theta)$	$\sigma^2(\theta)$
UnCondl Mean	μ_1	μ_2	μ
Process Variance	σ_1^2	σ_2^2	σ^2
Parameter Variance	τ_1^2	τ_2^2	τ^2
Total Variance	λ_1^2	λ_2^2	λ^2
Process Cov	ρ		
Parameter Cov	π		
Total Cov	κ		

Split Credibility Notation- Total Variances

- *Total Component Variances*
  - $\lambda_1^2 = \sigma_1^2 + \tau_1^2$ *and* $\lambda_2^2 = \sigma_2^2 + \tau_2^2$
- *Total Covariance*
  - $\kappa = \rho + \pi$
- *Total Variances*
  - *Total:* $\lambda^2 = \lambda_1^2 + \lambda_2^2 + 2\kappa$
  - *Process:* $\sigma^2 = \sigma_1^2 + \sigma_2^2 + 2\rho$
  - *Parameter:* $\tau^2 = \tau_1^2 + \tau_2^2 + 2\pi$

Optimal Split Credibilty Formulas

$$z_1 = \frac{\lambda_2^2(\tau_1^2 + \pi) - \kappa(\tau_2^2 + \pi)}{D}$$

$$z_2 = \frac{\lambda_1^2(\tau_2^2 + \pi) - \kappa(\tau_1^2 + \pi)}{D}$$

$$\text{where } D = \lambda_1^2\lambda_2^2 - \kappa^2$$

Mean Square Error with Optimal Credibilities

- It can be proved that minimal MSE is given as :

$$\varepsilon_0^2(SP) = (\tau_1^2 + \pi)(1 - z_1^*) + (\tau_2^2 + \pi)(1 - z_2^*)$$

- Error reduction interpretation
  - Each component starts with its own parameter risk plus the parameter covariance
  - The separate “z” are the factors by which the parameter error is reduced for each component

When Is a Split Plan Effective?

- Most extreme improvement is achieved if:
  - One component gets all the parameter risk
  - The other gets all the process risk
  - Covariances are zero
- Intuition: A Split works to the degree that it separates noise from signal!
- Split plan improves on No-split Plan when the Split Induces a Differential Allocation of Process and Parameter Risk.

Example-Split Plan A

Unsplit Plan			Split Plan A	Combined	Primary	Excess	CoVar
Process Var	300.0		***Process Var***	300.0	200.0	60.0	20.0
Parameter Var	100.0		***Parameter Var***	100.0	66.7	20.0	6.7
Total Var	400.0		***Total Var***	400.0	266.7	80.0	26.7
			D	20,622			
Credibility	25%		***Credibility***		25%	25%	
MSE	75.0		***MSE***	75.0			

Example-Split Plan B

Unsplit Plan	
Process Var	300.0
Parameter Var	100.0
Total Var	400.0
Credibility	25%
MSE	75.0

Split Plan B	Combined	Primary	Excess	CoVar
Process Var	300.0	150.0	130.0	10.0
Parameter Var	100.0	80.0	10.0	5.0
Total Var	400.0	230.0	140.0	15.0
D	31,975			
Credibility		37%	7%	
MSE	67.9			

Quiz 2

- Which was a more effective split?
  - A
  - B
  - Both equally effective
- Split Plan B does not work as well as a no-split plan because component 2 has a credibility of only 7% versus 25% for the non-split plan.
  - True
  - False

Quiz 3: True or False

Assume credibilities are determined by minimizing MSE:

- Q1. Splitting produces two layers each with higher z than the z of the original unsplit losses.
- Q2. $Z_P \leq Z_e$

Conclusions and Questions

- The Split Credibility approach works in practice
- With arbitrary frequency severity model and arbitrary priors, splitting does not necessarily offer much improvement.
- With the right model and reasonable data and a well chosen split point, split credibility can be superior to unsplit credibility.
- Having caps on large losses, raises credibility of the excess layer and improves overall performance
- Questions??