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STOCHASTIC RESERVING: THE THEORY AND THE PRACTICE

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Advance.

Objectives

An overview of common stochastic reserving methods
- *Bootstrapping*
- *Mack*
- *Merz and Wuthrich*

The theoretical basis and practical examples for implementing methods

A comparison to demonstrate some benefits and limitations of each

How stochastic reserving is used in practice in an insurance company.
- How do you use them?
- What do they mean for the business?
- How do you explain them to management and the Board?

Contents

01

Background

Polling question

What stochastic reserves do you currently calculate?

01 Bootstrapping

02 Mack, Merz & Wuthrich

03 Other

04 None

Polling question

What stochastic reserves do you think you will calculate in the next 1 to 2 years?

01 Bootstrapping

02 Mack, Merz & Wuthrich

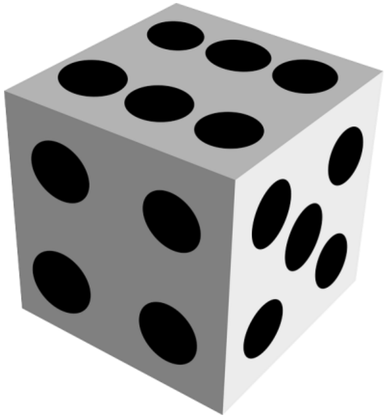
03 Other

04 None

Definition

Stochastic

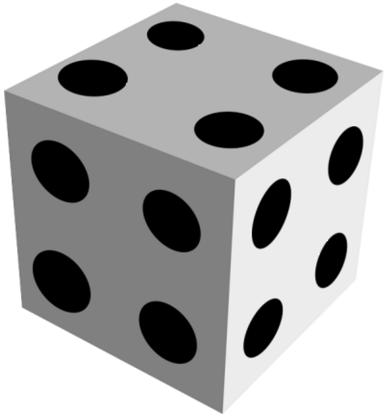
- Events or systems that are unpredictable due to the influence of a random variable
- Pertaining to chance
- Randomly determined
- Having a random probability distribution or pattern that may be analysed statistically but may not be predicted precisely



