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Credibility – An Incredibly Good Idea !

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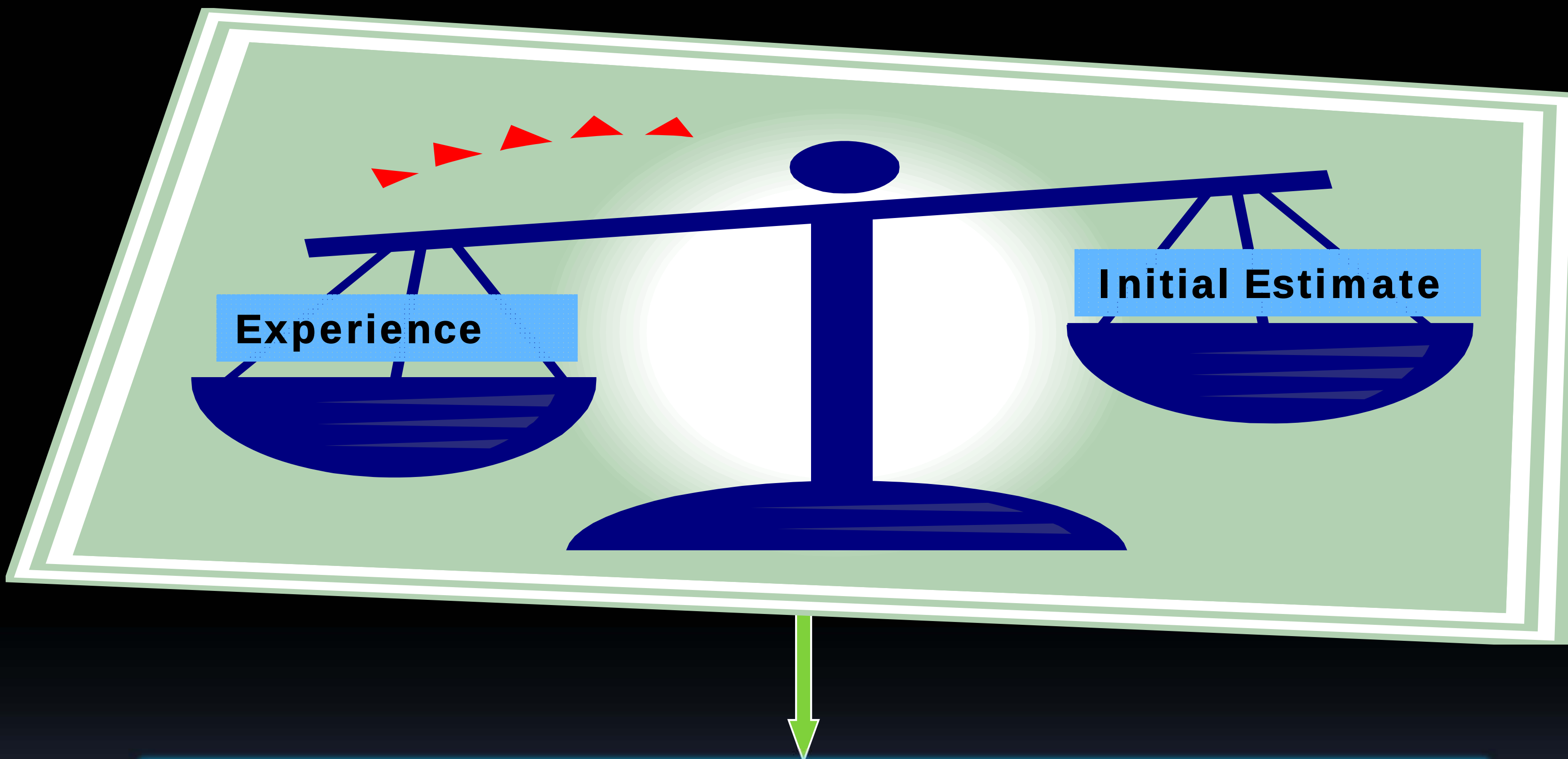
Balancing Experience vs Initial Estimate



Revised Estimate

Credibility Estimate

- Linear mix of actual and expected

$$\mu^* = zA + (1-z)E$$

  - E = initial (prior) mean= complement
  - A = mean of actual data
  - z = credibility

$$\text{Mod} = \frac{\mu^*}{E} = 1 + z\frac{(A-E)}{E}$$

Credibility: Our Big Idea

- Original meaning (1914): reliability of data for ratemaking
  - How much data is needed for it to be fully credible
- What happens if data is not 100% credible? Give it partial credibility
  - Credibility is the weight to be given to data-based estimate versus the complement of credibility
  - The complement is 0% change, the overall avg, ….
- Claimed as a unique contribution from American P&C actuaries
  - Contrasted with pure frequentist approaches taken by statisticians at the time