Definition

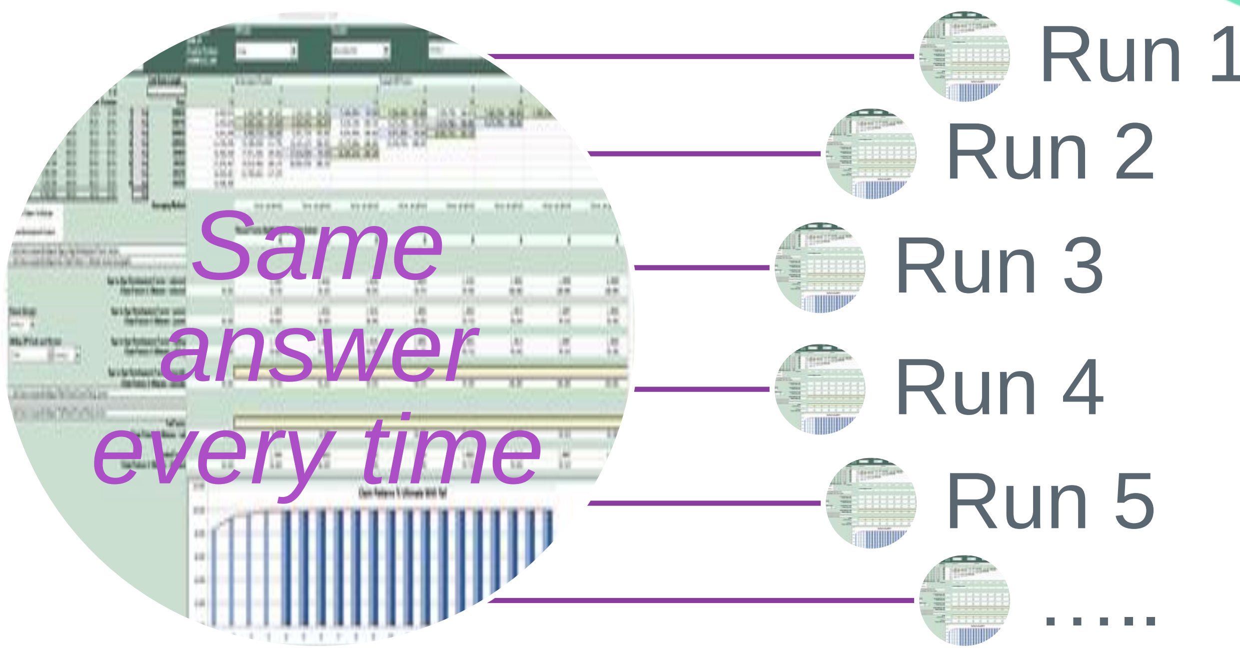
Deterministic

- A deterministic system is a system in which no randomness is involved in the development of future states of the system
- A deterministic model will thus always produce the same output from a given starting condition or initial state



Example

Deterministic Basic Chain Ladder



Development pattern is fixed		
Fixed proportional development	Fixed timing	Fixed tail development

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Example

Stochastic Basic Chain Ladder

Development pa is fixed		
Fixed proportional development	Fixe g	Fixed tail development

- Basic chain ladder assumes averages
- User often provides expert judgeme the process
- Sometimes curve fitting
- But then still just one view of the development pattern
- The only guarantee is that this will be wrong!

02

Bootstrapping The Theory

Bootstrapping

- Bootstrapping usually refers to a self-starting process that is supposed to proceed without external input
- In computers, shortened to booting, refers to the process of loading the basic software which will then take care of loading other software as needed.

Originally from putting on a boot with the help of the boot itself
A bootstrap

The Theory

- Bootstrapping can refer to any test or metric that relies on random sampling, with replacement.
- Bootstrapping allows assigning measures of accuracy (defined in terms of bias, variance, confidence intervals, prediction error or some other such measure) to sample estimates.
- This technique allows estimation of the sampling distribution of almost any statistic using random sampling methods.
- Generally, it falls in the broader class of resampling methods.

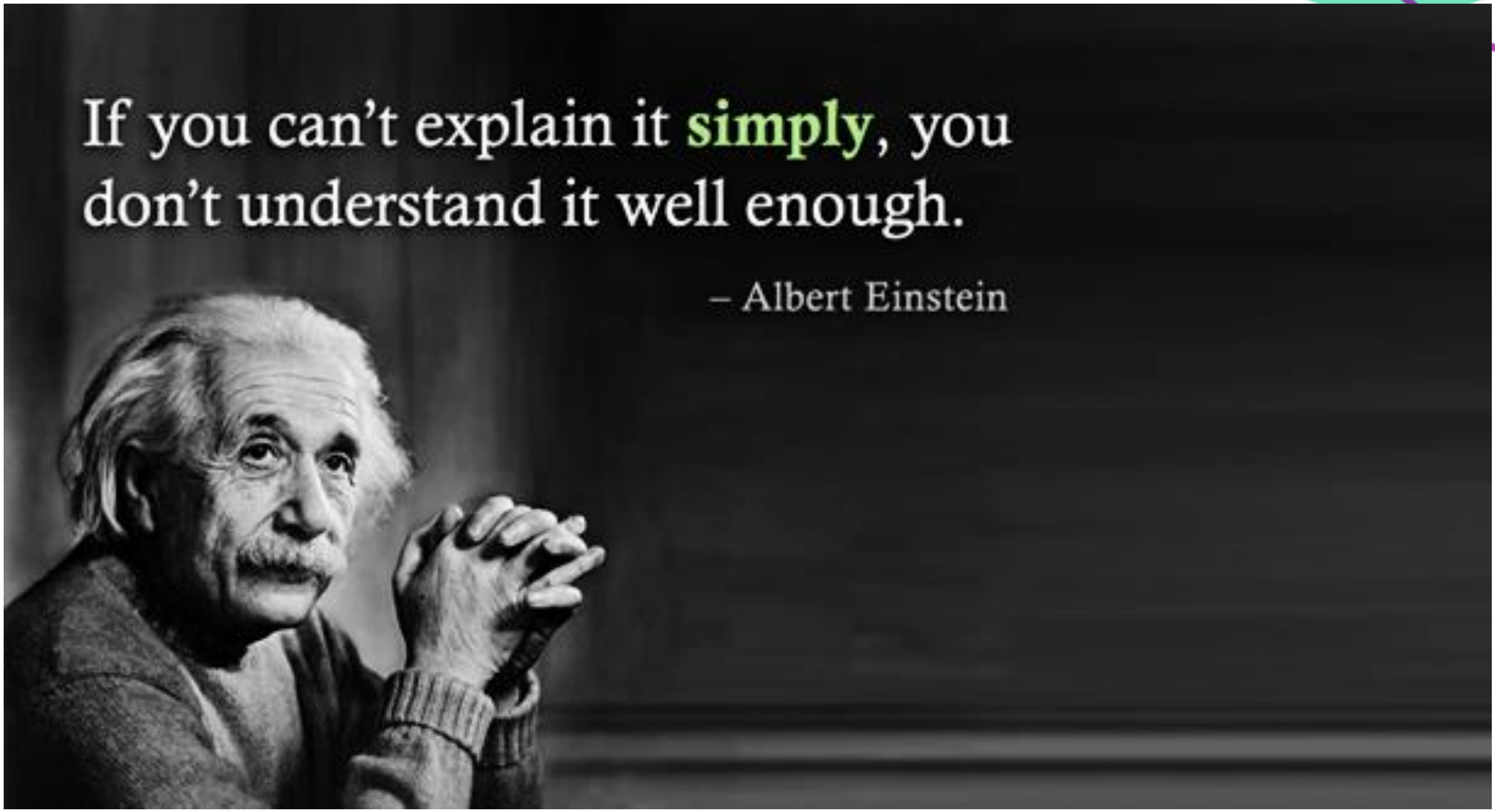
The Theory

- Bootstrapping is the practice of estimating properties of an estimator (such as its variance) by measuring those properties when sampling from an approximating distribution.

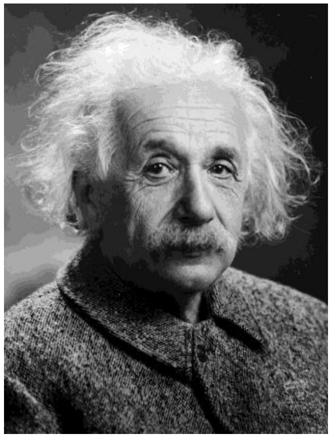
- One standard choice for an approximating distribution is the empirical distribution function of the observed data.

- In the case where a set of observations can be assumed to be from an independent and identically distributed population, this can be implemented by constructing a number of resamples with replacement, of the observed dataset

The Challenge



He wrote the book on it!



Common Bootstrap

ODP Model

An incremental loss type model

- Incremental losses $C_{i,j}$ by origin period i and development period j are independent
- Incremental losses $C_{i,j}$ are related:

$$C_{i,j} = \alpha_i . \beta_j$$

- There is a constant variance to mean ratio, σ^2

Note:

- Poisson distribution mean = variance
- Overdispersed Poisson distribution mean $\propto$ variance with the ratio being the dispersion parameter

Common Bootstrap

ODP Model

For the basic chain ladder, cumulative claims $D_{i,j}$:

$$D_{i,j} = D_{i,k_i}.\, d_j$$

To date amount times cumulative development factor

$$d_j = \prod_{m=k_i+1}^{j} aa_m$$

Cumulative development factor is the product of age to age, link ratio, factors aa_m