We Can’t Stop Writing About It

- Mowbray 1914 “How Extensive a Payroll Exposure is Necessary to Give a Dependable Pure Premium”
- Whitney 1918 “The Theory of Experience Rating”
- Perryman 1932 – “Notes on Credibility”
- Dorweiler 1934 “… Risk Credibility in Experience Rating”
- Bailey 1945 – “A Generalized Theory of Credibility”
- Bailey and Simon 1959 – “Credibility of … Private Passenger Car”
- Hurley 1954 – “ ..Credibility Framework for … Fire Classification… ”
- Longley- Cook 1962 - “An Introduction to Credibility Theory”
- Mayerson 1964 - “A Bayesian View of Credibility”
- Buhlman 1967- “Experience Rating and Credibility”
- Hewitt 1966- “Credibility- An American Idea”
- Philbrick – “Examination of Credibility Concepts”
- Dean 1996– “Introduction to Credibility”
- Venter 2003 – “Credibility Theory for Dummies”

Property Casualty Insurance Applications

- Overall Rate Change Indications
- Class Relativities
- Territorial Relativities
- Individual Risk Experience Rating
- Excess Layer Rating
- Reserving

Credibility Impact on Rates

Full z Standards

defines when a state/territory/ class group is large enough for self rating

Limited Fluctuation over time

tempers excessive year-to-year rate movement

Class Credibility

reduces instability in class rate differentials

Individual Risk Experience Rating

improves accuracy by capturing differences not reflected by class plan

Conceptual Virtues of Credibility

Balances Stability versus Responsiveness

- Prevents excessive volatility in rates
- Attempts to recognize signal and not mimic the noise of actual data.

Systematically reflects our beliefs

- How much risk classes differ
- Heterogeneity of individuals within a class

Provides realistic and fair incentives

- Gives classes and states reasonable credits/penalties
- Motivates efficient level of safety and loss control

Classic Credibility Full z standard

- Number of Claims needed to achieve z=100%
  - Longley-Cook derivation
    - Uses Normal Distrib approximation

$$\Pr ob\left(\left|N - E[N]\right| < kE[N]\right) > P$$

E[N]= Expected Number of Claims Required			
k = width of interval	P = Level of confidence		
	99%	95%	90%
2.5%	10,623	6,147	4,326
5.0%	2,656	1,537	1,082
7.5%	1,180	683	481
10.0%	664	384	271

Two Classic Options for Partial Z

$$z = C \cdot \frac{n}{n+k}$$

- n = Expected number of claims
  - k selected to hit desired "swing"
  - C chosen so z= 100% at full z standard

$$z = \sqrt{\frac{n}{N_{100\%}}}$$

- Square root rule

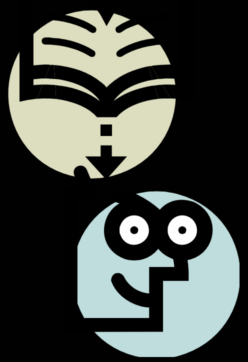
Classic Z Criticisms and Limitations

- Lack of coherent theoretical foundation
  - Importance of prior knowledge stressed but not used in derivation of full z standard
- Insurance losses are skewed and do not follow the Normal distribution
  - Need to reflect Severity, not just Frequency
- Insufficient awareness of Off-balance and possible bias.
- No valid conceptual rationale for use of loss capping and loss splitting procedures

- Over the years, actuaries addressed all these issues

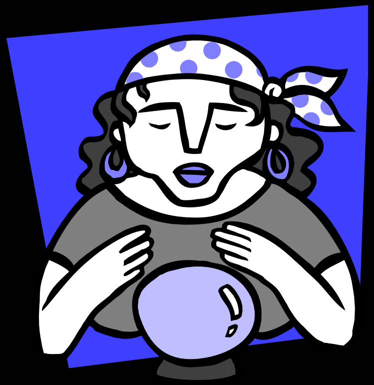
Modifying Our Beliefs- Bayes

$$h(\theta | x) = f(x | \theta)\frac{h(\theta)}{f(x)}$$



- X is RV parametrically dependent on θ
- Define h(θ) as the prior distribution of the parameter
- Define h(θ|x) as the posterior distribution of the parameter

The Mysterious Prior



- Captures the unknown
- Records what we think we know
  - How confident are we?
- Inherent uncertainty
  - Our knowledge is not exact
  - Sampling error
- How much the future could vary from the past
  - Variation beyond expected sampling error

Modifying the Expectation- Bayes

$$E[X \mid A] = \int E[X(\theta)] \cdot h(\theta \mid A)\, d\theta$$

- Parametric Model
- X (θ) is RV parametrically dependent on θ
- A = Actual result of an experiment

Bayesian Credibility

$$\varepsilon^2 = \int \left[\mu(\theta) - \left(zA + (1-z)\mu_0\right)\right]^2 \cdot f(A \mid \theta) \cdot h(\theta)\, d\theta$$

- Best linear fit
  - Optimal Z gives best fit to the parametric model
  - Mean Square Error fit minimizes ε^2
- Z never reaches 100% in theory