Hence incremental claims

$$C_{i,j} = D_{i,j} - D_{i,j-1} = D_{i,k_i}.\left(d_j - d_{j-1}\right) = \alpha_i.\, \beta_j$$

$$\alpha_i = D_{i,k_i} \quad \beta_j = \left(d_j - d_{j-1}\right)$$

Common Bootstrap

ODP Model

So $C_{i,j} = \alpha_i . \beta_j$ as required

$$E\left(C_{i,j}\right) = D_{i,k_i} . \left(d_j - d_{j-1}\right)$$
$$Var\left(C_{i,j}\right) = \sigma^2 . E\left(C_{i,j}\right)$$

- $\widehat{\alpha_i} = \widehat{D}_{i,k_i}$ and $\widehat{\beta_j} = \widehat{d_j} - \widehat{d_{j-1}}$ are estimated straight from the basic chain ladder calculations
- Variance proportional to the mean defines an ODP distribution, σ^2 being the dispersion parameter

Types of Bootstrap

Non-Parametric Empirical residuals distribution

- Does not require a distributional assumption
- Just uses the data itself
- Works best for larger samples

Parametric Theoretical residuals distribution

- e.g., ODP or Normal distributions are common
- Mean and variance parameters selected based on the original data

Residuals

Typical unscaled residual is based on Pearson's Residual

$$\varepsilon_{i,j} = \frac{C_{i,j} - \hat{C}_{i,j}}{\sqrt{\hat{C}_{i,j}}}$$

Actual less expected as a proportion of the square root of expected

The set of residuals $\{\varepsilon_{i,j}\}$ then forms the sample distribution:

- Used directly as the residuals distribution, an empirical distribution
- Fitted to an appropriate standard distribution, e.g. ODP or normal, using the sample mean and variance

Residuals

Bias Correction

Residuals can further be scaled to correct for bias

$$\varepsilon_{i,j}^{adj} = \varepsilon_{i,j} \cdot \sqrt{\frac{n}{n-p}}$$

Where:

- n is the number of observations, the size of the residuals triangle
- p the number of parameters estimated, the number of origin and development periods

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Residuals

Process Variance

Process variance can be estimated as:

$$\sigma^2 = \frac{1}{n-p} . \sum_{i,j} \varepsilon_{i,j}^2$$

Where:

- n is the number of observations
- p the number of parameters estimated

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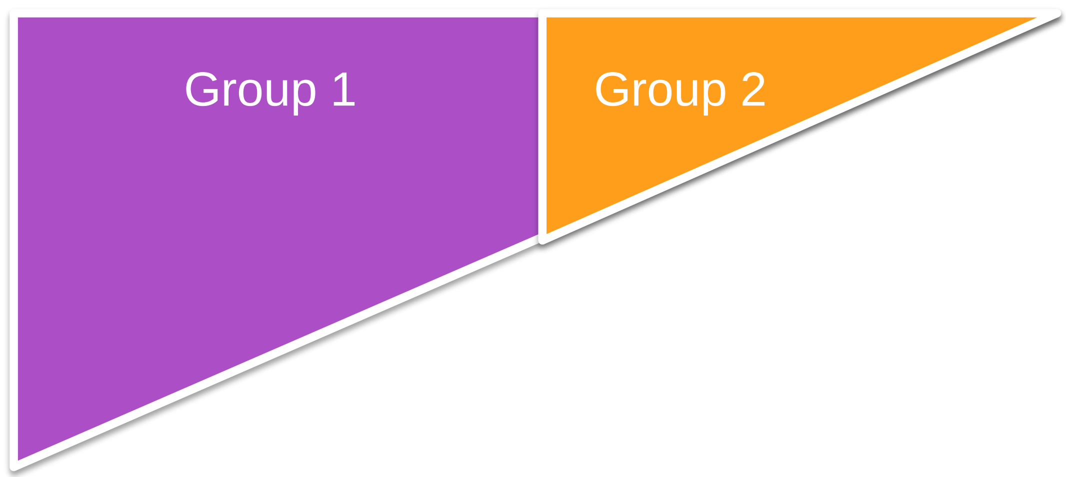
Residuals

Selections

Residuals can be selected, in turn adjusting the residual distribution

- Individual residuals can be ignored, for example, the maximum and minimum
- Residuals can be grouped for sampling, for example, use the tail group residuals for the tail only



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Bootstrapping The Practice

Bootstrapping Process

Cumulative Claims

Start with the cumulative claims triangle

Cumulative Claims

	0	1	2	3	4
1	100	120	125	130	132
2	75	90	105	115	
3	110	140	155		
4	105	130			
5	115				

Bootstrapping Process

Development Pattern

Calculate the usual development pattern

- Include any adjustments as usual
- Example here is a simple average of all values

Development Factors	0	1	2	3	4
1		1.200	1.042	1.040	1.015
2		1.200	1.167	1.095	
3		1.273	1.107		
4		1.238			
5					
Average		1.228	1.105	1.068	1.015

Bootstrapping Process

Developed Claims

Calculate developed claims in the usual way and hence the usual BCL reserves

Developed Claims

	0	1	2	3	4
1					132
2				115	117
3			155	165	168
4		130	144	153	156
5	115	141	156	167	169

Actual Reserve
0.0
1.8
13.0
25.7
54.1
94.7

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Bootstrapping Process

Expected Cumulative Claims

Calculate expected past cumulative claims using the development pattern

- Start with the latest, to date, value and apply the development pattern in reverse
- What previous claim would have given rise to this one
- For example 130 = 132 / 1.015

Expected Cumulative Claims

	0	1	2	3	4
1	90	110	122	130	132
2	79	97	108	115	
3	114	140	155		
4	106	130			
5	115				

Development Factors

	0	1	2	3	4
1		1.200	1.042	1.040	1.015
2		1.200	1.167	1.095	
3		1.273	1.107		
4		1.238			
5					
Average		1.228	1.105	1.068	1.015

Bootstrapping Process

Incremental Claims

Convert cumulative actual and expected claims to incremental claims

Actual Incremental Claims

	0	1	2	3	4
1	100	20	5	5	2
2	75	15	15	10	
3	110	30	15		
4	105	25			
5	115				

Expected Incremental Claims

	0	1	2	3	4
1	90	20	12	8	2
2	79	18	10	7	
3	114	26	15		
4	106	24			
5	115				

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Bootstrapping Process

Unscaled Pearson Residual

Calculate unscaled Pearson residual

$$\varepsilon_{i,j} = \frac{C_{i,j} - \hat{C}_{i,j}}{\sqrt{\hat{C}_{i,j}}}$$

Actual less expected divided by the square root of expected

Pearsons Unscaled Residual

	0	1	2	3	4
1	1.083	-0.096	-1.935	-1.127	
2	-0.493	-0.724	1.484		
3	-0.397	0.782			
4	-0.086				
5					

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Bootstrapping Process

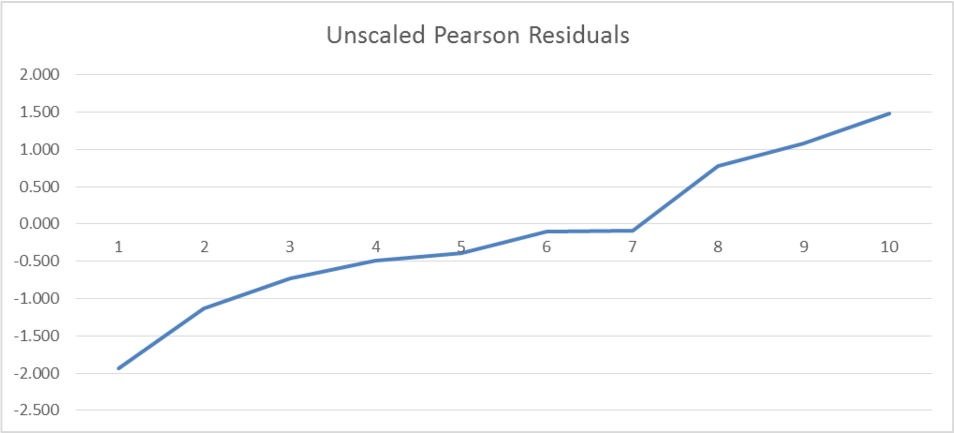
Empirical Residual Distribution

These 10 residuals then form our empirical distribution

Note you would usually look to have a fuller sample, e.g. 100 or more

Empirical Residual Distribution

1	1.083
2	-0.096
3	-1.935
4	-1.127
5	-0.493
6	-0.724
7	1.484
8	-0.397
9	0.782
10	-0.086



Bootstrapping Process

Simulated Residual Triangle

Through sampling from the residual distribution, with replacement, randomly populate a new simulated residual triangle

Empirical Residual Distribution

1	1.083
2	-0.096
3	-1.935
4	-1.127
5	-0.493
6	-0.724
7	1.484
8	-0.397
9	0.782
10	-0.086

Simulated Residuals

	0	1	2	3	4
1	-0.493	-1.935	-0.096	-1.127	
2	-0.086	-0.096	-0.397		
3	-0.086	-0.086			
4	1.083				
5					

Bootstrapping Process

Simulated Incremental Claims

Using the same expected claims, reverse the residual calculation to generate simulated actual claims

$$C_{i,j} = \hat{C}_{i,j} + \varepsilon_{i,j}.\sqrt{\hat{C}_{i,j}}$$

$$\varepsilon_{i,j} = \frac{C_{i,j} - \hat{C}_{i,j}}{\sqrt{\hat{C}_{i,j}}}$$

Simulated actual claims equals expected claims plus the residual times the square root of the expected claims

Simulated Incremental Claims

	0	1	2	3	4
1	85	12	11	5	2
2	79	18	9	7	
3	113	26	15		
4	117	24			
5	115				

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Bootstrapping Process

Simulated Cumulative Claims

Calculate cumulative simulated claims from the incremental claims

Simulated Cumulative Claims

	0	1	2	3	4
1	85	97	108	113	115
2	79	96	105	113	
3	113	139	154		
4	117	141			
5	115				

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Bootstrapping Process

Simulated Development Pattern

Calculate a usual development pattern

- Adjustments not typically included further
- Example is again a simple average

Development Factors	0	1	2	3	4
1		1.137	1.116	1.046	1.018
2		1.225	1.093	1.069	
3		1.226	1.106		
4		1.206			
5					
		1.198	1.105	1.058	1.018

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Bootstrapping Process

Developed Simulated Claims

Calculate developed simulated claims in the usual way and hence the usual BCL reserves

Developed Claims

	0	1	2	3	4
1					115.0245
2				112.5516	115
3			153.6366	163	165
4		141.1397	156	165	168
5	115	138	152	161	164

Simulated Reserve
0.0
2.0
11.7
26.8
49.0
89.5

Bootstrapping Process

We now have two results for claim reserves

- One from actual claims
- One from simulated claims

Actual Reserve	Simulated Reserve
0.0	0.0
1.8	2.0
13.0	11.7
25.7	26.8
54.1	49.0
94.7	89.5

Now repeat from the simulating residual triangle step to produce another simulated claim reserve, and another, and another…….

Bootstrapping Process

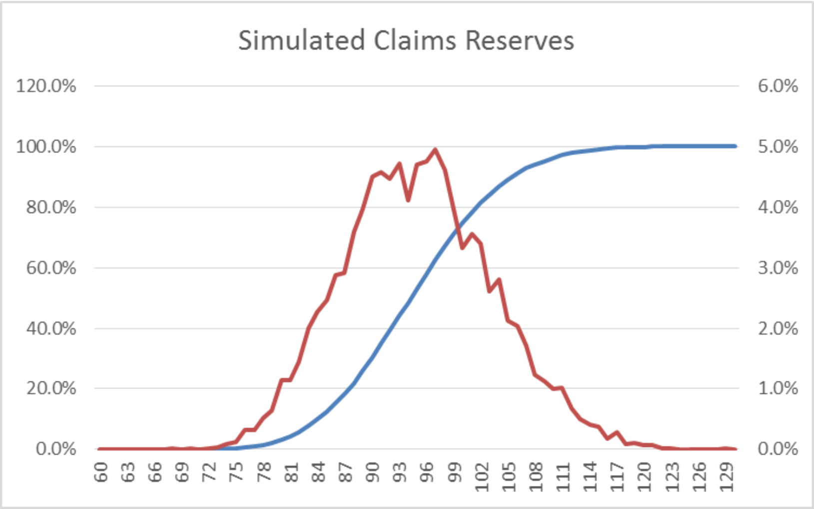
So what?

Bootstrapping Process

Using Results Examples

- Mean 94.6
- Coefficient of variance 8.7%
- 75th percentile 100.1
- 99.5th percentile 116.5

- 25th to 75th percentile confidence interval 88.8 to 100.1 or -6.1% to +5.9% of the mean



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Bootstrapping Process

So what?

Bootstrapping Process

Using Results Examples

The Science	The English
Mean 94.6	Best estimate, expected value
Coefficient of variance 8.7%, approx. 83rd percentile	Average difference to mean measure How stable is the estimate?
75th percentile 100.1, +5.9% of mean	Prudent estimate 1 in 4 year level
99.5th percentile 116.5, +23.2% mean	Extreme, 1 in 200 year, estimate How bad could it get?
25th to 75th percentile confidence interval 88.8 to 100.1, -6.1% to +5.9% of the mean	A likely range 50% chance it will be in this range

Use them as a tool to inform and assist with your work

Bootstrapping Process

Common Misuses

There are no guarantees, these are simulations only

- Don’t be fooled into a false sense of security
- Use them as an informative tool only, an extra piece of information to help you

The statistics, e.g. coefficient of variation, only apply to this data set

- A common mis-use is to run a simulation on one dataset and then use the statistics, e.g. coefficient of variation, on another data set

04

Other Stochastic Reserving Methods

Non-Bootstrap Methods

Mack

- Thomas Mack et al
- Single measure of ultimate variability, no distribution
- Formulaic, no simulation needed
- Builds on the standard ODP basic chain ladder model

- Well documented method

Non-Bootstrap Methods

Mack

- Mean squared error of reserve R by origin period i:

$$\widehat{mse(\hat{R}_i)} = \hat{C}_{i,I}^2 \sum_{k=I+1-i}^{I-1} \frac{\hat{\sigma}_k^2}{\hat{d}_k^2}\left(\frac{1}{\hat{C}_{i,k}} + \frac{1}{\sum_{j=1}^{I-k} C_{j,k}}\right)$$

- I represents the maximum origin, and development, period
- Measures the average of the squares of the errors, the difference between the estimator and what is estimated, actual versus expected
- Similar formula for mean squared error of total reserves by origin period including a covariance adjustment
- Extensions, e.g. Mack with Tail

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Non-Bootstrap Methods

Mack

- ‘Simple’ formulaic calculation, no need for simulations
- Useful non-complicating measure
- Only a single point value, no distribution or other values
- No specific defined percentile measure for the mean squared error

Non-Bootstrap Methods

Merz and Wuthrich

- Michael Merz and Mario V. Wuthrich
- Builds on the work of Mack et al

- Mean squared error of reserve over:
  - One development period, e.g. a year
  - Ultimate

- Measure of variability, no distribution
- Formulaic, no simulation needed
- By origin period and in total

- Process variance and estimation variance by origin period
- Aggregate with covariance adjustment to get total

- One year measure used for Solvency II reserve risk calibration and is the prescribed USP calculation

Non-Bootstrap Methods

Merz and Wuthrich

- ✓ ‘Simple’ formulaic calculation, no need for simulations
- ✓ Useful non-complicating measure
- ✓ Single development period and ultimate measures
- ✗ Only a point value, no distribution
- ✗ No specific defined percentile measure for the mean squared error

Non-Bootstrap Methods

Others

- Parodi
  - Frequency and severity model approach
  - Model a frequency distribution and a severity distribution
  - Simulate using a Monte Carlo approach

- Double Chain Ladder

Over to Richard……

STOCHASTIC RESERVING:
THE THEORY AND THE PRACTICE

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Questions.